

$$\sqrt{x^2+7x-7} - \sqrt{x^2+3x} = 1$$

$$\sqrt{x^2+7x-7} = 1 + \sqrt{x^2+3x}$$

$$x^2+7x-7 = 1 + 2\sqrt{x^2+3x} + x^2+3x$$

$$4x-8 = 2\sqrt{x^2+3x}$$

$$2x-4 = \sqrt{x^2+3x}$$

$$4x^2-16x+16 = x^2+3x$$

$$3x^2-19x+16 = 0$$

$$3x^2-19x = -16$$

$$x^2 - \frac{19}{3}x = -\frac{16}{3}$$

$$x^2 - \frac{19x}{3} - \left(\frac{19}{6}\right)^2 = -\frac{16}{3} - \left(\frac{19}{6}\right)^2$$

$$x^2 - \frac{19x}{3} + \frac{361}{36} = -\frac{16}{3} - \frac{361}{36} = -\frac{192}{36} - \frac{361}{36} = -\frac{553}{36}$$

$$x = \frac{19}{6} \pm \sqrt{\frac{553}{36}}$$

$$x = \pm \sqrt{\frac{553}{36}} + \frac{19}{6} = \pm \frac{\sqrt{553}}{6} + \frac{19}{6}$$

$$= \pm \sqrt{15.36} + 3.17 = \pm 3.92 + 3.17$$

$$= 7.09 \text{ or } -0.75$$

Check  $\sqrt{49.63+49.63-7} = \sqrt{49.63+21.27} = 1$

$$\sqrt{92.26} - \sqrt{70.90} = 1$$

$$9.60 - 8.42 = 1$$

$$\sqrt{.5625 + 5.25 - 7} = \sqrt{.5625 - 2.25} = 1$$

$$\sqrt{11.6875} - \sqrt{-1.6875} = 1$$

} ?



Pg. 74 No. 2

$$9x^4 - 37x^2 + 4 = 0$$

$$\frac{9}{x^4} - \frac{37}{x^2} + 4 = 0$$

$$9 - 37x^2 + 4x^4 = 0$$

$$4x^4 - 37x^2 = -9$$

$$x^4 - \frac{37x^2}{4} = -\frac{9}{4}$$

$$x^4 - \frac{37x^2}{4} + \left(\frac{37}{8}\right)^2 = -\frac{9}{4} + \left(\frac{37}{8}\right)^2$$

$$\left(x^2 - \frac{37}{8}\right)^2 = -\frac{9}{4} + \frac{1369}{64}$$

$$= -\frac{144}{64} + \frac{1369}{64} = +\frac{1513}{64}$$

$$x^2 - \frac{37}{8} = \sqrt{\frac{1513}{64}} = \frac{\sqrt{1513}}{8}$$

$$x^2 = \frac{\sqrt{1513} + 37}{8} = \frac{38.85 + 37}{8}$$

$$x = 3.08 \qquad = 9.475$$

Check

$$9 - 37x^2 + 4x^4 = 0$$

$$9 - 333 + 324 = 0$$

$x^2 - \frac{37}{8} = \frac{35}{8}$   $x^2 = \frac{35}{8} + \frac{37}{8} = \frac{72}{8} = 9$   $x = 3$   
 $x^2 - \frac{37}{8} = -\frac{35}{8}$   $x^2 = -\frac{35}{8} + \frac{37}{8} = \frac{2}{8} = \frac{1}{4}$   $x = \frac{1}{2}$

How about  
minus sign

12-25



Pg. 89 No. 9

$$\begin{aligned} 3x^2 + 2y^2 &= 11 \\ 3x^2 - 4y^2 &= 11 \end{aligned}$$

$$2y^2 + 4y = 0$$

$$y^2 + 2y = 0$$

$$y + 2 = 0$$

$$y = -2 \checkmark$$

Then  $3x^2 + 8 = 11$

$$3x^2 = 3$$

$$x = 1 \checkmark$$

---

Page 98 No. 5

Express as logarithm of single quantity

$$2 \log_7 76 + 3 \log_7 48 - 5 \log_7 59$$

$$= \log_7 76^2 + \log_7 48^3 - \log_7 59^5$$

$$= \log_7 \frac{76^2 \cdot 48^3}{59^5}$$

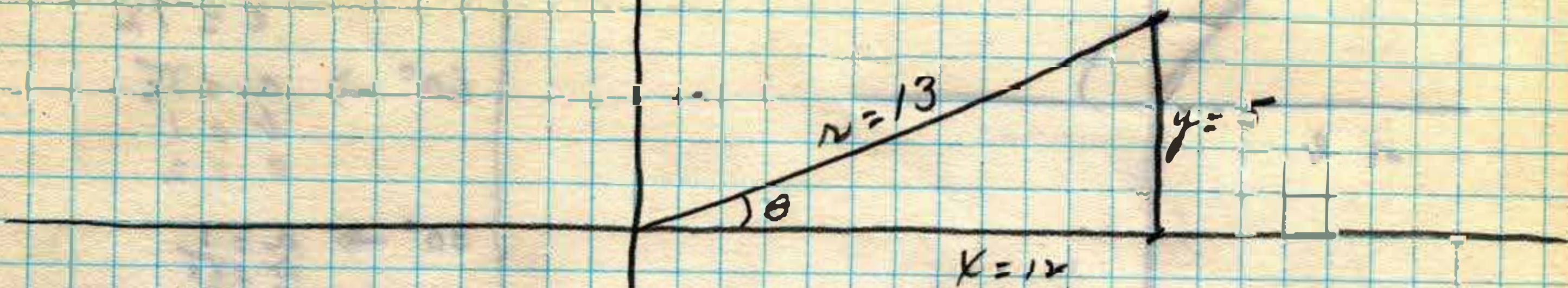






Page 117 No. 1

$\tan \theta = \frac{5}{12}$  (first quadrant)



$\sin \theta = \frac{5}{13}$   
 $\cos \theta = \frac{12}{13}$   
 $\tan \theta = \frac{5}{12}$   
 $\cot \theta = \frac{12}{5}$   
 $\sec \theta = \frac{13}{12}$   
 $\csc \theta = \frac{13}{5}$

Page 117 No. 3

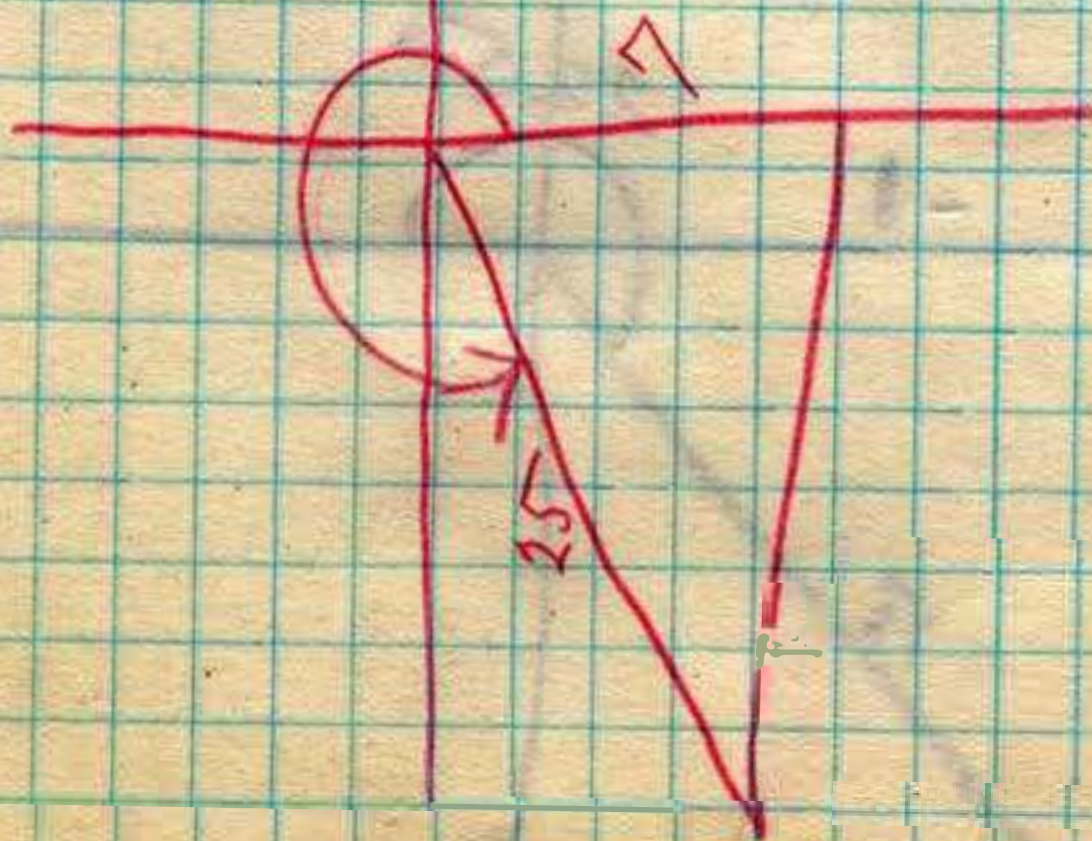
$\cos \theta = \frac{7}{25}$  fourth quadrant



$\sin \theta = -\frac{24}{25}$   
 $\cos \theta = \frac{7}{25}$   
 $\tan \theta = -\frac{24}{7}$   
 $\cot \theta = -\frac{7}{24}$   
 $\sec \theta = \frac{25}{7}$   
 $\csc \theta = -\frac{25}{24}$

$25^2 - 7^2 =$

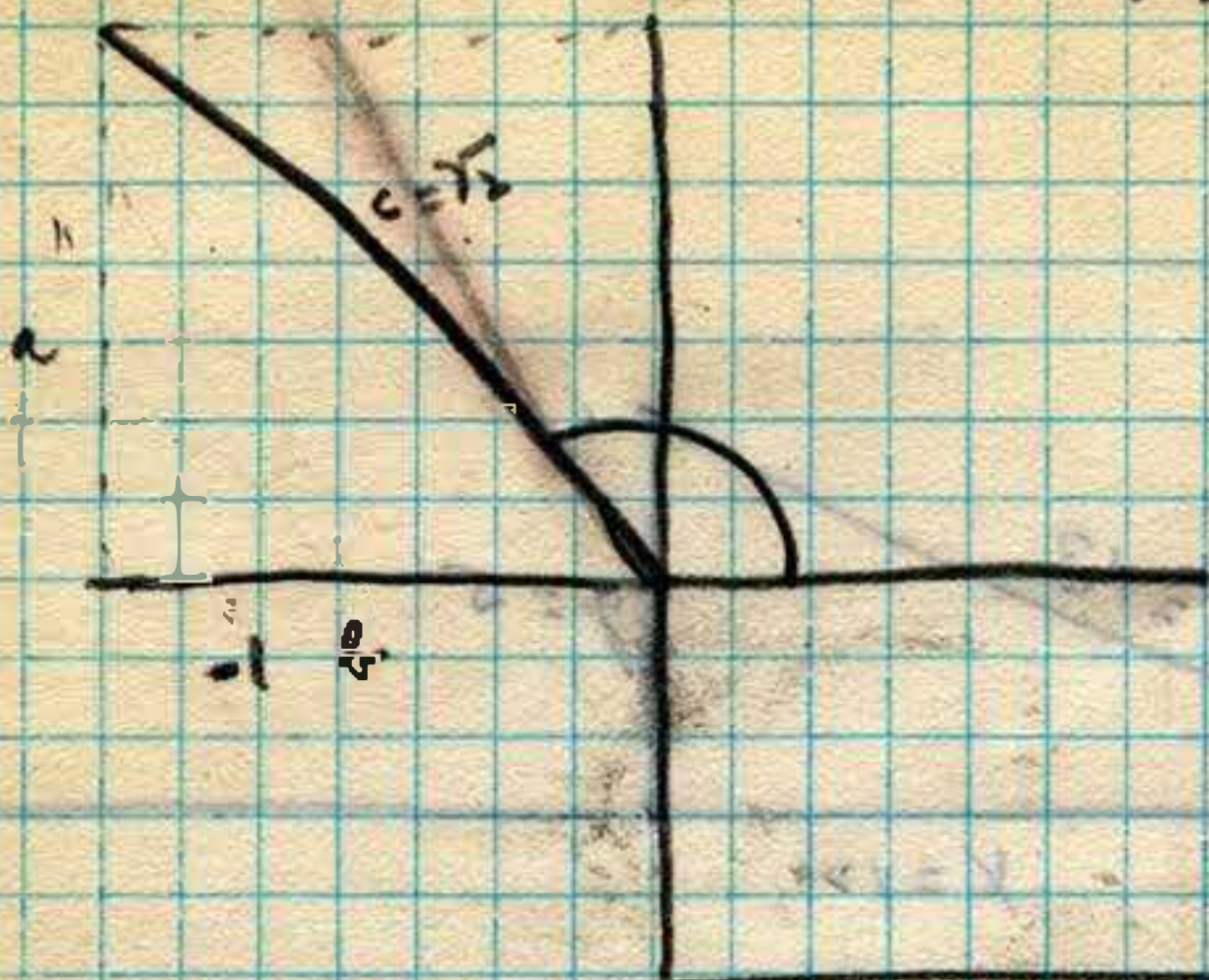
$$\frac{625}{49} = 24$$



$$\frac{25}{-24}$$



$$\sin 135^\circ = \frac{a}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



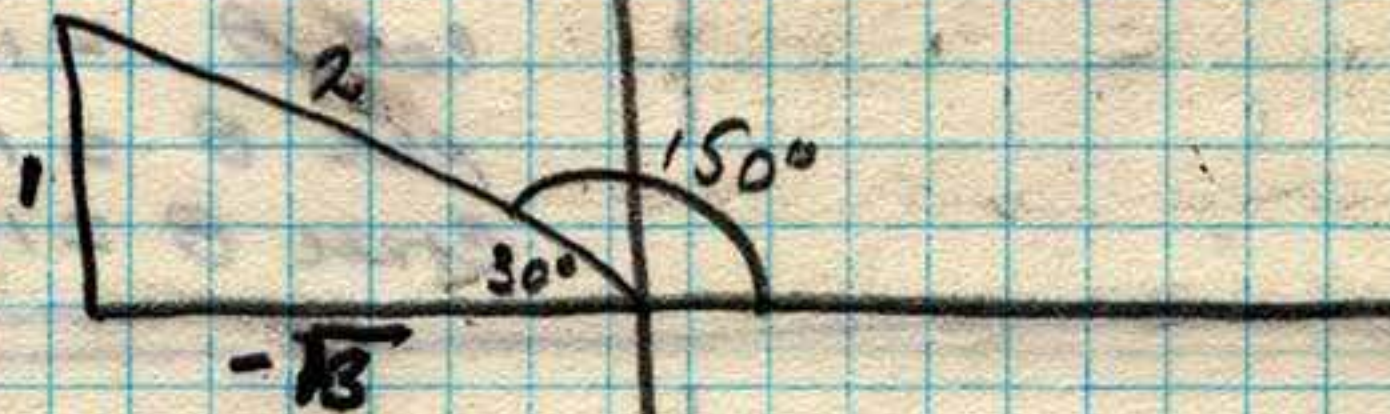
✓

$$45^\circ \rightarrow \begin{aligned} a &= 1 \\ b &= 1 \\ c &= \sqrt{2} \end{aligned}$$

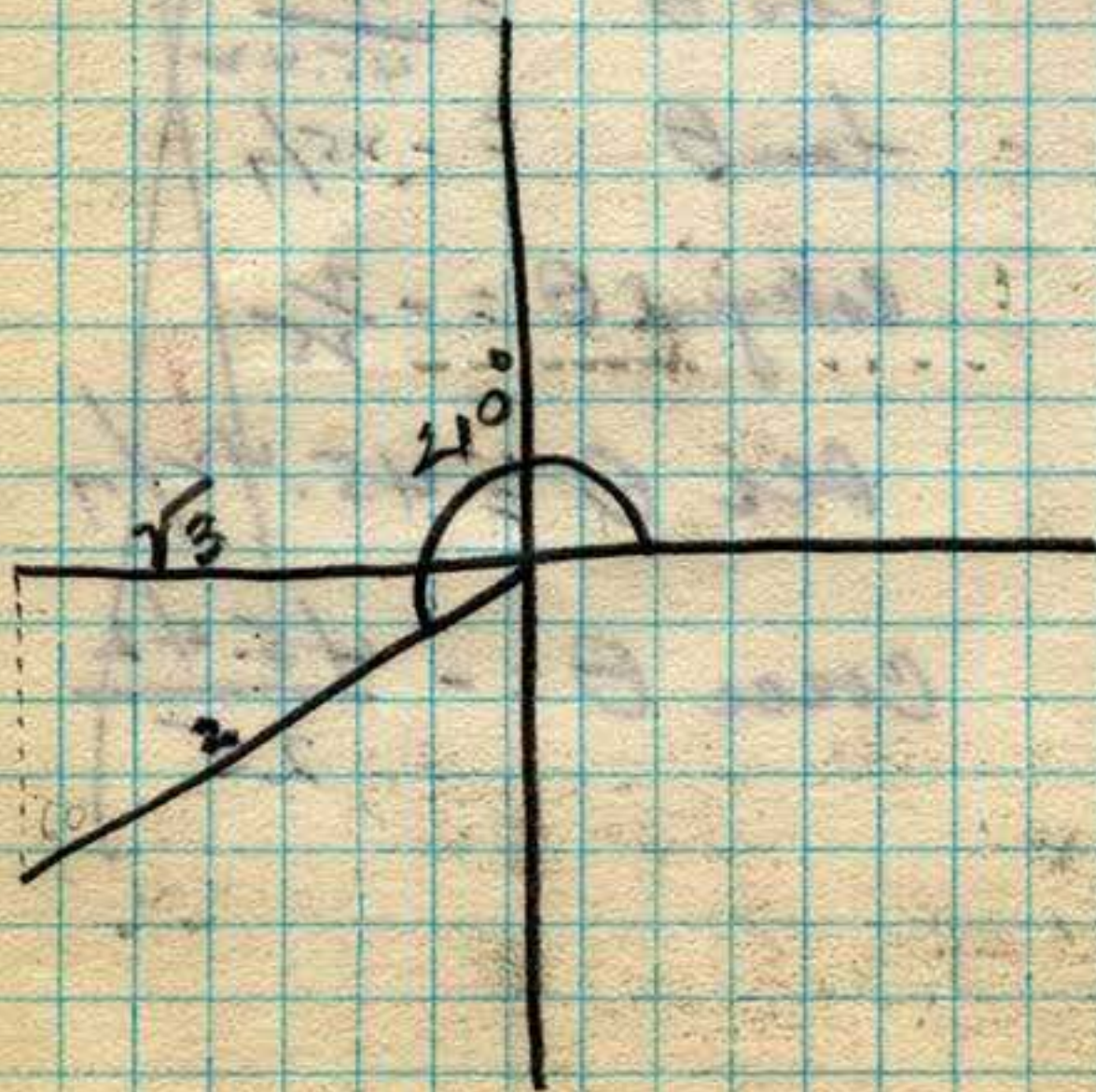
$$60^\circ \rightarrow \begin{aligned} a &= \sqrt{3} \\ b &= 1 \\ c &= 2 \end{aligned}$$

$$30^\circ \rightarrow \begin{aligned} a &= 1 \\ b &= \sqrt{3} \\ c &= 2 \end{aligned}$$

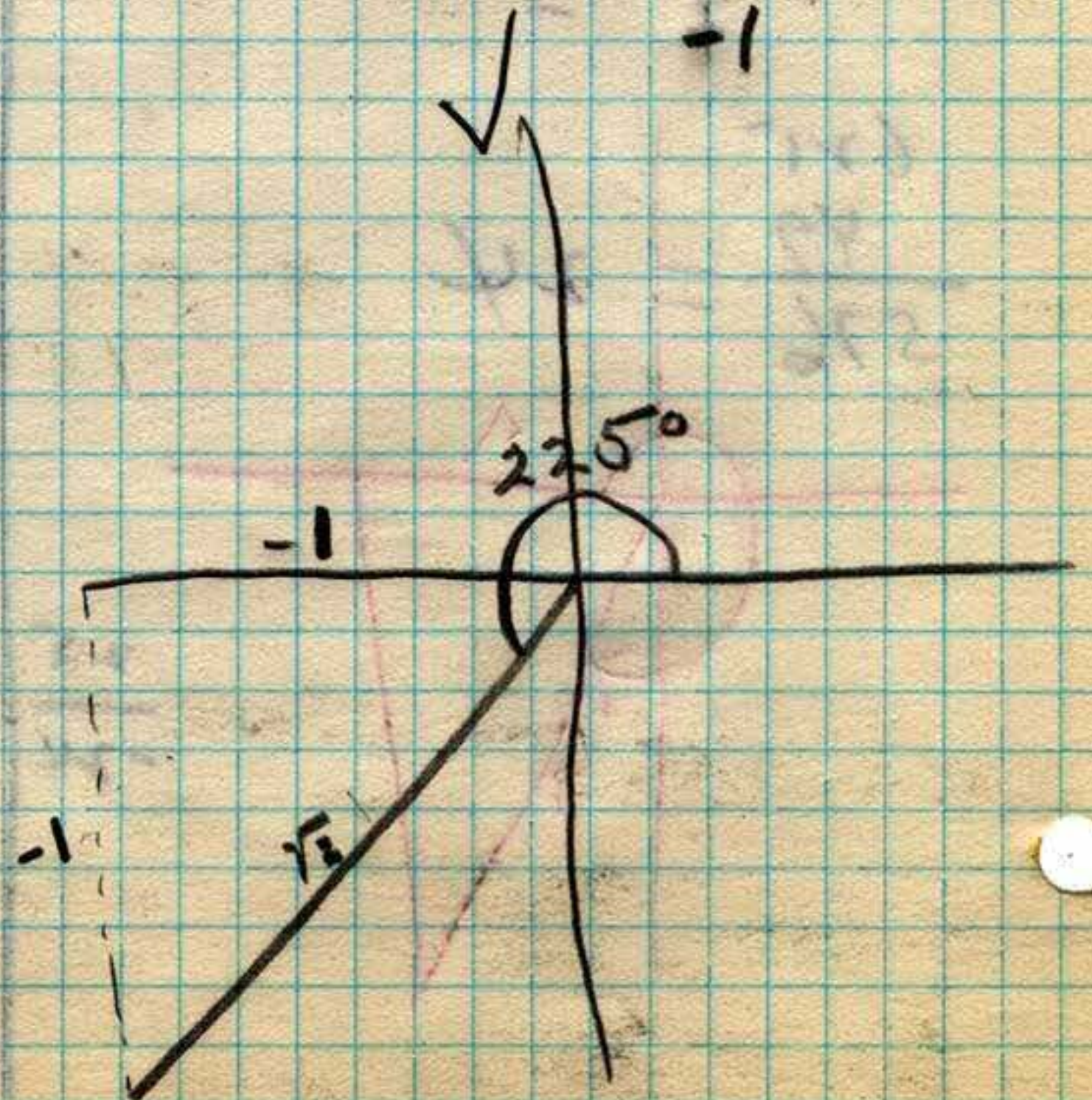
$$\begin{aligned} \sin 150^\circ &= \frac{a}{c} = \frac{1}{2} = \frac{1}{2} \\ \cos 150^\circ &= \frac{b}{c} = \frac{-\sqrt{3}}{2} \end{aligned}$$



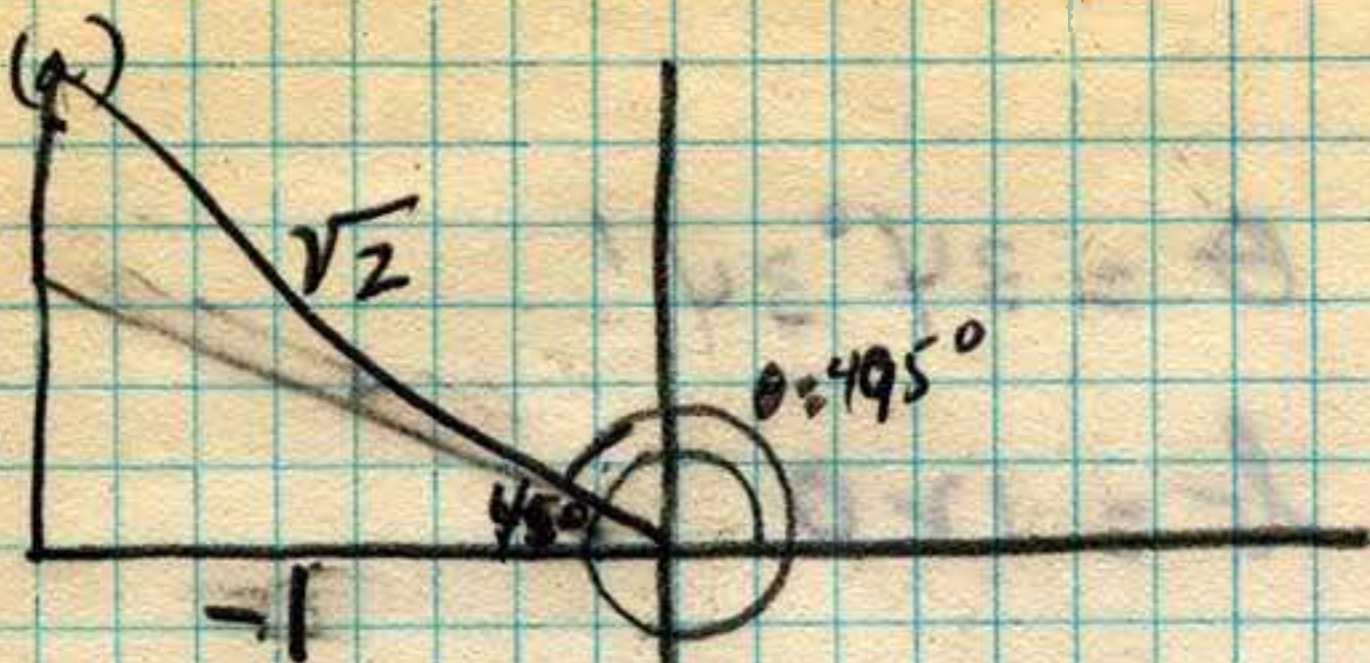
$$\cos 210^\circ = -\frac{\sqrt{3}}{2} \quad \checkmark$$



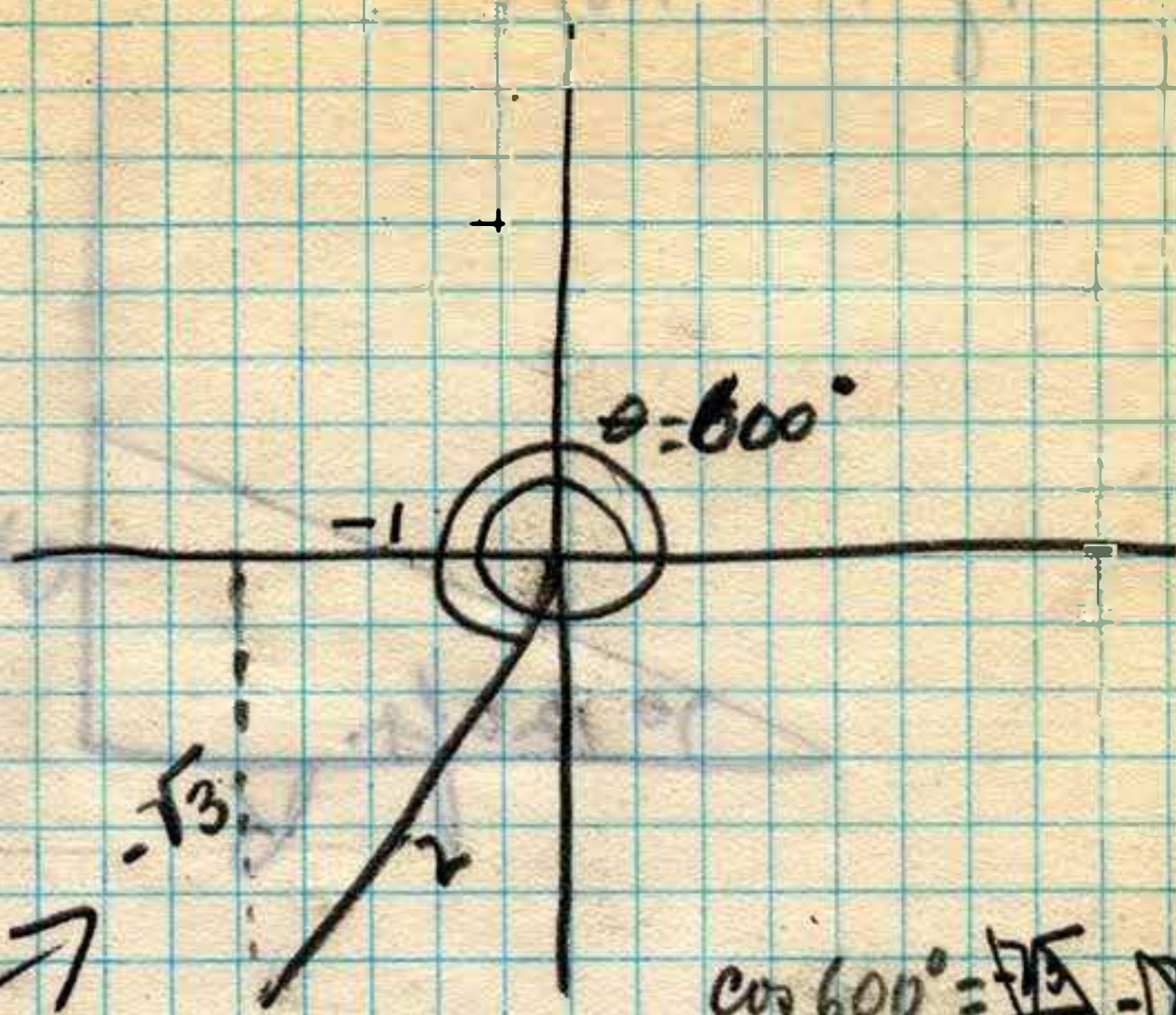
$$\tan 225^\circ = \frac{-1}{-1} = +1 \quad \checkmark$$







(b)



$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

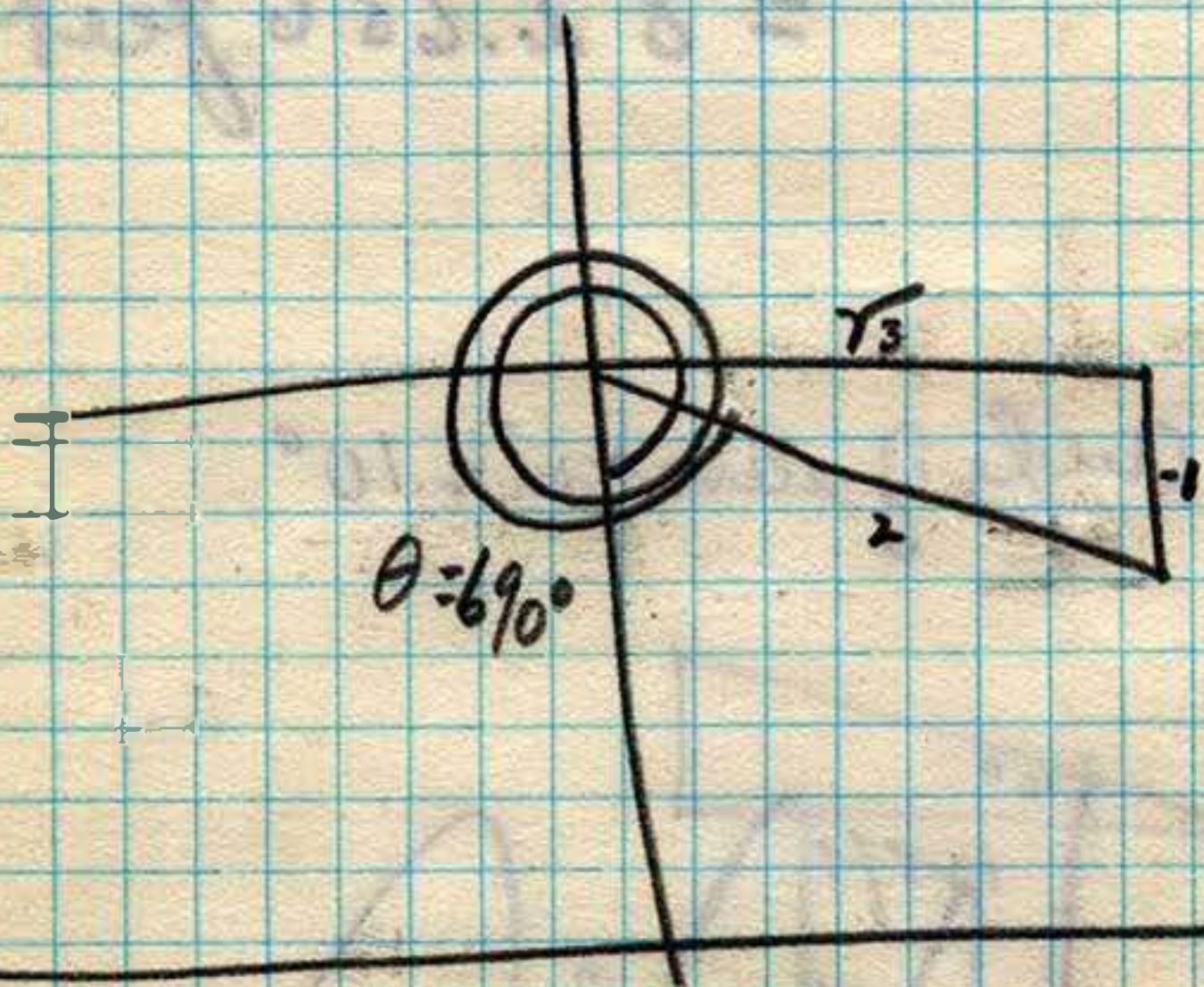
$$\cos 60^\circ = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{-1}{2} = -\frac{1}{2}$$

\* Question regarding Fig. 21 page 22



(c)



$$\tan 60^\circ = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Page 23 No 10

$$\begin{aligned} \sin 120^\circ \cos 30^\circ - \cos 120^\circ \sin 30^\circ \\ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right) \cdot \frac{1}{2} \\ = \frac{3}{4} + \frac{1}{4} = 1 \end{aligned}$$

Page 23 No 13

Prove  $\frac{\sin 150^\circ}{\sin 30^\circ} - \frac{\cos 150^\circ}{\cos 30^\circ} = 2$

$$\frac{\frac{1}{2}}{\frac{1}{2}} - \left(\frac{-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}\right) = 1 + 1 = 2$$

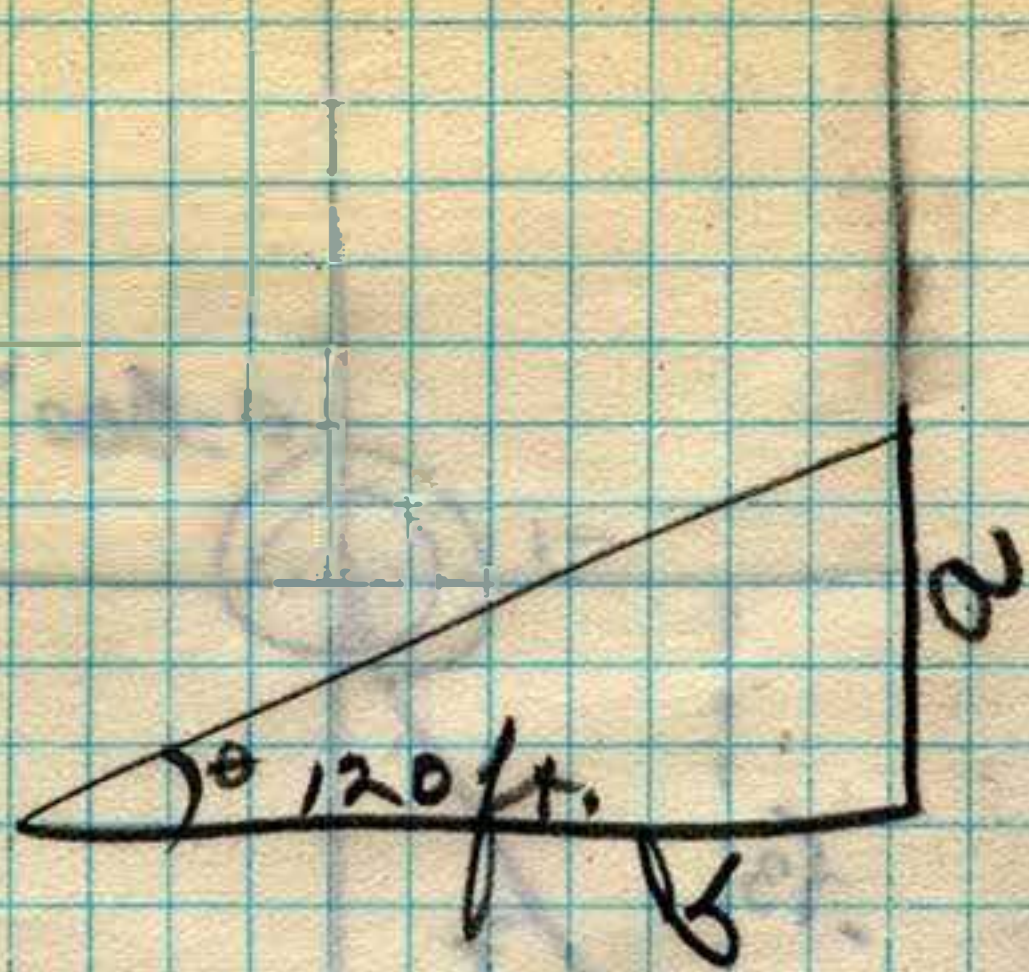
Page 23 No. 15

Prove that

$$\begin{aligned} \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} &= 2 + \sqrt{3} \\ &= \frac{1 + \frac{\sqrt{3}}{3}}{1 - \left(1 \cdot \frac{\sqrt{3}}{3}\right)} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \end{aligned}$$



Pg 116 No. 9



$$\theta = 34^{\circ}34'$$

$$b = 120$$

$$\tan \theta = \frac{a}{b}$$

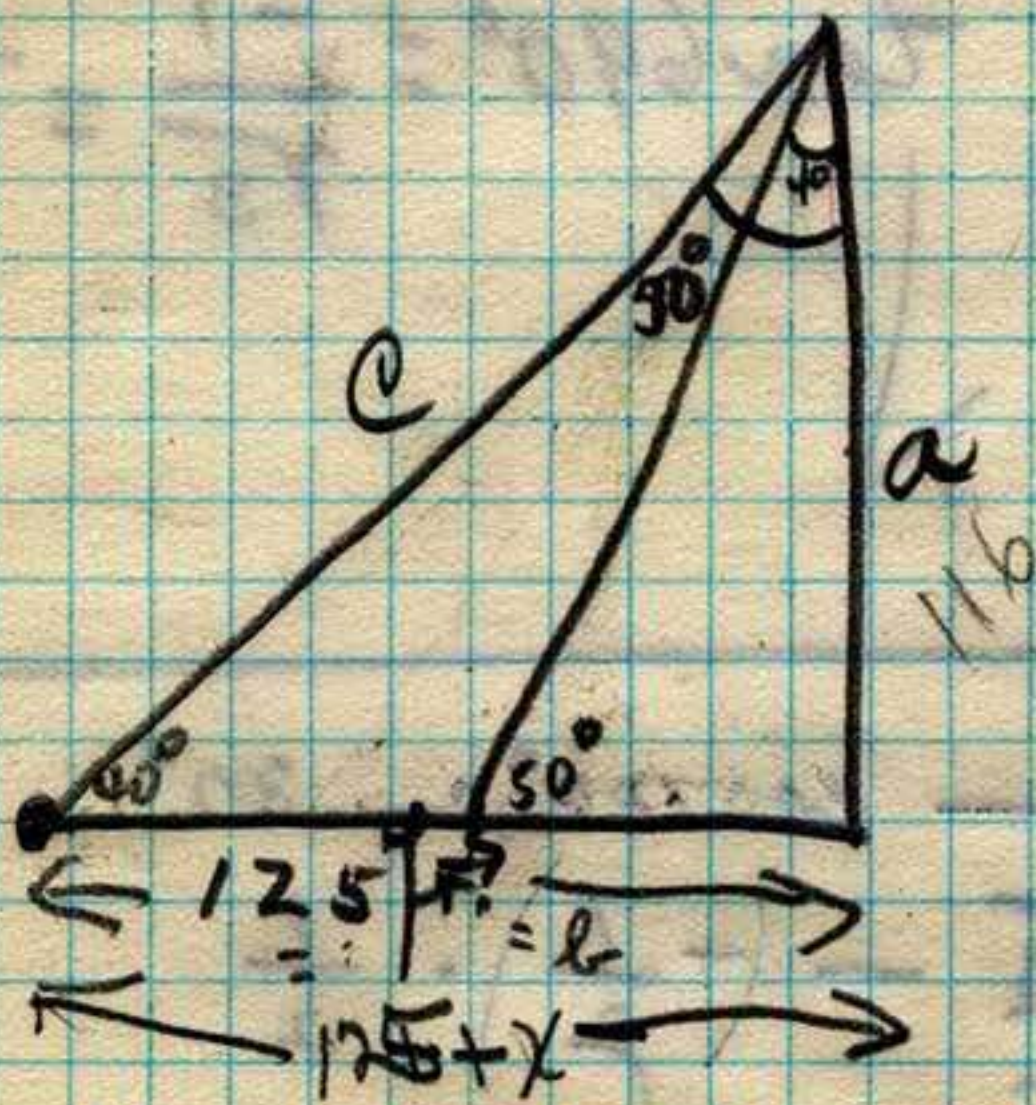
$$\tan 34^{\circ}34' = \frac{a}{b} = \frac{a}{120}$$

$$.6888 = \frac{a}{120}$$

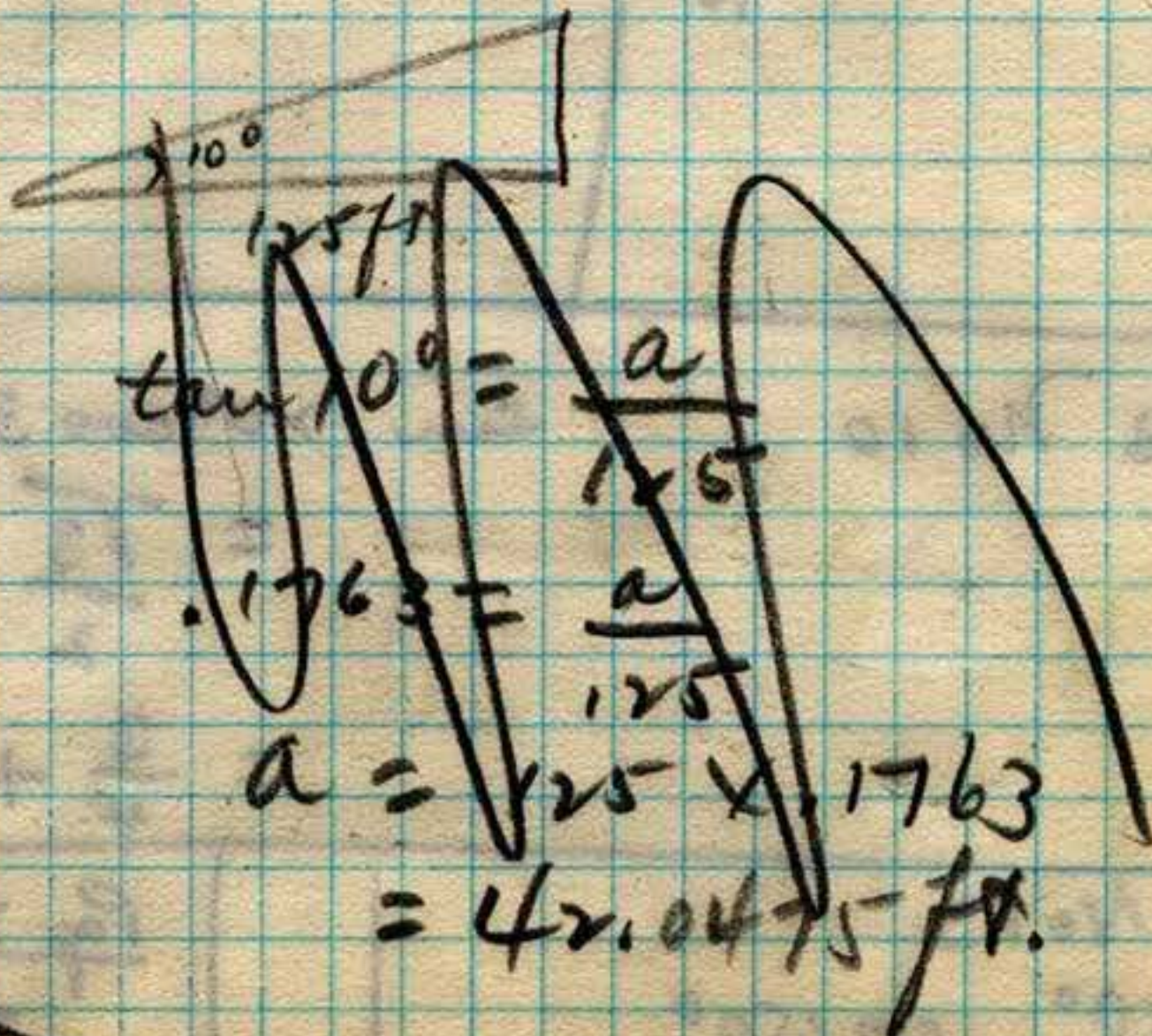
$$a = 120 \times .6888$$

$$= 82.656 \text{ feet}$$

Pg. 117 No. 12



Angle of elevation =  $10^{\circ}$



$$\tan 10^{\circ} = \frac{a}{125}$$

$$.1763 = \frac{a}{125}$$

$$a = 125 \times .1763$$

$$= 42.0475 \text{ ft.}$$

or  $\log a = \log \tan 10^{\circ} + \log 125$   
 $.09691$



Sisam - Page 140 - No. 8, 13, 21, 32

Page 142 - No. 7, 9, 11, 16, 19

Page 144 Formulas 1 + 2

143 " 3

146 " 5, 6, 7

Page 146 Nos. 4 + 5

147 No. 17 + 23

Page 44 nos. 13 + 14

Page 45 No. 7, 9, 22

Page 44 No. 13

$$\frac{\sqrt{6a^3b^2}}{\sqrt[4]{2ab^5}} = \frac{\sqrt{6a^3b^2} \cdot \sqrt[4]{2ab^5}}{2ab^5}$$

$$\frac{x}{a} = .83$$

$$a = .83x$$

$$\begin{array}{r} 1.19 \\ .83 \\ \hline 359 \\ 952 \\ \hline 9.877 \end{array}$$

$$\begin{array}{r} 13 \\ 888 \\ \hline 3620 \end{array}$$

$$13 + 125 = 138$$

$$\frac{a}{x} = \frac{1}{.83}$$

$$x + 2 = x + 2\sqrt{x} + 1$$

$$\frac{1}{2} = \sqrt{x}$$

$$\frac{1}{4} = x$$

$$\begin{array}{r} 1.19 \\ 116 \overline{) 138} \\ \underline{116} \\ 220 \\ \underline{116} \\ 104 \end{array}$$

$$\frac{138}{116} = 1.19$$

$$\begin{array}{r} 1.19 \\ 116 \overline{) 13800} \\ \underline{11900} \\ 1900 \\ \underline{11900} \\ 710 \end{array}$$

$$\frac{125 + x}{.83x} = 1.19$$

$$125 + x = 1.19 \cdot .83x$$

$$= 9.877x$$

$$8.88x = 125$$

$$x = 13.9$$

$$\frac{138}{a} = 1.19$$



$$(1 - \cos x)(2 + 3 \cos x)$$

$$(a/3)^{1/5} = 6^{1/5}$$

$$\sqrt[5]{15} = 2$$

$$\frac{2\sqrt{3}}{3}$$

$$\sqrt[5]{15} = 2$$

$$2 \cos - 3 = \frac{2}{\cos}$$

$$2 \cos = \frac{2}{\cos} + 3$$

$$\cos = \frac{\left(\frac{2}{\cos} + 3\right)}{2}$$

$$= \frac{2 + 3 \cos}{2 \cos} \cdot \frac{1}{2}$$

$$= \frac{2 + 3 \cos}{2 \cos}$$

$$\cos$$

$$\frac{2}{\sqrt{3}}$$

$$1.732 / 2$$



pg 44 no. 13

$$\frac{\sqrt[4]{6a^3b^2}}{\sqrt[4]{2ab^5}} = \frac{\sqrt[4]{36a^6b^4}}{\sqrt[4]{2ab^5}} = \frac{\sqrt[4]{2^2 \cdot 3^2 \cdot a^6 b^4}}{\sqrt[4]{2ab^5}} = \frac{\sqrt[4]{18a^5}}{\sqrt[4]{b}}$$

m. 4

$$\frac{\sqrt[5]{2a^7b^2c^5}}{\sqrt[5]{a^4b^3c^7}} = \frac{\sqrt[5]{2^5 a^8 b^7 c^{10}}}{\sqrt[5]{a^4 b^3 c^7}} = 2 \sqrt[5]{a^4 b^4 c^3} = 2a^2 b^2 c^3$$

correct?  $\sqrt[4]{18a^5/b}$

Page 45 no. 7

$$\frac{(7\sqrt{3} + 2\sqrt{5})(2\sqrt{5} + \sqrt{5})}{(2\sqrt{3} - \sqrt{5})(2\sqrt{3} + \sqrt{5})} = \frac{42 + 11\sqrt{15} + 10}{12 - 5} = \frac{52 + 11\sqrt{15}}{7}$$

Page 45 no. 9

$$\frac{\sqrt{\frac{7}{3}} + \sqrt{\frac{3}{5}}}{\sqrt{\frac{7}{3}} - \sqrt{\frac{3}{5}}} = \frac{\frac{7}{3} + 2\sqrt{\frac{7}{3} \cdot \frac{3}{5}} + \frac{3}{5}}{\frac{7}{3} - \frac{3}{5}} = \frac{\frac{35}{15} + 2\sqrt{\frac{21}{15}} + \frac{9}{15}}{\frac{26}{15}} = \left(\frac{44}{15} + 2\sqrt{\frac{21}{15}}\right) \cdot \frac{15}{26} = \left(\frac{44}{15} + \frac{2\sqrt{35}}{5}\right) \cdot \frac{15}{26} = \frac{44 + 6\sqrt{35}}{15} \cdot \frac{15}{26} = \frac{44 + 6\sqrt{35}}{26} = 22 + \frac{3\sqrt{35}}{13}$$



Pg 140 No. 8 Prove  $\tan^2 \alpha = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha}$

add 1 to each side

$$\tan^2 \alpha + 1 = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} + 1$$

$$\sec^2 \alpha = \frac{1}{\cos^2 \alpha} = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} + 1$$

$$1 = 1 - \cos^2 \alpha + \cos^2 \alpha$$

$$1 = 1$$

$$\frac{1 - \cos^2 \alpha + \cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha}$$

$$\tan^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha}$$

Page 140 No. 13

Prove  $\frac{1 + \cot^2 \beta}{1 + \tan^2 \beta} = \cot^2 \beta$

$$1 + \cot^2 \beta = \cot^2 \beta (1 + \tan^2 \beta)$$

$$= \cot^2 \beta \left(1 + \frac{1}{\cot^2 \beta}\right)$$

$$= \cot^2 \beta + \frac{\cot^2 \beta}{\cot^2 \beta} = \cot^2 \beta + 1$$

Page 140 No. 21

Prove  $\frac{1 - 2 \cos^2 x}{\sin x \cos x} = \tan x - \cot x$

$$= \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}$$

$$1 - 2 \cos^2 x = \sin^2 x - \cos^2 x$$

$$= 1 - \cos^2 x - \cos^2 x$$

$$= 1 - 2 \cos^2 x$$

Page 140 No. 32

Prove  $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \csc x$

$$= \frac{2}{\sin x}$$

$$\sin^2 x + (1 + \cos x)^2 = 2(1 + \cos x)$$

$$\sin^2 x + 1 + 2 \cos x + \cos^2 x = 2 + 2 \cos x$$

$$\sin^2 x + 1 + \cos^2 x = 2$$

$$1 + 1 = 2$$



Page 146 No. 5

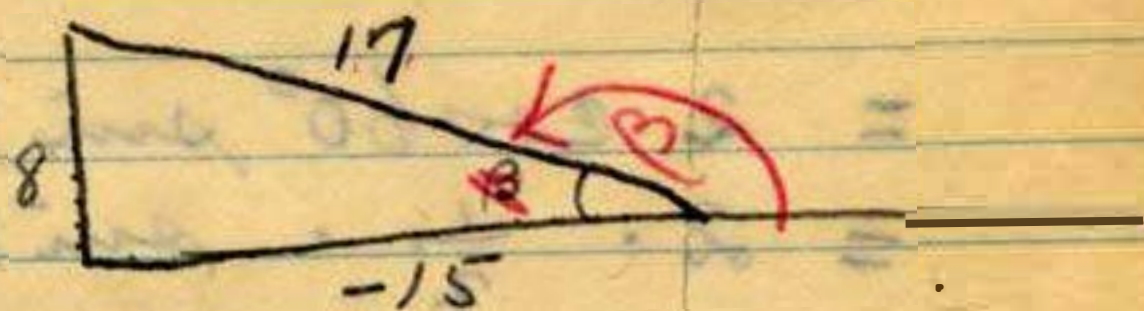
$$\sin \alpha = \frac{7}{25}$$

$$\tan \beta = -\frac{8}{15}$$



$$\begin{aligned} \text{For } \alpha, \quad a &= 7 \\ b &= -24 \\ c &= 25 \end{aligned}$$

$$\begin{aligned} \text{For } \beta, \quad a &= 8 \\ b &= -15 \\ c &= 17 \end{aligned}$$



$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \frac{7}{25} \cdot -\frac{15}{17} + -\frac{24}{25} \cdot \frac{8}{17} \\ &= \left( -\frac{105}{425} \right) + \left( -\frac{192}{425} \right) = -\frac{297}{425} \end{aligned}$$

$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= -\frac{24}{25} \cdot -\frac{15}{17} - \frac{7}{25} \cdot \frac{8}{17} \\ &= \frac{360}{425} - \frac{56}{425} = \frac{304}{425} \end{aligned}$$

~~$\sin(\alpha + \beta)$~~

$$\begin{aligned} \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \left( \frac{7}{25} \cdot -\frac{15}{17} \right) - \left( -\frac{24}{25} \cdot \frac{8}{17} \right) = \frac{-105}{425} + \frac{192}{425} \\ &= \frac{87}{425} \end{aligned}$$

$$\begin{aligned} \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \left( -\frac{24}{25} \cdot -\frac{15}{17} \right) + \left( \frac{7}{25} \cdot \frac{8}{17} \right) \\ &= \frac{360}{425} + \frac{56}{425} = \frac{416}{425} \end{aligned}$$



Page 147 No. 17

Prove  $\sin(60^\circ + x) - \cos(30^\circ + x) = \sin x$

$$\sin 60^\circ \cos x + \cos 60^\circ \sin x - (\cos 30^\circ \cos x - \sin 30^\circ \sin x)$$

$$= \sin 60^\circ \cos x + \cos 60^\circ \sin x - \cos 30^\circ \cos x + \sin 30^\circ \sin x$$

$$= 2 \cos 60^\circ \sin x$$

$$= 2 \cdot \frac{1}{2} \sin x = \sin x$$

Page 147 No. 23

Prove  $\sin 3\theta \cos \theta - \cos 3\theta \sin \theta = \sin 2\theta$

$$\sin(\alpha - \beta) = \sin(3\theta - 2\theta) = \sin 2\theta$$

Northcott

Page 85 — No. 2, 3, 10

88 — No. 1, 2

Read page 34

Page 36 No. 1, 2, 3, 6, 7

Again Page 165 No. 1, 4

170 No. 14

Smith, Longley, Wilson Page 8 No. 1 a, c, e

Page 9 c, e, g

2 c

3 a

7

8

9 c

10



Page 142 No. 7

$$5 \cos x + 2 = 3(2 - \cos x)$$
$$= 6 - 3 \cos x$$

$$8 \cos x = 4$$

$$\cos x = \frac{1}{2}$$

$$\angle x = 60^\circ \text{ or } 300^\circ$$

Page 142 No. 9

$$\sqrt{2} \tan x \sin x + \tan x = 0$$

$$\tan x (\sqrt{2} \sin x + 1) = 0$$

$$\text{If } \tan x = 0$$

$$\text{Then } x = 0^\circ \text{ or } 180^\circ$$

$$\sqrt{2} \sin x + 1 = 0$$

$$\sqrt{2} \sin x = -1$$

$$2 \sin x = -\sqrt{2}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$\text{Then } x = 225^\circ \text{ or } 315^\circ$$

Page 142 No. 11

$$2 \sin^2 x + 3 \cos x = 0$$

$$2(1 - \cos^2 x) + 3 \cos x = 0$$

$$2 - 2 \cos^2 x + 3 \cos x = 0$$

$$2 \cos^2 x - 3 \cos x - 2 = 0$$

$$(2 \cos x + 1)(\cos x - 2) = 0$$

$$\text{Thus } \cos x = -\frac{1}{2} \text{ or } 2$$

$$\text{for } \angle = 120^\circ \text{ or } 240^\circ \quad ? \text{ no such angle?}$$



Page 142 No. 16

$$3 \csc^2 x + \cot^2 x = 15$$

$$3(\cot^2 x + 1) + \cot^2 x = 15$$

$$3 \cot^2 x + 3 + \cot^2 x = 15$$

$$4 \cot^2 x = 12$$

$$\cot^2 x = 3$$

$$\cot x = \sqrt{3} \text{ or } -\sqrt{3}$$

$$\text{If } \cot x = \sqrt{3}, \angle x = 30^\circ \text{ or } 210^\circ$$

$$\text{If } \cot x = -\sqrt{3}, \angle x = 150^\circ \text{ or } 330^\circ$$

$$\begin{array}{r} 633 \\ \underline{114} \\ 378 \\ \underline{396} \\ 6 \end{array}$$



$$\sin \alpha = \frac{3}{5} \quad \cos \beta = \frac{5}{13}$$

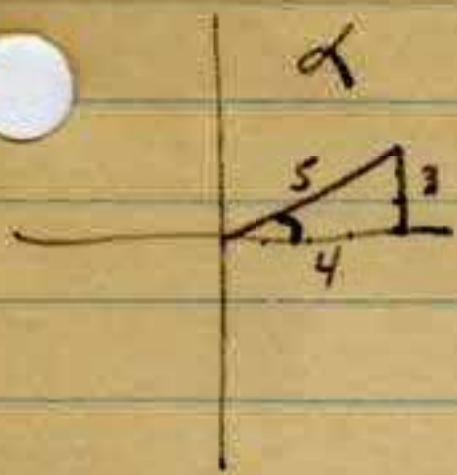
 $\alpha + \beta$  in first quadrant

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13} \end{aligned}$$

$$= \frac{3}{13} + \frac{48}{65} = \frac{\cancel{45} + 48}{65} = \frac{93}{65}$$

$$\begin{aligned} \text{for } \alpha, \quad a &= 3 \\ b &= 4 \\ c &= 5 \end{aligned}$$

$$\begin{aligned} \text{for } \beta \quad a &= 12 \\ b &= 5 \\ c &= 13 \end{aligned}$$



$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \frac{4}{5} \cdot \frac{5}{13} - \frac{3}{5} \cdot \frac{12}{13}$$

$$= \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$= \frac{3}{5} \cdot \frac{5}{13} - \frac{4}{5} \cdot \frac{12}{13}$$

$$= \frac{15}{65} - \frac{48}{65} = -\frac{33}{65}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

$$\begin{aligned} \sin^2(\alpha + \beta) + \cos^2(\alpha + \beta) &= \frac{3969}{4225} + \frac{256}{4225} \\ &= \frac{4225}{4225} \\ &= 1 \end{aligned}$$







a) Find  $\cos(\arcsin \frac{4}{5})$



$$\sin \theta = \frac{4}{5}$$

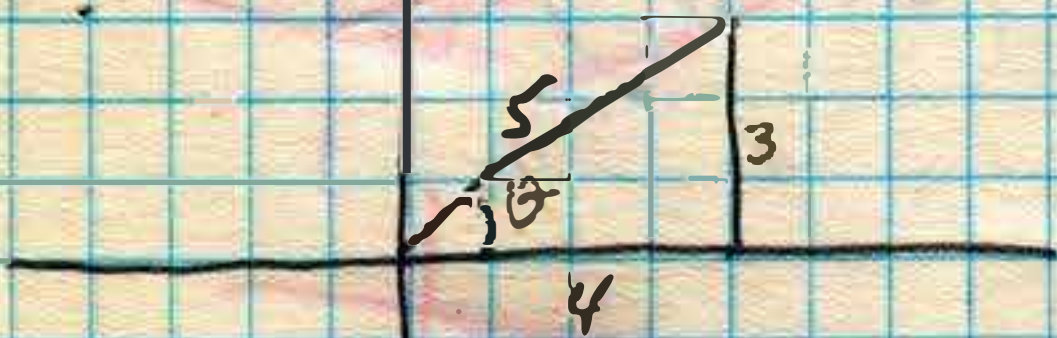
$$a = 4$$

$$c = 5$$

$$b = \sqrt{25 - 16} = 3$$

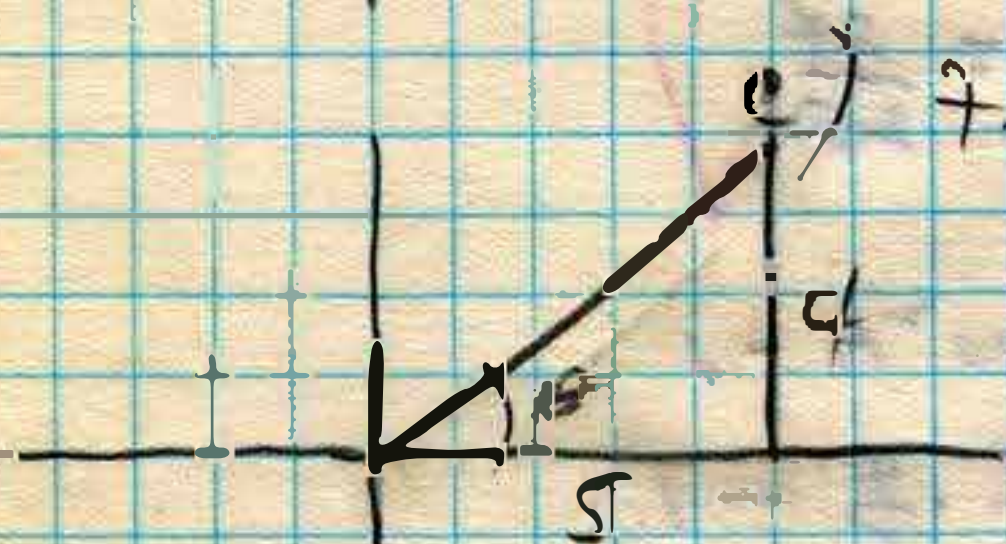
$$\cos \theta = \frac{3}{5} \checkmark$$

b) Find  $\sin(\arccos \frac{4}{5})$



$$\sin \theta = \frac{3}{5} \checkmark$$

c) Find  $\tan(\arccos \frac{3}{5})$



$$\tan \theta = \frac{4}{3} \checkmark$$

$$\text{prove } \arctan \frac{2}{3} + \arctan \frac{1}{5} = 45^\circ$$

$$\tan \alpha = \frac{2}{3}$$

$$\tan \beta = \frac{1}{5}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{\frac{2}{3} + \frac{1}{5}}{1 - (\frac{2}{3} \cdot \frac{1}{5})}$$

$$= \frac{\frac{10}{15} + \frac{3}{15}}{1 - \frac{2}{15}}$$

$$= \frac{\frac{13}{15}}{\frac{13}{15}} = 1$$

$$\therefore \alpha + \beta = 45^\circ$$

*Handwritten notes in red ink:*  
 $\tan 90^\circ = \frac{1}{0}$   
 $\tan 0^\circ = 0$   
 $\tan 45^\circ = 1$   
 $\tan 135^\circ = -1$   
 $\tan 225^\circ = 1$   
 $\tan 315^\circ = -1$   
 $\tan 180^\circ = 0$   
 $\tan 270^\circ = \frac{1}{0}$   
 $\tan 90^\circ = \frac{1}{0}$   
 $\tan 270^\circ = \frac{1}{0}$   
 $\tan 0^\circ = 0$   
 $\tan 180^\circ = 0$   
 $\tan 360^\circ = 0$   
 $\tan 540^\circ = 0$   
 $\tan 720^\circ = 0$   
 $\tan 900^\circ = 0$   
 $\tan 1080^\circ = 0$   
 $\tan 1260^\circ = 0$   
 $\tan 1440^\circ = 0$   
 $\tan 1620^\circ = 0$   
 $\tan 1800^\circ = 0$   
 $\tan 1980^\circ = 0$   
 $\tan 2160^\circ = 0$   
 $\tan 2340^\circ = 0$   
 $\tan 2520^\circ = 0$   
 $\tan 2700^\circ = 0$   
 $\tan 2880^\circ = 0$   
 $\tan 3060^\circ = 0$   
 $\tan 3240^\circ = 0$   
 $\tan 3420^\circ = 0$   
 $\tan 3600^\circ = 0$



Page 85 No. 10

Prove  $\arctan 2 + \arctan \frac{1}{2} = 90^\circ$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{2 + \frac{1}{2}}{1 - 2 \cdot \frac{1}{2}} = \frac{2\frac{1}{2}}{1-1} = \frac{2\frac{1}{2}}{0} = \infty$$

$$= \infty$$

$$\therefore \alpha + \beta = 90^\circ \text{ (or } 180^\circ)$$

Page 88 No. 1 Fund values

a)  $\sin(2 \arcsin \frac{4}{5})$

$$\cos = \frac{3}{5}$$

$$\begin{aligned} \sin 2A &= 2 \sin A \cos A \\ &= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25} \end{aligned}$$

b)  $\cos(2 \arccos \frac{4}{5})$

$$\begin{aligned} \cos 2A &= 2 \cos^2 A - 1 \\ &= 2 \cdot \frac{4}{5} \cdot \frac{4}{5} - 1 \end{aligned}$$

$$= \frac{32}{25} - 1 = \frac{7}{25}$$

c)  $\tan(2 \arctan \frac{4}{5})$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$= \frac{2 \cdot \frac{4}{5}}{1 - \frac{4}{5} \cdot \frac{4}{5}} = \frac{\frac{8}{5}}{\frac{9}{25}} = \frac{8}{5} \times \frac{25}{9} = \frac{40}{9}$$

d)  $\cos(2 \arctan \frac{5}{12})$

$$\sqrt{25+144} = 13$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$= 2 \cdot \left(\frac{12}{13}\right)^2 - 1 = \frac{288}{169} - 1 = \frac{119}{169}$$

e)  $\sin(\frac{1}{2} \arctan \frac{5}{12})$   
angle must be positive

$$\sin \frac{A}{2} = \frac{+ \sqrt{1 - \cos A}}{2} = \sqrt{\frac{1 - \frac{12}{13}}{2}}$$

$$= \sqrt{\frac{\frac{1}{13} \cdot \frac{1}{2}}{2}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$$



Northcott

Page 89 No. 2

Find values

a)  $\sin\left(\frac{1}{2} \text{ arc } \cos \frac{4}{5}\right)$

Note: Ask about (+ and - angles?)

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$= \sqrt{\frac{1 - \frac{4}{5}}{2}} = \sqrt{\frac{1}{10}}$$

$$= \frac{\sqrt{10}}{10} \checkmark$$

b)  $\cos\left(\frac{1}{2} \text{ arc } \sin \frac{4}{5}\right)$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} = + \sqrt{\frac{1 + \frac{3}{5}}{2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \checkmark$$

c)  $\tan(2 \text{ arc } \tan a)$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$= \frac{2a}{1 - a^2} \checkmark$$

d)  $\sin(2 \text{ arc } \tan a)$

$$\sin 2A = 2 \sin A \cos A$$

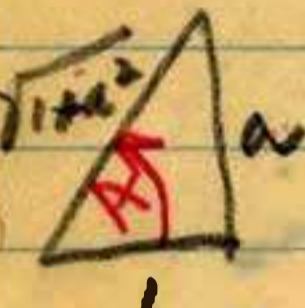
$$= \frac{2a \cdot 1}{\sqrt{1+a^2} \cdot \sqrt{1+a^2}}$$

$$= \frac{2a}{1+a^2} \checkmark$$

~~$\tan x = \frac{a}{1}$~~

~~$\cos x = \frac{1}{\sqrt{1+a^2}}$~~

~~$\sin x = \frac{a}{\sqrt{1+a^2}}$~~





Northcott

Page 36

1, 2, 3, 6, 7

change the following angles to degrees

1. a)  $\frac{\pi}{3}$  radians =  $\frac{180^\circ}{3} = 60^\circ$  ✓

b)  $\frac{3\pi}{4}$  radians =  $\frac{3}{4} \cdot 180^\circ = \frac{540^\circ}{4} = 135^\circ$  ✓

c)  $\frac{7\pi}{6}$  radians =  $\frac{7}{6} \cdot 180^\circ = 210^\circ$  ✓

d) 4 radians =  $\frac{4 \times 180^\circ}{\pi} = \frac{720^\circ}{\pi}$  ✓

e)  $\frac{\pi}{8}$  radians =  $\frac{180^\circ}{8} = 22.5^\circ$  ✓

$x$  radians =  $\frac{x \cdot 180^\circ}{\pi}$

$\frac{180^\circ}{\pi} = \frac{60^\circ}{\pi} = \frac{20^\circ}{\pi}$

2. Change the following angles to radian measure

a)  $105^\circ$

$\pi$  radians =  $180^\circ$

1 radian =  $\frac{180^\circ}{\pi}$

$1^\circ = \frac{\pi}{180}$  radians

$x^\circ = \frac{x \cdot \pi}{180}$  radians

$\therefore 105^\circ = \frac{105 \cdot \pi}{180}$  radians =  $\frac{7\pi}{12}$  radians ✓

b)  $225^\circ = \frac{225 \cdot \pi}{180}$  radians =  $\frac{15\pi}{12}$  radians =  $\frac{5\pi}{4}$  radians ✓

c)  $87^\circ 30' = \frac{87^\circ 30'}{180^\circ} \pi = \frac{5250\pi}{10800} = \frac{35\pi}{72}$  ✓

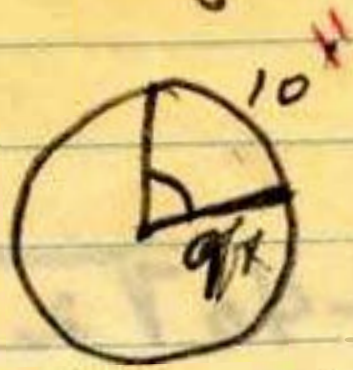
Wilson & Drake

d)  $12^\circ = \frac{12\pi}{180}$  radians =  $\frac{\pi}{15}$  radians ✓

e)  $270^\circ = \frac{270\pi}{180}$  radians =  $\frac{3\pi}{2}$  radians ✓



No. 3



$$\text{arc length} = \text{radius} \times \text{radians}$$

$$\text{radians} = \frac{\text{arc length}}{\text{radius}} = \frac{10}{108} = \frac{5}{54}$$

$$\frac{5}{54} \text{ radians} = \frac{5}{54} \cdot \frac{180^\circ}{\pi} = \left( \frac{50^\circ}{3\pi} \right) \checkmark$$

No. 6 Wheel = 24 inches in diameter  
5 revolutions per second

Circumference  
( $2\pi r$ )

$$5 \cdot 24\pi = 120\pi \text{ inches in 5 sec.}$$

$$\text{In one hour, distance} = 60 \cdot 60 \cdot 120\pi \text{ inches}$$

$$\text{No. miles} = \frac{60 \cdot 60 \cdot 120\pi}{5280 \cdot 12} = \frac{432000\pi}{63360}$$

$$= 6.8\pi = 21.36 \text{ miles} \checkmark$$

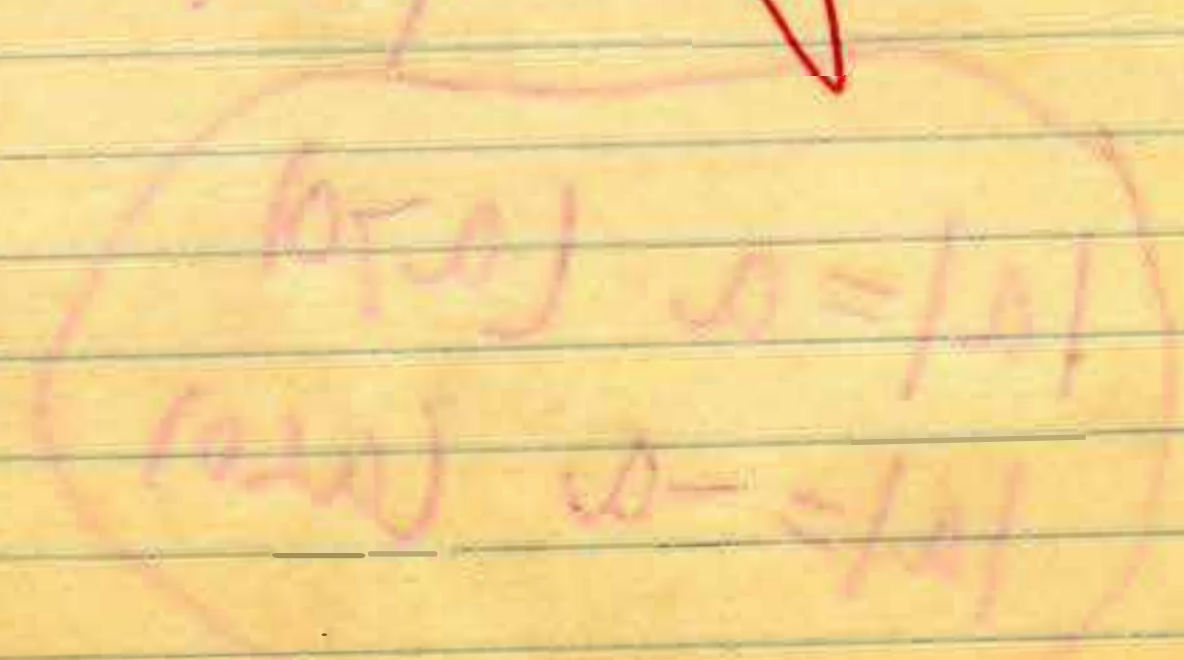
No. 7 Radian measure of two angles of  $\Delta$  are  $\frac{\pi}{4} + \frac{2\pi}{3}$ .

$$\text{Total radians in given angles} = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}$$

$$\pi \text{ radians} = 180^\circ$$

$$\pi - \frac{11\pi}{12} = \frac{\pi}{12}$$

$$\frac{\pi}{12} \text{ radians} = \frac{\pi \cdot 180^\circ}{12\pi} = 15^\circ$$





Asam Page 165 No. 1

$$\begin{cases} b = 5 \\ c = 8 \\ \alpha = 45^\circ \end{cases}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$= 25 + 64 - 2 \cdot 5 \cdot 8 \cdot \frac{\sqrt{2}}{2} = 89 - 40\sqrt{2}$$

$$a = \sqrt{89 - 40\sqrt{2}} = \sqrt{89 - (40 \times 1.41)} = \sqrt{89 - 56.4} = \sqrt{32.6} = 5.7$$

No. 4  $a = 9, b = 11, c = 14$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$a^2 - b^2 - c^2 = -2bc \cos \alpha$$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{121 + 196 - 81}{2 \times 11 \times 14}$$

$$= \frac{236}{308} = \frac{59}{77} \checkmark$$

Page 170 No. 14

$|a|$

$$|7| = 7$$

$$|-7| = 7$$

$$|2/5| = 2/5$$

$$|-2/5| = 2/5$$

~~2/5~~

$|a| = a \quad (a \geq 0)$   
 $|a| = -a \quad (a < 0)$

$$|0| = 0$$