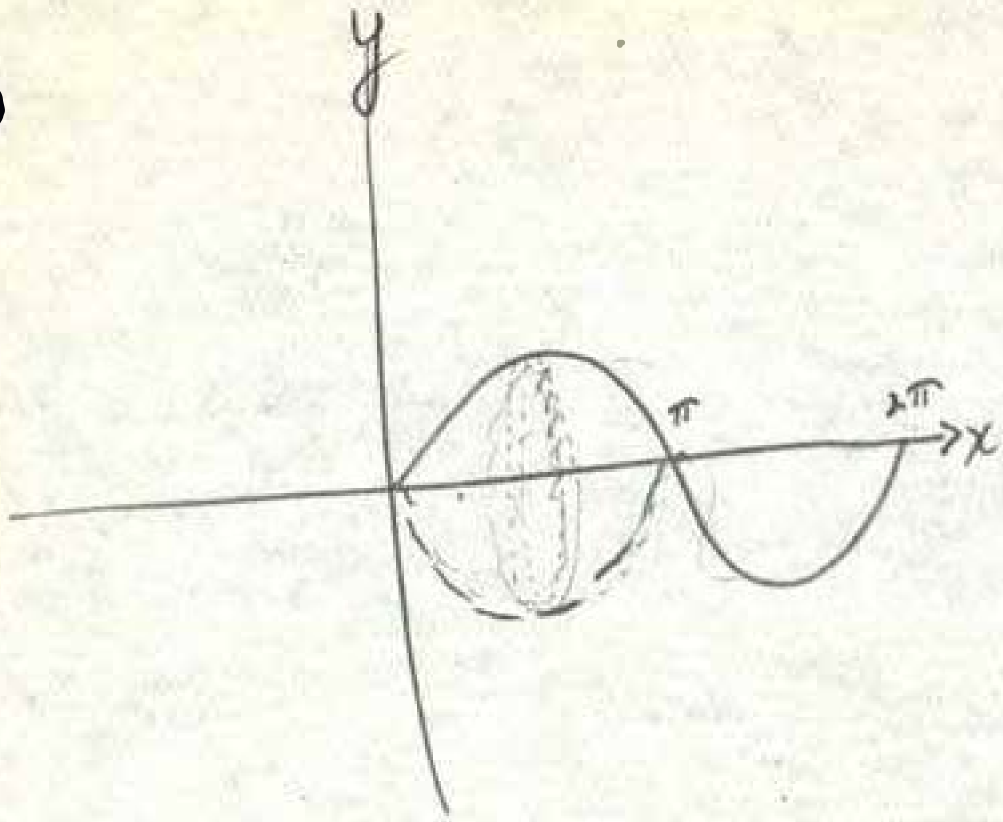
326) 4



$y = \sin x$

Element of Volume $= \pi r^2 a$

$= \pi y^2 a$

$= \pi \sin^2 x \, dx$

Total Volume $= \int_0^\pi \pi \sin^2 x \, dx$

$= \pi \int_0^\pi \sin^2 x \, dx$

$= \pi \int_0^\pi \frac{1-\cos 2x}{2} dx$

$= \frac{\pi}{2} \int_0^\pi (1-\cos 2x) dx$

$5 \sin A = \sin 5A$

$= \frac{\pi}{2}\left(x - \frac{\sin 2x}{2}\right)\Big]_0^\pi$

$= \frac{\pi}{2}\left(\pi - \frac{\sin 2\pi}{2}\right) - \cancel{\frac{\pi}{2}(0-0)}$

$= \frac{\pi}{2}(\pi - 0) = \pi^2/2$

($\sin 2\pi$ radians $= 0$

$\therefore$ every multiple of $\pi = 0$)

$\cancel{= \frac{\pi^2}{2} - \frac{\sin(2\pi^2)}{4}}$

$\cancel{= \frac{\pi^2}{2} - 0 = \frac{\pi^2}{2}}$

~~$\frac{\pi}{2}, \frac{2\pi}{2}, \frac{3\pi}{2}$ ---~~

~~$2\pi^2 = (2\pi)\pi = 2$ ---~~ ~~$\pi^2 = \pi(\pi) = 3.14\pi$~~

326) 8

$y' = \sec^2\theta + \tan\theta$

When $\theta = 0$, $\sec^2\theta + \tan\theta = 5$

$\sec^2\theta = 5 - \tan\theta$

$(5-\tan\theta)^2 + \tan\theta = 5$

$25 - 10\tan\theta + \tan^2\theta + \tan\theta = 5$

$\tan^2\theta - 9\tan\theta + 20 = 0$

Then $y = \int \tan^2\theta - 9\tan\theta + 20$

$= \int \sec^2\theta - 9\tan\theta + 19$

$= \tan\theta - \ln \sec\theta + 19\theta + C$

326) 8

$y' = \sec^2 + \tan\theta$

$y = \int (\sec^2\theta + \tan\theta)\,d\theta$

$y = \tan\theta + \ln \sec\theta + C$ ✓

When $\theta = 0$, $y = 5$

But when $\theta = 0$; $\tan\theta = 0$

$\sec\theta = 1.00$

$\ln\sec\theta = 0$

$\therefore\ y = 0 + 0 + C = 5$

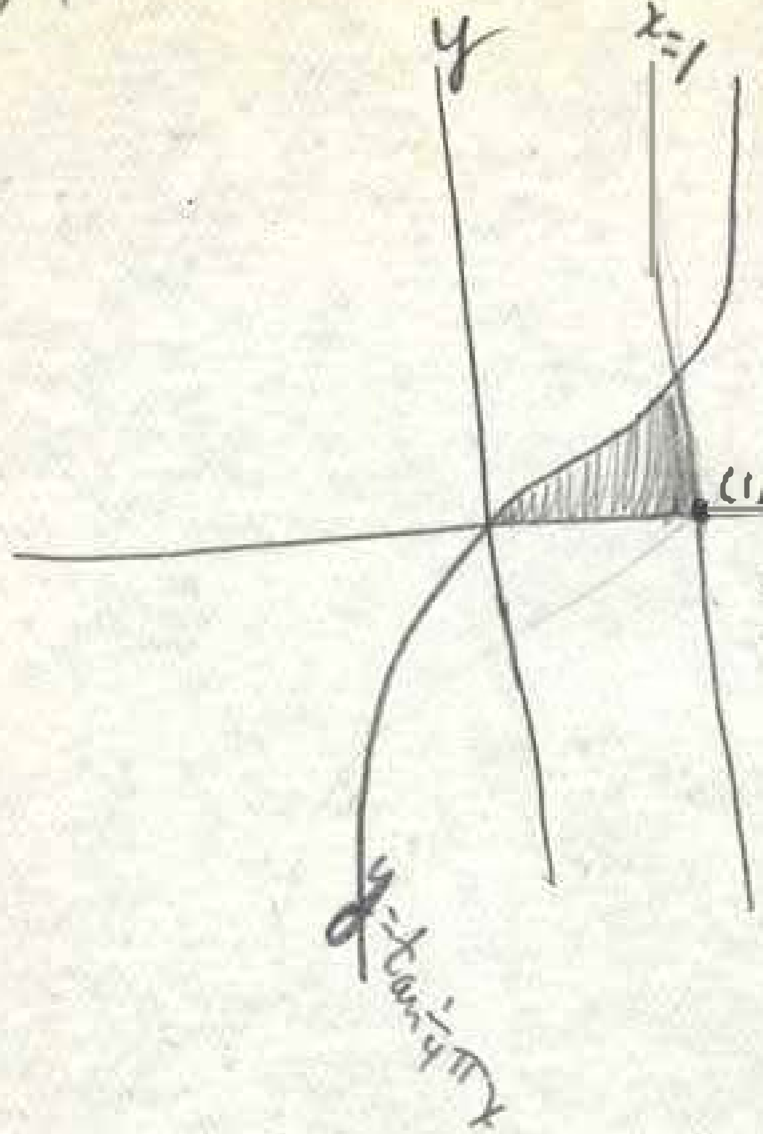
$\therefore\ C = 5$ ✓

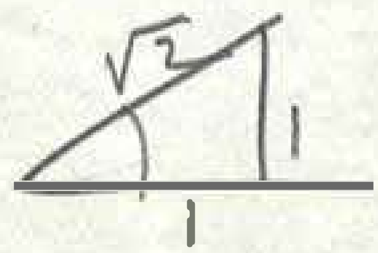
$\therefore\ y = \tan\theta + \ln\sec\theta + 5$

✓

3 2 6) 9

* means plotting of tangent





Element of area $= y\,dx$

Total area $= \int_0^1 y\,dx = \int_0^1 \underbrace{\tan\frac{1}{4}\pi x\,dx}_{\tan u\,\cdot\,\frac{4}{\pi}du}$

$\frac{1}{4}\pi x = u$

$\frac{1}{4}\pi\,dx = du$

$dx = \frac{4}{\pi}du$

$= \frac{4}{\pi}\ln\sec\frac{1}{4}\pi x\Big]_0^1 \qquad = \frac{4}{\pi}\int\tan u\,du$

$\int \tan u\,du = \cancel{\int\frac{\sin u}{\cos u}}$

$= \frac{4}{\pi}\left[\ln\sec\frac{1}{4}\pi\ \cancel{\text{radians}} - \ln\sec 0\right]$

$\frac{4}{\pi}\left[\ln\sqrt{2} - 0\right]$

$\cancel{= \ln\sec 45^\circ}$

$\cancel{= \ln\sqrt{2}} \qquad = \frac{4}{\pi}\ln\sqrt{2}$

$= \frac{4}{\pi}\cdot\frac{1}{2}\ln 2$

$= 2/\pi\,\ln 2$

326) 11

$v = \left(4\cos\frac{1}{2}t\right)$

$\longrightarrow v = 4\cos\frac{1}{2}t$

$t = 0 \qquad \frac{1}{2}\pi$

$$S = \int_0^{\frac{\pi}{2}} 4\cos\frac{1}{2}t\,(dt)$$

$$= 4\int_0^{\frac{\pi}{2}} \underbrace{\cos\frac{1}{2}t}_{\cos u}\,\underbrace{dt}_{2du}$$

$$= 8\sin\frac{1}{2}t\Big]_0^{\frac{\pi}{2}} = 8\sin\frac{\pi}{4} = 8\sin 45^\circ = 4\cdot\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$\frac{t}{2} = u$

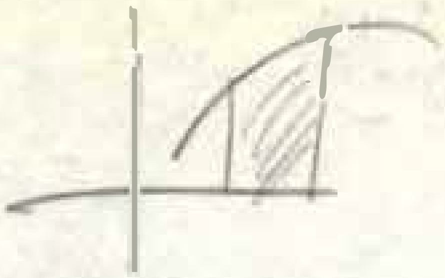
$\frac{1}{2}dt = du$

326) 12

327) 13

$y = \ln(\sec x)$

$dy/dx = \frac{\cancel{\sec x} \cdot \tan x}{\cancel{\sec x}}$



El Area $= y\,dx$

Total Area $= \int_0^{\frac{1}{3}\pi} y\,dx = \int_0^{\frac{1}{3}\pi} \ln\sec x\,dx$

$\boxed{\ln\sec x = \int \tan x}$

Let $u = \ln\sec x$

$du = \frac{1}{\sec x} \cdot 1 = \frac{1}{\sec x}$

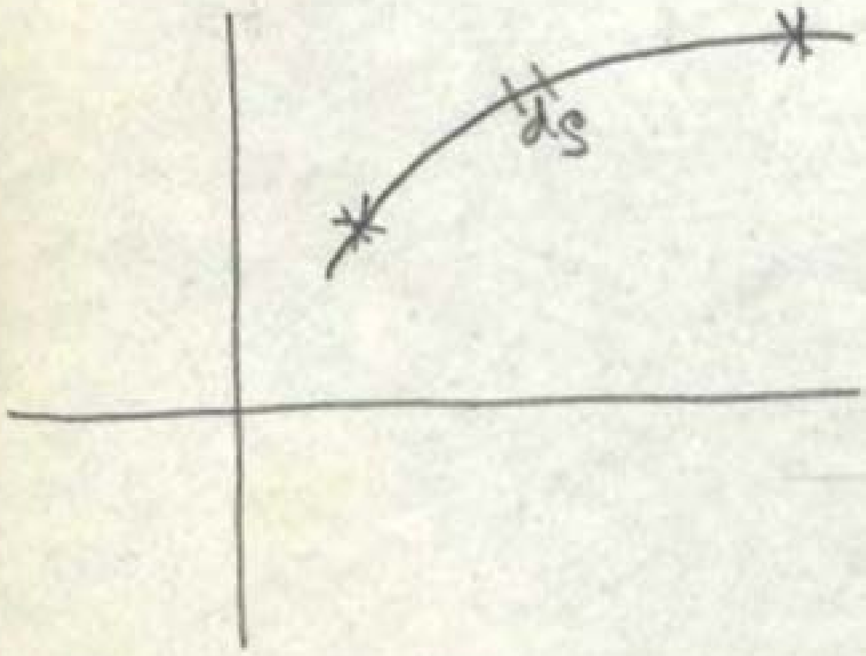
Then $\int_0^{\frac{1}{3}\pi} y\,dx = \sec x \int_0^{\frac{1}{3}\pi} u\,du$

$= \sec x \left.\frac{u^2}{2}\right]_0^{\frac{1}{3}\pi}$

$= \sec x \left.\frac{\ln^2 \sec x}{2}\right]_0^{\frac{1}{3}\pi}$

$= \sec\frac{1}{3}\pi \cdot \frac{\ln^2 \sec\frac{1}{3}\pi}{2}$

$= \sec 60° \cdot \frac{\ln^2 \sec 60°}{2}$



$ds = \sqrt{(dx)^2 + (dy)^2}$

$= \sqrt{\cancel{(dx)^2}\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\,dx$

$ds = \sqrt{1 + \tan^2 x}\,dx = \sec x\,dx$

$s = \int_0^{\pi/3} \sec x\,dx = \ln(\sec x + \tan x)\Big]_0^{\pi/3} =$

330) 1a

$$\int \frac{dx}{x^2-25} = \boxed{\int \frac{du}{u^2-a^2}} = \frac{1}{10} \ln\left(\frac{x-5}{x+5}\right) + C$$

Copied

1e) $\int \frac{dx}{9x^2+4} = \frac{1}{3}\frac{1}{3\cdot 2} \arctan \frac{3x}{2} + C$ $\int \frac{du}{u^2+a^2} = \frac{1}{a} \arctan \frac{u}{a}$

$= \frac{1}{3}\int \frac{du}{u^2+a^2}$ $u^2 = 9x^2$, $u = 3x$ $du = 3dx$

1f) $\int \frac{dx}{9x^2-4} = \frac{1}{3}\frac{1}{4} \ln\left(\frac{3x-2}{3x+2}\right) + C$

$u = 3x$
$du = 3dx$ $\frac{1}{3}\int \frac{du}{u^2-a^2}$

(Book gives $\frac{1}{12} \ln\left(\frac{3x-2}{3x+2}\right) + C$

1i) $\int \frac{dx}{x\sqrt{4x^2-9}} = \int \frac{\frac{1}{2}du}{\frac{u}{2}\sqrt{u^2-9}}$

Let $u^2 = 4x^2$
$u = 2x$
$x = \frac{u}{2}$
$dx = \frac{1}{2} du$

$\int \frac{du}{u\sqrt{u^2-a^2}} =$ $= \frac{1}{2}\int \frac{du}{\frac{u}{2}\sqrt{u^2-9}}$

$= \frac{1}{3} \operatorname{arcsec} \frac{u}{3} + C$ $= \int \frac{du}{u\sqrt{u^2-9}}$

$= \frac{1}{3} \operatorname{arcsec} \frac{2x}{3} + C$

$\cancel{= \int \frac{du}{\sqrt{u^4-9u^2}} = \frac{1}{6u} \ln \frac{u^2-3u}{u^2+3u}} + C = \frac{1}{12x} \ln \frac{4x^2-6x}{4x^2+6x} + C$

$= \frac{1}{12x} \ln \frac{2x-3}{2x+3} + C$

1o) $\int \frac{ds}{\sqrt{16-(s-3)^2}} = \arcsin \frac{(s-3)}{4} + C$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$

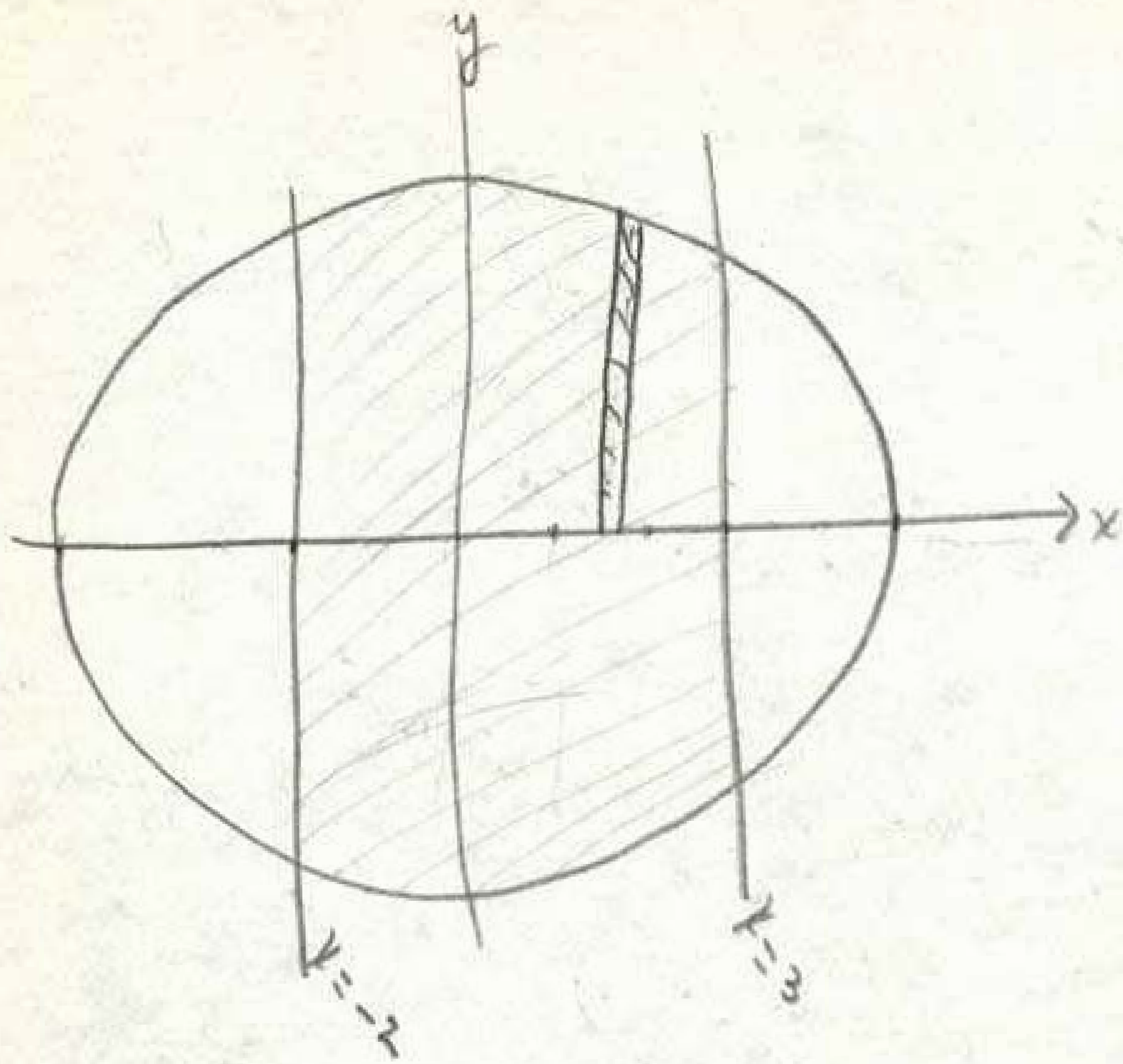
$a = 4$
$u = s-3$
$du = ds$

330) 3

$x^2+y^2=25$

Radius = 5

Center at origin



Element of area $= 2y\,dx$

Total Area $= \int_{-2}^{3} 2y\,dx$

arcsin $(.4) = .42$

arcsin $(-.4) = -.42$

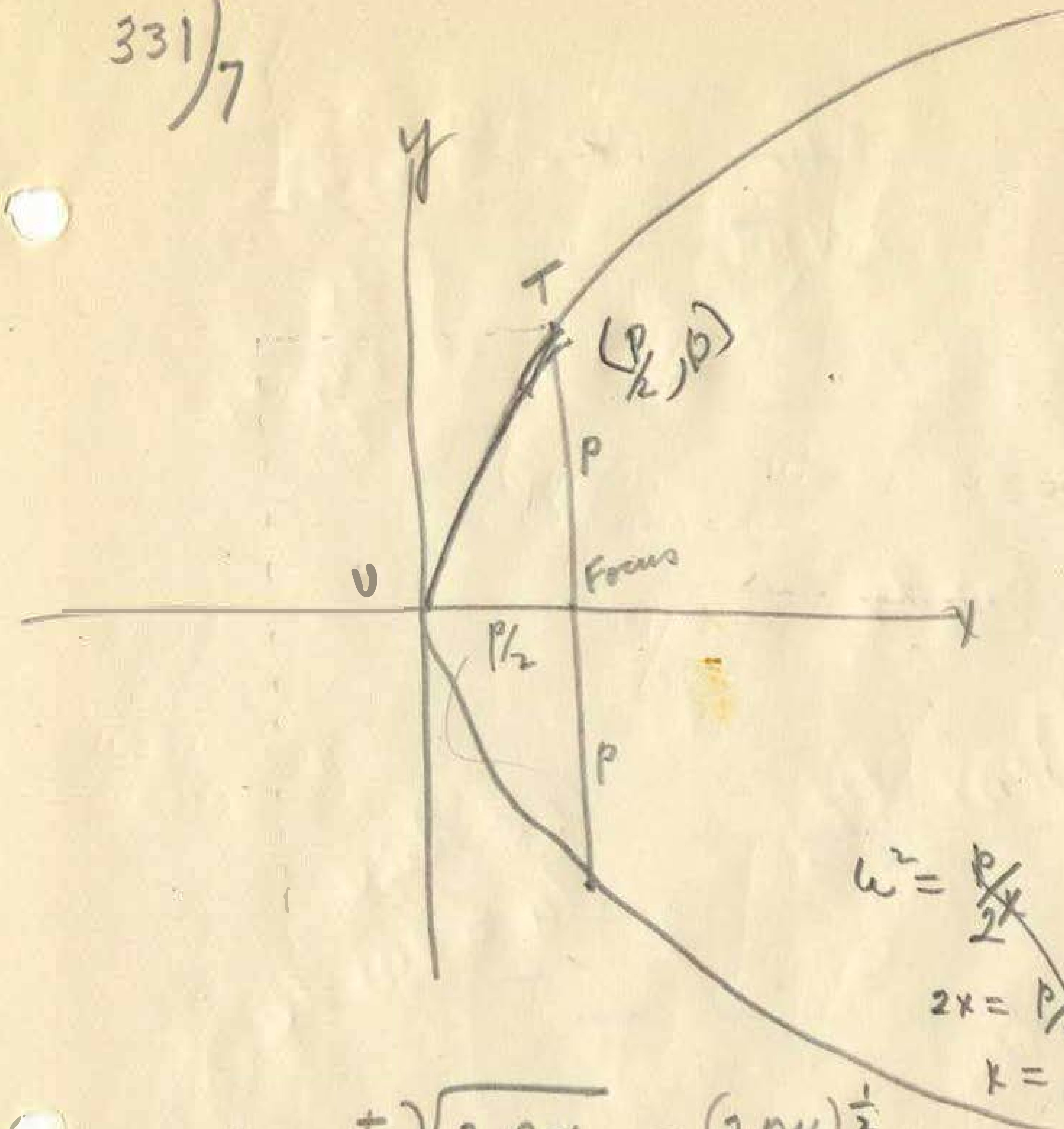
$$= \int_{-2}^{3} 2\sqrt{25-x^2}\,dx$$

$$= 2\left(\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\arcsin\frac{x}{5}\right)\Bigg]_{-2}^{3}$$

$$= 2\left[\left(\frac{3}{2}\sqrt{16} + \frac{25}{2}\arcsin\frac{3}{5}\right) - \left(-1\sqrt{21} + \frac{25}{2}\arcsin(-2/5)\right)\right]$$

$$= 2\left[6 + \frac{25}{2}(.65) + \sqrt{21} - \frac{25}{2}(-.42)\right]$$

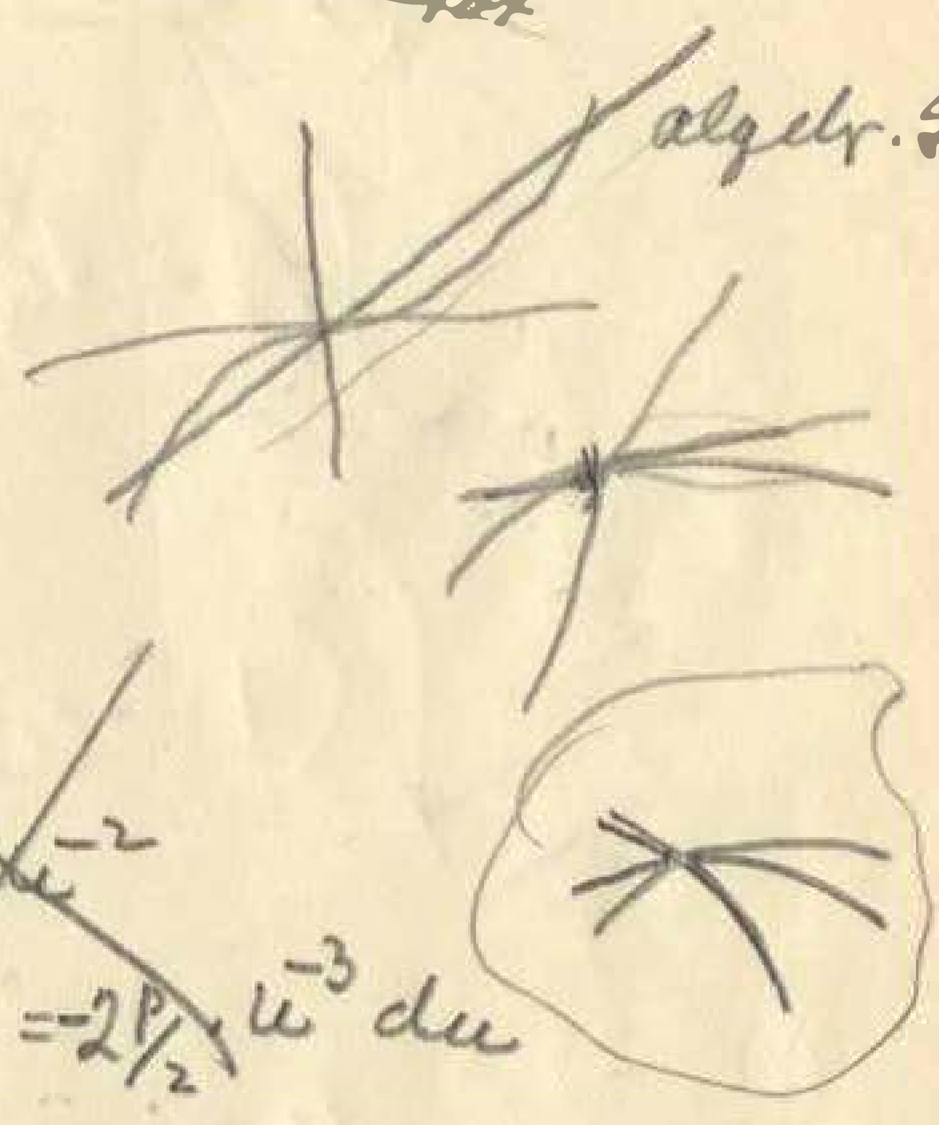
331)7



$y^2 = 2px$

To find length of UT = S

~~$m = dy/dx$~~



$u^2 = \frac{p}{2x}$

$2x = p/u^2$

$x = \frac{p}{2} u^{-2}$

$dx = -2\frac{p}{2} u^{-3} du$

$$y = \pm\sqrt{2px} = (2px)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(2px)^{-\frac{1}{2}}(2p) = \frac{p}{\sqrt{2px}} \checkmark$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + \frac{p^2}{2px}}\, dx$$

$$= \sqrt{1 + \frac{p}{2x}}\, dx$$

Let $u^2 = \frac{p}{2x}$

$$S = \int_0^{p/2} \sqrt{1 + \frac{p}{2x}}\, dx = \left[\int \sqrt{(u^2 \pm a^2)}\, du = \frac{1}{2}\sqrt{\frac{p}{2x}}\sqrt{\frac{p}{2x}+1} + \frac{1}{2}\ln\left(\sqrt{\frac{p}{2x}} + \sqrt{\frac{p}{2x}+1}\right)\right]_0^{p/2}$$

$$= \frac{1}{2}\cdot 1 \cdot \sqrt{2} + \frac{1}{2}\ln(1+\sqrt{2}) = \frac{1}{2}\left(\sqrt{2} + \ln(1+\sqrt{2})\right)$$

Book adds p

$$S = \int_0^{p/2} \sqrt{1 + \frac{p}{2x}}\, dx$$

$$= \int_0^{p/2} \sqrt{\frac{2x+p}{2x}}\, dx = \int_0^{p/2} \frac{\sqrt{2x+p}}{\sqrt{2x}}\, dx$$

$v = 2x + p, \quad a' = p, \quad b' = 2$

$u = 2x, \quad a = 0 \quad b = 2$

$K = -2p$

$$= \frac{1}{2}\sqrt{2x(2x+p)} + \frac{2p}{4}\int$$

$$= \frac{1}{2}\sqrt{2x(2x+p)} + \cancel{\frac{p}{2}}\frac{2}{2}\log\left(\sqrt{8x} + 2\sqrt{2x+p}\right)\Bigg]_0^{p/2}$$

$$= \left[\frac{1}{2}\sqrt{p \cdot 2p} + \frac{p}{2}\log\left(\sqrt{4p} + 2\sqrt{2p}\right)\right]$$

$$- \left[\cancel{0} + \frac{p}{2}\log\left(0 + 2\sqrt{p}\right)\right]$$

$\log a - \log b = \log a/b$

$$\frac{p}{2}\sqrt{2} + \frac{p}{2}\log\left(2\sqrt{p} + 2\sqrt{2p}\right) - \frac{p}{2}\log 2\sqrt{p}$$

$$= \frac{p}{2}\left[\sqrt{2} + \log\frac{2\sqrt{p} + 2\sqrt{2p}}{2\sqrt{p}}\right]$$

$$\int \frac{x^2}{\sqrt{9x^2+16}}$$

$u^2 = 9x^2$

$u = 3x$

$$= \frac{p}{2}\left[\sqrt{2} + \log\frac{2 + 2\sqrt{2}}{2}\right]$$

$$= p/2\left[\sqrt{2} + \log(1+\sqrt{2})\right]$$

$$\int x\sqrt{2 - 5x + 3x^2}$$

$$\int \frac{du}{\sqrt{u^2+a^2}} \longleftarrow \int \frac{dx}{\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{\sqrt{4x^2+9}}$$

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \underline{\cos x \, dx}$$

$$= \int (1-\sin^2 x) \underline{\cos x \, dx}$$

$u = \sin x$
$du = \cos x \, dx$

$$= \int (1-u^2) \, du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

$u = \tan x$
$du = \sec^2 x \, dx$

$$\int \tan^4 x \, dx$$

$$= \int \tan^2 x \cdot \frac{(\sec^2 x - 1)}{\cancel{\tan^2 x}} \, dx = \int (\tan^2 x \cdot \sec^2 x - \tan^2 x) \, dx$$

$$= \int \tan^2 x \cdot \underline{\sec^2 x \, dx} - \int \frac{\cancel{\tan^2 x}}{(\sec^2 x - 1)} \, dx = \int \tan^2 x \sec^2 x \, dx - \int \sec^2 x \, dx + \int dx$$

$$= \int u^2 \, du - \int du + x$$

$$\int d(uv) = \int u\,dv + \int v\,du$$

$\int dz =$

$$uv = \underline{\int u\,dv} + \int v\,du$$

integration by parts

$$\underline{\int u\,dv = uv - \int v\,du}$$

$$\int x \underline{\sin x\,dx} = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$$

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$$\int xe^x\,dx = xe^x - \int e^x dx$$

$u = x \qquad dv = e^x dx \qquad = xe^x - e^x + C$

$du = dx \qquad v = e^x$

Page 332)1a

$$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(x+1)^2+4} \quad \left[\text{Form } \int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a}+C\right]$$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+C$$

Form

1g) $$\int \frac{dx}{4x^2+4x+5} = \frac{1}{2}\int \frac{2dx}{(2x+1)^2+4} = \frac{1}{4}\arctan\left(\frac{2x+1}{2}\right)+C$$

$\int \frac{du}{u^2+a^2}$ $a=2$, $u=2x+1$, $du=2dx$

1i) $$\int \frac{dt}{\sqrt{15+6t-9t^2}} = \int \frac{dt}{\sqrt{15-(-6t+9t^2)}} = \int \frac{dt}{\sqrt{16-(9t^2-6t+1)}}$$

$\left[\text{Form } \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+C\right]$ $a=4$, $u=3t-1$, $du=3dt$

$$= \frac{1}{3}\int \frac{3dt}{\sqrt{16-(3t-1)^2}} = \frac{1}{3}\arcsin\frac{3t-1}{4}+C$$

Page 333)1 $$\int \frac{(2+x)dx}{9+x^2} = \int \frac{(2+x)dx}{(3+x)^2-6x} = \int \frac{2+u-3}{u^2-6u+18}du$$

Let $u^2 = (3+x)^2$
$u = 3+x$
$x = u-3$
$dx = du$

$$= \int \frac{u-1}{u^2-6u+18}du = \int \frac{u\,du}{u^2-6u+18} - \frac{du}{u^2-6u+18}$$

$\int \frac{2dx}{9+x^2} + \int \frac{x\,dx}{9+x^2}$

$$= \int \frac{u\,du}{(u-3)^2+9} - \frac{du}{(u-3)^2+9} \quad (\text{Form as in Pg. 332)1a}$$

$u=x$, $a=3$, $du=dx$: $2\int \frac{du}{u^2+a^2} = \frac{2}{a}\tan^{-1}\frac{u}{a} = \frac{2}{3}\arctan\frac{x}{3}$

$u=9+x^2$, $du=2x\,dx$: $\frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln u = \frac{1}{2}\ln(9+x^2)$

$$= \frac{1}{2}\ln\left[\cancel{(u-3)^2+9}\right] - \frac{1}{3}\arctan\frac{u}{3}+C$$

$$= \frac{1}{2}\ln\cancel{(9+x^2)} - \frac{1}{3}\arctan\frac{x+3}{3}+C$$

333) 4

$$\int \frac{(x+3)\,dx}{\sqrt{x^2+4}} = \int \frac{(x+3)\,dx}{\sqrt{(x+2)^2-4x}} = \int \frac{(u+1)\;du}{\sqrt{u^2-4u-8}}$$

Let $u^2 = (x+2)^2$

$u = x+2$

$x = u-2$

$dx = du$

$$= \int \frac{u\,du}{\sqrt{(u-2)^2-12}} + \int \frac{du}{\sqrt{(u-2)^2-12}}$$

$=$

$$\int \frac{x\,dx}{\sqrt{x^2+4}} + \int \frac{3\,dx}{\sqrt{x^2+4}}$$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$\frac{1}{2}\int u^{-1/2}\,du$

$3\ln(x+\sqrt{x^2+4})$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2})$$

$u = x$

$a = 2$

$du = dx$

333) 8

$$\int \frac{(x+3)\,dx}{\sqrt{2x-x^2}} = \int \frac{(x+3)\,dx}{\sqrt{-(1-x)^2+1}} = \int \frac{(x+3)\,dx}{\sqrt{1-(1-x)^2}}$$

Let $u^2 = (1-x)^2$

$u = 1-x$

$du = -dx$

$dx = -du$

$x = 1-u$

$$= \int \frac{(1-u+3)(-du)}{\sqrt{1-u^2}} = \int \frac{-u+4}{\sqrt{1-u^2}}\,du$$

$$= \int \frac{-u\,du}{\sqrt{1-u^2}} + \int \frac{4\,du}{\sqrt{1-u^2}}$$

$v = 1-u^2$

$dv = -2u\,du$

$= \frac{1}{2}\int \frac{dv}{\sqrt{v}}$

$= \frac{1}{2}\int v^{-1/2}\,dv$

$= \frac{1}{2}\cdot v^{1/2}\cdot 2 = \sqrt{1-u^2} = \sqrt{1-(1-x)^2} = \sqrt{2x-x^2}$

$+4\arcsin\frac{u}{a}$

$+4\arcsin(1-x)$

$$\frac{1}{2}\,\frac{u^{1/2}}{1/2}$$

$\sqrt{x^2+4}$

333) 8

$$\int \frac{(x+3)\,dx}{\sqrt{2x-x^2}} = \int \frac{(x+3)\,dx}{\sqrt{-(1-2x+x^2)+1}}$$

$$= \int \frac{(x+3)\,dx}{\sqrt{1-(1-x)^2}}$$

Let $u^2 = (1-x)^2$

$u = 1-x$

$x = 1-u$

$dx = -du$

$$= \int \frac{(1-u)\,du}{\sqrt{1-(u)^2}} + \int \frac{-3\,du}{\sqrt{1-(u)^2}}$$

$$= \int \frac{(u-1)\,du}{\sqrt{1-(u)^2}} - \int \frac{3\,du}{\sqrt{1-(u)^2}}$$

$$= \int \frac{u\,du}{\sqrt{1-u^2}} - \int \frac{du}{\sqrt{1-u^2}} - \int \frac{3\,du}{\sqrt{1-u^2}}$$

$$= \int \frac{u\,du}{\sqrt{1-u^2}} - \int \frac{4\,du}{\sqrt{1-u^2}}$$

$=$

336) 1

$$\int \sin^3 x \, dx = \int (1-\cos^2 x)\sin x \, dx$$

$$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$$

Let $u = \cos x$

$du = -\sin x \, dx$

$$= \int \sin x \, dx - \int u^2(-du)$$

$$= -\cos x + \frac{1}{3}\cos^3 x + C$$

336) 4

$$\int \cos^2 x \, dx = \int (\sin^2 x + \cos 2x)\, dx$$

$$\cancel{= \int (\sin^2 x + 2\cos^2 x - 1)\, dx}$$

$$\int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\sin 2x$$

$$\cancel{\int (1 - \sin^2 x)\, dx}$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$\cos 2x = 1 - 2\sin^2 x$

$-2\sin^2 x = \cos 2x - 1$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$= \int \left(\frac{1}{2} - \frac{\cos 2x}{2} + \cos 2x\right) dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

336) 9

$$\int \operatorname{ctn}^2\theta \, d\theta =$$

$$\int (\csc^2\theta - 1)\, d\theta$$

$$= \int \csc^2\theta \, d\theta - \int d\theta$$

$$= -\cot\theta - \theta + C$$

$$\int x^2 dx = x^3/3 + C$$

340) 1

$\left(\int u\,dv = uv - \int v\,du\right)$

$$\int x \cos x\,dx = x(\sin x + \cancel{C_1}) - \int (\sin x + \cancel{C_1})\,dx$$

Let $u = x$

$du = dx$

$dv = \cos x\,dx$

$v = \sin x + \cancel{C_1}$

$$= x \sin x + \cancel{C_1 x} + \cos x - \cancel{C_1 x} + C$$

$$= x \sin x + \cos x + C$$

340) 2

$$\int x^2 e^{-x}\,dx = -x^2(e^{-x} + \cancel{C_1}) + \int (e^{-x} + \cancel{C_1})\,2x\,dx$$

Let $u = x^2$

$du = 2x\,dx$

$dv = e^{-x}\,dx$

$v = -e^{-x} + \cancel{C_1}$

$-v\,du = e^{-x}\,2x\,dx$

$$= -x^2 e^{-x} + \cancel{C_1 x^2} + 2\int x e^{-x} dx - C_1$$

Let $u = x$

$du = dx$

$dv = e^{-x} dx$

$v = -e^{-x}$

$\int x e^{-x} dx - \cancel{C}$

$= -x e^{-x} + \int (e^{-x} + \cancel{C}) dx$

$= -x e^{-x} - e^{-x} - \cancel{C}$

$$= -x^2 e^{-x} + \cancel{C_1 x^2} - 2x e^{-x} - \cancel{e^{-x}}$$

$C_1 x\, x\, C$

?

$$\cancel{= x^2 e^{-x} - 2x e^{-x} - e^{-x} + C}$$

$$\cancel{= e^{-x}(x^2 - 2x - 1) + C}$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$= e^{-x}(-x^2 - 2x - 2) + C$$

340) 18

$\int u\,dv = uv - \int v\,du$ $\frac{\pi}{3}$ rad

$$\int \arccos x \, dx =$$

$u = \arccos x$

$dv = dx$

$du = -\frac{1}{\sqrt{1-x^2}}\,dx$

$v = x$

$$= x \arccos x + \int x\left(\frac{1}{\sqrt{1-x^2}}\right)dx$$

$z = 1 - x^2$

$dz = -2x\,dx$

$-\frac{dz}{2} = x\,dx$

$$\int x\left(\frac{1}{\sqrt{1-x^2}}\right)dx = \int \frac{-\frac{dz}{2}}{\sqrt{z}}$$

$$= -\frac{1}{2}\int z^{-\frac{1}{2}}\,dz$$

$$= -\frac{1}{2}\cdot \frac{z^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= -\sqrt{z} = -\sqrt{1-x^2}$$

$$= x \arccos x - \sqrt{1-x^2} + C$$

Li 1-2846 (6A)

340) 1p

$$\int e^{2t}\cos 3t\,dt = e^{2t}\frac{\sin 3t}{3} - \frac{2}{3}\int \frac{\sin 3t}{\cancel{3}}(\cancel{2}e^{2t}\,dt)$$

Let $u = e^{2t}$

$dv = \cos 3t\,dt$

$du = 2e^{2t}\,dt$

$v = \frac{\sin 3t}{3}$

Let $u = e^{2t}$

$du = 2e^{2t}\,dt$

$dv = \sin 3t$

$v = \frac{-\cos 3t}{3}$

$$\int \sin 3t\, e^{2t}\,dt = e^{2t}\left(\frac{-\cos 3t}{3}\right) + \int \frac{+\cos 3t}{3}\,2e^{2t}\,dt$$

$$\int e^{2t}\cos 3t\,dt = \frac{1}{3}e^{2t}\sin 3t - \frac{2}{3}\left[-\frac{e^{2t}\cos 3t}{3} + \frac{2}{3}\int \cos 3t\, e^{2t}\,dt\right]$$

$$\int e^{2t}\cos 3t\,dt = \frac{1}{3}e^{2t}\sin 3t + \frac{2}{9}e^{2t}\cos 3t - \frac{4}{9}\int \cos 3t\, e^{2t}\,dt$$

$$\frac{13}{9}\int e^{2t}\cos 3t\,dt = e^{2t}\left(\frac{1}{3}\sin 3t + \frac{2}{9}\cos 3t\right)$$

$$\int e^{2t}\cos 3t\,dt = \frac{9}{13}e^{2t}\left(\frac{1}{3}\sin 3t + \frac{2}{9}\cos 3t\right) + C$$

$$\int \frac{2x+1}{x^2-1}\,dx \qquad \frac{2x+1}{x^2-1}$$

$$\frac{2x+1}{x^2-1} =$$

$$\frac{2x+1}{x^2-1} = \frac{2x+1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$2x+1 = A(x-1) + B(x+1)$$

$$3 = 2B, \quad B = 3/2$$

$$+1 = +2A, \quad A = 1/2$$

$$\frac{2x+1}{x^2-1} = \frac{1/2}{x+1} + \frac{3/2}{x-1}$$

$$\int \frac{2x+1}{x^2-1}\,dx = \frac{1}{2}\int \frac{dx}{x+1} + \frac{3}{2}\int \frac{dx}{x-1}$$

$$= \frac{1}{2}\ln(x+1) + \frac{3}{2}\ln(x-1) + C$$

$$= \ln\sqrt{x+1} + \ln(x-1)^{3/2} + C$$

$$= \ln\left(\sqrt{x+1}\,(x-1)^{3/2}\right) + \ln c$$

$$= \ln\left[c\sqrt{x+1}\,(x-1)^{3/2}\right]$$

$$\int \frac{2x-4}{x(4x^2-9)}\,dx =$$

$$\frac{2x-4}{x(4x^2-9)} = \frac{A}{x} + \frac{B}{(2x-3)} + \frac{C}{(2x+3)}$$

$$2x-4 = (2x-3)(2x+3)A + x(2x+3)B + x(2x-3)C$$

If $x=0$, $-4 = (-3)(+3)A$

$$-4 = -9A$$

$$A = \frac{4}{9}$$

If $x = \frac{3}{2}$, $3-4 = -1 = 0 + (9)B + 0$

$$9B = -1$$

$$B = -\frac{1}{9}$$

If $x = -\frac{3}{2}$, $-3-4 = -7 = 0 + 0 + -\frac{3}{2}(-3-3)C$

$$-7 = 9C$$

$$C = -\frac{7}{9}$$

2 sin x 9 cos x

3 sin x

$$\int \frac{2x^2+x-4}{x^2-1}\,dx$$

$$x^2-1 \overline{)\,2x^2+x-4\,}\,(2$$
$$\underline{-2x^2 \quad \mp 2}$$
$$x-2$$

$$\int \frac{2x^2+x-4}{x^2-1}\,dx = 2+\frac{x-2}{x^2-1} = \int 2\,dx + \int \frac{\checkmark}{x+1}\,dx + \int \frac{\checkmark}{x-1}\,dx$$

$$\frac{x-2}{x^2-1} = \frac{\checkmark}{x+1}\quad\frac{\checkmark}{x-1}$$

ask

$$\int \frac{2x-4}{x(4x^2-9)}dx = \int \frac{\frac{4}{9}}{x}dx - \int \frac{\frac{1}{9}}{(2x-3)}dx - \int \frac{\frac{7}{9}}{(2x+3)}dx$$

$$= \frac{4}{9}\ln x - \frac{1}{18}\ln(2x-3) - \frac{7}{18}\ln(2x+3) + C$$

$$\int \frac{3x-4}{(x-3)^2\,x}dx$$

$$\frac{3x-4}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \_\_\_\_$$

$$\int \frac{2x+7}{(x^2+4)x}dx$$

$$\frac{2x+7}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \_\_\_\_$$

Page 348

350

Page 336) 2

$$\int \sin^5 x\,dx = \int (1-\cos^2 x)^2 \sin x\,dx$$

$$= \int (1 - 2\cos^2 x + \cos^4 x)\sin x\,dx$$

$$= \int \sin x\,dx - 2\int \cos^2 x \sin x\,dx + \int \cos^4 x \sin x\,dx$$

Let $u = \cos x$
$du = -\sin x\,dx$

$$= \int \sin x\,dx + 2\int u^2\,du - \int u^4\,du$$

$$= -\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + C$$

$$\frac{d}{dx}\left(\qquad \right) = \ln x$$

$$\cancel{\frac{d}{dx}\left(\frac{1}{x}\right)}$$

logaritm Naturalis
ln

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx = x\ln x - \int dx$$
$$= x\ln x - x$$

$u = \ln x$
$dv = dx$
$v = x$
$du = \frac{1}{x}dx$

336) 5

$$-2\sin^2 A = \cos 2A - 1$$
$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\int \sin^4 x\,dx = \int\left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \int\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right)dx$$

$$= \frac{1}{4}\int(1-2\cos 2x+\cos^2 2x)\,dx$$

$\int \cos u\,du = \sin u$

$$= \frac{1}{4}\int\left(1-2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos 4x\right)dx$$

$$= \frac{1}{4}\int\left(\frac{2-4\cos 2x+1+\cos 4x}{2}\right)dx$$

$\int \cos 2x\,dx = \frac{1}{2}\sin 2x$

$$= \frac{1}{8}\int(3-4\cos 2x+\cos 4x)\,dx$$

$$= \frac{3}{8}x - \frac{4}{8\cdot 2}\sin 2x + \frac{1}{8\cdot 4}\sin 4x + C$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

336) 10

$$\int \tan^3 x\,dx = \int(\sec^2 x - 1)\tan x\,dx$$

$$= \int \underbrace{\sec^2 x}_{du}\,\underbrace{\tan x}_{u}\,dx - \int \tan x\,dx$$

let $u = \cancel{\sec^2 x}$ $\tan x$

$du = \cancel{\tan x\,dx}$ $\sec^2 x\,dx$

$$= \frac{\tan^2 x}{2} - \ln \sec x + C$$

$\int \sec^2 x\,dx = \tan x$ $\quad \int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = -\ln\cos x$

$-\ln\left(\frac{1}{\sec x}\right)$

$= -(\ln 1 - \ln\sec x)$

Book gives $\ln\cos x$

337) 1 d

$$\int \frac{\sqrt{4-x^2}\,dx}{x^2} = -\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\frac{x}{2} + C$$

(Form #3) or 76 in LSM (pg. 554)

$$\boxed{\begin{array}{l} \int \frac{(a^2-u^2)^{\frac{1}{2}}}{u^2}du = -\frac{\sqrt{a^2-u^2}}{u} - \arcsin\frac{u}{a} + C \\ a = 2 \\ u = x \\ du = dx \end{array}}$$

$$= -\frac{\sqrt{4-x^2}}{x} - \arcsin\frac{1}{2}x + C$$

337) 1 e

$u = s$
$a = 4$
$du = ds$

$$\int \frac{\sqrt{s^2-16}}{s}ds = \sqrt{s^2-16} - 4\,\mathrm{arc\,sec}\,\frac{s}{4} + C$$

Form 55 (LSM)

$$\int \frac{(u^2-a^2)^{\frac{1}{2}}}{u}du = \sqrt{u^2-a^2} - a\,\mathrm{arc\,sec}\,\frac{u}{a} + C$$

337) 1 i

Form 28 (page 550)

$$\int \frac{u\,du}{\sqrt{a+bu}} = -\frac{2(2a-bu)(\sqrt{a+bu})}{3b^2} + C$$

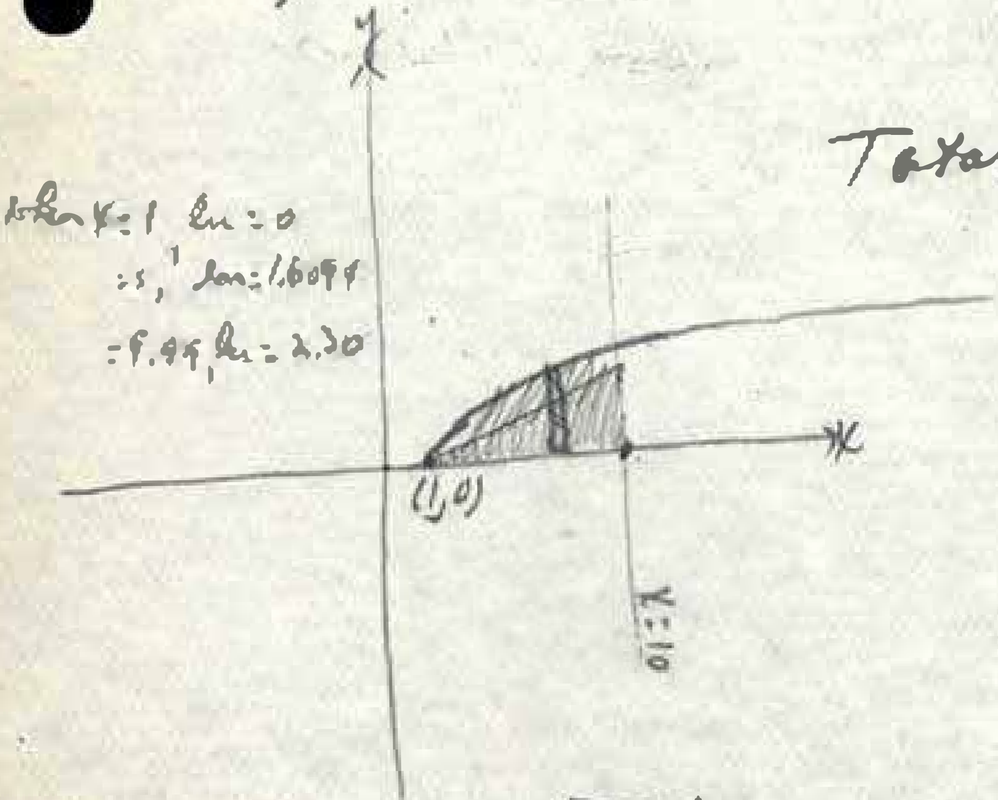
$\cancel{u=t^2}$
$\cancel{b=4}$
$a=1$
$du \neq dt$

$$\int \frac{t^2\,dt}{2\sqrt{4(t^2+1/4)}} = -\frac{2(2-4t^2)(\sqrt{4t^2+1})}{48} + C$$

$$\frac{1}{2}\int \frac{t^2\,dt}{\sqrt{t^2+\frac{1}{4}}} = \frac{1}{4}\left[\frac{t}{2}\sqrt{t^2+1/4} - \frac{1}{16}\log\left(t+\sqrt{t^2+1/4}\right)\right] + C = -\frac{(4-8t^2)(\sqrt{4t^2+1})}{48} + C$$

---

340) 2



Element of area $= y\,dx$

$$\text{Total Area} = \int_1^{10} y\,dx = \int_1^{10} \ln x\,dx$$

$\cancel{u=\ln x}$
$\cancel{du=dx}$

$$= \cancel{\frac{\ln^2 x}{2}\Big]_1^{10}}$$

$$= \frac{\ln^2(10)}{2} - \frac{\ln^2(1)}{2}$$

$$= \frac{\quad}{2}$$

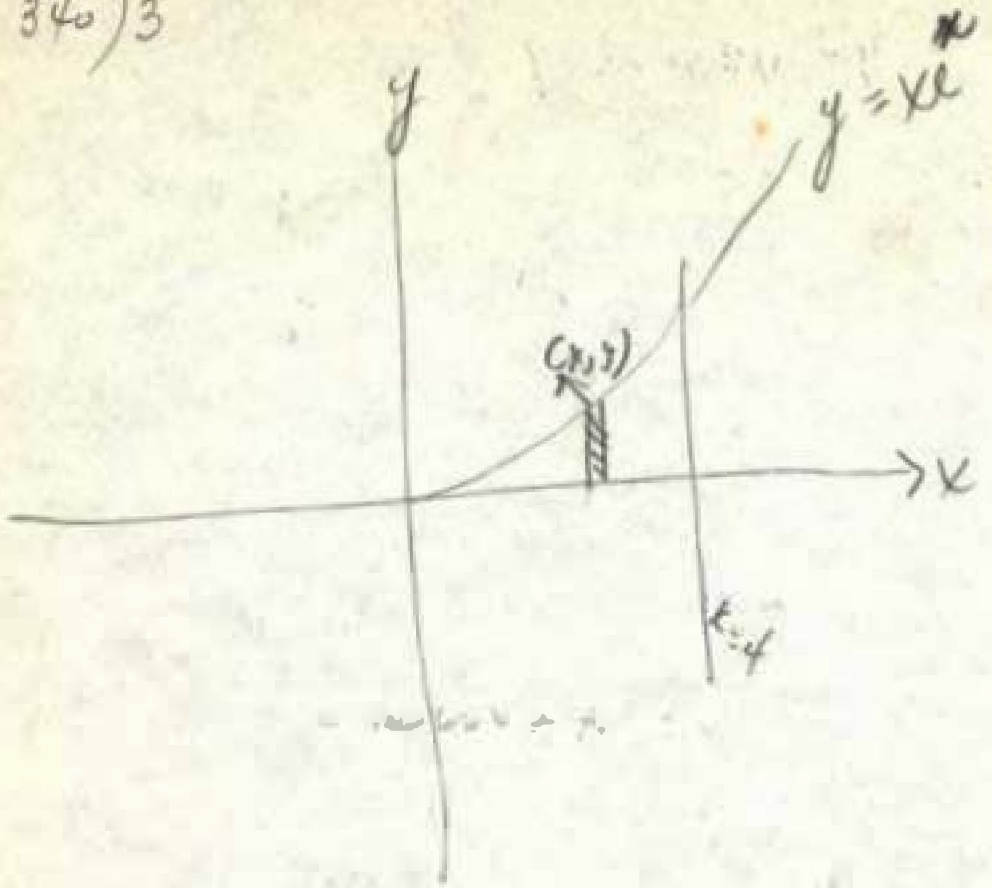
$$x\ln x - x\Big]_1^{10}$$

$= (10\ln 10 - 10) - (0-1)$

$10\ln 10 - 10 + 1$

$= \underline{10\ln 10 - 9}$

340) 3



$$\int u\,dv = uv - \int v\,du$$

El. of area = $y\,dx$

$$\text{Area} = \int_0^4 y\,dx = \int_0^4 \underline{xe^x}\,dx = e^x(x-1)\Big]_0^4$$

$$= x\frac{de^x}{dx} + e^x\frac{dx}{dx}\Big]_0^4$$

$$= e^4\cdot 3 - 1\cdot(-1)$$

$$= \underline{3e^4+1}$$

$$\int u\,dv = uv - \int v\,du$$

$$= xe^x + e^x\Big]_0^4$$

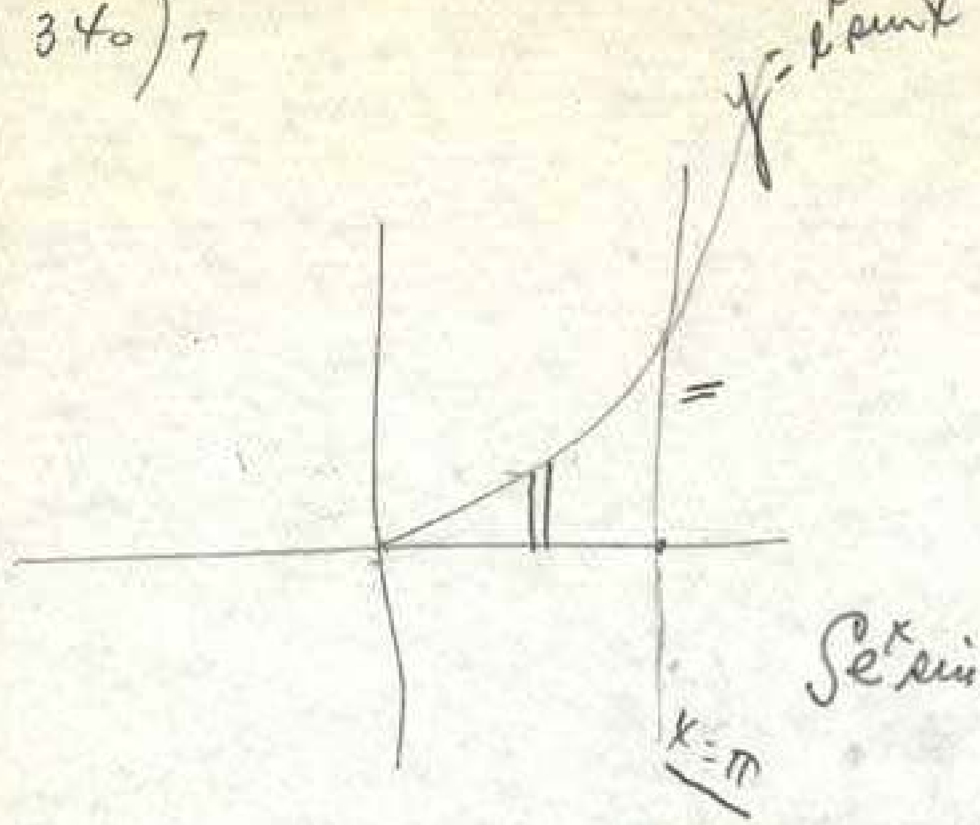
$$= e^x(x+1)\Big]_0^4 = 5e^4$$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x$$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

340)7



El. of area $= y\,dx$

Total area $= \int_0^{\pi} y\,dx = \int_0^{\pi} e^x \sin x\,dx$

$= e^x \frac{d}{dx}\sin x + \sin x \frac{d}{dx}e^x \Big]_0^{\pi}$

$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$

$= \cancel{e^x \cos x} + \sin x\, e^x \Big]_0^{\pi}$

$= e^x(\cos x + \sin x)\Big]_0^{\pi}$

$= e^{\pi}(\cos\pi + \sin\pi)$

$\frac{e^x(\sin x - \cos x)}{2}$

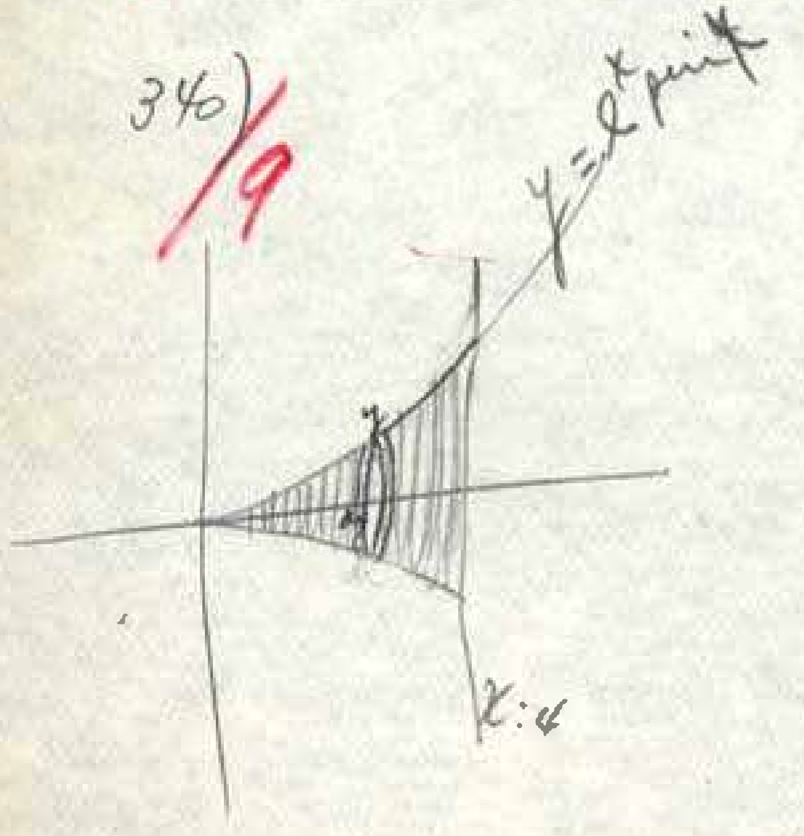
$u = e^x \quad dv = \sin x\,dx$
$du = e^x dx \quad v = -\cos x$

$u = e^x \quad dv = \cos x\,dx$
$du = e^x dx \quad v = \sin x$

$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \cancel{\int e^x \sin x\,dx}$

$\int_0^{\pi} e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2}\Big]_0^{\pi} = \frac{0 + e^{\pi}}{2} - \frac{0 - 1 \cdot 1}{2} = \frac{e^{\pi}}{2} + \frac{1}{2} = \frac{e^{\pi}+1}{2}$

340)9



El. of vol. $= \pi r^2 dx = \pi y^2 dx$

Total Vol. $= \int_0^4 (e^x \sin x)^2 dx$

$= \int_0^4 e^{2x} \sin^2 x\,dx$

$= e^{2x} \frac{d}{dx}\sin^2 x + \sin^2 x \frac{d}{dx} e^{2x} \Big]_0^4$

$= e^{2x}\, 2 \sin x \cos x + \sin^2 x\, 2e^{2x} \Big]_0^4$

$= 2e^{2x}(\sin x \cos x + \sin^2 x)\Big]_0^4$

$$\int u\,dv = uv - \int v\,du$$

$$\int \sin 2x \sin 3x \, dx$$

let $u = \sin 2x$

$dv = \sin 3x \, dx$

$du = 2\cos 2x \, dx$

$v = \dfrac{-\cos 3x}{3}$

$$= \frac{-\sin 2x \cos 3x}{3} + \frac{2}{3}\int + \frac{\cos 3x}{\cancel{3}} \cdot \cancel{2} \cos 2x \, dx$$

343) 3

$$\int \frac{dx}{x(3-2x)^2} = \frac{1}{3(3-2x)} - \frac{1}{9} \ln\left(\frac{3-2x}{x}\right) + C$$

✓

Form 15, Page 549: $u = x$, $a = 3$, $b = -2$, $du = dx$

See Form 204 Page 230 in Handbook

343) 10

$$\int \frac{dx}{5+4\sin x} = \frac{2}{3} \tan^{-1} \frac{3\tan\frac{1}{2}x + 4}{3}$$

✓

$a = 5$
$b = 4$

343) 11

$$\int \sin 2x \sin 3x \, dx = \cancel{\sin 2x \frac{d}{dx}\sin 3x + \sin 3x \frac{d}{dx}\sin 2x + C}$$

$m = 3$
$n = 2$

$$\frac{\sin(x)}{2} - \frac{\sin 5x}{10}$$

$$\cancel{= \sin 2x \, 3\cos 3x \, dx + \sin 3x \, 2\cos 2x \, dx + C}$$

343) 16

Page 554 Form 71 $\int \frac{du}{u^2\sqrt{a^2-u^2}} = \frac{\sqrt{a^2-u^2}}{a^2 u} + C$

$$\int \frac{dx}{x^2\sqrt{9-4x^2}} = \cancel{\frac{\sqrt{9-4x^2}}{18x} + C}$$

$u = x$
$a = 3/2$
$du = dx$

$$\frac{1}{2}\int \frac{dx}{x^2\sqrt{\cancel{4}(\frac{9}{4}-x^2)}} = \frac{1}{2} \cdot \frac{\sqrt{\frac{9}{4}-x^2}}{\frac{9}{4}x} + c = \frac{1}{\cancel{2}} \frac{\sqrt{9-4x^2}}{\cancel{2} \cdot 9x} \cdot \cancel{4} = \frac{\sqrt{9-4x^2}}{9x} + C$$

343) 17

$$\int \frac{\sqrt{a+bu}}{u^m} du$$

$m = 3$
$a = 2$
$b = -1$
$u = x$
$du = dx$

$$\frac{\sqrt{a+bx}}{x^3}$$

$a = 2$
$b = -1$

$$\int \frac{\sqrt{a+bx}}{x^3} dx$$

$$\int \frac{\sqrt{2-x}}{x^3} dx = -\frac{(2-x)^{3/2}}{2 \cdot 2x^2} + \frac{1}{4 \cdot 2}\int \text{---}$$

$$= -\frac{(2-x)^{3/2}}{4x^2} + \frac{1}{8}\left[-\frac{(2-x)^{3/2}}{2x} + \frac{-1}{4}\left(2\sqrt{2-x} + \frac{2}{\sqrt{2}} \ln \frac{\sqrt{2-x}-\sqrt{2}}{\sqrt{2-x}+\sqrt{2}}\right)\right] + C$$

Page 348) 1

$$\frac{(5x^2-15x+12)\,dx}{x^3-5x^2+6x} = \frac{(5x^2-15x+12)\,dx}{x(x-2)(x-3)}$$

$$\frac{5x^2-15x+12}{x(x-2)(x-3)} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$5x^2-15x+12 = A(x-2)(x-3) + Bx(x-3) + Cx(x-2)$$

If $x=0$, $12 = 6A$

$A = 2$ ✓

If $x=2$, $20-30+12 = B\,2(-1)$

$2 = -2B$

$B = -1$ ✓

$\int x^2 dx$

If $x=3$, $45-45+12 = C\,3(1)$

$12 = 3C$

$C = 4$ ✓

then $\frac{(5x^2-15x+12)\,\cancel{dx}}{x^3-5x^2+6x} = \frac{2}{x} - \frac{1}{(x-2)} + \frac{4}{(x-3)}$

and $\int \frac{(5x^2-15x+12)\,dx}{x^3-5x^2+6x} = \left[2\int \frac{1\,dx}{x} \int \frac{1\,dx}{(x-2)} + 4\int \frac{1\,dx}{(x-3)}\right] \cancel{dx}$

$= \ln x^2$

$= 2\ln x - \ln(x-2) + 4\ln(x-3) + C$

$\ln \cancel{\#} \frac{x^2(x-3)^4}{(x-2)} + C$ ✓

348) 9 $\displaystyle\int \frac{(x^4-1)\,dx}{x^3-9x} = \int \frac{(x^4-1)\,dx}{x(x+3)(x+3)}$

Complete $\displaystyle\frac{x^4-1}{x^3-9x} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3}$

$x^4-1 = A(x+3)(x-3) + Bx(x-3) + Cx(x+3)$

If $x=0$, $-1=-9A$; $A=\frac{1}{9}$ ✓

If $x=-3$, $80=18B$; $B=5$ ✓

If $x=3$, $80=18C$; $C=5$ ✓

Then $\displaystyle\int \frac{(x^4-1)\,dx}{x^3-9x} = \left[\frac{1}{9}\int x\,dx + 5\int \frac{1}{x+3} + 5\int \frac{1}{x-3}\right]dx$

$\frac{7}{5} = 1\frac{2}{5}$ $\qquad = \frac{1}{9}\ln x + 5\ln(x+3) + 5\ln(x-3) + C$

?

$\frac{12}{5} = 2\frac{2}{5}$

$\displaystyle\frac{\frac{1}{9}}{x} + \frac{5}{x+3} + \frac{5}{x-3} =$

$$\begin{array}{r|l} x^3-9x & x^4-1 \quad (x \\ & -x^4 \mp 9x^2 \\ \hline & 9x^2-1 \end{array}$$

$\displaystyle\frac{x^4-1}{x^3-9x} = x + \frac{9x^2-1}{x^3-9x} = x + \frac{\quad}{x} + \frac{\quad}{x-3} + \frac{\quad}{x+3}$

$\displaystyle\frac{9x^2-1}{x(x-3)(x+3)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+3}$

348) 15 $\displaystyle\int_3^5 \frac{dx}{(x-1)^2(x-2)}$

(What is $\displaystyle\int \frac{\frac{1}{9}}{(2x-3)}\,dx$)

Complete

$$\frac{1}{(x-1)^2(x-2)} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x-2)}$$

~~$1 = A(x-1)(x-2) + B$~~

(ask about clearing of fractions, top page 348)

$$1 = A(x-2) + B(x-2)(x-1) + C(x-1)^2$$

If $x=2$, $1 = A + C$

If $x=1$, $1 = A - B$

Then $A = 1 - C$

$A = 1 + B$

$1 - C = 1 + B$

$B = -C$

~~Then $1 = (\ +B) - B(x-2) + C(x-1)$~~

~~If $x = 1$, $A = \ + B$~~

$1 = A - C(x-2) + C(x-1)$

If $x=2$, $1 = A + C$

If $x=1$, $1 = A + C$

ask page 349 ⊛

$\displaystyle\int \frac{du}{u} = \ln u$

$\frac{du}{dx} = 2$

$u = 2x-3$

$du = 2dx$

$$\frac{1}{9}\int \frac{dx}{2x-3} = \frac{1}{9}\int \frac{\frac{1}{2}du}{u} = \frac{1}{18}\ln u + C$$

$$= \frac{1}{18}\ln(2x-3) + C$$

$C.D = x^2(x+1)^2$

$\cancel{2x^3 - x}$

$$\frac{2x^3-1}{x^2(x+1)^2} = \frac{A}{x^2} + \frac{C}{(x+1)^2}$$

$$\frac{2x^3-1}{x^2(x+1)^2} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(x+1)^2} + \frac{D}{x+1}$$

$$2x^3 - 1 = A(x+1)^2 + Bx(x+1)^2 + Cx^2 + Dx^2(x+1)$$

$$= A(x^2+2x+1) + B(x^3+2x^2+x) + Cx^2 + Dx^3 + Dx^2$$

$$2x^3 - 1 \equiv (B+D)x^3 + (A+2B+C+D)x^2 + (2A+B)x + A$$

$B+D = 2$

$A+2B+C+D = 0$

$2A+B = 0$

$A = -1$

$A = -1$

$B = 2$

$D = 0$

$C = -3$

$-1 + 4 + C = 0$

$$\int \frac{2x^3-1}{x^2(x+1)^2}\,dx = \int -\frac{1}{x^2}\,dx + \int \frac{2}{x}\,dx - \int \frac{3}{(x+1)^2}\,dx$$

Morris & Brown

Page 279 — 1
280 — 2, 9, 11, 16, 20
281 — 42, 45
282 — 50
282 — 74
283 — 80, 83, 84

$$-\frac{2x}{1+x^2}$$

$$= -2(1+x^2)^{-\frac{1}{2}} = \cancel{2u^{-\frac{1}{2}}}$$

$$\cancel{2u^{-\frac{3}{2}}}$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$(1+x^2)^{-\frac{3}{2}} \cdot 2x = \frac{2x}{(1+x^2)^{3/2}}$$

$$\frac{d}{du} \ln u = \frac{1}{u}$$

$$\frac{2x\sqrt{1+x^2}}{(1+x^2)^2} = \frac{\sqrt{4x^2+4x^4}}{(1+x^2)^2}$$

$$\frac{d}{du} \ln(-u) = -\frac{1}{u}(-1) = \frac{1}{u}$$

$$-\frac{2(2x)+(1+x^2)}{(1+x^2)^2}$$

$$\int \frac{1}{u} du = \ln u + C$$
$$= \ln(-u) + C$$
$$= \ln|u|$$

$$\frac{4x}{(1+x^2)^2}$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$u^2+10 \overline{) u^2} \quad 1$$
$$\underline{u^2+10}$$
$$-10$$

279) 1

$$\int \frac{x^2\,dx}{1+x^6} = \int \frac{x^2\,dx}{(1+x^3)^2 - 2x^3}$$

Let $u = x^3$ ✓

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$$= \frac{1}{3}\int \cancel{\frac{du}{(1+u)^2 - 2u}} = \frac{1}{3}\int \frac{du}{1+u^2}$$

this is unnecessary

Formula $\overline{XVII}$ (page 324)

$$\boxed{\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C}$$

$$= \frac{1}{3}\cdot\frac{1}{1}\arctan\frac{u}{a} + C$$

$$= \frac{1}{3}\arctan x^3 + C$$

✓

---

280) 2

$\int \frac{du}{1-u^2} = \frac{1}{2}\ln$

$$\int \frac{x^3\,dx}{1-x^8} = \frac{1}{4}\int \frac{du}{1-u^2}$$

Let $u = x^4$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

Form. $\overline{XVIII}$-a

Page 324

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\frac{a+u}{a-u} + C$$

$$= \frac{1}{4}\cdot\frac{1}{2}\ln\frac{1+x^4}{1-\cancel{u^4}} + C$$

$$= \frac{1}{8}\ln\frac{1+x^4}{1-x^4} + C$$

(see ans. in book)

✓

---

280) a

$$\int \frac{\sqrt{3x-4}}{x+2}\,dx = \cancel{\int \frac{\sqrt{3x-4}\,dx}{\frac{3x-4+10}{3}}} = \frac{2}{3}\cancel{\int \frac{u^2\,du}{\left(\frac{u^2+10}{3}\right)}}$$

Let $u = \sqrt{3x-4}$

Then $u^2 = 3x-4$

$= \frac{2}{3}\int u^2\,du$

$2u\,du = 3\,dx$

$dx = \frac{2u\,du}{3}$

$$= 2\int \frac{u^2\,du}{u^2+10} = 2\int\left(1 - \frac{10}{u^2+10}\right)du$$

$$= 2\left[u - \frac{10}{\sqrt{10}}\cdot\tan^{-1}\frac{u}{\sqrt{10}}\right] + C$$

$$= 2\left[\sqrt{3x-4} - \sqrt{10}\tan^{-1}\frac{\sqrt{3x-4}}{\sqrt{10}}\right] + C$$

$$x+2 = \frac{3x-4+10}{3} = \frac{1}{3}(u^2+10)$$

(see over
for another soln)

280) 9 $\displaystyle\int \frac{\sqrt{3x-4}}{x+2}\,dx = \int \frac{\sqrt{3u-10}\,du}{u}$

Let $u = x+2$

$du = dx$

$3u = 3x+6$

$3x-4 = 3x+6-10 = 3u-10$

Formula 34 Page 551

$$\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu} + a\int\frac{du}{u\sqrt{a+bu}}$$

$a = -10$

$b = 3$

$$= 2\sqrt{3u-10} + \left[-10\int\frac{du}{u\sqrt{a+bu}}\right]$$

Formula 32 Page 551

$$\int\frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bu}{-a}} + C$$

(for $a<0$)

$$= 2\sqrt{3u-10} + \left[-10\left(\frac{2}{\sqrt{10}}\arctan\sqrt{\frac{a+bu}{-a}}\right) + C\right]$$

$$= 2\sqrt{3x+6-10} - \frac{20}{\sqrt{10}}\arctan\sqrt{\frac{3x+6-10}{10}} + C$$

$$= 2\sqrt{3x-4} - \frac{20}{\sqrt{10}}\arctan\sqrt{\frac{3x-4}{10}} + C$$

280) A

Let $u = \sqrt{3x-4}$

$u^2 = 3x-4$

$2u\,du = 3\,dx$

$x = \frac{u^2+4}{3}$

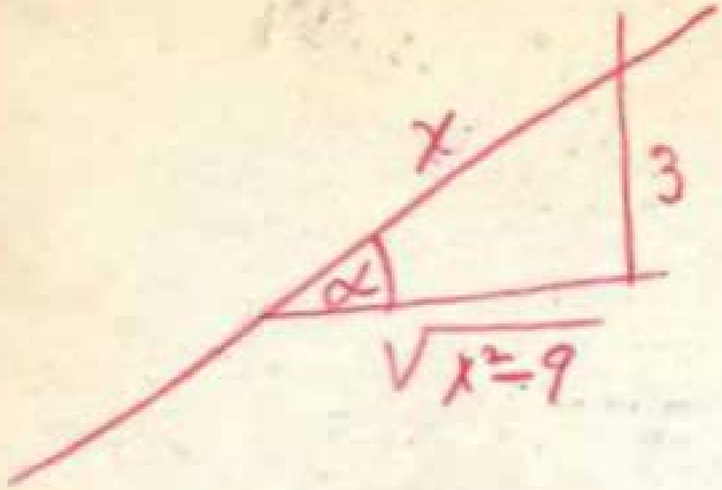
$$\int \frac{\sqrt{3x-4}}{x+2}\,dx$$

$$= \int \frac{u \cdot \frac{2u\,du}{3}}{\frac{u^2+4}{3}+2}$$

$$= \frac{2}{3}\int \frac{\cancel{3}\,u^2\,du}{\frac{u^2+4+6}{3}} = 3\int \frac{2u^2\,du}{u^2+10}$$

$$= 6\int \frac{u^2\,du}{u^2+10}$$

280/11



$\csc\alpha = \frac{x}{3}$, $\quad x = 3\csc\alpha$

$\alpha = \csc^{-1} x/3 \quad dx = -3\csc\alpha \cdot \cot\alpha \, d\alpha$

$$\int \frac{\sqrt{x^2-9}}{x^2}\,dx = -3\int \frac{3\cot\alpha \cdot \csc\alpha \cot\alpha \, d\alpha}{9\csc^2\alpha}$$

$$= -\int \frac{\cot^2\alpha}{\csc\alpha}\,d\alpha$$

$$= -\int \frac{\cos^2\alpha \cdot \sin\alpha}{\sin^2\alpha}\,d\alpha = -\int \frac{\cos^2\alpha}{\sin\alpha}\,d\alpha$$

$$= -\int \frac{1-\sin^2\alpha}{\sin\alpha}\,d\alpha$$

$$= -\left[\int \csc\alpha\,d\alpha - \int \sin\alpha\,d\alpha\right]$$

$$= \cancel{\int}$$

$$-\log\tan\frac{\alpha}{2} - \cos\alpha + C$$

$\therefore -\log\tan\left(\frac{1}{2}\csc^{-1} x/3\right) - \frac{\sqrt{x^2-9}}{x} + C$

280) 11 $\displaystyle\int \frac{\sqrt{x^2-9}}{x^2}dx = -\frac{\sqrt{x^2-9}}{x} + \ln\left(x+\sqrt{x^2-9}\right) + C$

$u = x$

$a = 3$

let $\mu = \sqrt{\ }$ when only 1st degree is present

See 259 Problem 8 M+B

Page 553 — No. 56

$$\int \frac{\sqrt{u^2-a^2}}{u^2} = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left(u+\sqrt{u^2-a^2}\right) + C$$

---

280) 16 $\displaystyle\int \frac{x^3\,dx}{\sqrt{4-3x^2}} = \int \frac{u^3\,du}{\sqrt{3}\sqrt{\frac{4}{3}-u^2}} = \frac{1}{\sqrt{3}}\int \frac{u^3\,du}{\sqrt{\frac{4}{3}-u^2}}$

$m=3$

Let $u = x$

$du = dx$

Note formulas 68 + 69 on page 553 (why both?)

see alternate method as in 280) 20

Formula 69

$m = 3$

$n = 1$

$a^2 = \frac{4}{3}$

$$= \frac{1}{\sqrt{3}}\left[\frac{u^4}{\frac{4}{3}(1-2)\left(\frac{4}{3}-u^2\right)^{-\frac{1}{2}}} - \frac{3-1+3}{\frac{4}{3}(1-2)}\int\frac{u^3\,du}{\left(\frac{4}{3}-u^2\right)^{-\frac{1}{2}}}\right]$$

$$\int\frac{u^3\,du}{\left(\frac{4}{3}-u^2\right)^{-\frac{1}{2}}} = \int\sqrt{\frac{4}{3}-u^2}\,u^3\,du = \sqrt{\frac{4}{3}-u^2}\cdot 3u^2 - \int 3u^2\cdot u\,du$$

$\boxed{uv - \int v\,du}$

let $u = \sqrt{\frac{4}{3}-u^2}$

$dv = u^3\,du$

$v = 3u^2$

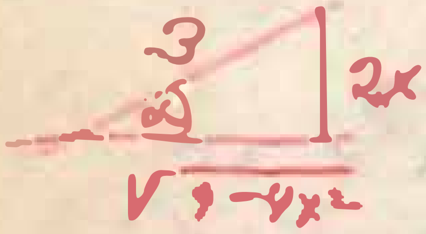
$du = -2u\cdot -\frac{1}{2} = u\,du$

$$= \sqrt{\frac{4}{3}-u^2}\cdot 3u^2 - \frac{3}{4}u^4$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^4\sqrt{\frac{4}{3}-x^2}}{-\frac{4}{3}} - \frac{-1}{-\frac{4}{3}}\sqrt{\frac{4}{3}-x^2}\cdot 3x^2 - \frac{3}{4}x^4\right]$$

$$= \frac{1}{\sqrt{3}}\left[-\frac{3x^4\sqrt{\frac{4}{3}-x^2}}{4} - \frac{9x^2\sqrt{\frac{4}{3}-x^2}}{4} - \frac{3}{4}x^4\right]$$

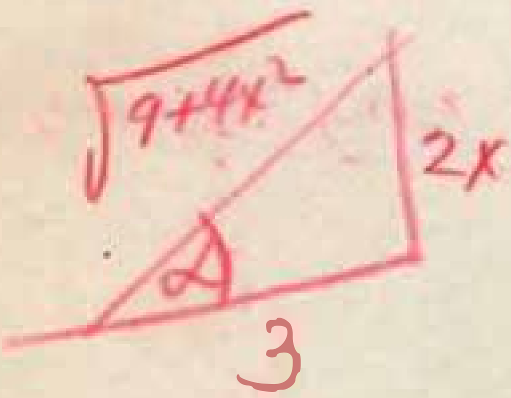
$$\int \sqrt{9-4x^2}\, dx = \int 3\cos\alpha \cdot \frac{3}{2}\cos\alpha\, d\alpha$$



$\sin\alpha = \frac{2x}{3}$, $x = \frac{3}{2}\sin\alpha$, $dx = \frac{3}{2}\cos\alpha\, d\alpha$

M+B
260
Problem 9

$$\int \sqrt{9+4x^2}\, dx = \int 3\sec\alpha \cdot \frac{3}{2}\sec^3\alpha\, d\alpha$$



$\frac{2x}{3} = \tan\alpha$, $x = \frac{3}{2}\tan\alpha$, $dx = \frac{3}{2}\sec^2\alpha\, d\alpha$

$-30) \, 20$

$$\int \frac{x^3\,dx}{\sqrt{2x+3}} = \int \frac{\left(\frac{v^2-3}{2}\right)^3 \cancel{v}\,dv}{\cancel{v}} = \int \left(\frac{v^2-3}{2}\right)^3 dv$$

$$\boxed{\begin{aligned} &\text{Let } v = \sqrt{2x+3} \\ &v^2 = 2x+3 \\ &x = \frac{v^2-3}{2} \\ &dx = v\,dv \end{aligned}}$$

$$= \frac{1}{8}\int \left(v^6 - 9v^4 + 27v^2 - 27\right) dv$$

$$= \frac{1}{8}\left(\frac{v^7}{7} - \frac{9}{5}v^5 + 9v^3 - 27v\right) + C$$

$$= \frac{v^7}{56} - \frac{9}{40}v^5 + \frac{9}{8}v^3 - \frac{27}{8}v + C$$

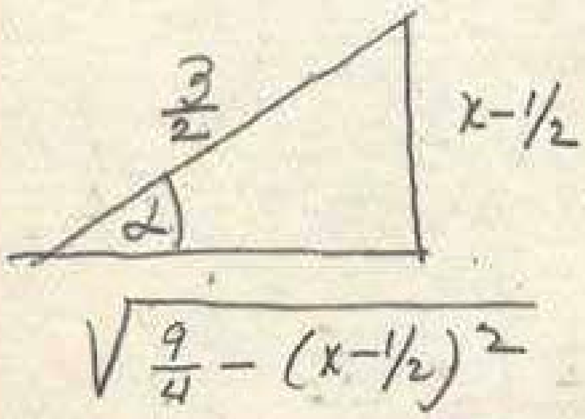
$$= v\left[\frac{v^6}{56} - \frac{9}{40}v^4 + \frac{9}{8}v^2 - \frac{27}{8}\right] + C$$

$$= \sqrt{2x+3}\left[\frac{(2x+3)^3}{56} - \frac{9}{40}(2x+3)^2 + \frac{9}{8}(2x+3) - \frac{27}{8}\right] + C$$

281/42

$$\int \frac{(3x-1)\,dx}{\sqrt{2\tfrac{+1}{4}-(x^2-x+\frac{1}{4})}} -$$

$$= \int \frac{(3x-1)\,dx}{\sqrt{\frac{9}{4}-(x-\frac{1}{2})^2}} = \int \frac{\left(\frac{9\sin\alpha+3}{2}-1\right)\cdot \cancel{\frac{3}{2}\cos\alpha}\,d\alpha}{\cancel{\frac{3}{2}\cos\alpha}}$$



$$= \int \frac{9\sin\alpha+3-2}{2}\,d\alpha$$

$$= \frac{1}{2}\left[-9\cos\alpha+\alpha\right]+C$$

$$= \frac{1}{2}\left[6\sqrt{2+x-x^2}+\sin^{-1}\frac{2x-1}{3}\right]+C$$

$$\sin\alpha = \frac{x-\frac{1}{2}}{3/2} = \frac{2x-1}{3}$$

$$2x-1 = 3\sin\alpha$$

$$x = \frac{3\sin\alpha+1}{2}$$

$$dx = \frac{3}{2}\cos\alpha\,d\alpha$$

$$\frac{\text{this}}{3/2} = \cos\alpha$$

this =

281) 42 $$\int \frac{(3x-1)\,dx}{\sqrt{2+x-x^2}} = \frac{3}{2}\ln\left(2+x-x^2\right)+\left(-1\left(\frac{3}{2}\right)\right)\int\frac{du}{2+x-x^2}$$

See Formula 108 Page 557

$a=2$
$b=1$
$c=1$
$M=3$
$N=-1$

See M+B 258 Problem 7

Formula 105 Page 556

$$\int\frac{du}{2+x-x^2} = \frac{2}{\sqrt{8-1}}\arctan\left(\frac{2x^2+1}{\sqrt{8-1}}\right)+C$$

$$= \frac{3}{2}\ln\left(2+x-x^2\right) - \frac{3}{\sqrt{7}}\arctan\frac{2x^2+7}{\sqrt{7}} + C$$

281) 40

$$\int \frac{\cos 3x\,dx}{e^{4x}} = \int \frac{1}{e^{4x}} \cdot \cos 3x\,dx$$

Let $u = \frac{1}{e^{4x}} = e^{-4x}$

$dv = \cos 3x\,dx$

$v = \frac{\sin 3x}{3}$

$du = \cancel{\frac{4}{e^{4x}}}\ -4e^{-4x}dx$

$$= \frac{1}{e^{4x}} \cdot \frac{\sin 3x}{3} + \int \frac{\sin 3x}{3} \cdot \frac{4}{e^{4x}}\ dx$$

$$= \frac{\sin 3x}{3e^{4x}} + \frac{4}{3}\int \frac{\cancel{4}\sin 3x}{\cancel{3}e^{4x}}\,dx$$

$\int u\,dv = uv - \int v\,du$

$u = e^{-4x}$ $\quad dv = \sin 3x\,dx$

$du = -4e^{-4x}dx$ $\quad v = -\frac{\cos 3x}{3}$

$\int e^{-4x}\cos 3x\,dx$

$$\int e^{-4x}\cos 3x\,dx = \frac{1}{3}e^{-4x}\sin 3x + \frac{4}{3}\left(-e^{-4x}\frac{\cos 3x}{3} - \frac{4}{3}\int e^{-4x}\cos 3x\,dx\right)$$

$$\int e^{-4x}\cos 3x\,dx = \frac{1}{3}e^{-4x}\sin 3x - \frac{4}{9}e^{-4x}\cos 3x - \frac{16}{9}\int e^{-4x}\cos 3x\,dx$$

$$\frac{25}{9}\int e^{-4x}\cos 3x\,dx = e^{-4x}\left(\frac{\sin 3x}{3} - \frac{4\cos 3x}{9}\right)$$

$$\int \quad " \quad = \frac{9}{25}e^{-4x}\left(\quad " \quad " \quad \right) + C$$

282) 50

$$\int \frac{(1+\cos x)^2}{\sin x}\,dx =$$

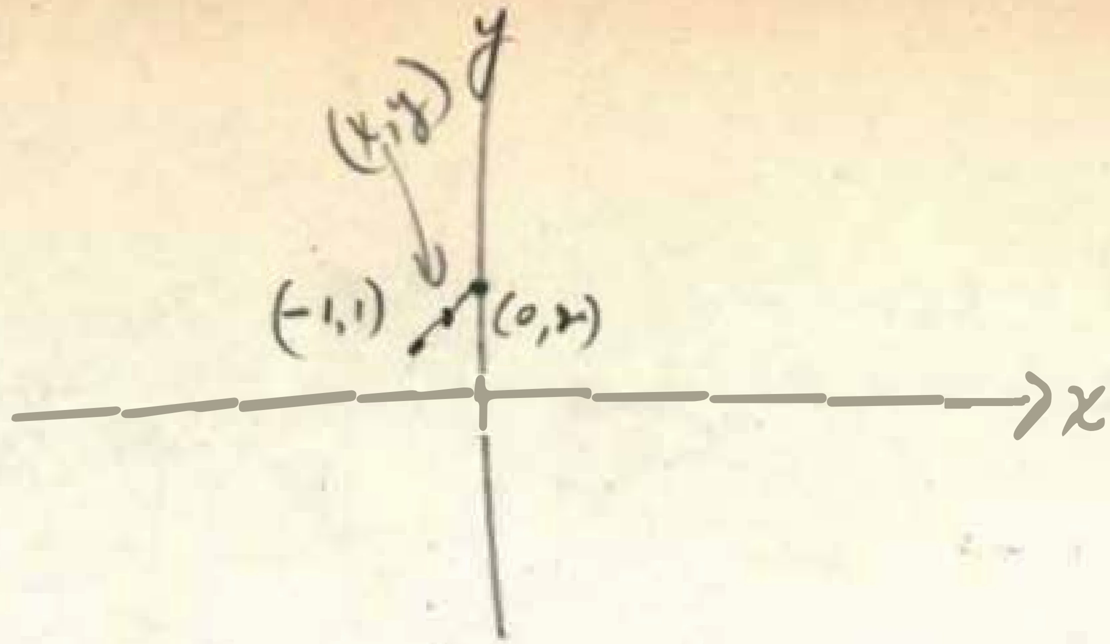
$$\int \frac{1+2\cos x+\cos^2 x}{\sin x}\,dx$$

$$= \int \csc x\,dx + 2\int \frac{\cos x}{\sin x}\,dx + \int \frac{1-\sin^2 x}{\sin x}\,dx$$

$$= 2\int \csc x\,dx + 2\int \frac{\cos x}{\sin x}\,dx - \int \sin x\,dx$$

$$= 2\,\overset{\ln}{\cancel{\log}}\tan\frac{x}{2} + 2\ln\sin x + \cos x + C$$

282) 74



$$\frac{d^2y}{dx^2} = 2-4x$$

$$\frac{dy}{dx} = \int(2-4x)\,dx$$

$$= 2x-2x^2+C$$

~~at (0,2), C = 0~~

~~at (-1,1), C = -2-2 = -4~~

~~$y = \int 2x-2x^2\,dx = x^2-\frac{2}{3}x^3+C$~~

~~at (-1,1) $y = 1+\frac{2}{3} = \frac{5}{3}$~~

~~$\therefore$ Slope = $2x-2x^2$~~

at point (x, y),

$$\frac{y-2}{x-0} = 2x-2x^2$$

$$\frac{y-1}{x+1} = 2x-2x^2$$

$$\frac{dy}{dx} = (2x-2x^2+C)\,dx$$

$$y = x^2-\frac{2x^3}{3}+Cx+K$$

~~$y-2 = 2x^2-2x^3$~~

~~$y = 2x^2-2x^3+2$~~

~~$y-1 = 2x^2-2x^3+2x-2x^2$~~

~~$y = -2x^3+2x+1$~~

~~$y-2 = x^3-\frac{2}{3}x^4$~~

~~$y = x^3-\frac{2}{3}x^4+2$~~

~~$\frac{2-1}{0+1} = 2x-2x^2-4 = x+x^2-2$~~

~~$-1 = x-x^2-2$~~

~~$x^2-x+1 = 0$~~

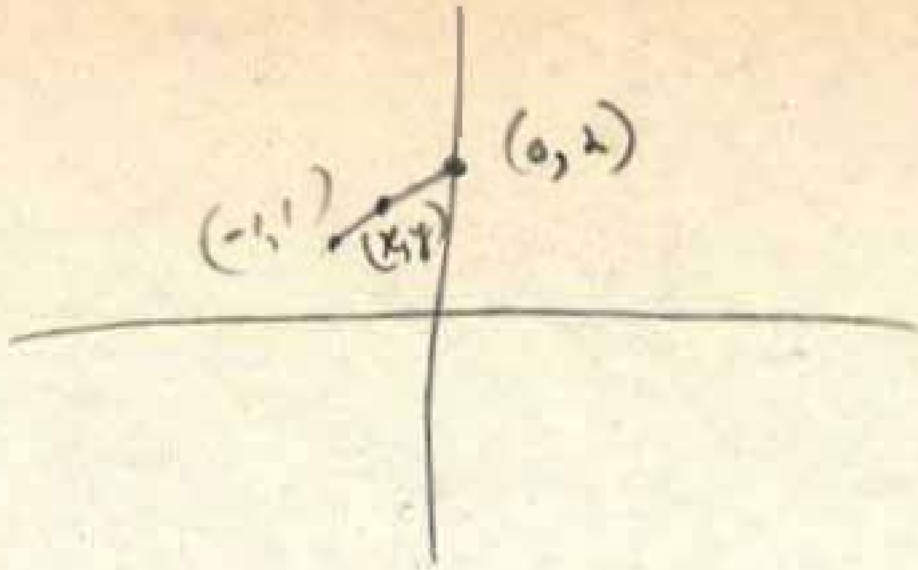
$2 = 0-0+0+K$, $K = 2$

$1 = 1+\frac{2}{3}-C+2$

$C = 8/3$

$$\therefore y = x^2-\frac{2}{3}x^3+\frac{8}{3}x+2$$

282) 24



$$\frac{d^2y}{dx^2} = 2 - 4x$$

$$\frac{dy}{dx} = \int 2 - 4x \, dx$$

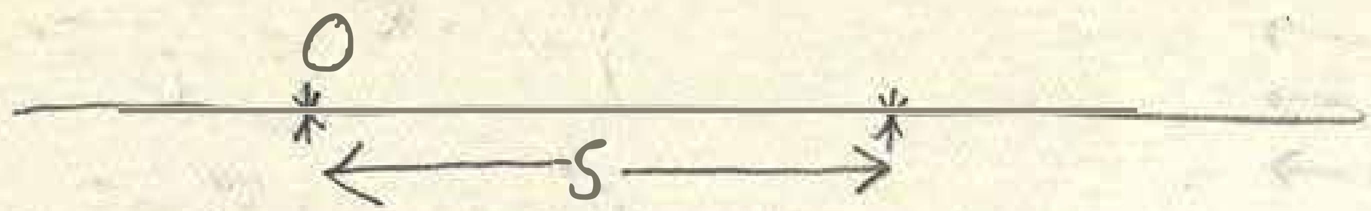
$$= 2x - 2x^2 + C$$

When $x = 0$, $C = 0$

$$\frac{y-2}{x-0} = 2x - 2x^2$$

$$y - 2 = 2x^2 - 2x^3$$

$$y = 2x^2 - 2x^3 + 2$$



$S = f(t)$

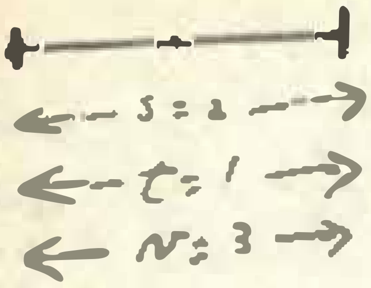
$v = ds/dt$

$a = dv/dt$

283) 80

$$a = 2t - 3$$

when $t = 1$, $s = 2$ + $v = 3$



$$v = \int a\,dt = \int (2t-3)\,dt$$

$$v = t^2 - 3t + C$$

when $t=1$, $v=3$, $\quad 3 = 1 - 3 + C$

$$C = 3 + 3 - 1 = 5$$

$$\therefore \underline{v = t^2 - 3t + 5}$$

$$s = \int v\,dt$$

$$s = \int (t^2 - 3t + 5)\cancel{dt} = \frac{t^3}{3} - \frac{3t^2}{2} + 5t + C$$

when $t = 1$, $s = 2$, $\quad 2 = \frac{t^3}{3} - \frac{3t^2}{2} + 5t + C$

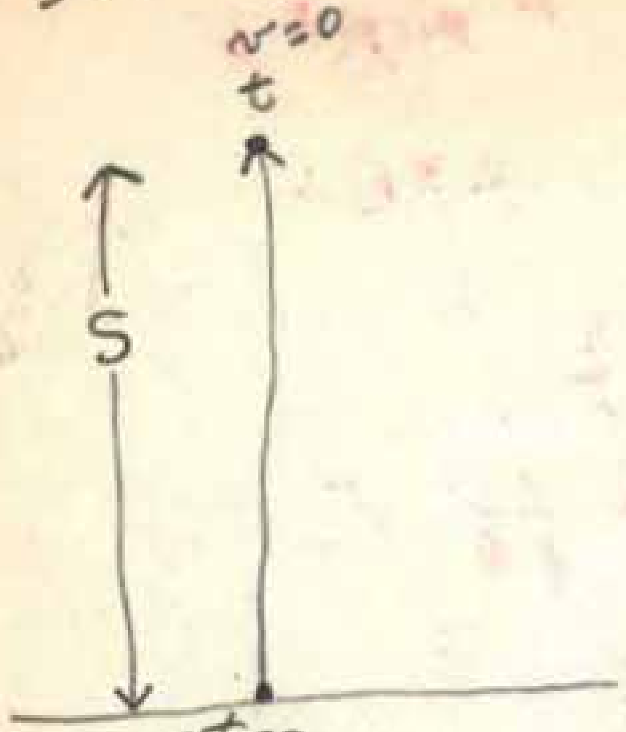
$$2 = \frac{1}{3} - \frac{3}{2} + 5 + C$$

$$C = 2 - \frac{1}{3} + \frac{3}{5} - 5 = \frac{30}{15} - \frac{5}{15} + \frac{9}{15} - \frac{75}{15}$$

$$C = -\frac{41}{15}$$

Equation $s = \frac{t^3}{3} - \frac{3t^2}{2} + 5t - \frac{41}{15}$

283) 83



$t=0$
$v=20$ ft/sec.
$s=0$

$$a = \frac{dv}{dt} = \frac{d}{dt}20 = 20t$$

$$S = \int v\,dt$$

$$0 = \int 20\,dt = 20t + C$$

when $t=0$, $C=0$

$$t = 1 + \frac{12}{20} = 1\tfrac{3}{5} \text{ sec.}$$

(at $\frac{20 \text{ ft}}{\text{sec}}$ initial vel. against 32 ft./sec^2)

Then $S = 20 \times \frac{32}{20} = 32$ ft.

Going down, [redacted]

S of 32 ft. will be traversed in 1 sec. (at rate of 32 ft/sec).

Therefore total time before striking ground $= 1\tfrac{3}{5} + 1 = 2\tfrac{3}{5}$ sec.

$\frac{dv}{dt} = -32$, $v = \int -32\,dt$, $v = -32t + C$ $\quad$ $t=0$, $v=20$

$20 = 0 + C$,

$v = -32t + 20$

$\frac{ds}{dt} = (-32t+20)\,dt$

$S = -16t^2 + 20t$

$S = -16t^2 + 20t + K$ $\quad$ $S=0$, $t=0$

$0 = 0 + 0 + K$

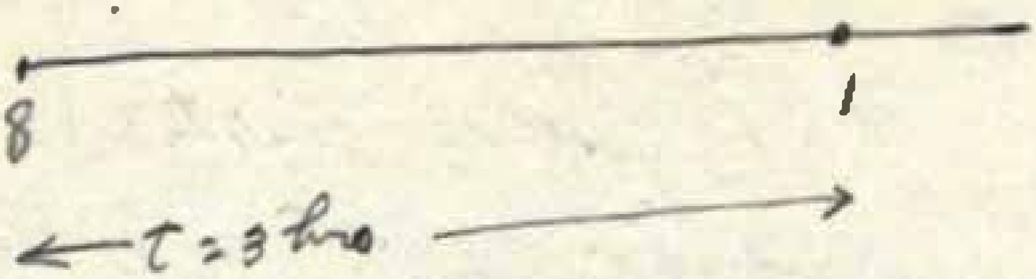
1) $-32t + 20 = 0$, $t = 5/8$ sec.
2) $S = -16 \cdot \frac{25}{64} + 20 \cdot 5/8$ ft.
3) $S=0$, $4t(5-4t)=0$, $t=0$, $t=5/4$ sec.

283)84

$\ln(a^m) = m\ln a$

u prop. to v

$u = cv$



$8^{1/3} = 2$

$8^{-1/3} = \frac{1}{8^{1/3}} = 1/2$

$v = \frac{ds}{dt}$ $\quad v = \frac{d}{dt}7 = 7t + C$

at 3 hrs., $\frac{7}{3} = 7\cdot 3 + C$

am't subst. S lb.

$\frac{ds}{dt} = cs$

$C = \frac{7}{3} - 21 = -\frac{56}{3}$

$\frac{ds}{s} = c\,dt$

Then $v = 7t - \frac{56}{3}$

$\ln S = ct + K$

$S = \int v\,dt = \int 7t - \frac{56}{3} = \frac{7t^2}{2} - \frac{56t}{3} + C$

$t=0, S=8$

$t=3, S=1$

$\ln 8 = K$

$\ln 1 = 3c + \ln 8$

$0 = 3c + \ln 8$

$c = -\frac{1}{3}\ln 8$

$\ln S = (-\frac{1}{3}\ln 8)t + \ln 8$

$7 = \frac{7t^2}{2} - \frac{56}{3}t + C$

at 3 hrs. $7 = \frac{63}{2} - \frac{56}{3} + C$

$C = 7 - \frac{63}{2} + \frac{56}{3} = \frac{42 - 189 + 112}{6} = -\frac{33}{6} = -\frac{11}{3}$

Then $S = \frac{7t^2}{2} - \frac{56t}{3} - \frac{11}{3}$

When $t = 4$, $S = \frac{7\cdot 16}{2} - \frac{56\cdot 4}{3} - \frac{11}{3} = 56 - \frac{224}{3} - \frac{11}{3}$

$S = \frac{168 - 224 - 11}{3} = -\frac{67}{3}$

$t=4,$

$\ln S = -\frac{4}{3}\ln 8 + \ln 8$

$\ln S = -\frac{1}{3}\ln 8$

$\ln S = \ln(8^{-1/3})$

$\ln S = \ln\frac{1}{2}$

$S = \frac{1}{2}$

~~$y=f(t)$~~

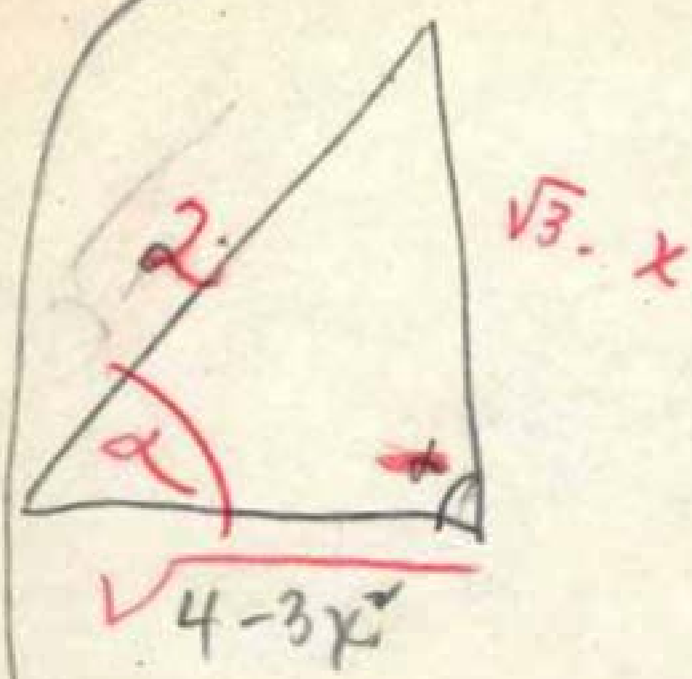
$$\left.\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}\right\}$$

$$\left\{\begin{aligned} x &= t+2 \\ y &= t^2-2t \end{aligned}\right\}$$

t	x	y
0	—	—
1		
2		
3		
−1		
−2		
−3		

280/16 (m.B.)

$$\int \frac{x^3\,dx}{\sqrt{4-3x^2}} = \frac{8}{3\sqrt{3}}\int \frac{\sin^3\alpha \cdot \frac{2}{\sqrt{3}}\cancel{\cos\alpha}\,d\alpha}{2\cancel{\cos\alpha}}$$



$$\sin\alpha = \frac{\sqrt{3}\cdot x}{2}$$

$$x = \frac{2\sin\alpha}{\sqrt{3}}$$

$$dx = \frac{2}{\sqrt{3}}\cos\alpha$$

$$\sqrt{4-3x^2} = 2\cos\alpha$$

$$\frac{8}{9}\int \sin^3\alpha\,d\alpha = \frac{8}{9}\int \sin^2\alpha\,\sin\alpha\,d\alpha$$

$$= \frac{8}{9}\int (1-\cos^2\alpha)\sin\alpha\,d\alpha$$

$u = \cos\alpha$

$du = -\sin\alpha\,d\alpha$

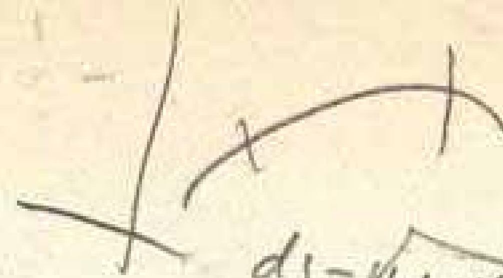
$$= -\frac{8}{9}\int (1-u^2)\,du$$

$$= -\frac{8}{9}\left(u - \frac{u^3}{3}\right) + C$$

$$= -\frac{8}{9}\left(\cos\alpha - \frac{\cos^3\alpha}{3}\right) + C$$

$$-\frac{8}{9}\left[\frac{\sqrt{4-3x^2}}{2} - \frac{(4-3x^2)\sqrt{4-3x^2}}{24}\right] + C$$

Morris & Brown



$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

282) 75, 76

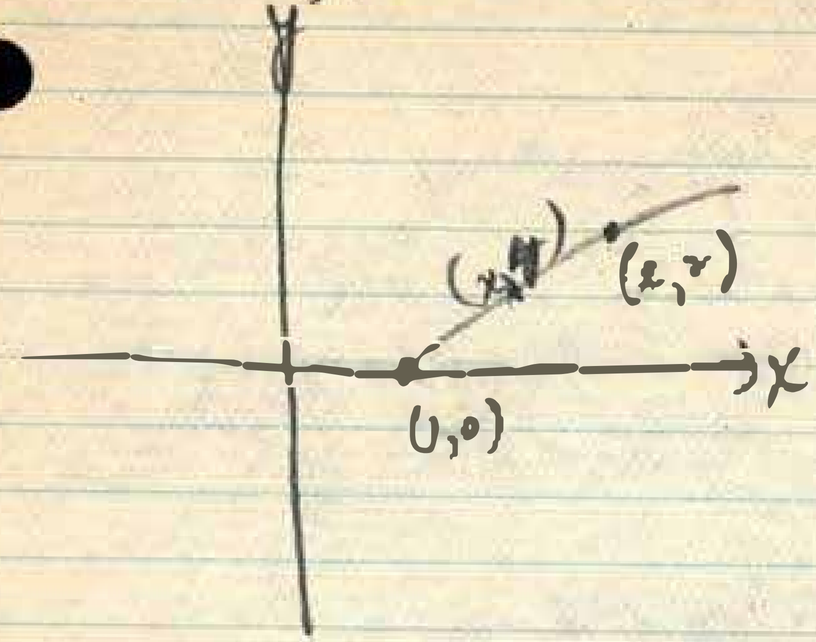
283) 77, 81, 82

297) 1i, 2e, 2h

303) 1a, 1e, 1g

309) 3

282)75



$$\frac{d^2y}{dx^2} = 1 + 2x$$

$$\frac{dy}{dx} = \int (1+2x)\,dx$$

$$= x + x^2 + C$$

$$y = \int (x + x^2 + C)\,dx$$

$$y = \frac{x^2}{2} + \frac{x^3}{3} + Cx + K$$

at (1,0) → $0 = \frac{x^2}{2} + \frac{x^3}{3} + Cx + K$

$$0 = 3x^2 + 2x^3 + 6Cx + 6K$$

$$0 = 3 + 2 + 6C + 6K$$

$$6K = -4 - 6C$$

$$3K = -2 - 3C$$

$$K = \frac{-2-3C}{3} = -\frac{2}{3} - C$$

At (e,2) → $2 = \frac{e^2}{2} + \frac{e^3}{3} + Ce + \left(\frac{-2-3C}{3}\right)$

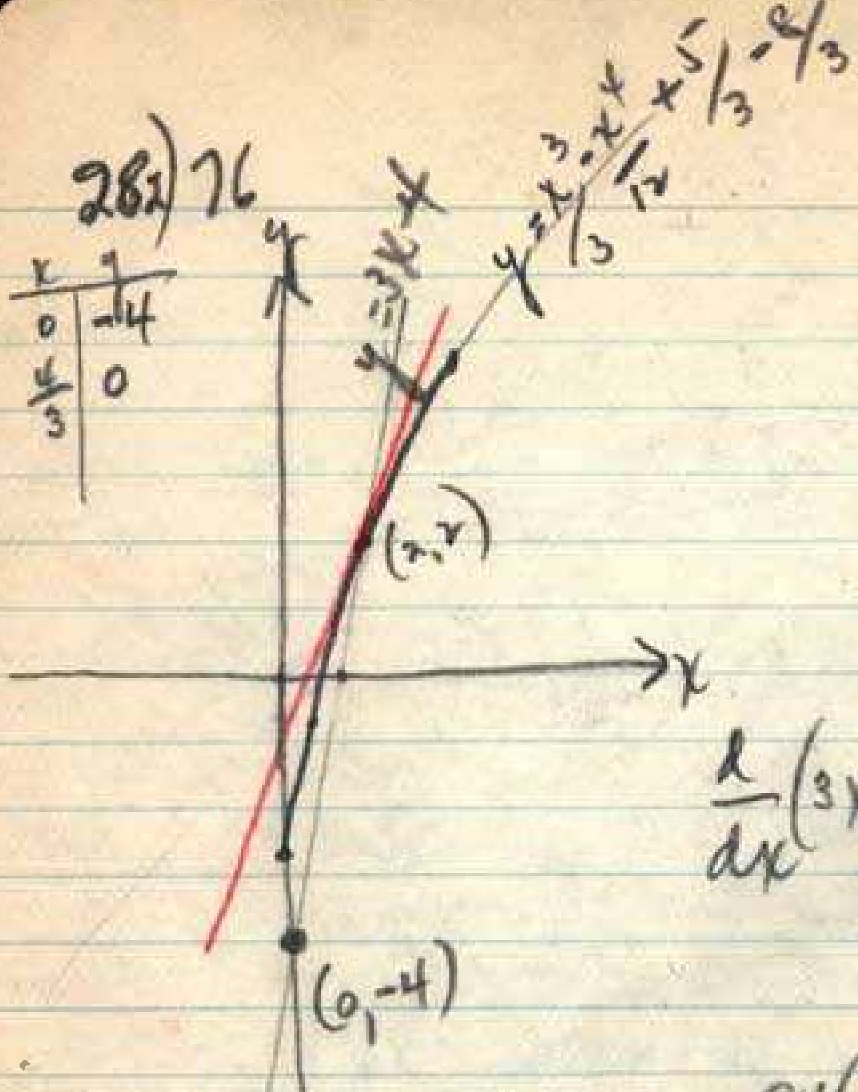
$$12 = 3e^2 + 2e^3 + 6Ce - 4 - 6C$$

$$3e^2 + 2e^3 - 16 = 6C - 6Ce \qquad 6C - 6Ce = 3e^2 + 2e^3 - 16$$

$$C = \frac{3e^2 + 2e^3 - 16}{6(1-e)}$$

Equation $y = \frac{x^2}{2} + \frac{x^3}{3} + \left(\frac{3e^2 + 2e^3 - 16}{6(1-e)}\right)x - \frac{2}{3} - \left(\frac{3e^2 + 2e^3 - 16}{6(1-e)}\right)$

28x) 26



$$\frac{d}{dx}(3x-4) = 3$$

$$\frac{d^2y}{dx^2} = 2x - x^2$$

$$\frac{dy}{dx} = \int (2x - x^2)\,dx = x^2 - \frac{x^3}{3} + C$$

$$y = \int \left(x^2 - \frac{x^3}{3} + C\right) dx$$

$$= \frac{x^3}{3} - \frac{x^4}{12} + Cx + K$$

$$\text{at } (2,2) \rightarrow x^2 - \frac{x^3}{3} + C = 3$$

$$4 - \frac{8}{3} + C = 3$$

$$C = 3 - 4 + \frac{8}{3}$$

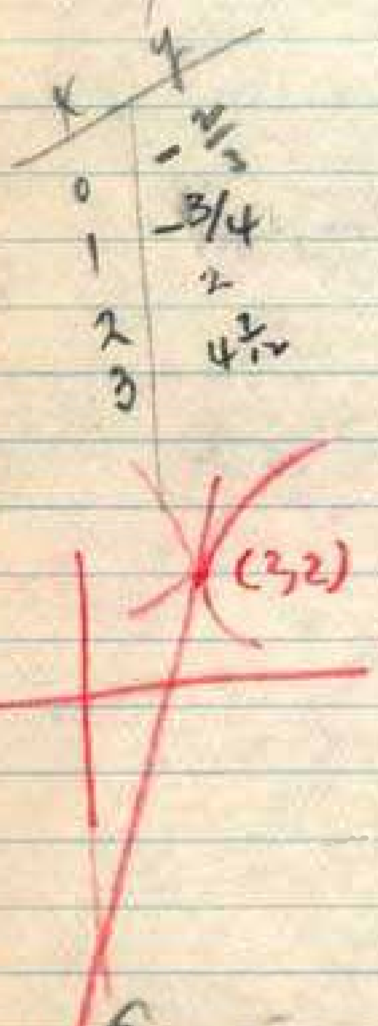
$$C = \frac{5}{3}$$

$$2 = \frac{8}{3} - \frac{16}{12} + \frac{10}{3} + K$$

$$-K = \frac{8}{3} - \frac{16}{12} + \frac{10}{3} - 2$$

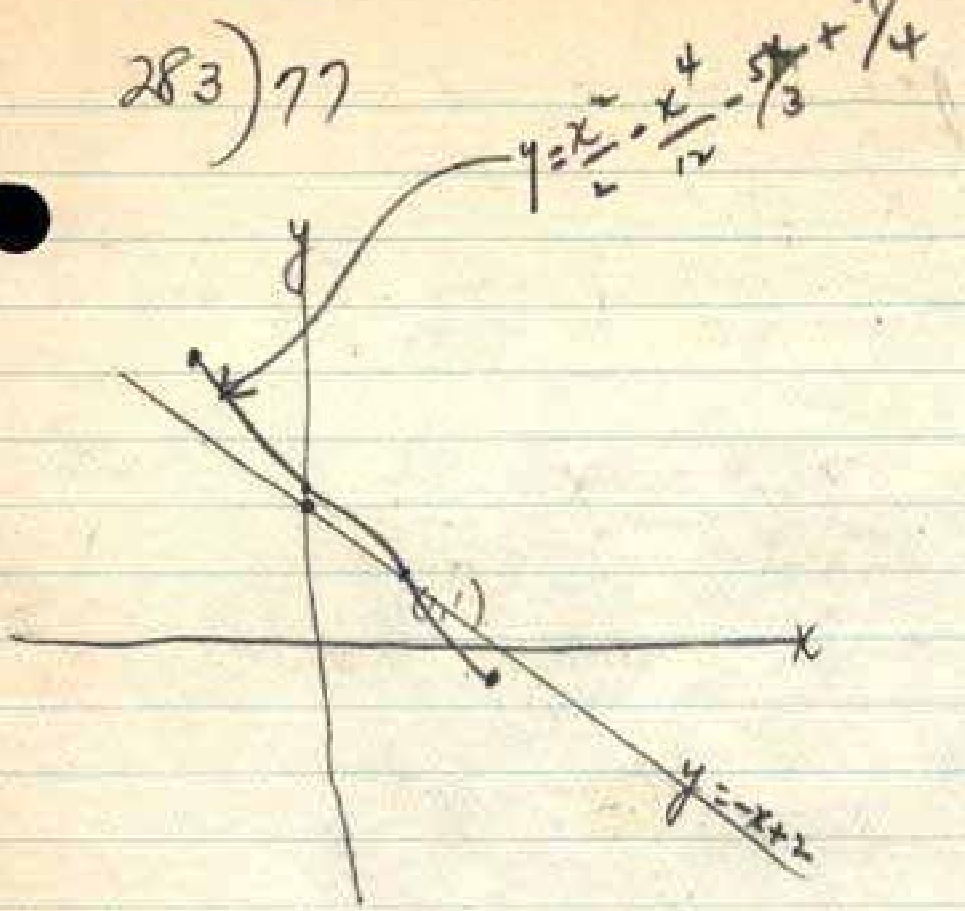
$$12K = -32 + 16 - 40 + 24$$

$$K = -\frac{32}{12} = -\frac{8}{3}$$



Equation = $y = \frac{x^3}{3} - \frac{x^4}{12} + \frac{5}{3}x - \frac{8}{3}$

283) 77

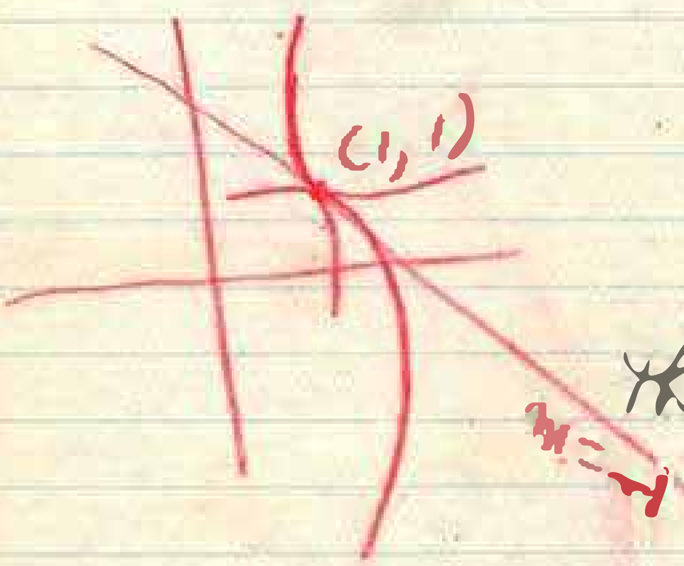


$$\frac{d^2y}{dx^2}=1-x^2$$

$$\frac{dy}{dx}=\int(1-x^2)\,dx=x-\frac{x^3}{3}+C$$

$$y=\int\left(x-\frac{x^3}{3}+C\right)dx$$

$$y=\frac{x^2}{2}-\frac{x^4}{12}+Cx+K$$



$$\frac{d}{dx}(-x+2)=-1$$

then $x-\frac{x^3}{3}+C=-1$

$$C=-1-x+\frac{x^3}{3}$$

at (1,1) $\qquad C=-1-1+\frac{1}{3}=-\frac{5}{3}$

x	y
0	$\frac{9}{4}$
1	1
2	$-5/12$
-1	

$$1=\frac{1}{2}-\frac{1}{12}-\frac{5}{3}+K$$

$$-K=\frac{1}{2}-\frac{1}{12}-\frac{5}{3}-1$$

$$-12K=6-1-20-12=-27$$

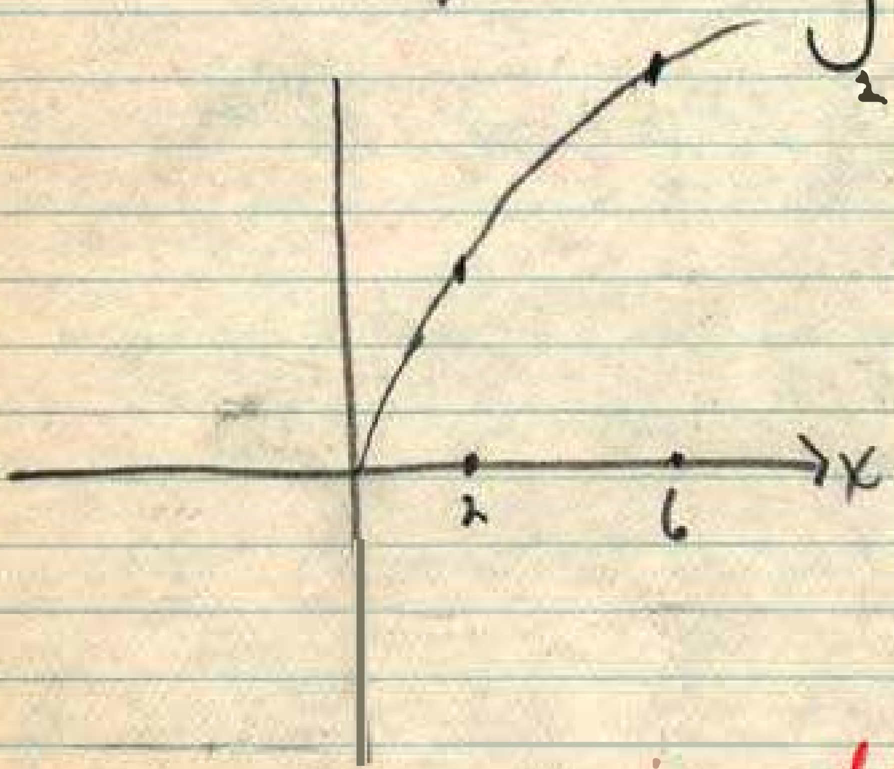
$$K=\frac{27}{12}=\frac{9}{4}$$

Equation = $y=\frac{x^2}{2}-\frac{x^4}{12}-\frac{5x}{3}+\frac{9}{4}$

253) 81 Slope of Curve = (Slope of Tangent) = $2 + x^3$ $\left(= \frac{dy}{dx}\right)$

Amount of change of y when x changes from 2 to 6 is equal to

$$\int_2^6 (2 + x^3)\,dx = 2x + \frac{x^4}{4}\Big]_2^6$$



$= (12 + 328) - (4 + 4)$

$= 340 - 8 = 332$

$$\frac{dy}{dx} = 2 + x^3$$

$$y = 2x + \frac{x^4}{4} + C$$

$$ch = (12 + 328 + \cancel{C}) - (4 + 4 + \cancel{C})$$

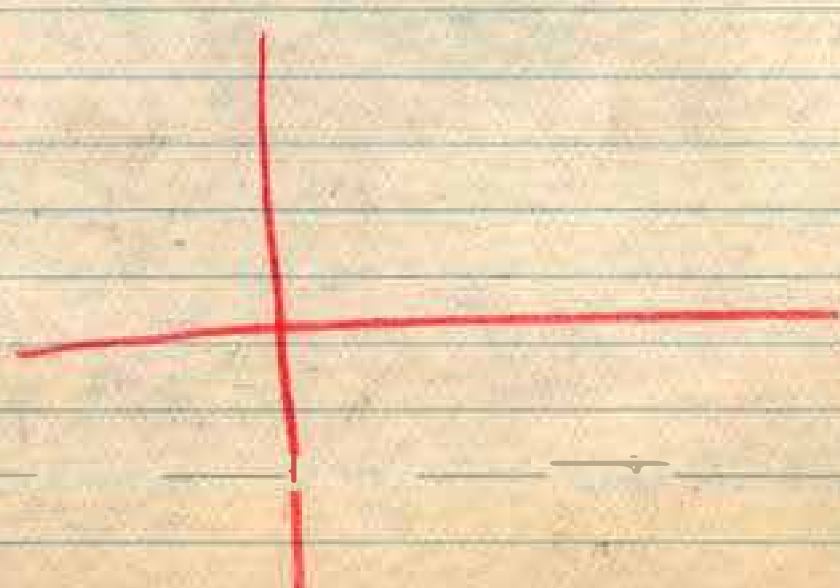
---

curve: $y = f(x)$, $f(x, y) = 0$

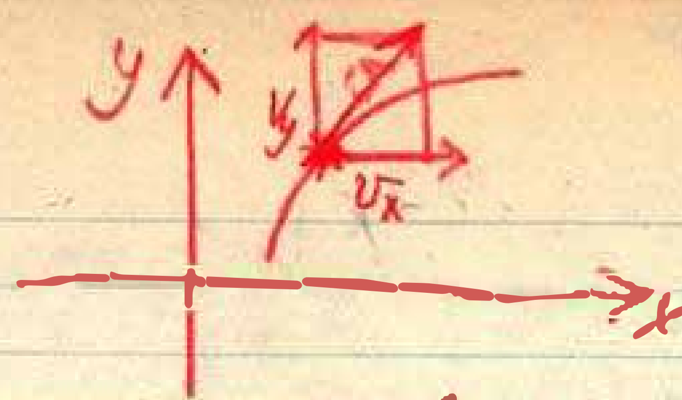
s	x	y
0	—	—
1	—	—
2	—	—
3		
-1		

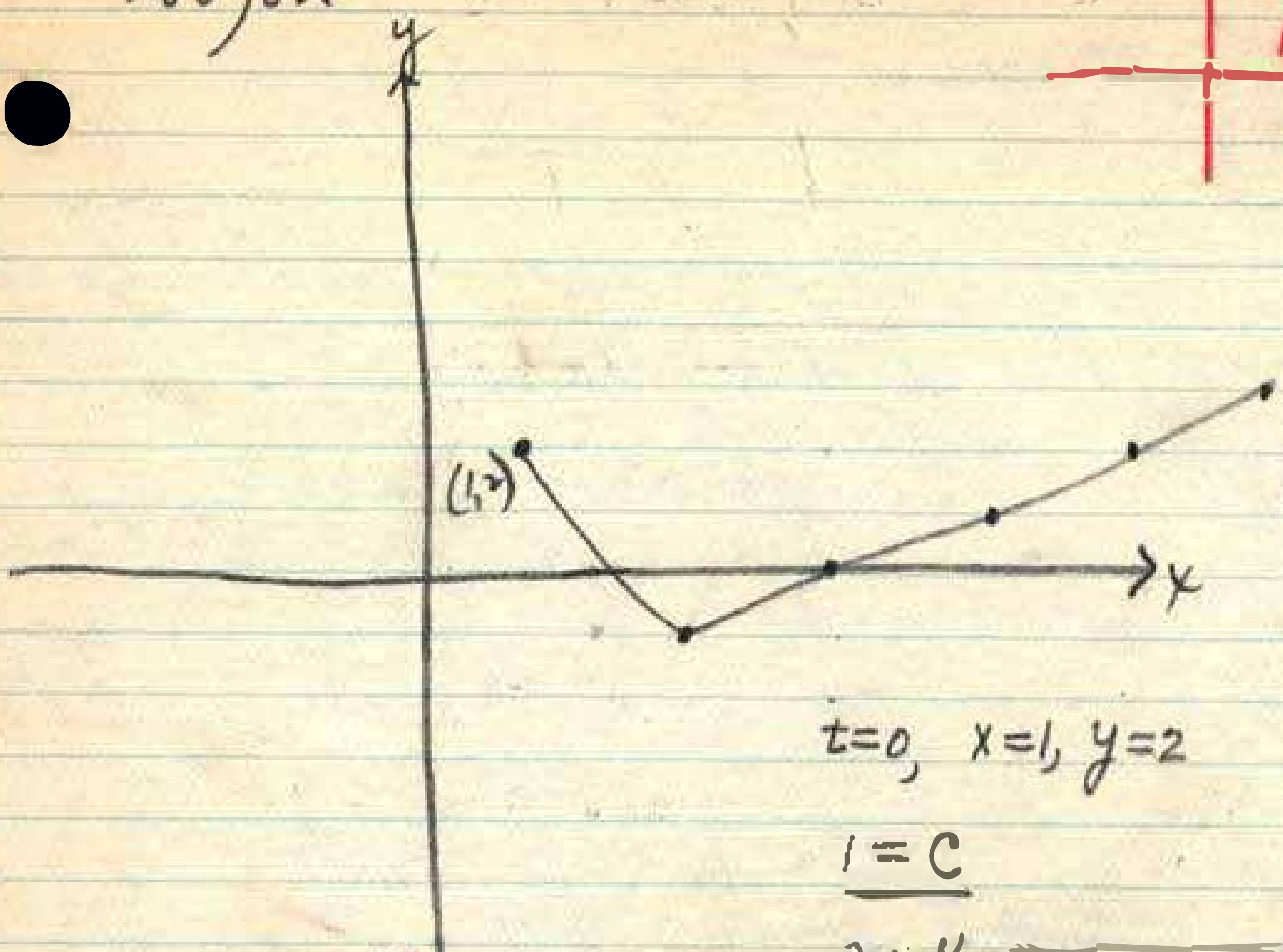
$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

$$\begin{cases} x = 2s + 1 \\ y = 3s^2 - 2s^3 \end{cases}$$



283) 82

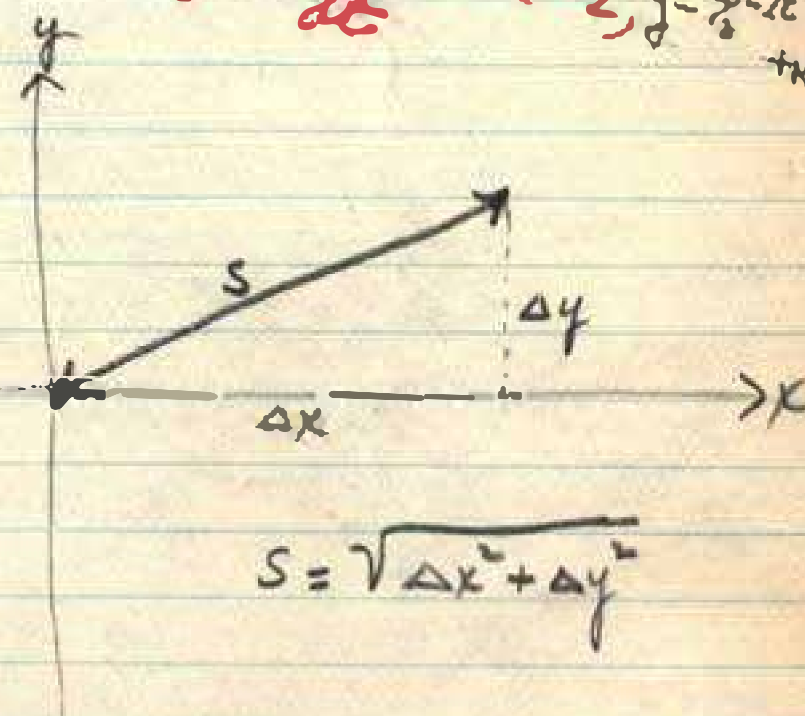




$v^2 = v_x^2 + v_y^2$

$v_x = \frac{dx}{dt} = t + 2, \quad x = \frac{t^2}{2} + 2t + C$

$v_y = \frac{dy}{dt} = t - 2, \quad y = \frac{t^2}{2} - 2t + K$



$t = 0, \quad x = 1, \quad y = 2$

$1 = C$

$2 = K$

t	v_x	v_y
1	3	-1
2	4	0
3	5	1
4	6	2
5	7	3

$x = \frac{t^2}{2} + 2t + 1$

$y = \frac{t^2}{2} - 2t + 2$

Ans.

$s = \sqrt{\Delta x^2 + \Delta y^2}$

x comp. of velocity $= t + 2$

y . . . . . . $= t - 2$

$$s = \int v\,dt = \int \sqrt{(t+2)^2 + (t-2)^2}\,dt$$

$$\boxed{\int \sqrt{u^2+a^2} = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left(u+\sqrt{u^2+a^2}\right) + C} = \int \sqrt{t^2+4t+4+t^2-4t+4}\,dt$$

$$S = \int \sqrt{2t^2+8}\,dt = \sqrt{2}\int \sqrt{t^2+4}\,dt$$

$dx/dt, \quad dy/dt$

$$S = \frac{t}{2}\sqrt{t^2+4} + 2\ln\left(t+\sqrt{t^2+4}\right) + C$$

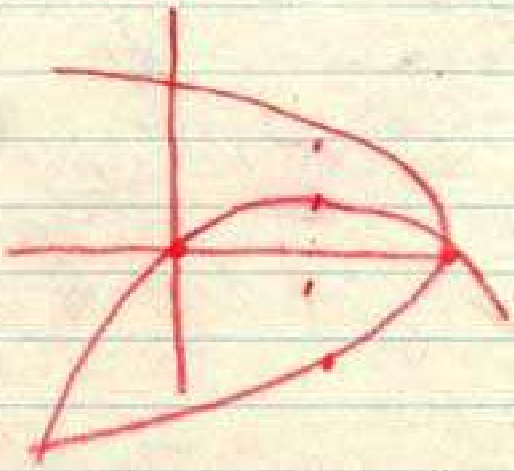
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t-2}{t+2}$$

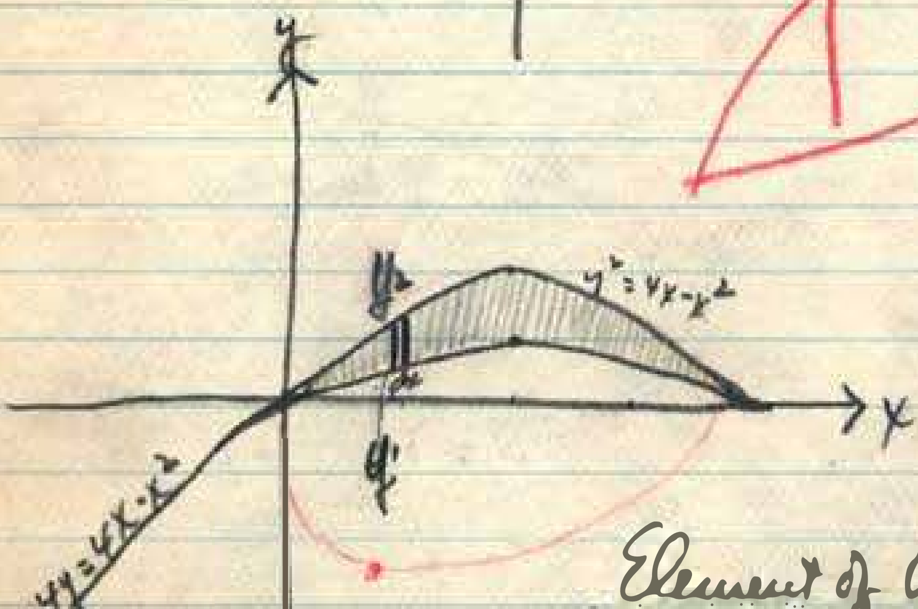
297) 1i $\quad y^2 = 4x - x^2 \quad + \quad 4y = 4x - x^2$

$$y = \pm\sqrt{4x - x^2} \qquad\qquad y = \frac{4x - x^2}{4}$$

x	y
0	0
2	± 2
4	0

x	y
0	0
2	± 1
4	0
-1	$-\frac{5}{4}$
-2	-3





~~Let $v = \sqrt{4-x}$~~

~~then $xv^2 = 4x - x^2$~~

$$\cancel{v^2 = \frac{4x - x^2}{x} = 4 - x}$$

$$\cancel{2v\,dv = -dx}$$

$$\cancel{dx = -2v\,dv}$$

$$\cancel{-x = v^2 - 4}$$

$$\cancel{x = 4 - v^2}$$

$$\cancel{\int_0^4 (4 - v^2)^{\frac{1}{2}} v \cdot (-2v)\,dv}$$

$$\cancel{= \int_0^4}$$

Element of Area $= (y_2 - y_1)\,dx$

Total Area $= \int_0^4 (y_2 - y_1)\,dx$

$$= \int_0^4 \left(\sqrt{4x - x^2} - \left(\frac{4x - x^2}{4}\right)\right) dx$$

$$= \int_0^4 \sqrt{4x - x^2}\,dx - \int_0^4 \left(\frac{4x - x^2}{4}\right) dx$$

$$= \int_0^4 \sqrt{4x - x^2}\,dx - \int_0^4 x\,dx + \int_0^4 \frac{x^2}{4}\,dx$$

$$= 2\pi \quad - x^2\Big]_0^4 + \frac{x^3}{12}\Big]_0^4$$

$$= 2\pi \quad \cancel{-16 + 64}$$

$$= \cancel{2\pi + 48} \quad 2\pi - 8/3$$

297) 12

$$\sqrt{4x-x^2} = \sqrt{-(x^2-4x+4)+4}$$

$$= \sqrt{4-(x^2-4x+4)}$$

$$= \sqrt{4-(x-2)^2}$$

Let $u = x-2$

$a = 2$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\frac{u}{a} + C$$

$$\int_0^4 \sqrt{a^2-u^2}\,du = \frac{x-2}{2}\sqrt{4-(x-2)^2} + 2\arcsin\frac{x-2}{2}\Big]_0^4$$

$$= \Big[1\sqrt{0} + 2\arcsin 1\Big] - \Big[-1\sqrt{0} + 2\arcsin -1\Big]$$

$$\left[\frac{x-2}{2}\sqrt{4x-x^2} + 2\arcsin\frac{x-2}{2} - \frac{x^2}{2} + \frac{x^3}{12}\right]_0^4$$

$$\left(0 + \pi - 8 + \frac{16}{3}\right) - \left(0 - \pi\right)$$

$$=$$

$$= 2\arcsin 1 - 2\arcsin -1$$

$$= 2\cdot\frac{\pi}{2} - \left(2\cdot -\frac{\pi}{2}\right)$$

$$= \pi + \pi$$

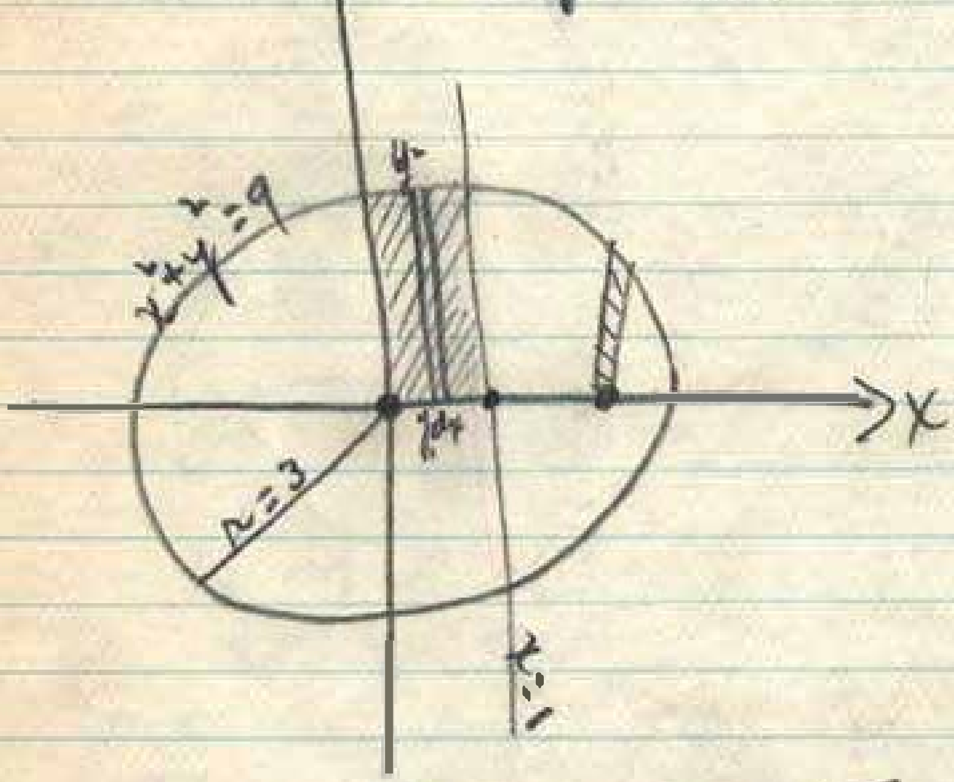
$$= 2\pi$$

297) 2c

$x^2+y^2=9$ & $x=1$

(circle with center at origin & radius $=3$)

$y=\sqrt{9-x^2}$



Element of area $=(y_2-y_1)\,dx$

Total area $=2\int_1^3\left[\sqrt{9-x^2}-0\right]dx$

$=$ (see formula in 297) 1i

$a=3$

$u=x$

Total Area $=2\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\arcsin\frac{x}{3}\right]_1^3$

$$\cancel{=\frac{1}{2}\sqrt{9-1}+\frac{3}{2}\arcsin\frac{1}{2}}$$

$$\cancel{=\sqrt{2}+\frac{3}{2}\cdot\frac{\pi}{6}}$$

$$\cancel{=\sqrt{2}+\frac{\pi}{4}}$$

arcsin 1 =

$\frac{1}{3}=.3333-$

$$=2\left[\left(\frac{9}{2}\cdot\frac{\pi}{2}\right)-\left(\frac{1}{2}\sqrt{8}+\frac{9}{2}\arcsin\frac{1}{3}\right)\right]$$

$$=2\left(\frac{9\pi}{4}-\sqrt{2}-\frac{9}{2}\arcsin\frac{1}{3}\right)$$