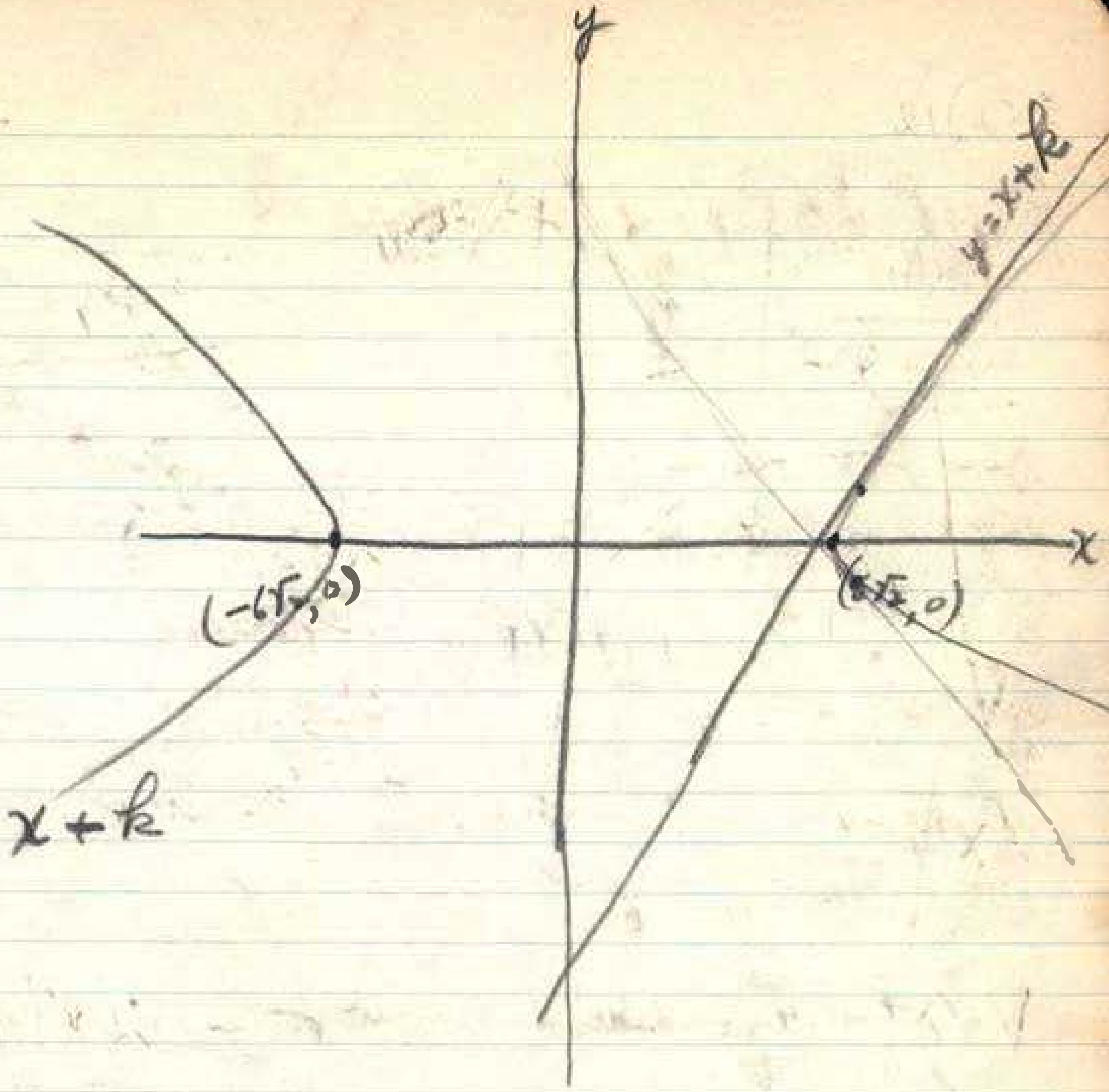
252) 11

$x^2 - 9y^2 = 72$

$\frac{x^2}{72} - \frac{y^2}{8} = 1$

Center at origin
Foci on x-axis
$a = \pm 6\sqrt{2} = \pm 8.49$

When $x = \pm 9$, $y = \pm 1$



Given line $\rightarrow y = x + k$
slope ± 1

$x^2 - 9y^2 = 72$

Differentiating,

$2x - 18y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-2x}{-18y} = \frac{x}{9y}$$

Slope of tangent to hyperbola at any point $P = \frac{x}{9y}$

Then $\frac{x}{9y} = 1$, $x = 9y$ at point of tangency.

Since $x^2 - 9y^2 = 72$

$81y^2 - 9y^2 = 72$

$72y^2 = 72$

$y^2 = 1$

$y = \pm 1$

Then $x = \pm 9$

$y=1$ $x=9$ $y=-1$ $x=-9$

Point of tangency is $(9,1)$ or $(-9,-1)$

$y = x + k$

$k = y - x = 1 - 9 = -8$

or $k = -1 + 9 = 8$

Then y-intercept of line $= \pm 8$

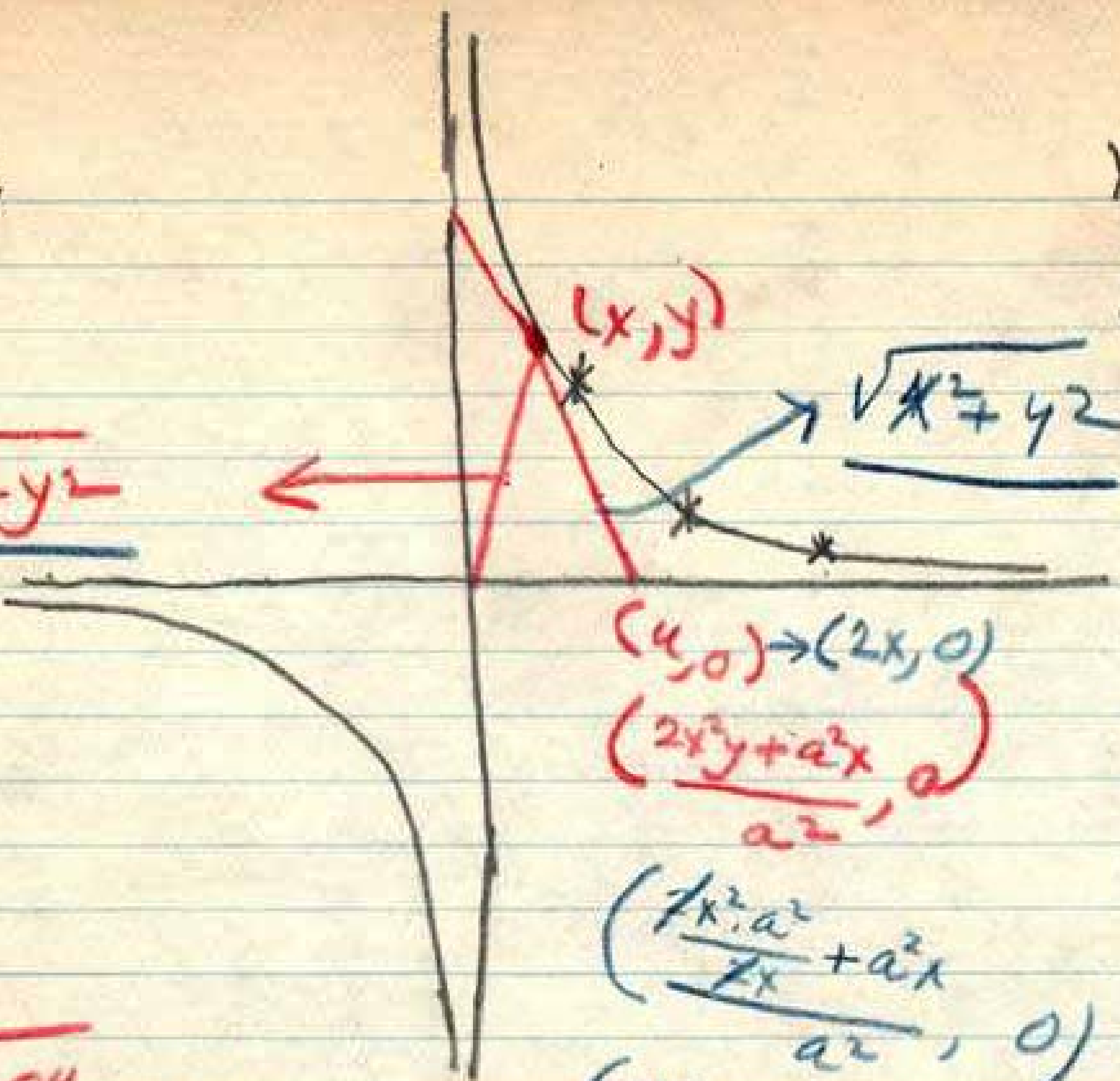
$y = x + 8$ or $y = x - 8$

252) 12

$xy = a^2/2$

$y = \frac{a^2}{2x}$

x	y
a	$a/2$
$2a$	$a/4$
$\frac{a}{2}$	a
$-a$	$-a/2$
$-2a$	$-a/4$
$-a/2$	$-a$



$\sqrt{x^2+y^2}$

$\sqrt{x^2+y^2}$

$\sqrt{x^2+\frac{a^4}{4x^2}}$

$\sqrt{\frac{4x^4+a^4}{4x^2}}$

$= \frac{\sqrt{4x^4+a^4}}{2x}$

$(u,0) \to (2x,0)$

$\left(\frac{2x^2y+a^2x}{a^2}, 0\right)$

$\left(\frac{\frac{\cancel{2}x^2 \cdot a^2}{\cancel{2}x}+a^2x}{a^2}, 0\right)$

$(2x)$

$$\frac{dy}{dx} = \frac{2x(0) - a^2\cdot 2}{4x^2}$$

$$= \frac{\cancel{2x}-2a^2}{4x^2} = \frac{-a^2}{2x^2}$$

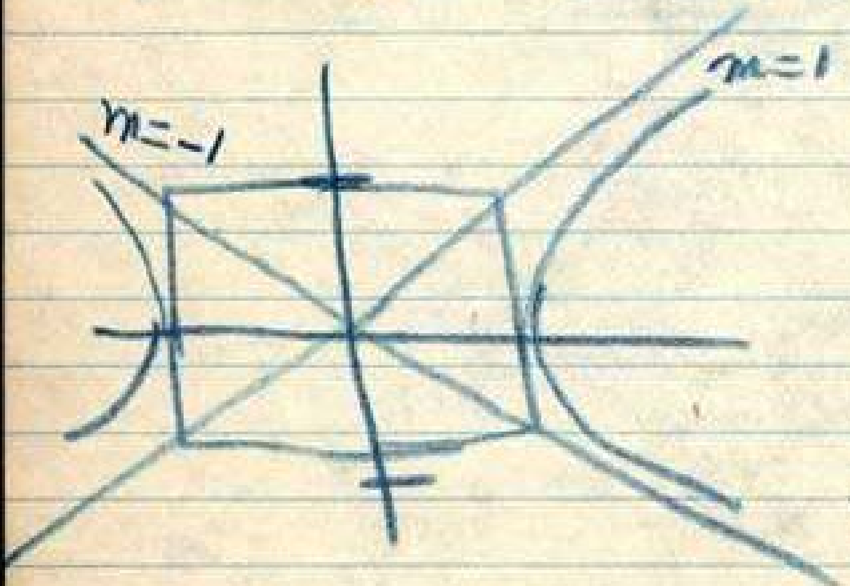
$$\frac{-a^2}{2x^2} = \frac{y-0}{x-u}$$

$$-a^2x + a^2u = 2x^2y$$

$$a^2u = 2x^2y + a^2x$$

$$u = \frac{2x^2y + a^2x}{a^2}$$

Equil. hyp. $\boxed{a=b}$



$$\begin{cases} \frac{x^2}{9} - y^2/9 = 1 \\ xy = 15 \end{cases}$$

252) 15 e

$$16x^2 - 9y^2 + 36y + 108 = 0$$
$$16x^2 - 9y^2 + 36y = -108$$
$$9y^2 - 36y - 16x^2 = 108$$
$$9(y^2 - 4y + 4) - 16x^2 = 108 + 36 = 144$$
$$\frac{(y-2)^2}{16} - \frac{x^2}{9} = 1$$

Center of Hyperbola is at $(0, 2)$

$a = \pm 4$

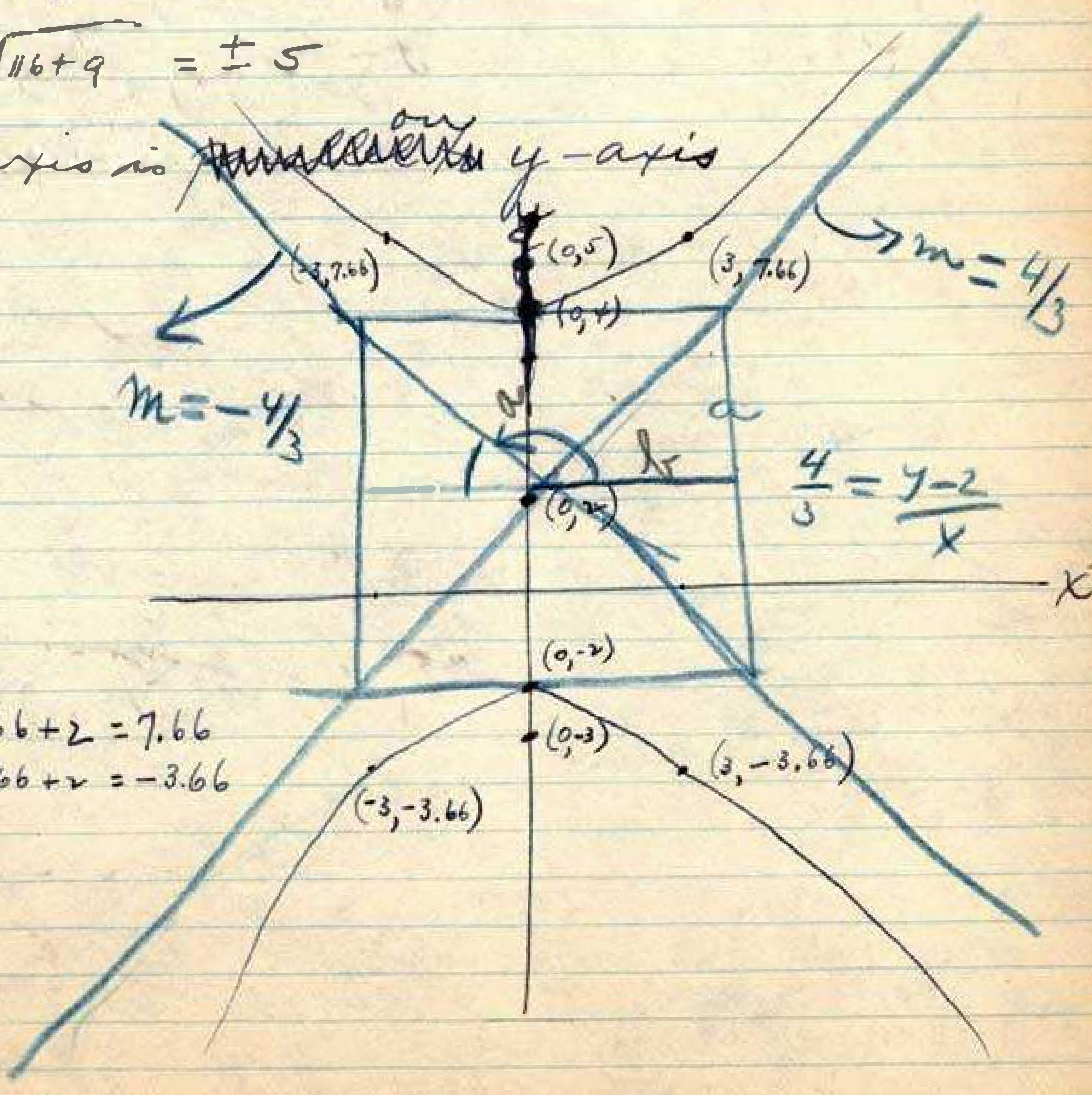
$b = \pm 3$

$e = \sqrt{16+9} = \pm 5$

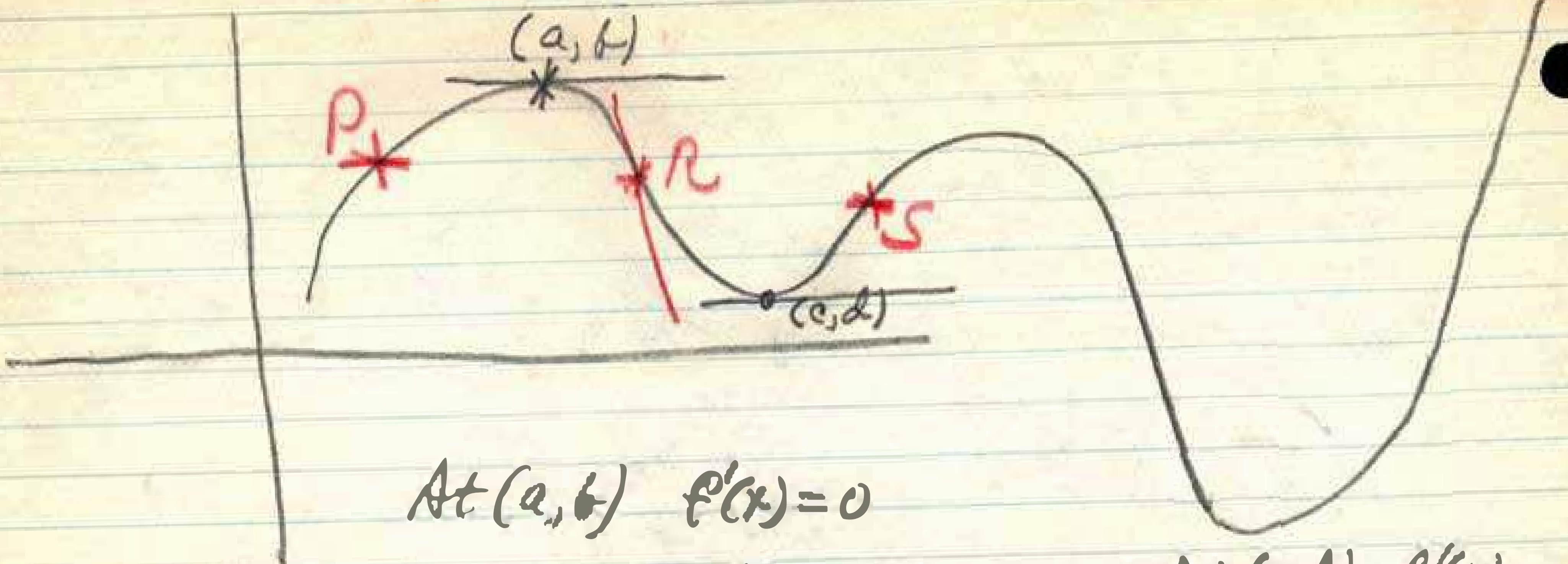
Principal axis is ~~parallel to~~ on y-axis

when $x = \pm 3$, $y =$

$$\frac{(y-2)^2}{16} - 1 = 1$$
$$\frac{(y-2)^2}{16} = 2$$
$$(y-2)^2 = 32$$
$$y - 2 = \pm\sqrt{32}$$
$$y = \pm\sqrt{32} + 2 = \begin{cases} 5.66 + 2 = 7.66 \\ -5.66 + 2 = -3.66 \end{cases}$$



$4, 2, 1, \ldots 0, -1, -2$



At (a,b) $f'(x)=0$

	$f'(x)$
$x<a$	>0
$x>a$	<0

At (c,d) $f'(x)=0$

	$f'(x)$
$x<c$	<0
$x>c$	>0

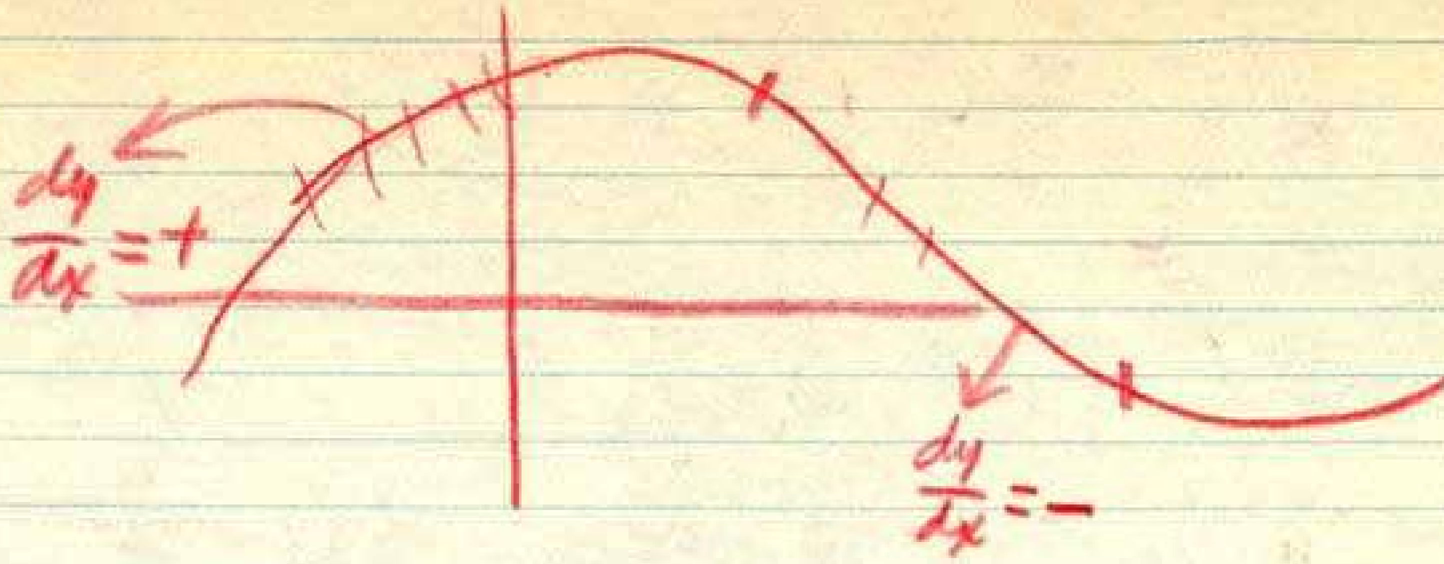
$f'(x)$ decr. throughout PR, (concave downward) $f''(x)$ is negat.

$f'(x)$ incr. " RS, (concave upward) $f''(x)$ is posit.

A pt. where $f''(x)$ changes from posit. to negat. (or vice versa) is called a pt. of inflect.

~~y decr. as x incr.~~ $\cancel{\frac{dy}{dx}}$

At a pt. of infl. $f''(x)=0$



26 r) 1

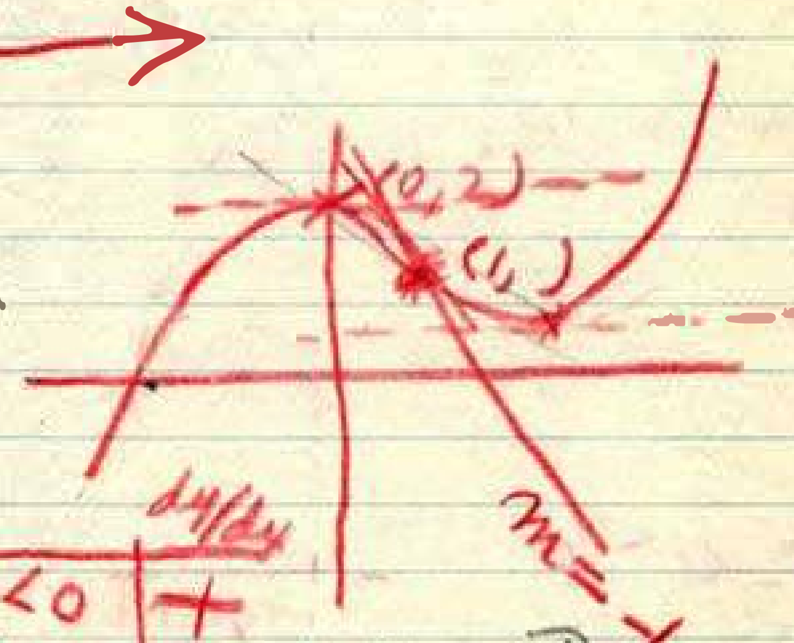
$$y = \frac{x^3}{3} - x^2 + 2$$

$$\frac{dy}{dx} = x^2 - 2x$$

Setting $\frac{dy}{dy}$ at 0, $x^2 - 2x = 0$

$$x(x-2) = 0$$

$$\underline{x = 2, 0}$$



	dy/dx
$x<0$	+
$2>x>0$	−
$x>2$	+

$(1 - 2 = -1$

$$f'(x) = x^2 - 2x$$

$$f''(x) = \underline{2x - 2}$$

$$2x - 2 = 0$$
$$2x = 2$$
$$x = 1$$

	$f''(x)$
$x<1$	−
$x>1$	+

26 2) 7

$$y = \frac{x^4}{4} - 2x^2 + 2$$

$$\frac{dy}{dx} = x^3 - 4x$$

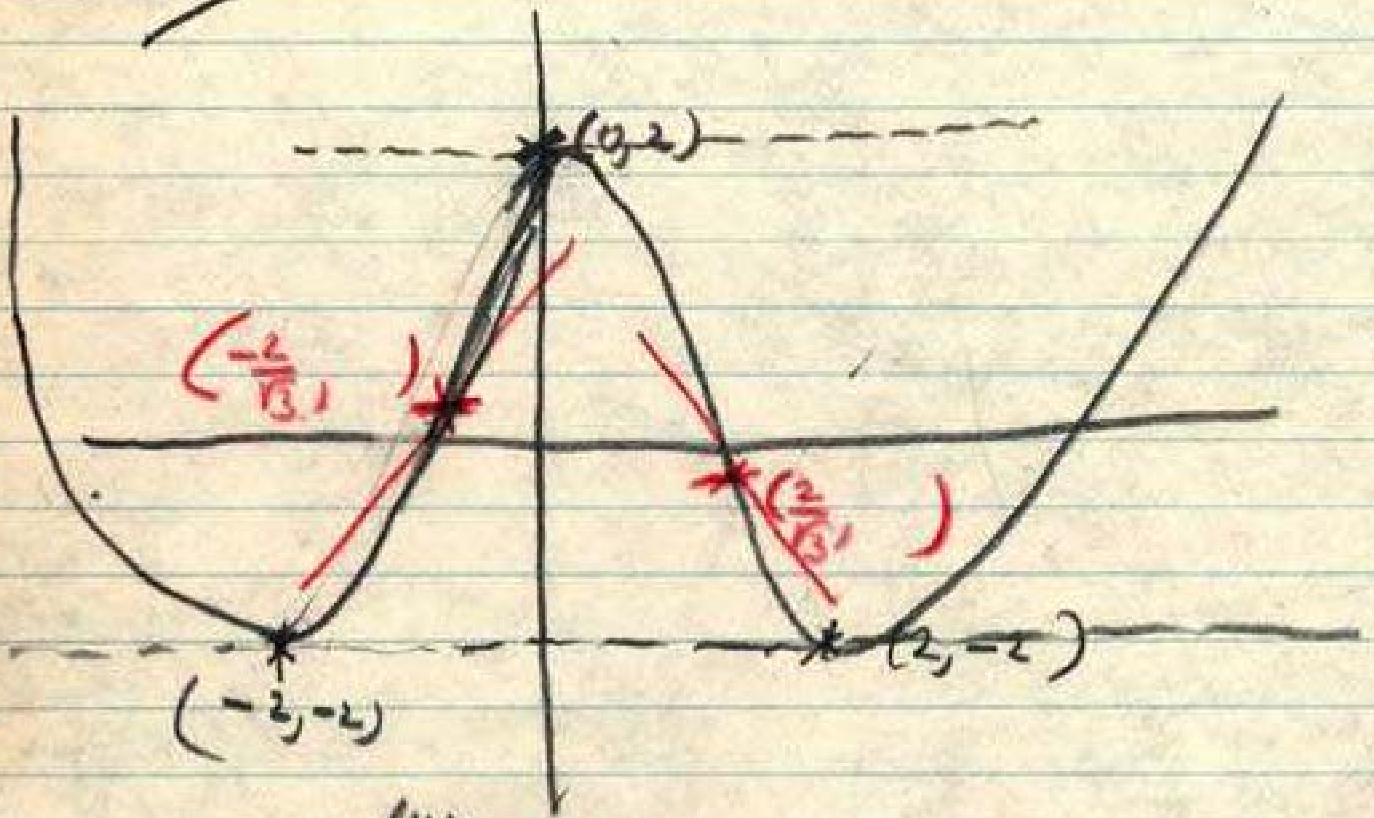
Setting $\frac{dy}{dx}$ at 0, $x^3 - 4x = 0$

$$x(x^2 - 4) = 0$$

$x = 0, \pm 2$ $\qquad$ $x^2 = 4$

$\qquad\qquad$ $x = \pm 2$

$\longrightarrow$



$f''(x) = 3x^2 - 4$

Setting $f''(x)$ at 0

$$3x^2 - 4 = 0$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \pm\frac{2}{\sqrt{3}}$$

	dy/dx
$x < -2$	$-$
$0 > x > -2$	$+$
$2 > x > 0$	$-$
$x > 2$	$+$

	$f''(x)$
$x < -\frac{2}{\sqrt{3}}$	$+$
$\frac{2}{\sqrt{3}} > x > -\frac{2}{\sqrt{3}}$	$-$
$x > \frac{2}{\sqrt{3}}$	$+$

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no. 18, 19

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276 nos. 11, 12, 14
21d, f, 22a, e.
23, 24a, c

$v = \underline{\log_a u}$, $\underline{a^v = u}$

$$\left\{\begin{array}{l} \log_a (r \cdot s) = \log_a r + \log_a s \\ \log_a \left(\frac{r}{s}\right) = \log_a r - \log_a s \\ \log_a (r^m) = m \log_a r \\ \log_a \sqrt[m]{r} = \frac{1}{m} \log_a r \end{array}\right.$$

$m . \log K = \log K^m$

$\log_b b = 1$

$\log_b 1 = 0$

$\log_e a$

$f(t) = (1+t)^{1/t}$

t	$f(t)$
1	2
$\frac{1}{2}$	2.25
$1/10$	2.2594
$1/100$	2.704

2.71828 1829

$\cancel{\left(\frac{3}{2}\right)^2}$

$(1.1)^{10}$

.04139

0.41390

$(1.01)^{100}$

.00432

.43200

$$\lim_{t\to 0} (1+t)^{1/t} = e \qquad e = 2.71828 \ldots$$

$$\left\{\begin{aligned} y &= \log_a u \\ y+\Delta y &= \log_a(u+\Delta u) \\ \hline \Delta y &= \log_a(u+\Delta u) - \log_a u \end{aligned}\right.$$

$u = x^5 + \sqrt{x} - 7$

$a = 10, 12, 2/9, \ldots$

x, u, y

$x+\Delta x, u+\Delta u, y+\Delta y$

$$\Delta y = \log_a\left(\frac{u+\Delta u}{u}\right) = \log_a\left(1+\frac{\Delta u}{u}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{\Delta x}\cdot \log_a\left(1+\frac{\Delta u}{u}\right) = \frac{u}{\Delta u}\cdot\frac{\Delta u}{u}\cdot\frac{1}{\Delta x}\log_a\left(1+\frac{\Delta u}{u}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{u}\cdot\left(\frac{\Delta u}{\Delta x}\right)\log_a\left(1+\frac{\Delta u}{u}\right)^{\frac{u}{\Delta u}}$$

$$\left\{\begin{aligned} t &= \frac{\Delta u}{u} \\ \frac{1}{t} &= \frac{u}{\Delta u} \end{aligned}\right.$$

$$\frac{dy}{dx} = \frac{1}{u}\cdot\frac{du}{dx}\cdot\log_a e$$

$\Delta u \to 0, \ t \to 0$

$$\therefore \boxed{\begin{aligned} \frac{d}{dx}(\log_a u) &= \frac{1}{u}\cdot\frac{du}{dx}\cdot\log_a e \\ \frac{d}{dx}(\log_e u) &= \frac{1}{u}\cdot\frac{du}{dx} \\ \frac{d}{dx}(\ln u) &= \frac{1}{u}\cdot\frac{du}{dx} \\ \frac{d}{dx}(\ln x) &= \frac{1}{x} \end{aligned}}$$

$$\frac{d}{dx}(\ln(x^2+2)) = \frac{1}{x^2+2}\cdot 2x = \frac{2x}{x^2+2}$$

$$\log_e u = \ln u$$

$$\frac{d}{dx}(\log_5 x^3) = \frac{1}{x^3}\cdot 3x^2\cdot \log_5 e$$
$$= \frac{3}{x}\cdot\log_5 e$$

275) 3

$$y = \ln(x^2 + 2x)$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$$

Let $u = (x^2 + 2x)$

$$\frac{du}{dx} = \frac{1}{x^2+2x}\cdot(2x+2)$$

$$= \frac{2x+2}{x^2+2x}$$

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275) 5

$$y = x^2 \ln x$$

$$\frac{d}{dx}(x^2 \ln x) = x^2\cdot\frac{d}{dx}(\ln x) + \ln x\cdot\frac{d}{dx}(x^2)$$

$$= x^2\cdot\frac{1}{x} + \cancel{\frac{1}{x}}^{\ln x}\cdot 2x$$

$$= \frac{x^2}{x} + \frac{2x\cdot\ln x}{\cancel{x}} = \cancel{x+2}$$

$= x + 2x\cdot\ln x$

---

275) 7

$$f(x) = \ln^3 x = (\ln x)^3$$

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}(\ln x)^3 = 3(\ln x)^2\cdot\frac{d}{dx}\ln x$$

$$= 3(\ln x)^2\cdot\frac{1}{x} = \frac{3(\ln x^2)}{x}$$

---

275) 8

$$f(x) = \ln\left[\frac{(a-x)}{(a+x)}\right]$$

$$\frac{d}{dx}f(x) = \frac{d}{dx}\ln\left[\frac{(a-x)}{(a+x)}\right] = \frac{d}{dx}\left[\ln(a-x) - \ln(a+x)\right]$$

$$= \left[\frac{1}{(a-x)}\cdot -1\right] - \left[\frac{1}{(a+x)}\cdot 1\right] = \frac{-1}{(a-x)} - \frac{1}{(a+x)}$$

$$= \frac{-(a+x)-(a-x)}{(a-x)(a+x)} = \frac{-\cancel{2(a+x)}}{(a-x)(a+x)}\quad \cancel{= -2}$$

276) 11

$$s = t \ln \sqrt{t} = t \cdot \tfrac{1}{2} \ln t$$

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[illegible]

$$\frac{ds}{dt} = \left(t \cdot \frac{1}{2} \cdot \frac{1}{t}\right) + \frac{1}{2} \ln t \cdot 1 = \frac{t}{2t} + \frac{1}{2} \ln t$$

$$= \frac{1}{2} + \frac{1}{2} \ln t = \frac{1 + \ln t}{2}$$

$$= \frac{\ln e + \ln t}{2} = \frac{\ln(et)}{2} = \ln \sqrt{et}$$

---

276) 12

$$s = \ln\left(\frac{t^2}{\sqrt{3-2t}}\right) = \ln t^2 - \ln \sqrt{3-2t}$$

$$s = \ln t^2 - \frac{1}{2} \ln(3-2t) = 2 \ln t - \frac{1}{2} \ln(3-2t)$$

$$\frac{ds}{dt} = 2 \cdot \frac{1}{t} - \left[\frac{1}{2} \cdot \frac{1}{(3-2t)} \cdot (-2)\right] = \frac{2}{t} - \left(\frac{-1}{3-2t}\right)$$

$$= \frac{2(3-2t) + t}{t(3-2t)} = \frac{6-4t+t}{t(3-2t)} = \frac{6-3t}{t(3-2t)}$$

---

276) 14

$$y = \ln \sqrt{(2x-1)(2x^2-1)} = \frac{1}{2} \ln (2x-1)(2x^2-1)$$

$$y = \frac{1}{2}\left[\ln(2x-1) + \ln(2x^2-1)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\left(\frac{1}{2x-1} \cdot 2\right) + \left(\frac{1}{2x^2-1} \cdot 4x\right)\right]$$

$$= \frac{1}{2}\left[\frac{2}{(2x-1)} + \frac{4x}{(2x^2-1)}\right]$$

$$= \frac{1}{2x-1} + \frac{2x}{2x^2-1} = \frac{2x^2-1+4x^2-2x}{(2x-1)(2x^2-1)} = \frac{-2x+6x^2-1}{(2x-1)(2x^2-1)}$$

276) 21 d $\quad y = \frac{\ln x}{x^2}$, $x=2$

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$$\frac{dy}{dx} = \frac{\left(x^2 \cdot \frac{d}{dx} \ln x\right) - \ln x \cdot \frac{d}{dx} x^2}{(x^2)^2}$$

$$= \frac{\left(x^2 \cdot \frac{1}{x}\right) - (\ln x \cdot 2x)}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1 - 2\ln x}{x^3}$$

If $x=2$, $\frac{dy}{dx} = \frac{2 - 4(0.6931)}{16}$, $\quad$ & $y = \frac{\ln 2}{4} = \frac{0.6931}{4} = 0.173$

$$= \frac{2 - 2.7724}{16}$$

$$= -\frac{.7724}{16} = -.0483$$

---

276) 21 f $\quad y = \log_{10}\left(x\sqrt{20-7x}\right)$, $x = 2$

$$\frac{dy}{dx} = \frac{0.434}{x\sqrt{20-7x}} \cdot \frac{d}{dx}\left(x\sqrt{20-7x}\right)$$

$\frac{d}{dx} \log_{10} u = \frac{1}{u} \cdot \frac{du}{dx} \cdot \log_{10} e$

$$= \frac{0.434}{x\sqrt{(20-7x)}} \cdot \frac{d}{dx}\sqrt{20x^2 - 7x^3}$$

$$= \frac{0.434}{x\sqrt{20-7x}} \cdot \left[\frac{1\,(40x - 21x^2)}{2\,(20x^2 - 7x^3)^{1/2}}\right]$$

$$= \frac{0.434\,(40x - 21x^2)}{2x\sqrt{20-7x}\,\sqrt{(20x^2 - 7x^3)}}$$

If $x = 2$, $y = \log\left(2\sqrt{6}\right) = \log\sqrt{24} = \frac{1}{2}\log 24 = \frac{1.3802}{2}$

$$= 0.6901$$

$$\frac{dy}{dx} = \frac{0.434 \cdot (80-84)}{4\sqrt{6}\,\sqrt{(80-56)}} = \frac{-1.736}{2\sqrt{24}\cdot\sqrt{24}} = -\cancel{.0330}$$

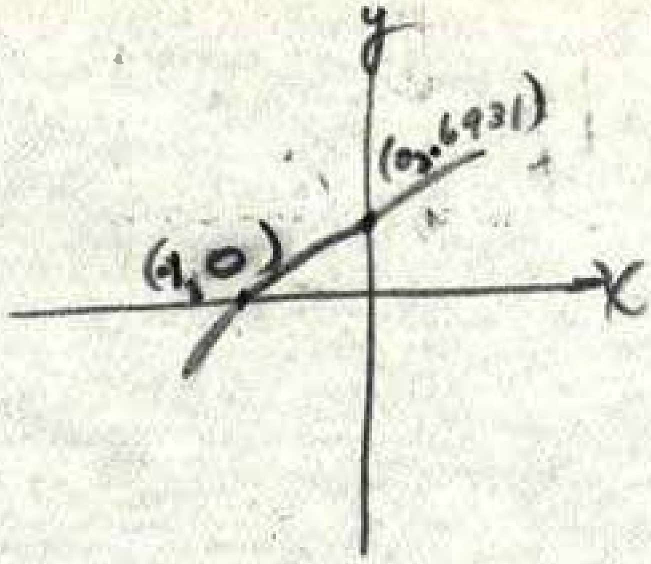
276) 22 a

$y = \ln_e(x+2) \rightarrow x+2 = e^y$ (=1 when $y=0$)

If $x=0$, $y = \ln 2 = 0.6931$

If $y=0$, $x+2=1$, $x=-1$
(ln 1 = 0)

$\frac{dy}{dx} = \frac{1}{x+2} \cdot 1 = \frac{1}{x+2}$



At intersection with y-axis,

$\frac{dy}{dx} = \frac{1}{0+2} = \frac{1}{2}$

At intersection ē x-axis

$\frac{dy}{dx} = \frac{1}{-1+2} = 1$

---

276) 22 e

$y = \ln(4-x)$

If $x=0$, $y = \ln 4 = 1.3863$

If $y=0$, $4-x=1$, $x=3$

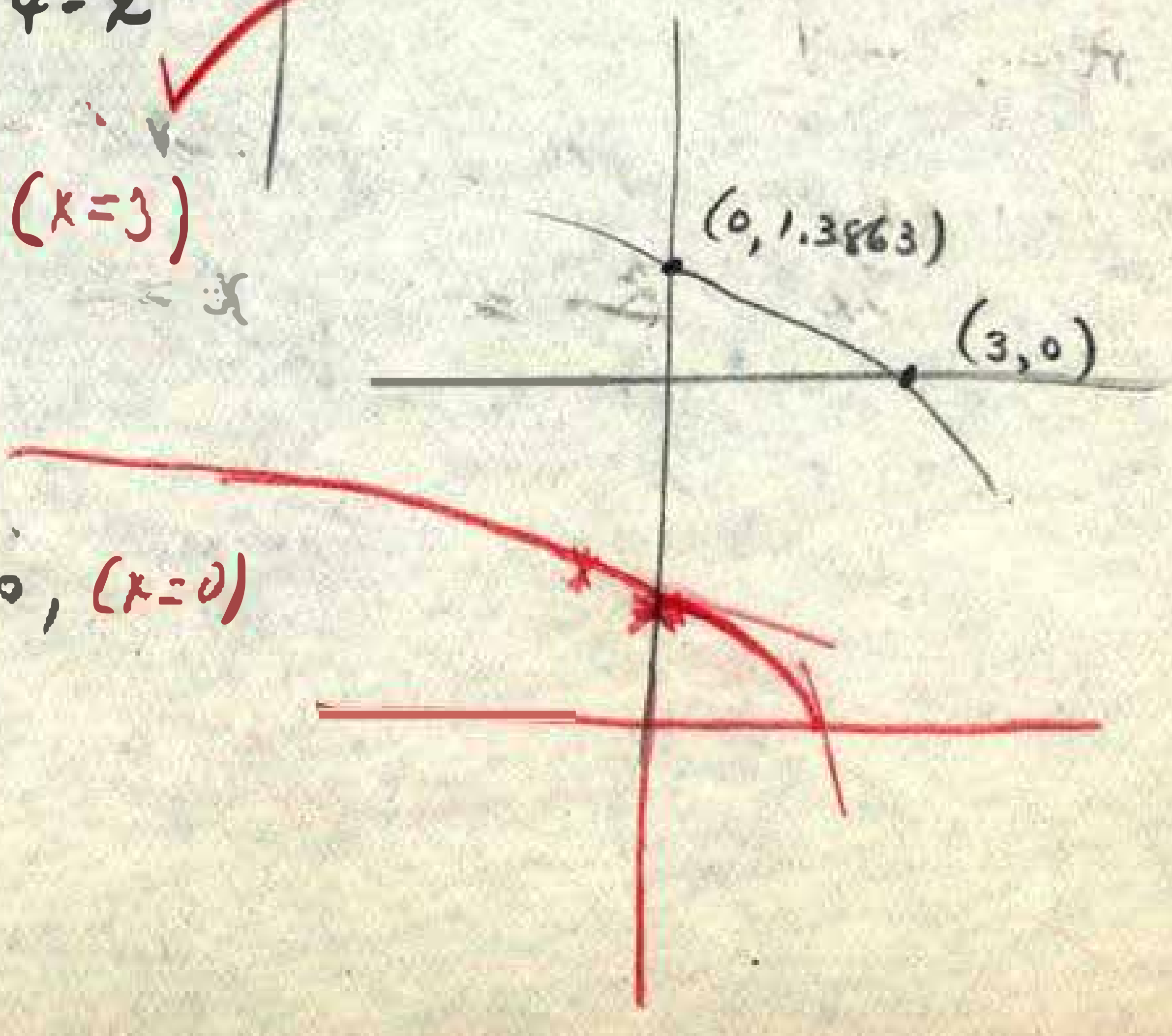
$\frac{dy}{dx} = \frac{1}{4-x} \cdot -1 = \frac{-1}{4-x}$

At intersection ē ~~y~~ X-axis (x=3)

$\frac{dy}{dx} = \frac{-1}{4-3} = -1$

At intersection ē ~~X~~ Y-axis, (x=0)

$\frac{dy}{dx} = \frac{1}{4-0} = -\frac{1}{4}$



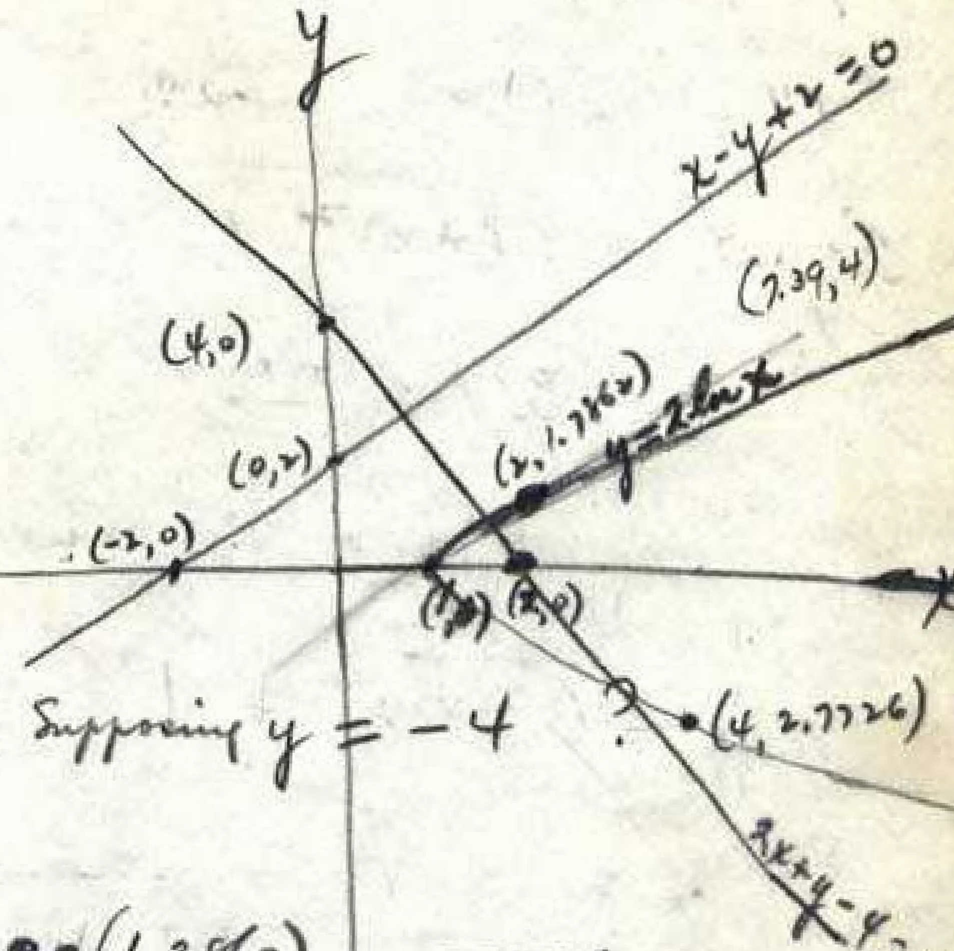
276) 23

$y = 2 \ln x$ | when $x = 1$, $y = 0$

$\frac{dy}{dx} = \frac{2}{x}$ ✓ | " $x = 2$, $y = 2 \times 0.6931 = 1.7862$

If $y = 4$, $\ln x = 2$, + $x = 7.39$

$2x+y-4=0$	$x-y+2=0$
$2x+y=4$	$x-y=-2$
If $x=0$, $y=4$	when $x=0$, $y=2$
If $y=0$, $x=2$	" $y=0$, $x=-2$



Question: Supposing $y = -4$?

$2x+y-4=0$

$y = -2x+4$

Slope $= -2$ ✓

Then at point where tangent to curve is perpendicular to line $\frac{2}{x} = \frac{1}{2}$, $x = 4$, + $y = 2 \cdot (1.3863) = 2.7726$ ✓

$x-y+2=0$

$-y = -x-2$

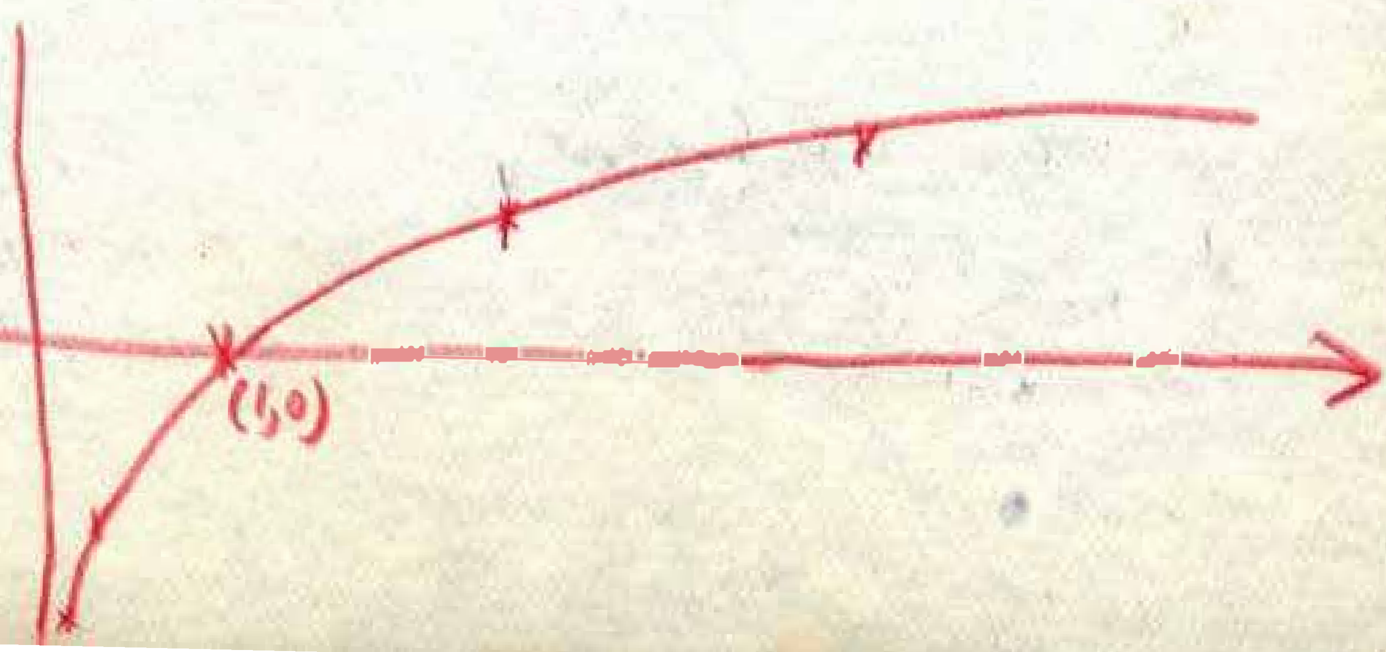
$y = x+2$

Slope $= 1$

Then at point where tangent to curve is parallel to line, $\frac{2}{x} = 1$, $x = 2$, + $y = 2 \cdot (0.6931) = 1.3862$

$x = 1/e = e^{-1}$, $\ln x = -1$

$x = 1/e^2$, $\ln x = -2$



276) 24a

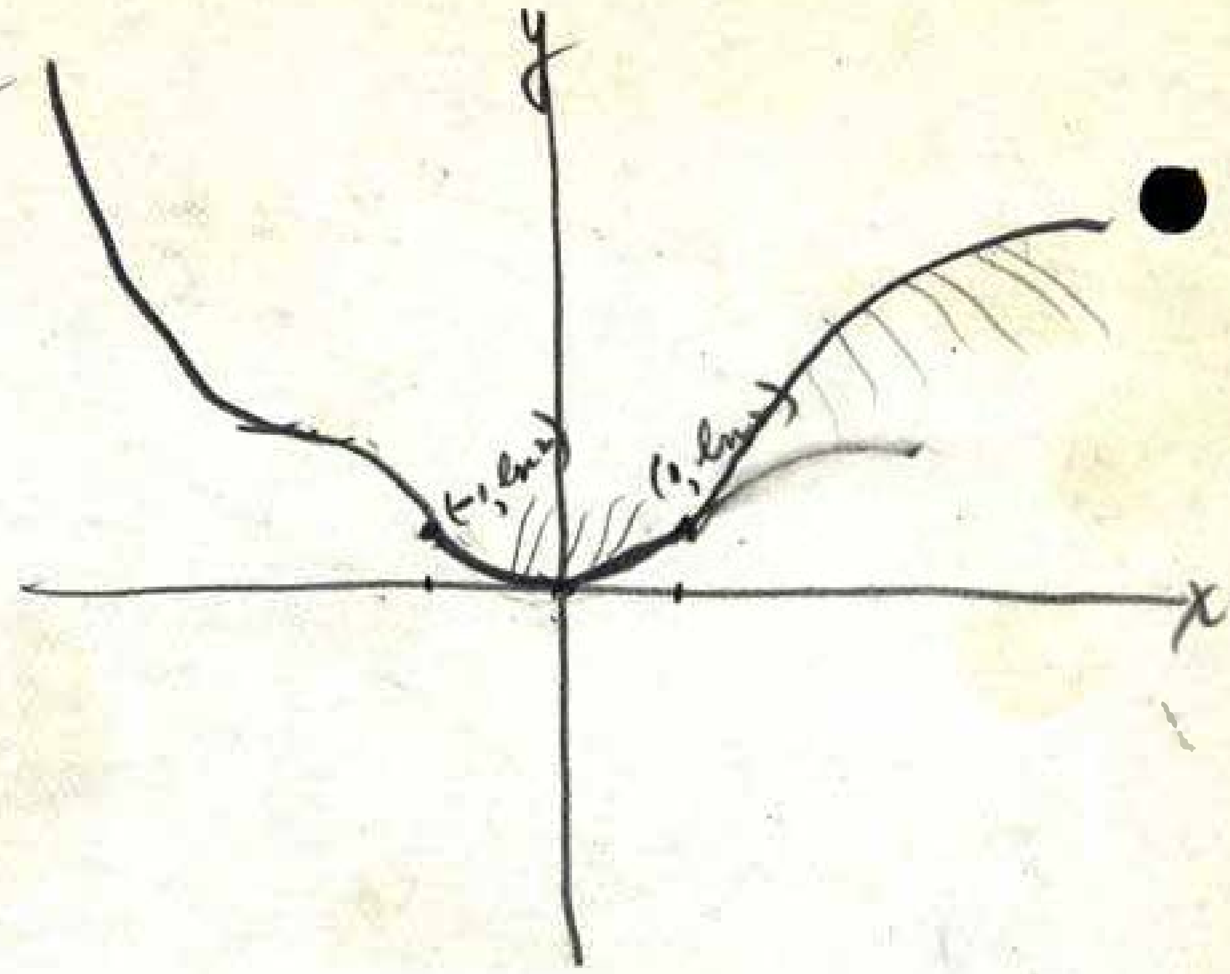
$y = \ln(1+x^2)$

$y' = \frac{1}{(1+x^2)} \cdot 2x = \frac{2x}{1+x^2}$ ✓

$y'' = \frac{[(1+x^2)\cdot 2] - [2x \cdot 2x]}{(1+x^2)^2}$

$= \frac{2 + 2x^2 - 4x^2}{(1+x^2)^2}$

$= \frac{2(1-x^2)}{(1+x^2)^2}$ ✓



Setting $y' = 0$, $\frac{2x}{1+x^2} = 0$, $\underline{x = 0}$, + $y = \ln 1 = \underline{0}$

$\therefore$ minimum pt. $= (0,0)$

	dy/dx
$x < 0$	$-$
$x > 0$	$+$

$\therefore$ y has ~~max~~ min. value when $x = 0$

Setting $y'' = 0$, $\frac{2(1-x^2)}{(1+x^2)^2} = 0$

$2(1-x^2) = 0$

$2 - 2x^2 = 0$

$2x^2 = 2$

$x^2 = 1$

$\underline{x = \pm 1}$, + $y = \ln(1+1) = \ln 2$

$= 0.6931$

when $x = \pm 5$, $y = \ln(26) =$

	d^2y/dx^2
$x < -1$	$-$
$1 > x > -1$	$+$
$x > 1$	$-$

Pt. inflect. at $x = -1$ and at $x = 1$

276) 24c

$y = x \ln x$

$y' = \frac{dy}{dx} = x \frac{d}{dx} \ln x + \ln x \frac{dx}{dx}$

$= x \cdot \frac{1}{x} + \ln x \cdot 1$

$= 1 + \ln x$ ✓

$y'' = \frac{1}{x}$ ✓

Setting $y' = 0$, $1 + \ln x = 0$

$\ln_e x = -1$ ✓

$x = \cancel{-2.718 = -e}$ ✓

~~$(-e, e)$~~

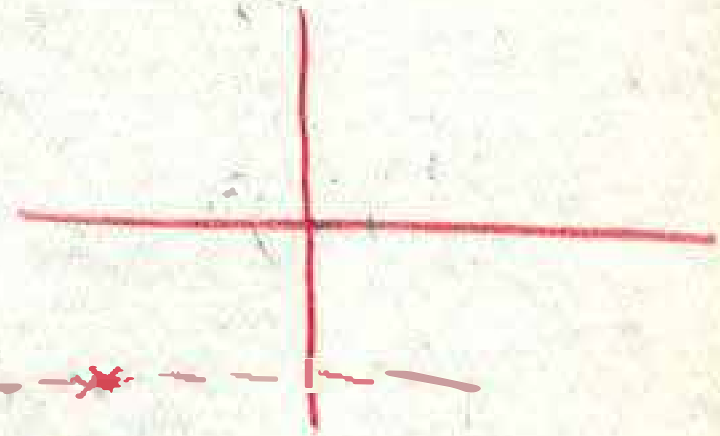
then $y = \cancel{-e \cdot (1) = e}$ } ~~[illegible]~~

Setting $\frac{1}{x} = 0$, $\cancel{x = 0}$

$e^{-1} = 1/e$

$\ln_e(1/e) = -1$

$\ln_e 0 = -\infty$



~~dy/dx~~

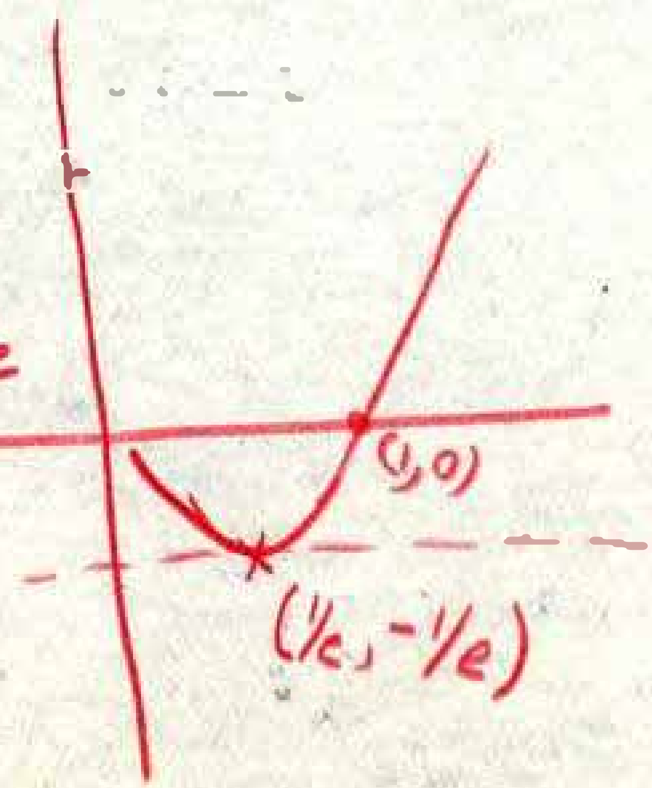
~~$x < -e$~~

~~$x > -e$~~

$\ln_e(-e)$

$\begin{cases} x = 1/e \\ y = \frac{1}{e}(-1) = -1/e \end{cases}$

	dy/dx
$x < 1/e$	$-$
$x > 1/e$	$+$



$e^{-10} = \frac{1}{e^{10}}$, $e^{-100} = \frac{1}{e^{100}}$

$e^{-\infty} = 0$

2.6 2) 10

$y = (x^2-1)(x^2-4)$

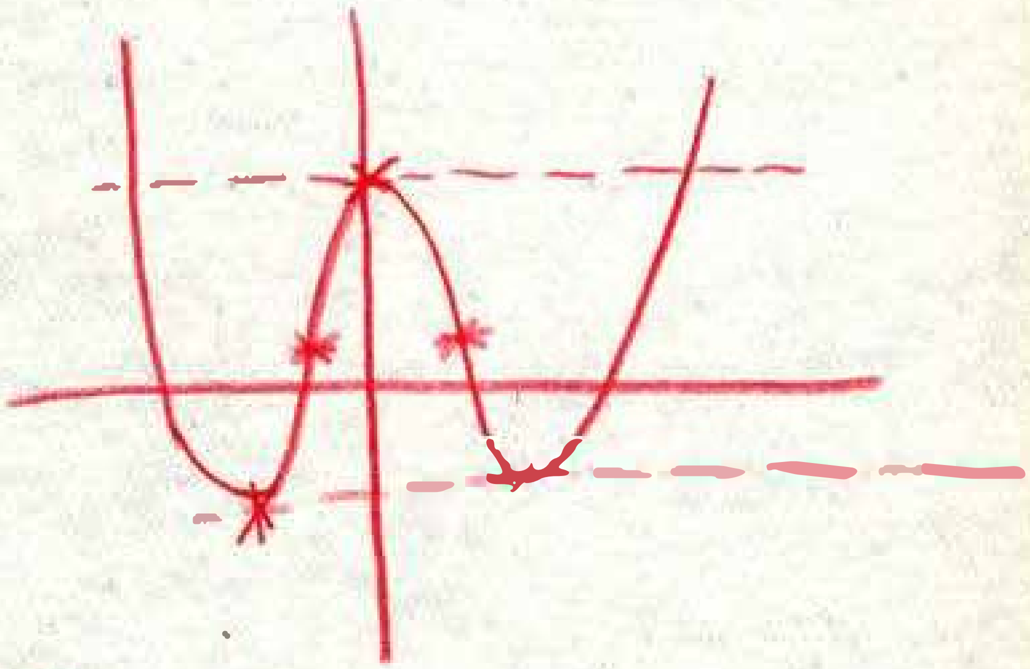
$y = x^4 - 5x^2 + 4$

$y' = 4x^3 - 10x$

$y'' = 12x^2 - 10$

$\left(\frac{5}{2}-1\right)\left(\frac{5}{2}-4\right)$

$\frac{3}{2}\left(-\frac{3}{2}\right)$



Setting $y' = 0$, $4x^3 - 10x = 0$

$\underline{x(4x^2-10)} = 0$

$x = 0$

$4x^2 - 10 = 0$

$4x^2 = 10$

$x^2 = \frac{10}{4} = \frac{5}{2}$

$x = \pm \frac{1}{2}\sqrt{10}$

$x = \underline{0}, \underline{+\frac{1}{2}\sqrt{10}}, \underline{-\frac{1}{2}\sqrt{10}}$

	dy/dx
$x < -\frac{1}{2}\sqrt{10}$	$-$
$0 > x > -\frac{1}{2}\sqrt{10}$	$+$
$\frac{1}{2}\sqrt{10} > x > 0$	$-$
$x > \frac{1}{2}\sqrt{10}$	$+$

Setting $y'' = 0$, $12x^2 - 10 = 0$

$12x^2 = 10$

$x^2 = \frac{10}{12}$

$x = \pm\sqrt{\frac{5}{6}}$

$= \pm\frac{1}{6}\sqrt{30}$

	d^2y/dx^2
$x < -\sqrt{5/6}$	$+$
$\sqrt{5/6} > x > -\sqrt{5/6}$	$-$
$x > \sqrt{5/6}$	$+$

262) 18

$$y = \frac{x}{x^2+1}$$

$$y' = \frac{(x^2+1)\cdot 1 - x(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{1-x^2}{x^4+2x^2+1}$$

$$y'' = \frac{\left[(x^2+1)^2(-2x)\right] - \left[(1-x^2)\cancel{(4x^3+4x)}^{\,2(x^2+1)\cdot 2x}\right]}{(x^2+1)^4}$$

$$= \frac{-2x(x^2+1) - (1-x^2)4x}{(x^2+1)^3}$$

$$= \frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3}$$

$$= \frac{2x^3-6x}{(x^2+1)^3}$$

$$= \frac{(-2x^5-4x^3-2x) - \left[4x^3+4x-4x^5-4x^3\right]}{(x^2+1)^4}$$

$$= \frac{-2x^5-4x^3-2x-4x^3-4x+4x^5+4x^3}{(x^2+1)^4}$$

$$= \frac{2x^5-4x^3-6x}{(x^2+1)^4} = \frac{2(x^5-2x^3-3x)}{(x^2+1)^4}$$

Setting $y''=0$, $\cancel{2(x^5-2x^3-3x)=0}$

$2x^3-6x=0$

$2x(x^2-6)=0$

$\cancel{x(x^4-2x^2-3)=0}$, $\cancel{x=0}$

$\cancel{(x^2-3)(x^2+1)=0}$

$x=0, \sqrt{6}, -\sqrt{6}$

$\cancel{x^2-3=0}$

$\cancel{x^2=3, \quad x=\pm\sqrt{3}}$

If $x=0$, $y=0$

If $x=\cancel{\sqrt{3}}$, $\cancel{y}=\frac{\sqrt{3}}{3+1}=\frac{\sqrt{3}}{4}$

If $x=-\sqrt{3}$, $y=\frac{-\sqrt{3}}{4}$

$\therefore$ Points of inflection are on straight line passing thru $(\sqrt{3}, \cancel{\tfrac{\sqrt{3}}{4}})$ & $(-\sqrt{3}, -\tfrac{\sqrt{3}}{4})$ & thru origin

262)19

$x^3+y^3=1$

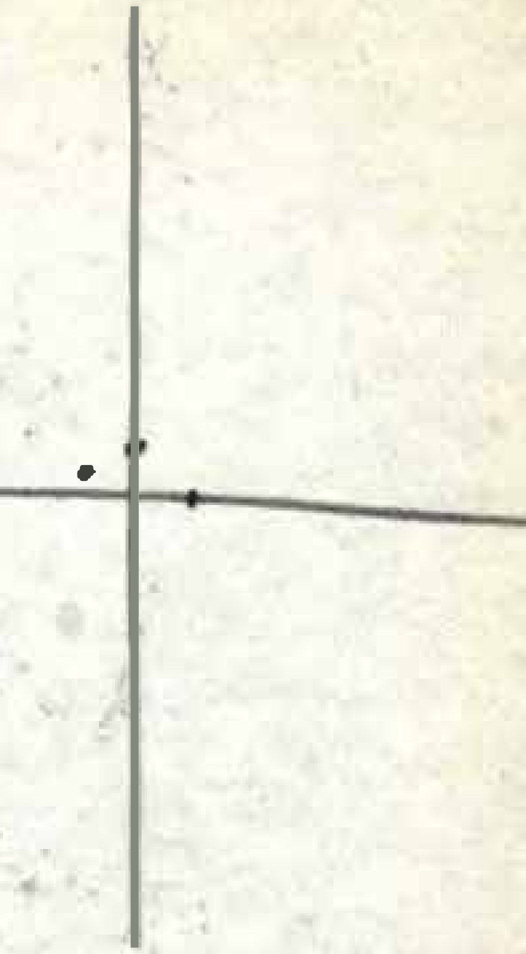
$y^3=1-x^3$

$y=\sqrt[3]{1-x^3}=(1-x^3)^{1/3}$

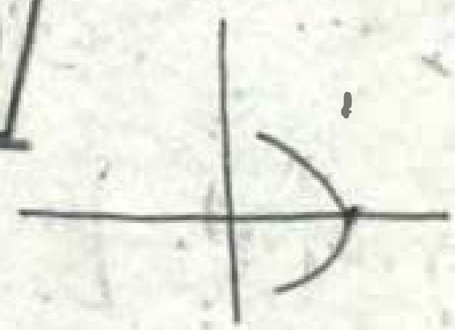
x	y
0	1
1	0
-1	$\sqrt[3]{2}$
-2	$\sqrt[3]{3}$

$3x^2+3y^2\frac{dy}{dx}=0$

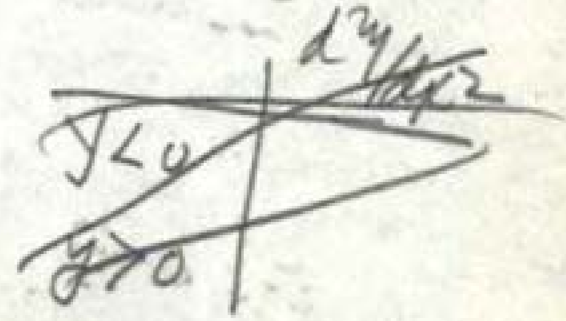
$\frac{dy}{dx}=\frac{-3x^2}{3y^2}=\frac{-x^2}{y^2}$ ✓



$y''=\frac{\left[y^2(-2x)\right]-\left[-x^2\left(2y\frac{dy}{dx}\right)\right]}{y^4}$



$=\frac{-2xy^2+2x^2y\frac{dy}{dx}}{y^4}$



Setting $y''=0$, $-2\left(xy^2 \pm x^2y\frac{dy}{dx}\right)=0$

$y''=0$, $-\frac{2x}{y}=0$, $x=0$

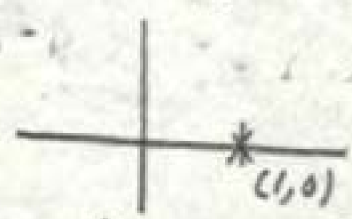
$y''=\infty$, $-\frac{2x}{y}=\infty$, $y=0$

$y''=-2xy\left(y-x\left(-\frac{x^2}{y^2}\right)\right)$

$=-2xy\left(y+\frac{x^3}{y^2}\right)=-2xy\cdot\frac{y^3+x^3}{y^2}=\frac{-2xy}{y^2}$

$=-2x/y$

$xy\left(y \pm x\frac{dy}{dx}\right)=0$



	y''
$y<0$	+
$y>0$	−

$xy=0$ $(x=0, y=0)$

$\therefore$ one point of inflection must be when curve crosses y-axis (or when $y=0$)

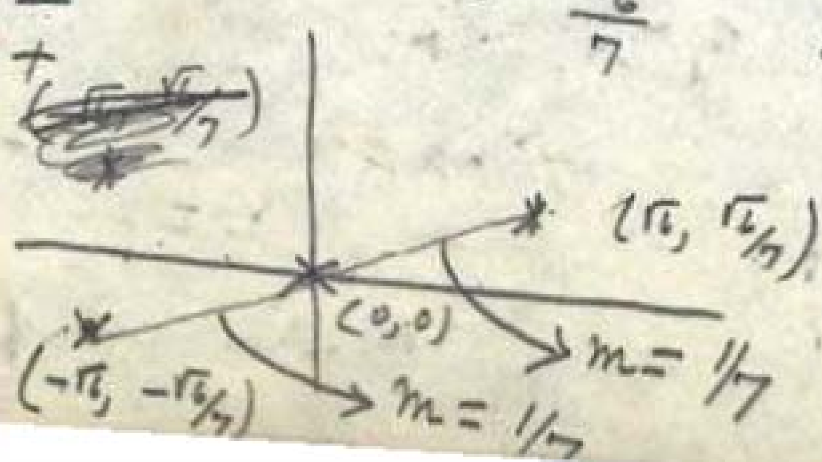
262/18 (cont.)

	d^2y/dx^2
$x<-\sqrt{6}$	−
$0>x>-\sqrt{6}$	+
$\sqrt{6}>x>0$	−
$x>\sqrt{6}$	+

$x=-\sqrt{6}$
$y=\frac{-\sqrt{6}}{7}$

$x=0$
$y=0$

$x=\sqrt{6}$
$y=\sqrt{6}/7$



$\ln_{10}(-10) = x$

$10^x = -10$

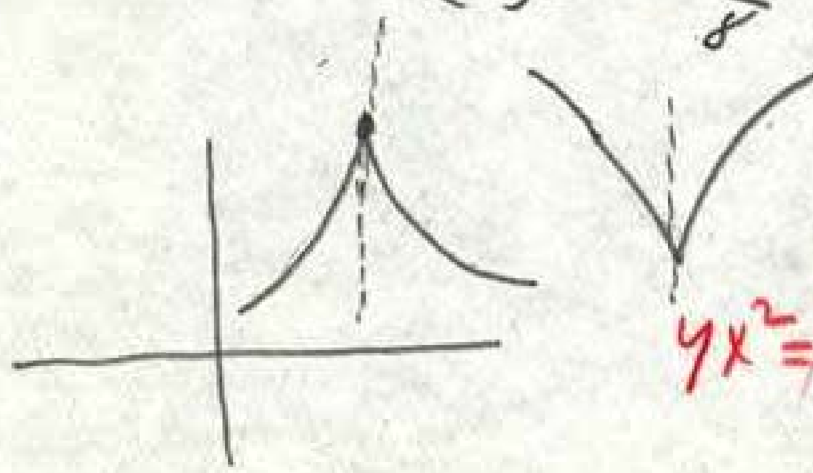
negat. numbers have no real logs to a posit. base

$\cancel{10^{-5} = \frac{1}{10^5}}$

$\log_{-2}(-8) = 3$

$\log_{-2}(-1/8) = -3$

$(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$



$yx^2 = y \times x$

$y = \frac{x}{x^2+1}$

$yx^2 + y = x$ 3rd degree curve

cubic curve

a cubic curve has 3 pts. of infl. which lie on a str. line

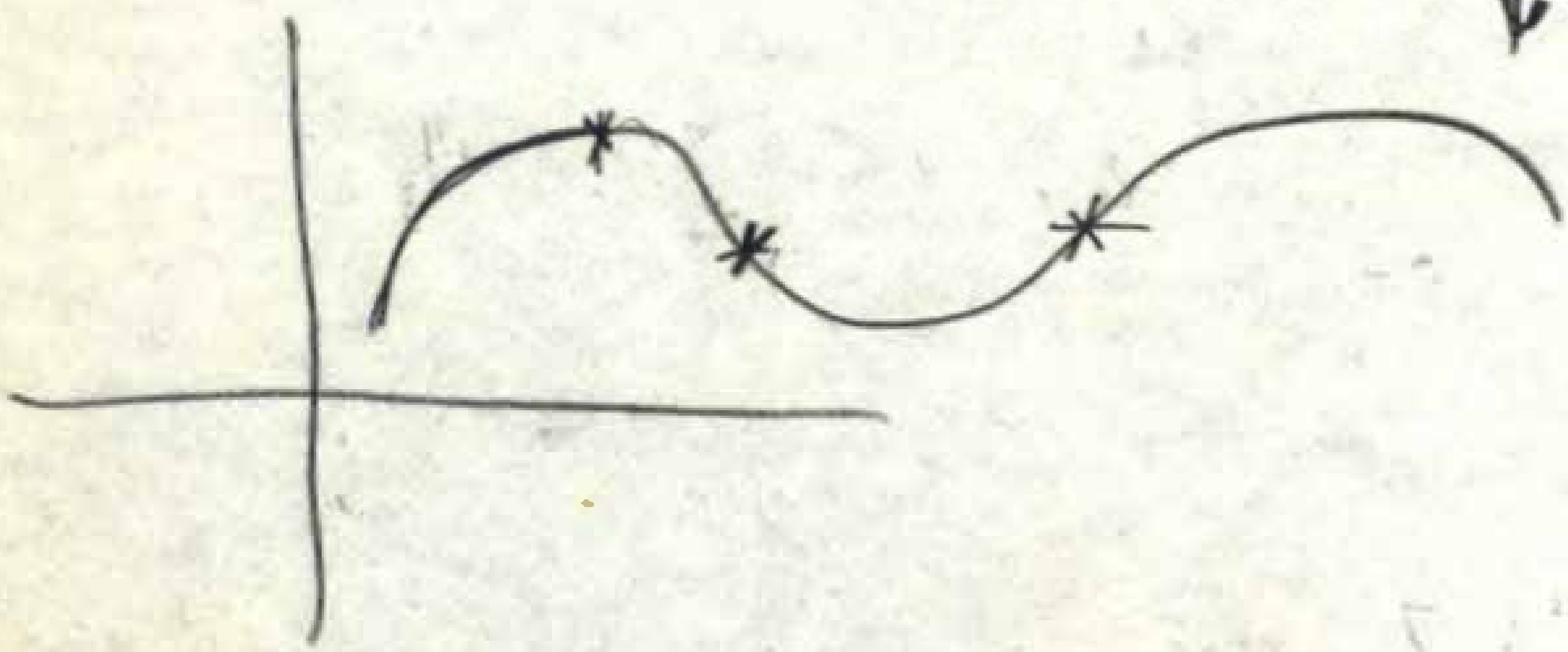
$x^2 \cdot \frac{de^{ax}}{dx} + e^{ax} \cdot 2x$

$x^2 \cdot e^{ax} + e^{ax} \cdot 2x$

$e^{ax}(x^2 + 2x)$

$x^2 \cdot e^{ax} \cdot a + e^{ax} \cdot 2x$

$e^{ax}(ax^2 + 2x)$



$\frac{d}{dx} \ln u = \frac{1}{u}$, $d(\ln u) = \frac{1}{u} du$, $\int \frac{1}{u} du = \ln u + C$

$\frac{d}{du}(e^u) = e^u$, $d(e^u) = e^u du$, $\int e^u du = e^u + C$

$\int e^{3x} dx = \frac{e^{3x}}{3} + C$

$\frac{d}{dx}(e^{3x}) = 3e^{3x}$

$\frac{d}{dx}\left(\frac{1}{3}e^{3x}\right) = e^{3x}$

$\frac{1}{2}\int \frac{2x\,dx}{x^2+1} = \frac{1}{2}\int \frac{du}{u}$

$u = x^2 + 1$

$du = 2x\,dx$

$= \frac{1}{2} \ln u + C$

$= \frac{1}{2} \ln(x^2+1) + C$

$y = a^x \quad (a>0)$

$\ln y = x \cdot (\ln a)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \ln a$

$\frac{dy}{dx} = y \cdot \ln a$

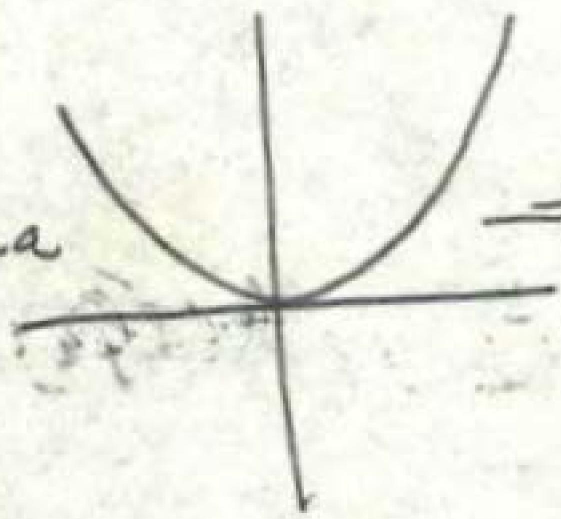
$\frac{dy}{dx} = a^x \cdot \ln a$

$y = 7^x$

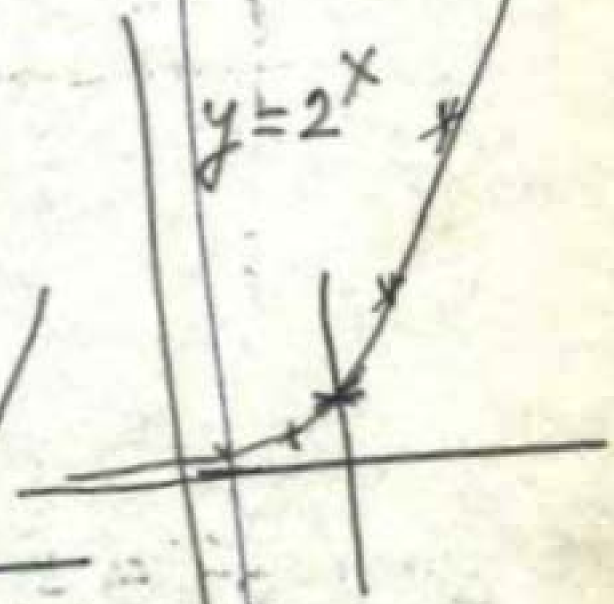
3^x = exponential function

$\frac{d}{dx}(a^x) = a^x \cdot \ln a$

$y = x^2$



$y = 2^x$



$\frac{d}{dx}(7^x) = 7^x \cdot \ln 7$

$\frac{d}{dx}(a^u) = a^u \cdot \frac{du}{dx} \cdot \ln a$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$

$y = e^{3x-2} \rightarrow \frac{dy}{dx} = e^{(3x-2)} \cdot 3 = 3e^{3x-2}$

$y = 10^{x^2} \rightarrow \frac{dy}{dx} = 10^{x^2} \cdot 2x \cdot \ln 10$

278) 5

279) 7, 8, 9, 11, 13, 15, 16, 17

280) 2, 3, 8, 9, 10, 11, 13, 16

281) 27, 30, 33, 37

$$\begin{cases} \dfrac{d}{dx} e^{u} = e^{u} \cdot \dfrac{du}{dx} \cdot \cancel{\ln e} \\[2ex] \dfrac{d}{dx} a^{u} = a^{u} \cdot \dfrac{du}{dx} \cdot \ln a \end{cases}$$

$$\begin{cases} \dfrac{d}{dx} \log_{e} u = \dfrac{1}{u} \cdot \dfrac{du}{dx} \\[2ex] \dfrac{d}{dx} \log_{a} u = \dfrac{1}{u} \cdot \dfrac{du}{dx} \cdot \log_{a} e \end{cases}$$

$$\frac{d}{dx}\left(e^{-x}\right) = -e^{-x}$$

278) 5 $y = x^2 e^{ax}$

$$\frac{dy}{dx} = (x^2 \cdot e^{ax} \cdot a) + e^{ax} \cdot 2x = e^{ax}(ax^2 + 2x)$$

---

279) 7 $f(x) = \frac{\ln x}{e^x}$

$$\frac{d f(x)}{dx} = \frac{\left(e^x \cdot \frac{1}{x}\right) - \ln x \cdot 1}{(e^x)^2} = \frac{\frac{e^x}{x} - \ln x}{(e^x)^2} = \boxed{\frac{e^x - x\ln x}{(e^x)^2 x}}$$

---

279) 8 $y = \ln\left(\frac{e^x}{1+e^x}\right) = \ln e^x - \ln(1+e^x)$

$$\frac{dy}{dx} = 1 - \left(\frac{1}{1+e^x}\right) \cdot e^x = 1 - \frac{e^x}{1+e^x} = \frac{1+e^x-e^x}{1+e^x}$$

$$= \frac{1}{1+e^x}$$

---

279) 9 $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$$\frac{dy}{dx} = \frac{\left[(e^x+e^{-x}) \cdot \frac{d}{dx}(e^x - e^{-x})\right] - \left[(e^x - e^{-x}) \cdot \frac{d}{dx}(e^x + e^{-x})\right]}{(e^x+e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{(e^x+e^{-x})(e^x + \cancel{\frac{1}{e^x}}\ e^{-x}) - (e^x - e^{-x})(e^x \cancel{+} - \cancel{\frac{1}{e^x}}\ e^{-x})}{(e^x+e^{-x})^2}$$

$$= \frac{(e^{2x}+2+e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x+e^{-x})^2} =$$

$$= \frac{(e^x+e^{-x})\,2\,(e^x - e^{-x})}{(e^x+e^{-x})^2} = \cancel{\frac{2(e^x - e^{-x})}{e^x + e^{-x}}}$$

$$\frac{4}{(e^x+e^{-x})^2}$$

279) 13

$$y = \frac{1}{2}a\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}a\left(\left[e^{\frac{x}{a}} \cdot \frac{1}{a}\right] + \left[e^{-\frac{x}{a}} \cdot -\frac{1}{a}\right]\right)$$

$$= \left[\frac{1}{2}a \cdot e^{\frac{x}{a}} \cdot \frac{1}{a}\right] + \left[\frac{1}{2}a \cdot e^{-\frac{x}{a}} \cdot -\frac{1}{a}\right]$$

$$= \frac{e^{\frac{x}{a}}}{2} - \frac{e^{-\frac{x}{a}}}{2} = \frac{e^{\frac{x}{a}} - e^{-\frac{x}{a}}}{2}$$

$$= \frac{1}{2}\left(e^{x/a} - e^{-x/a}\right)$$

$$y'' = \frac{2\left(\left[e^{\frac{x}{a}} \cdot \frac{1}{a}\right] - \left[e^{-\frac{x}{a}} \cdot -\frac{1}{a}\right]\right) - \cancel{\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right) \cdot 0}}{4}$$

$$= \frac{2\left(\frac{e^{\frac{x}{a}}}{a} + \frac{e^{-\frac{x}{a}}}{a}\right)}{4} = \frac{1}{2}\left(\frac{e^{\frac{x}{a}} + e^{-\frac{x}{a}}}{a}\right)$$

Multiply by $\frac{a}{a}$

$$\frac{1}{2}a\,\frac{\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)}{a^2} = \frac{y}{a^2}$$

$$y \cdot \frac{1}{a^2} = \frac{e^{x/a} + e^{-x/a}}{2a}$$

279) 15

$y = (e^{-x^2})$

$y' = \frac{dy}{dx} = (e^{-x^2})(-2x) = -2xy\,?$

$y'' = [e^{-x^2} \cdot (-2)] + [-2x(-2xe^{-x^2})] = -2e^{-x^2} + 4x^2e^{-x^2} = e^{-x^2}(4x^2-2)$

$= 4x^2y - 2y$

$= y(4x^2-2)$

Setting $y' = 0$, $\cancel{e^{-x^2}}(-2x) = 0$

$x = 0$, $y = e^0 = 1$

Setting $y'' = 0$, $y(4x^2-2) = 0$

~~Either y = 0~~

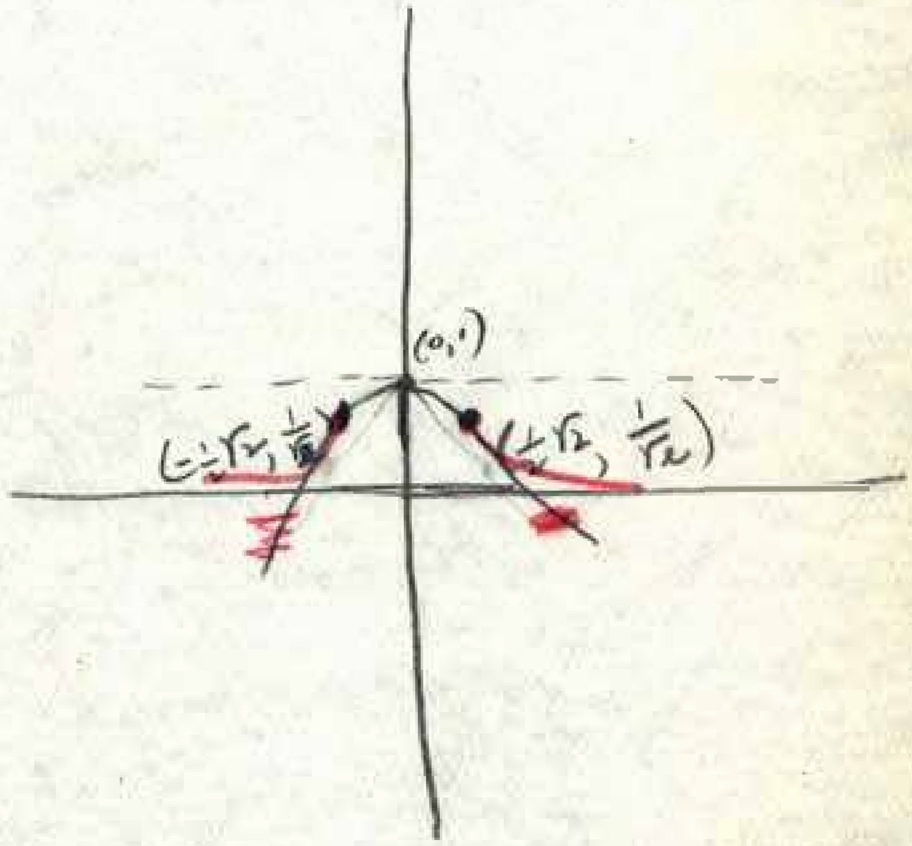
or $4x^2 - 2 = 0$

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{2}\sqrt{2}$



$y = e^{-x^2} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} = \frac{1}{e}\sqrt{e}$

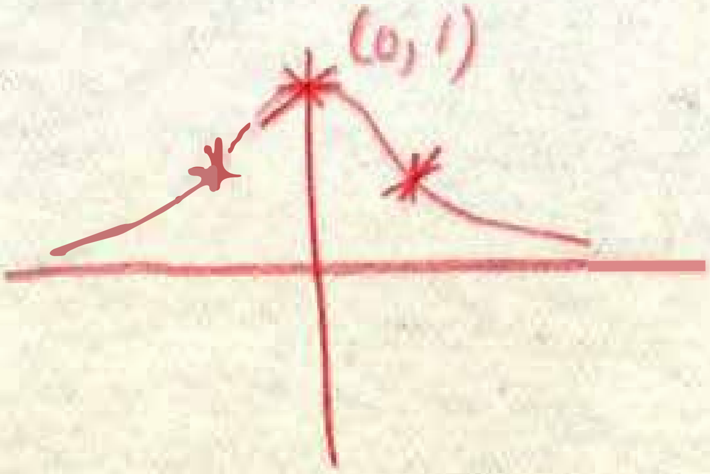
	y'
$x < 0$	+
$x > 0$	−

$\therefore$ maximum point $= (0,1)$

$(2.7)^{-10} = \frac{1}{(2.7)^{10}}$

$e^{-100} = \frac{1}{e^{100}}$

	y''
$x < -\frac{1}{2}\sqrt{2}$	+
$\frac{1}{2}\sqrt{2} > x > -\frac{1}{2}\sqrt{2}$	−
~~$\frac{1}{2}\sqrt{2} > x > 0$~~	− (?)
$x > \frac{1}{2}\sqrt{2}$	+



$$\int f(x)\,dx = g(x) + C$$

$$\frac{d}{dx}\left(-e^{-x}\right) = +e^{-x}$$

$$\frac{x+1}{x^2} = \frac{x}{x^2} + \frac{1}{x^2} = \frac{1}{x} + x^{-2}$$

$$\frac{x^2}{x+1}$$

280) 2

$$\int e^{-x}dx = -e^{-x} + C$$

280) 3

$$\int e^{2s}ds = \frac{e^{2s}}{2} + C$$

Let $u = 2s$
$du = 2$

$$\frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$

280) 8

$$\int \frac{x^2 dx}{x+1} = \int x \frac{dx}{\cancel{dx}} - dx + \frac{dx}{x+1}$$

$$= \frac{x^2}{2} - x + \ln(x+1) + C$$

$$\int \left(x - 1 + \frac{1}{x+1}\right) dx$$

$$\begin{array}{r} x-1 \\ x+1 \overline{)\, x^2 \qquad} \\ x^2 + x \\ \hline -x \\ -x-1 \\ \hline +1 \end{array}$$

280) 9

$$\int \frac{(x-1)dx}{x^2-2x-5} = \frac{1}{2}\int \frac{du}{u}$$

Let $u = x^2 - 2x - 5$
$\frac{du}{dx} = 2x - 2 = 2(x-1)$
$du = (2x-2)dx$

$$= \frac{1}{2}\ln u + C$$

$\frac{1}{2}\ln a$
$= \ln\sqrt{a}$

$$= \frac{\ln(x^2-2x-5)}{2} + C$$

(Book gives $\ln\sqrt{x^2-2x-5} + C$)

280) 10

$$\int \frac{5x^2 dx}{10x^3+6} = \frac{1}{6}\int \frac{du}{u}$$

Let $u = 10x^3 + 6$
$du = 30x^2 dx = 6(5x^2)dx$

$$= \frac{1}{6}\ln u + C$$

$\cancel{\ln \sqrt[6]{20}}$

$$= \frac{1}{6}\ln(10x^3+6) + C$$

$$= \ln\sqrt[6]{10x^3+6} + C$$

280) 11

$$\int \frac{(y^2-2)^3}{y^5}\,dy = \int \frac{y^6-6y^4+12y^2-8}{y^5}\,dy$$

$$= \int \left(\frac{y^6-6y^4+12y^2-8}{y^5} \right) dy$$

$$= \int \left(y^{-5}\left[y^6-6y^4+12y^2-8 \right] \right) dy$$

$$= \int \left(y - \cancel{6y^{-1}} + 12y^{-3} - 8y^{-5} \right) dy \quad \checkmark$$

$-6/y$

$\int y^n\,dy = \frac{y^{n+1}}{n+1}$ $(n \neq -1)$

$$= \frac{y^2}{2} - \cancel{6y^0} + 12\frac{y^{-2}}{-2} - 8\frac{y^{-4}}{-4} + C$$

$-6\ln y$

$\int \frac{1}{y}\,dy = \ln y$

$$= \frac{y^2}{2} - 6\ln y - 6y^{-2} + 2y^{-4} + C \quad \checkmark$$

Book gives $\frac{y^2}{2} - \ln y^6 - 6y^{-2} + 2y^{-4} + C$

$6 \ln a = \ln a^6$

$\frac{d}{dy}(e^{2y}) = 2e^{2y}$

$\frac{d}{dy}(e^{-2y}) = -2e^{-2y}$ $\cancel{2^{2y}}$

$\frac{d}{dy}\left(\frac{1}{2}e^{2y}\right) = e^{2y}$

$d/dy(\quad) = e^{-2y}$

280) 13

$$\int \frac{(\ln x)^3}{x}dx = \int \cancel{(\ln x)^3 \cdot \frac{3(\ln x)^2}{x} \cdot \frac{1}{x}}$$

Let $u = (\ln x)$

$du = \cancel{2(\ln x)^2 \cdot \frac{1}{x} = \frac{9(\ln x)^2}{x}}$ $\frac{1}{x}dx$

$$\cancel{= \int \frac{3(\ln x)^5}{x^2}}$$

$$\cancel{= 3\int \frac{(\ln x)^5}{x^2}}$$

$$\int u^3 du = \frac{u^4}{4} + C$$

$$= \frac{1}{4}(\ln x)^4 + C$$

$17 = (1.7)(10)$

$\ln 17 = \ln(1.7) + \ln 10$

$= 0.5306$

2.302

2.8332

---

280) 16 $\int (e^y + e^{-y})^2 dy$

$$= \int (e^{2y} + 2 + e^{-2y})dy$$

$$= \frac{1}{2}e^{2y} + 2y - \frac{1}{2}e^{-2y} + C \quad \cancel{= \frac{(e^y + e^{-y})^2}{\ln(e^y + e^{-y})} + C}$$

$$\int e^u du = e^u + C$$

$$\int e^{2y} dy = \frac{1}{2}\int e^u du$$

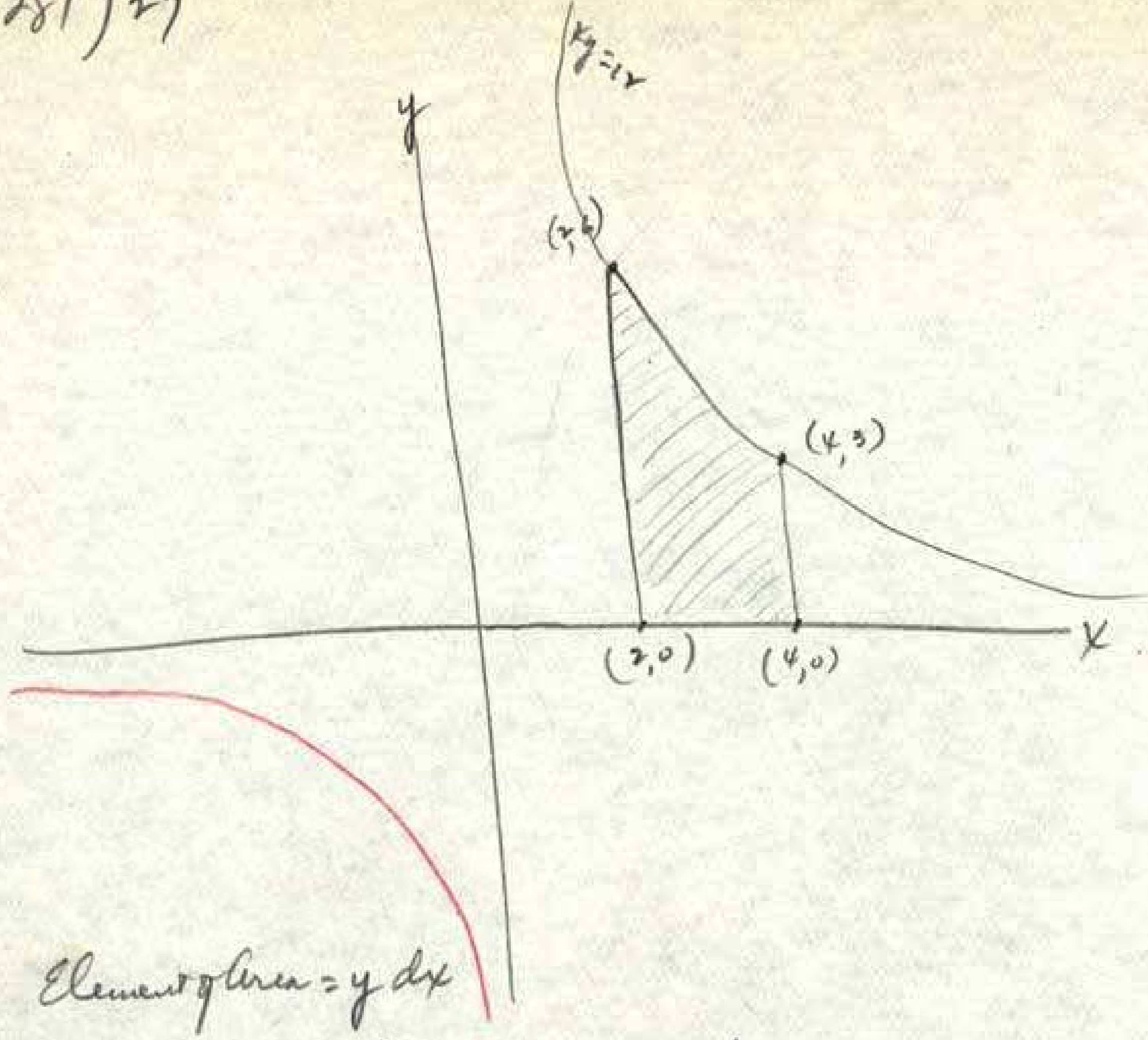
$u = 2y$

$du = 2dy$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{2y} + C$

281) 27



Element of Area = $y\,dx$

$$\text{Area} = \int_2^4 y\,dx = \int_2^4 \frac{12}{x}\,dx$$

$$= 12 \ln x \Big]_2^4$$

$\ln 4 - \ln 2 = \ln 2$

$$= 12(\ln 4 - \ln 2) = 12 \ln 2$$

$$= 12(1.3863 - 0.6931)$$

$$= 8.3184$$

281) 30

$y(1+x^2) = x$

$y = \frac{x}{1+x^2}$

when $x=0$, $y=0$

$x=2$, $y=\frac{2}{5}$

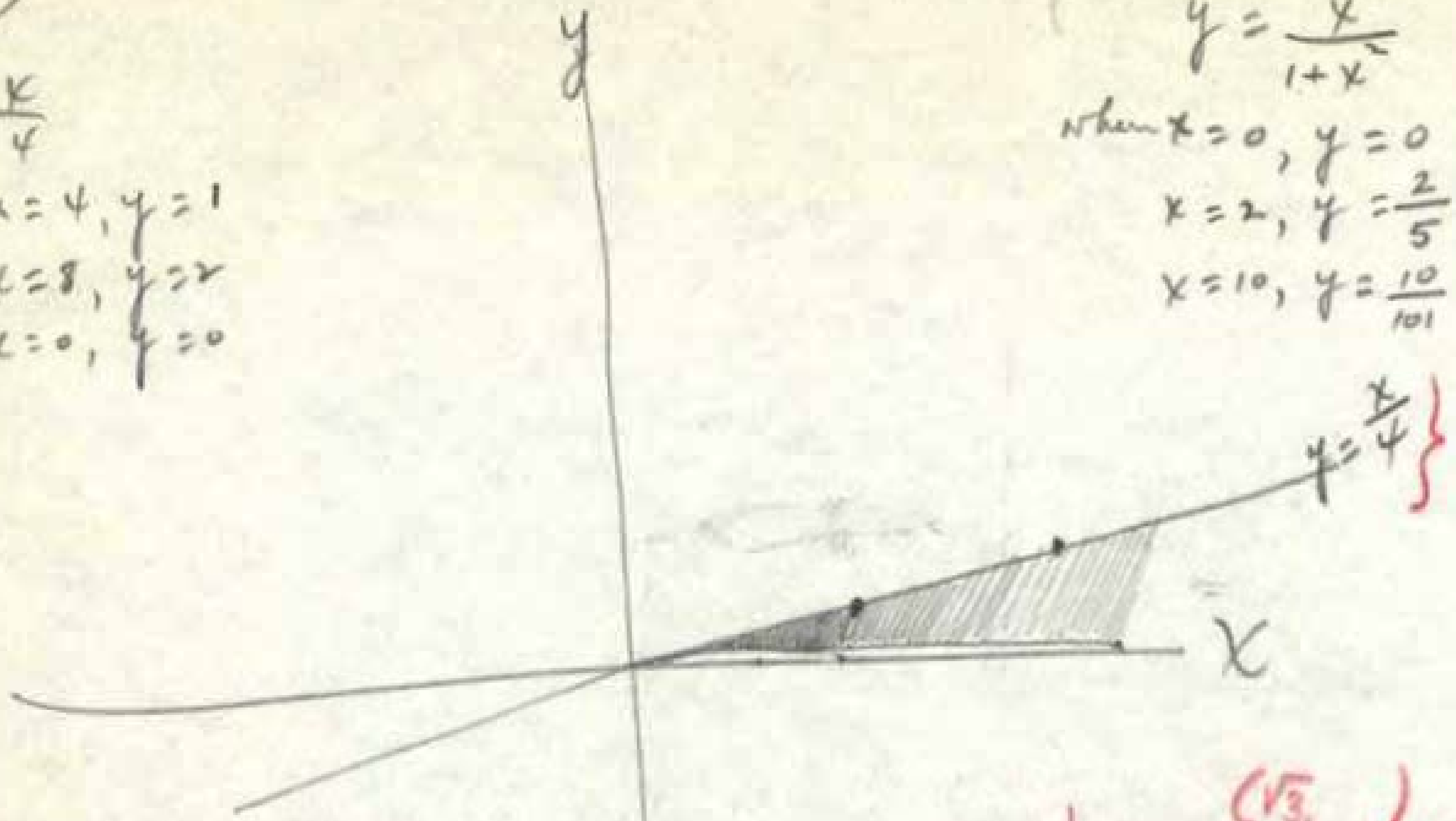
$x=10$, $y=\frac{10}{101}$

$y = \frac{x}{4}$

when $x=4$, $y=1$

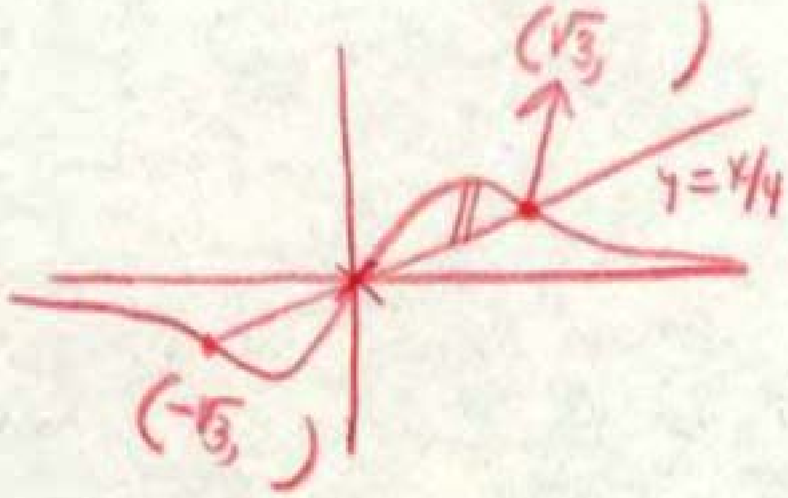
$x=8$, $y=2$

$x=0$, $y=0$



$\frac{x}{4}(1+x^2) - x = 0$

$x\left[\frac{1+x^2}{4} - 1\right] = 0$



El. of area $= (y'' - y')\,dx$

what is upper limit of x

Total area $= \int_0^{\sqrt{3}} \left(-\frac{x}{4} + \frac{x}{1+x^2}\right) dx = -\frac{x^2}{8} + \int \frac{x}{1+x^2}\,dx$

$\frac{1+x^2}{4} = 1$

$1+x^2 = 4$

$x^2 = 3$

$= -x^2/8 + \frac{1}{2}\int \frac{du}{u}$

$\cancel{= \int_0 \frac{x+x^3-4x}{4+4x^2}\,dx = \int_0 \frac{x^3-3x}{4+4x}\,dx}$

$u = 1+x^2$

$du = 2x\,dx$

$= -x^2/8 + \frac{1}{2}\ln u$

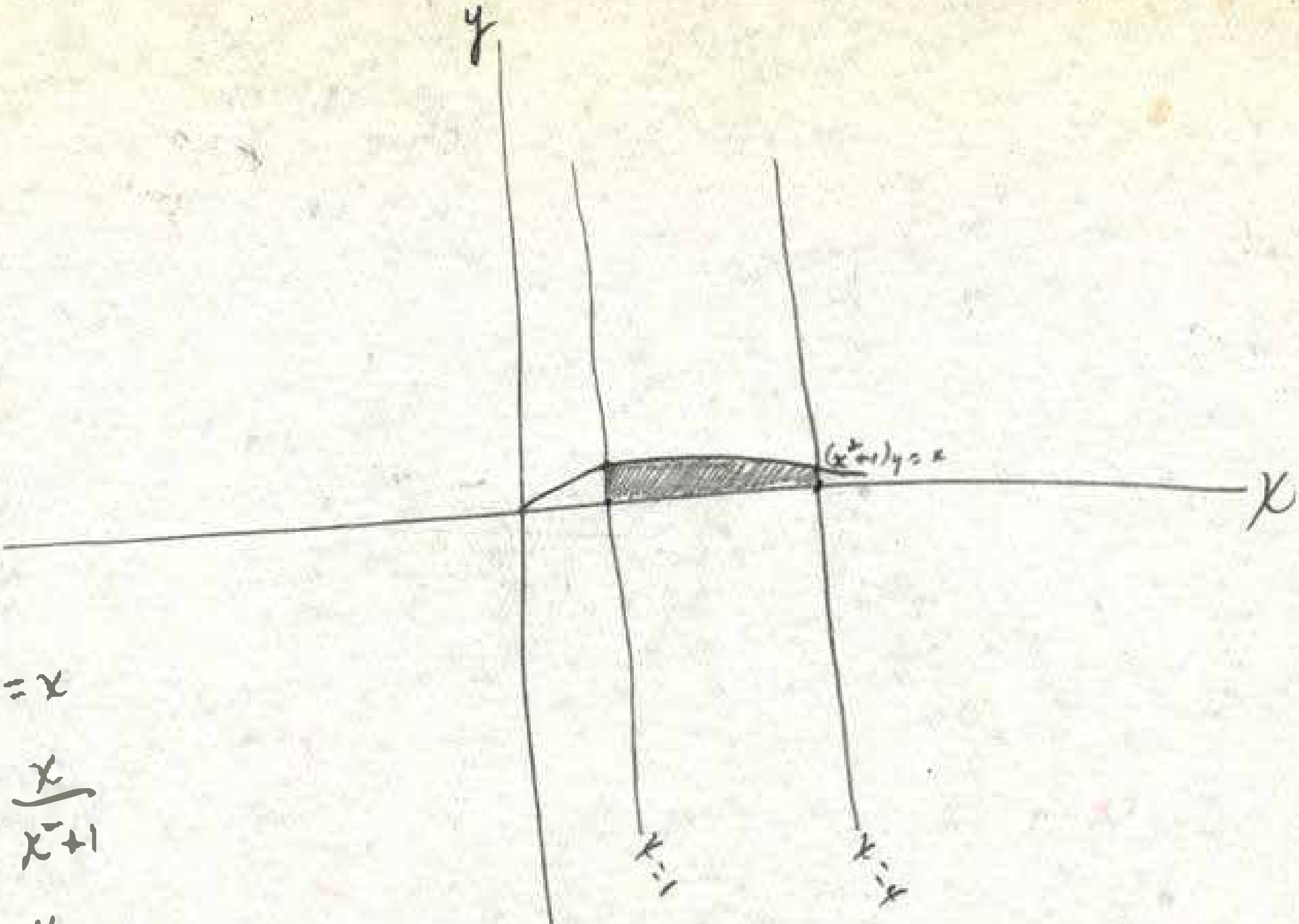
$= -x^2/8 + \frac{1}{2}\ln(1+x^2)\Big]_0^{\sqrt{3}}$

$= \left(-\frac{3}{8} + \frac{1}{2}\ln 4\right) - \cancel{\left(0 + \frac{1}{2}\ln 1\right)}$

$= -\frac{3}{8} + \frac{1}{2}\ln 4$

281) 33

y



$(x^2+1)y = x$

$y = \dfrac{x}{x^2+1}$

If $x=0$, $y=0$

$x=1$, $y=\frac{1}{2}$

$x=4$, $y=\frac{4}{17}$

Element of area $= y\,dx$

$$\text{Area} = \int_1^4 y\,dx = \int_1^4 \left(\frac{x}{x^2+1}\right) dx$$

Let $u = x^2+1$

$du = 2x$

$x\,dx = \frac{1}{2}du$

When $x=1$, $u=2$

$x=4$, $u=17$

$$= \frac{1}{2}\int_2^{17} \frac{du}{u}$$

$$= \frac{1}{2} \ln u \Big]_2^{17}$$

$$= \frac{1}{2}(\ln 17 - \ln 2)$$

$$= \frac{2.8332 - 0.6931}{2}$$

281) 37

$y^2(6-x) = x$

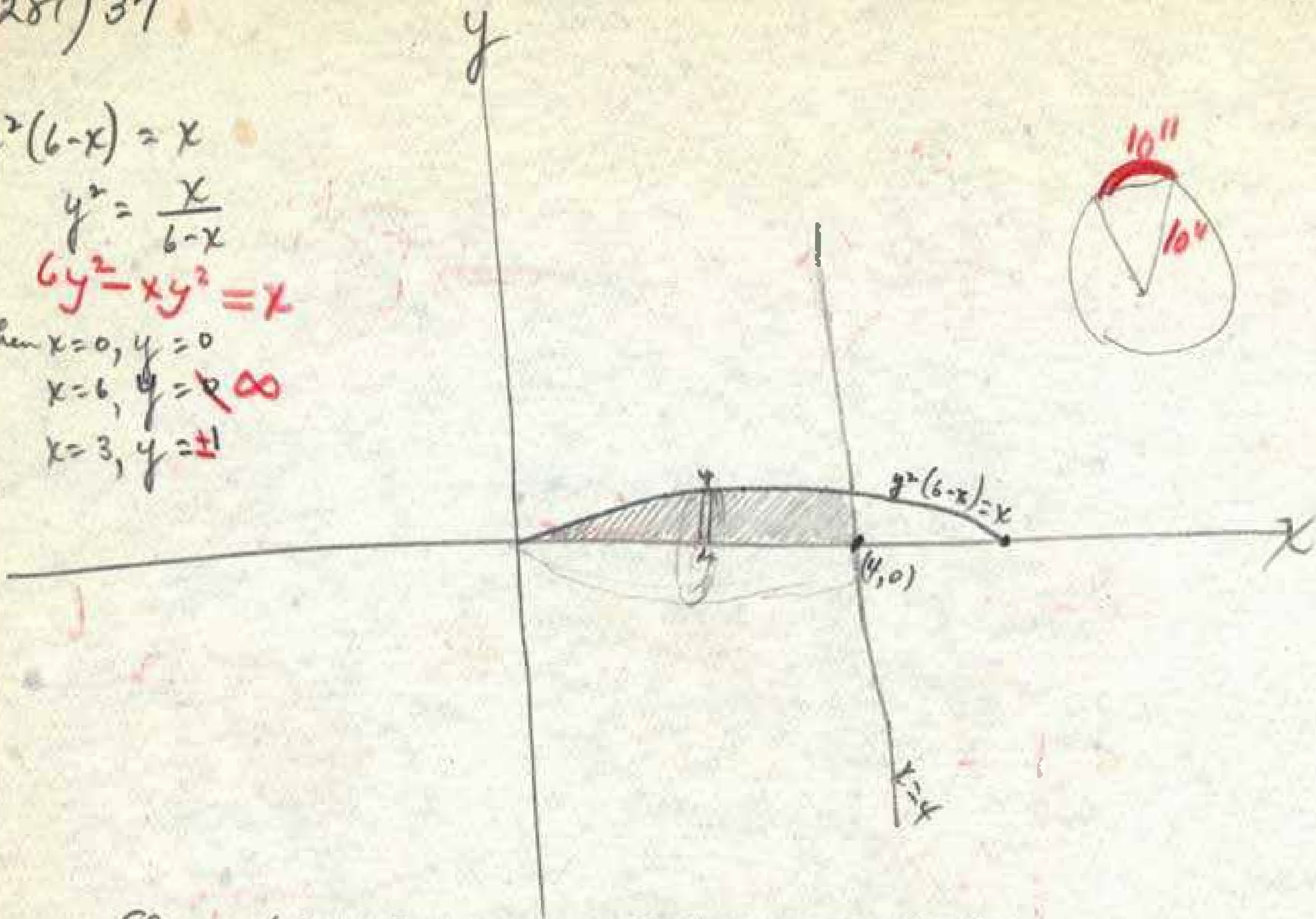
$y^2 = \frac{x}{6-x}$

$6y^2 - xy^2 = x$

when $x=0,\ y=0$

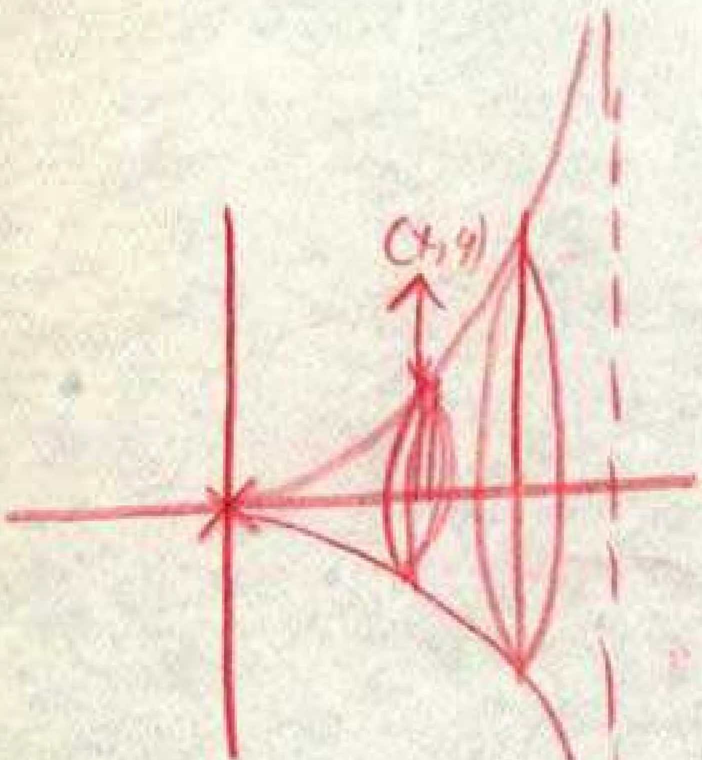
$x=6,\ y=\infty$

$x=3,\ y=\pm 1$



Element of Volume $= \pi r^2\,dx = \pi y^2\,dx$

Total Volume $= \int_0^4 \pi \frac{x}{6-x}\,dx$



$= \pi \int_0^4 \frac{x}{6-x}\,dx$

$= \pi \int_0^4 \left(\frac{6}{6-x} - 1\right) dx$

$= \pi \int_0^4 -6\ln(6-x) - x$

$= \pi \left(\left[-6\ln 2 - 4 \right] - \left[-6\ln 6 - 0 \right] \right)$

$\pi\left(-6\ln 2 - 4 + 6\ln 6\right)$

$\pi\left(6\ln 3 - 4\right)$

$= \pi\left(6(1.0986) - 4\right)$

$= 3.1416\left((4.3386 - 4) - 6(1.7918)\right)$

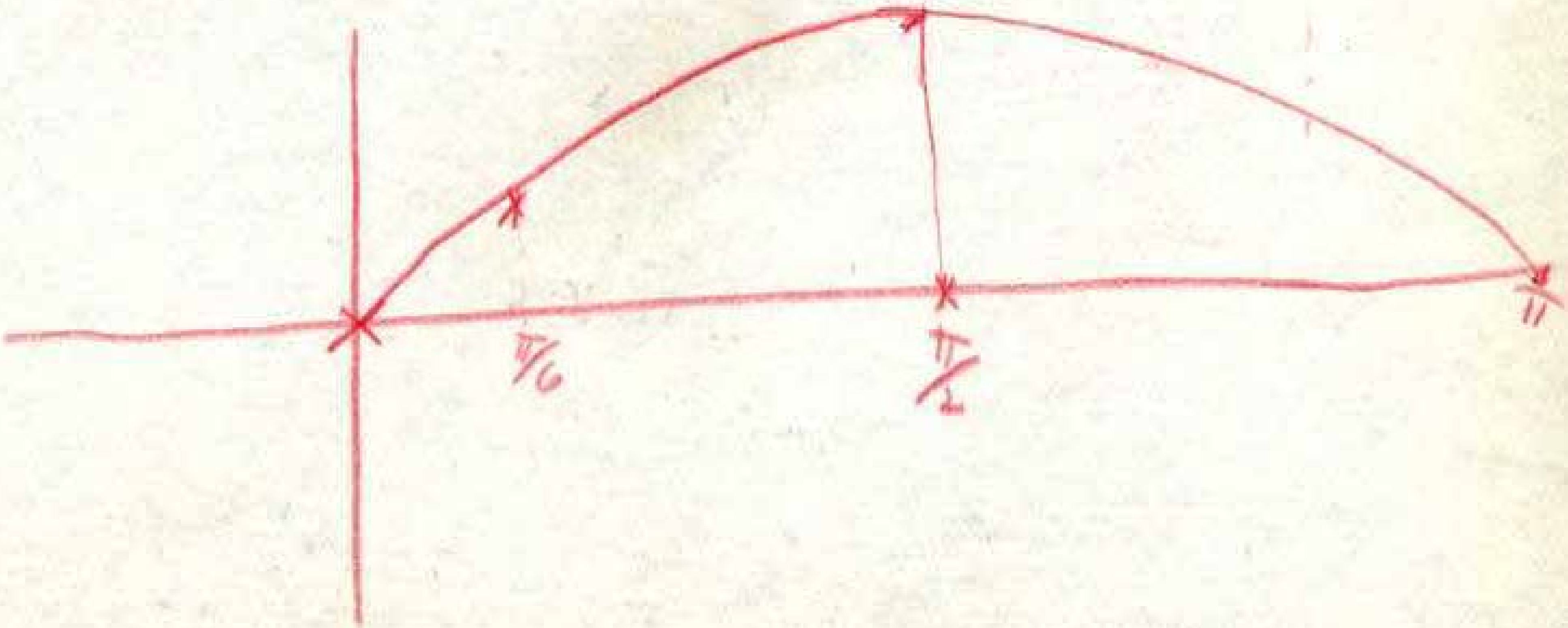
$= 3.1416\left(.3386 - 11.3508\right) = (3.1416)(-11.0122) =$

$f(x+p) = f(x)$

$\sin(x+2\pi) = \sin x$
$\cos(x+2\pi) = \cos x$

$y = \sin x$

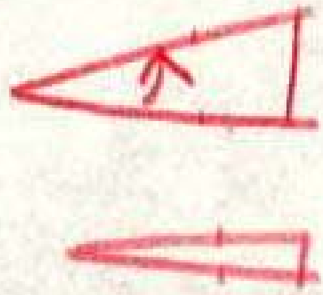
x	y
0	0
$\pi/6$	$\frac{1}{2}$
$\pi/2$	1
π	0
$7\pi/6$	$-1/2$
$3\pi/2$	-1

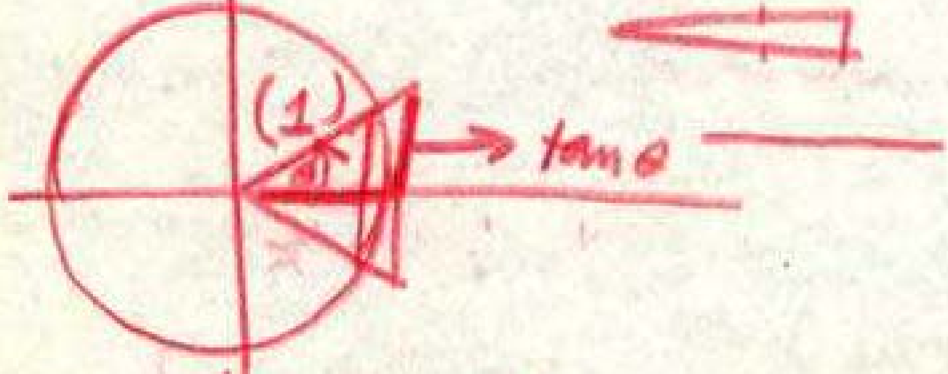


$$\lim_{\substack{u\to 0\\ v\to 0}}\left(\frac{u}{v}\right)$$

$$\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right)=1$$

$$\begin{cases}\frac{.2}{.1},\ \frac{.002}{.001},\ \frac{.0000002}{.0000001},\ — \\ \frac{.15}{.1},\ \frac{.0015}{.001},\ \frac{.0000015}{.000001},\ —\end{cases}$$





$$\cancel{\tfrac{1}{2}}\sin\theta < \cancel{\tfrac{1}{2}}\theta < \cancel{\tfrac{1}{2}}\tan\theta$$

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

$$\frac{dy}{dx}=\lim_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\left(\frac{\Delta y}{\Delta x}\right)$$

$$\frac{\theta}{\sin\theta}\to 1 \quad \text{as } \theta\to 0$$

$$\cancel{\sin}\ \theta$$

$$\frac{\sin\theta}{\cos\theta}.$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$y = \sin x$

$y + \Delta y = \sin(x + \Delta x)$

$\Delta y = \sin(x+\Delta x) - \sin x$

~~$\sin(A+B) = \cdots$~~

$\sin A - \sin B$

$= 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$$\frac{\Delta y}{\Delta x} = \frac{\sin(x+\Delta x) - \sin x}{\Delta x} = \frac{\cancel{2}\cos\frac{2x+\Delta x}{2}\cdot\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$= \cos\left(\frac{2x+\Delta x}{2}\right)\left(\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}\right)$$

$$\lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \cos x \cdot 1 = \cos x$$

$$\frac{d}{dx}(\sin x) = \cos x \longrightarrow \frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$$

294) 1a $y = 3 \sin \left(\frac{1}{2}x\right)$ $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$

$\frac{dy}{dx} = 3 \cos\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)$

$= \frac{3}{2} \cos\left(\frac{1}{2}x\right)$

g) $y = x \sin x$

$\frac{dy}{dx} = x \cdot \cos x + \sin x \cdot 1$

$= x \cos x + \sin x$

m) $y = e^x \ln \sin x$

$\frac{dy}{dx} = \left[e^x \cdot \frac{d}{dx} \ln \sin x\right] + \left[\ln \sin x \cdot e^x\right]$

$= e^x \cdot \frac{1}{\sin x} \cdot \cos x + \ln \sin x \cdot e^x$

$= e^x \left(\cot x + \ln \sin x\right)$

Let $y = (\sin x)^3$

$$\frac{dy}{dx} = 3(\sin x)^2 \cdot \cos x$$

---

l) $r = \sin^3 \frac{1}{3}\theta = \left(\sin \frac{1}{3}\theta\right)^3$

$$\frac{dr}{d\theta} = \cancel{3}\left(\sin \frac{1}{3}\theta\right)^2 \cdot \cos \frac{1}{3}\theta \cdot \frac{1}{\cancel{3}}$$

---

n) $y = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}} = \frac{1}{2} \ln \frac{1 + \sin x}{1 - \sin x}$

$$= \frac{1}{2}\left[\ln(1 + \sin x) - \ln(1 - \sin x)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1 + \sin x} \cdot \cos x + \frac{1}{1 - \sin x} \cdot (+\cos x)\right]$$

$$= \frac{1}{2}\left[\frac{(1 - \sin x)(\cos x) + (1 + \sin x)(\cos x)}{(1 + \sin x)(1 - \sin x)}\right]$$

$$= \frac{\cos x}{2}\left(\frac{1 - \sin x + 1 + \sin x}{\cos^2 x}\right) = \frac{1}{\cos x} = \sec x$$

295) 7

$y = x$

$y = x - \sin 2x$

$\sin 2x = 0$

$2x = 0, \pi, 2\pi$

$x = 0, \frac{\pi}{2}, \pi$

Int. at $(0,0)$ and at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y = x$

$\frac{dy}{dx} = 1$

at $0,0$ $= 1$

$y = x - \sin 2x$

$\frac{dy}{dx} = 1 - (\cos 2x \cdot 2) = 1 - 2\cos 2x$

$= 1 - 2\cos 0 = -1$

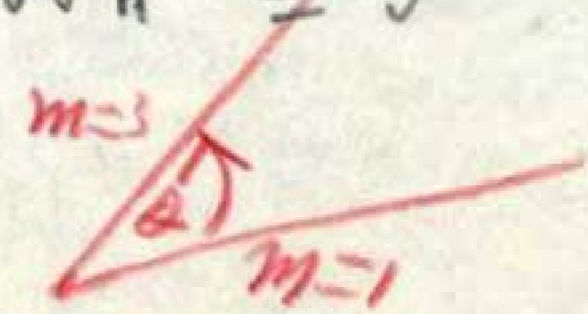
at $(0,0)$ angle $= 90^\circ$

at $\frac{\pi}{2}, \frac{\pi}{2}$ $= 1$

$1 - 2\cos \pi = 3$

$\tan\theta = \frac{m - m'}{1 + m m'}$

$= \frac{3 - 1}{1 + 3} = \frac{1}{2}$



Page 294 — k, k, h, j, g, u

305

311

294) 1e

$$r = \tan \frac{1}{2}\theta$$

$$\frac{dr}{d\theta} = \sec^2\left(\frac{1}{2}\theta\right) \frac{d}{d\theta}\left(\frac{1}{2}\theta\right)$$

$$= \sec^2\left(\frac{1}{2}\theta\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2} \sec^2\left(\frac{1}{2}\theta\right)$$

1k)

$$s = \cos \frac{a}{t}$$

$$\frac{ds}{dt} = -\sin \frac{a}{t} \cdot \frac{d}{dt}\left(\frac{a}{t}\right) = -\sin \frac{a}{t} \cdot \left(\frac{-a}{t^2}\right)$$

$$= \frac{a}{t^2}\left(\sin \frac{a}{t}\right)$$

1l)

$$s = e^{-2t} \cos t$$

$$\frac{ds}{dt} = \left[e^{-2t} \cdot \frac{d}{dt} \cos t\right] + \left[\cos t \cdot \frac{d}{dt} e^{-2t}\right]$$

$$= \left[e^{-2t} \cdot \left(-\sin t \frac{dt}{dt}\right)\right] + \left[\cos t \cdot e^{-2t} \cdot (-2)\right]$$

$$= e^{-2t}\left(-\sin t - 2\cos t\right)$$

$$= -e^{-2t}\left(\sin t + 2\cos t\right)$$

294) 1 j $\quad y = \dfrac{\cos x}{x}$

$$\frac{dy}{dx} = \frac{x\left(\frac{d}{dx}\cos x\right) - \cos x\left(\frac{dx}{dx}\right)}{x^2}$$

$$= \frac{\left[x \cdot \left(-\sin x \frac{dx}{dx}\right)\right] - \left[\cos x \cdot 1\right]}{x^2}$$

$$= \frac{x\left(-\sin x\right) - \cos x}{x^2} \quad = \frac{-x\sin x - \cos x}{x^2}$$

294) 1 g $\quad y = \cos\sqrt{5x}$

$$\frac{dy}{dx} = -\sin\sqrt{5x} \cdot \frac{d}{dx}\sqrt{5x}$$

$$= -\sin\sqrt{5x} \cdot \left[\frac{1}{2}(5x)^{-\frac{1}{2}}(5)\right]$$

$$= -\sin\sqrt{5x} \cdot \frac{5}{2\sqrt{5x}}$$

$$= \frac{-5\sin\sqrt{5x}}{2\sqrt{5x}}$$

294) 1 u

$$s = e^t \tan t$$

$$\frac{ds}{dt} = \left[e^t \cdot \frac{d}{dt} \tan t\right] + \left[\tan t \cdot \frac{d}{dt} e^t\right]$$

$$= \left[e^t \cdot \sec^2 t \frac{dt}{dt}\right] + \left[\tan t \cdot e^t\right]$$

$$= e^t (\sec^2 t + \tan t)$$

---

249) 1 v

$$s = e^{-\frac{1}{5}t} \sin 5t$$

$$\frac{ds}{dt} = \left[e^{-\frac{1}{5}t} \cdot \frac{d}{dt}(\sin 5t)\right] + \left[\sin 5t \cdot \frac{d}{dt} e^{-\frac{1}{5}t}\right]$$

$$= \left[e^{-\frac{1}{5}t} \cdot \left(\cos 5t \cdot \frac{d}{dt} 5t\right)\right] + \left[\sin 5t \cdot e^{-\frac{1}{5}t} \cdot \left(-\frac{1}{5}\right)\right]$$

$$= e^{-\frac{1}{5}t} \left(5 \cos 5t - \frac{1}{5} \sin 5t\right)$$

$$= e^{-\frac{1}{5}t} \left(\frac{25 \cos 5t - \sin 5t}{5}\right)$$

295) 3 e

$$\rho = \theta \cos\theta$$

$$\frac{d\rho}{d\theta} = \left[\theta \cdot \frac{d}{d\theta}\cos\theta\right] + \left[\cos\theta \cdot \frac{d\theta}{d\theta}\right]$$

$$= \left[\theta \cdot (-\sin\theta) \cdot \frac{d\theta}{d\theta}\right] + \left[\cos\theta \cdot (1)\right]$$

$$= \theta(-\sin\theta) + \cos\theta$$

$$= -\theta\sin\theta + \cos\theta$$

$$\frac{d_2\rho}{d\theta} = \left[\left(\theta \cdot \frac{d}{d\theta}(-\sin\theta) \cdot \frac{d\theta}{d\theta}\right) + \left(-\sin\theta \cdot \frac{d\theta}{d\theta}\right)\right] + \frac{d}{d\theta}\cos\theta$$

$$= \left[\left(\theta \cdot (-\cos\theta) \cdot \frac{d\theta}{d\theta} \cdot 1\right) + \left(-\sin\theta \cdot 1\right)\right] + \left(-\sin\theta \cdot \frac{d\theta}{d\theta}\right]$$

$$= \theta(-\cos\theta) - \sin\theta - \sin\theta$$

$$= \theta(-\cos\theta) - 2\sin\theta$$

$$= -\theta\cos\theta - 2\sin\theta$$

295) 3 j

$$s = e^{-\frac{1}{3}t} \sin \pi t$$

$$\frac{ds}{dt} = \left[e^{-\frac{1}{3}t} \cdot \frac{d}{dt} \sin \pi t\right] + \left[\sin \pi t \cdot \frac{d}{dt} e^{-\frac{1}{3}t}\right]$$

$$= \left[e^{-\frac{1}{3}t} \cdot \cos \pi t \cdot \frac{d}{dt} \pi t\right] + \left[\sin \pi t \cdot e^{-\frac{1}{3}t} \cdot \left(-\frac{1}{3}\right)\right]$$

$$= \left[e^{-\frac{1}{3}t} \cdot \cos \pi t \cdot \pi\right] + \left[\sin \pi t \cdot e^{-\frac{1}{3}t} \cdot \left(-\frac{1}{3}\right)\right]$$

$$= e^{-\frac{1}{3}t}\left(\pi \cos \pi t - \frac{\sin \pi t}{3}\right) = \frac{e^{-\frac{1}{3}t}}{3}(3\pi \cos \pi t - \sin \pi t)$$

$$\frac{d_2 s}{dt} = \frac{e^{-\frac{1}{3}t}}{3}\left[\frac{d}{dt}(3\pi \cos \pi t - \sin \pi t)\right] + \left[(3\pi \cos \pi t - \sin \pi t) \cdot \frac{d}{dt} \frac{e^{-\frac{1}{3}t}}{3}\right]$$

$$= \frac{e^{-\frac{1}{3}t}}{3}\left[\left(3\pi(-\sin \pi t) \cdot \frac{d}{dt} \pi t\right) - \left(\cos \pi t \cdot \frac{d}{dt} \pi t\right)\right] + \left[(3\pi \cos \pi t - \sin \pi t) \cdot \frac{e^{-\frac{1}{3}t}}{3} \cdot \left(-\frac{1}{3}\right)\right]$$

$$= \frac{e^{-\frac{1}{3}t}}{3}\left(-3\pi^2 \sin \pi t - \pi \cos \pi t - \pi \cos \pi t + \frac{\sin \pi t}{3}\right)$$

$$= \frac{e^{-\frac{1}{3}t}}{3}\left(\frac{-9\pi^2 \sin \pi t - 6\pi \cos \pi t + \sin \pi t}{3}\right)$$

$$= \frac{e^{-\frac{1}{3}t}(-9\pi^2 \sin \pi t - 6\pi \cos \pi t + \sin \pi t)}{9}$$

295) 8

$$\begin{cases} y = \cos x \\ y = \sin 2x \end{cases}$$

$y = \cos x$

x	y
0	1
$\pi/2$	0

* Discuss plotting of curves of this character

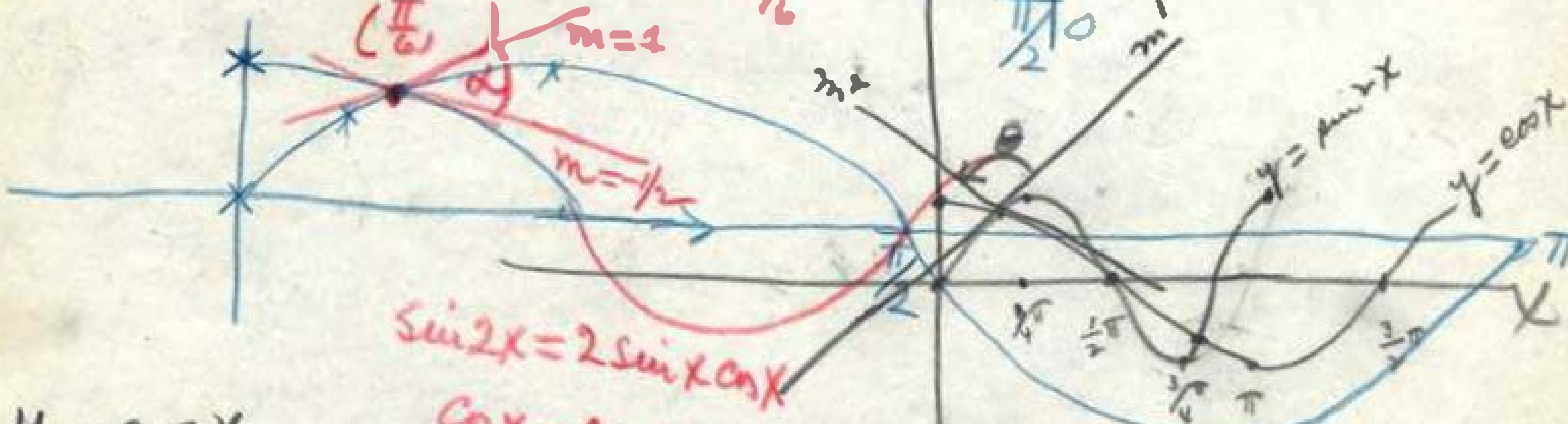
$y = \sin 2x$

x	y
0	0
$\pi/12$	$\frac{1}{2}$
$\pi/4$	1
$\pi/2$	0

$[\cos \underline{x} = \sin \underline{2x}]$ is point of intersection

$x + 2x = 90°$

$x = 30° = \pi/6$



$\sin 2x = 2 \sin x \cos x$

$\cos x - 2 \sin x \cos x = 0$

$\cos x (1 - 2 \sin x) = 0 \rightarrow \cos x = 0, \ x = \pi/2$
or $\sin x = 1/2, \ x = \pi/6$

$y = \cos x$

$$\frac{dy}{dx} = m = \frac{d}{dx} \cos x = -\sin x \cdot (1) = -\sin x = -1/2$$

At $\pi/2$, $m_1 = -1$
$m_2 = -2$

$y = \sin 2x$

$$\frac{dy}{dx} = m_1 = \frac{d}{dx} \sin 2x = \cos 2x \cdot \frac{d}{dx} 2x = 2 \cos 2x = 1$$

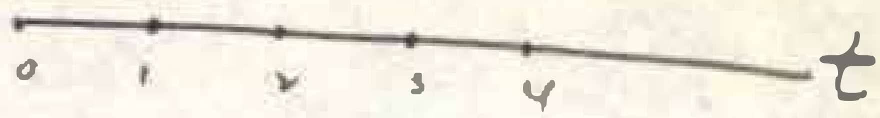
$$\tan \theta = \frac{-\sin x - 2 \cos 2x}{1 \ \cancel{+}\ 2 \sin x \cos 2x}$$

$\cos x = \sin 2x$

$$\tan \theta = \frac{1 + 1/2}{1 + (-1/2)} = \frac{3/2}{1/2} = 3$$

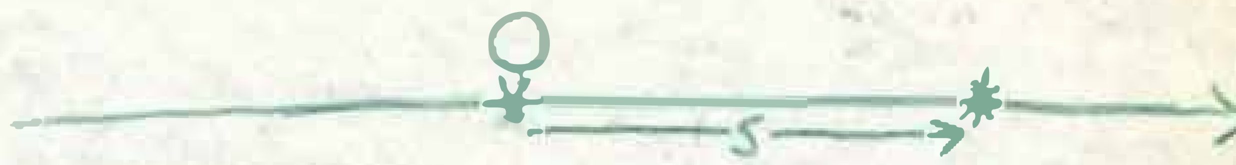
295/9

$S = 4 \sin \frac{1}{2}\pi t$



$v = \frac{ds}{dt}$

$a = \frac{dv}{dt}$



$v = \frac{d}{dt}\left(4 \sin \frac{1}{2}\pi t\right) = 4 \cos \frac{1}{2}\pi t \left(\frac{1}{2}\pi\right) = 2\pi \cos \frac{1}{2}\pi t$ ✓

$a = \frac{d}{dt}\ 2\pi \cos \frac{1}{2}\pi t = -2\pi \sin \frac{1}{2}\pi t \left(\frac{1}{2}\pi\right) = -\pi^2 \sin \frac{1}{2}\pi t$ ✓

When $t = 0$, $v = 2\pi \cos 0 = 2\pi \cdot 1 = 2\pi$

$a = -\pi^2 \sin 0 = 0$ ✓

$S = 0$

When $t = 1$, $v = 2\pi \cos \frac{1}{2}\pi = 2\pi \cdot 0 = 0$ ✓

$a = -\pi^2 \sin \frac{1}{2}\pi = -\pi^2 \cdot 1 = -\pi^2$ ✓

$S = 4 \sin \frac{1}{2}\pi = 4 \cdot 1 = 4$ ✓

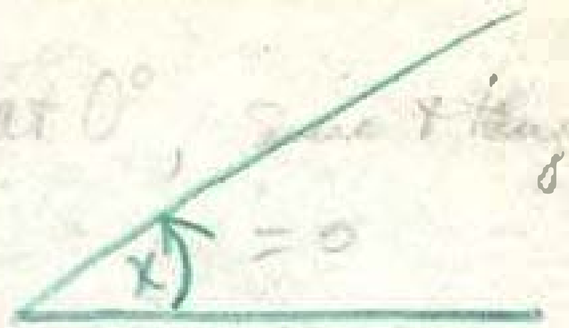
When $t = 2$, $v = 2\pi \cos \pi = -2\pi$

$a = -\pi^2 \sin \pi = 0$

$S = 4 \sin \pi = 0$

295) 11 Let x = angle

$$\frac{d}{dt}\tan x = \sec^2 x \frac{dx}{dt}$$



$$\frac{dx}{dt} = C$$

$$\frac{d}{dt}\sin x = \cos x \frac{dx}{dt}$$

at 0°, $x = 0$, $\therefore \sec^2 x(0) = 1$
$\cos x(0) = 1$

$\therefore$ at 0°, sine + tangent increase at same rate

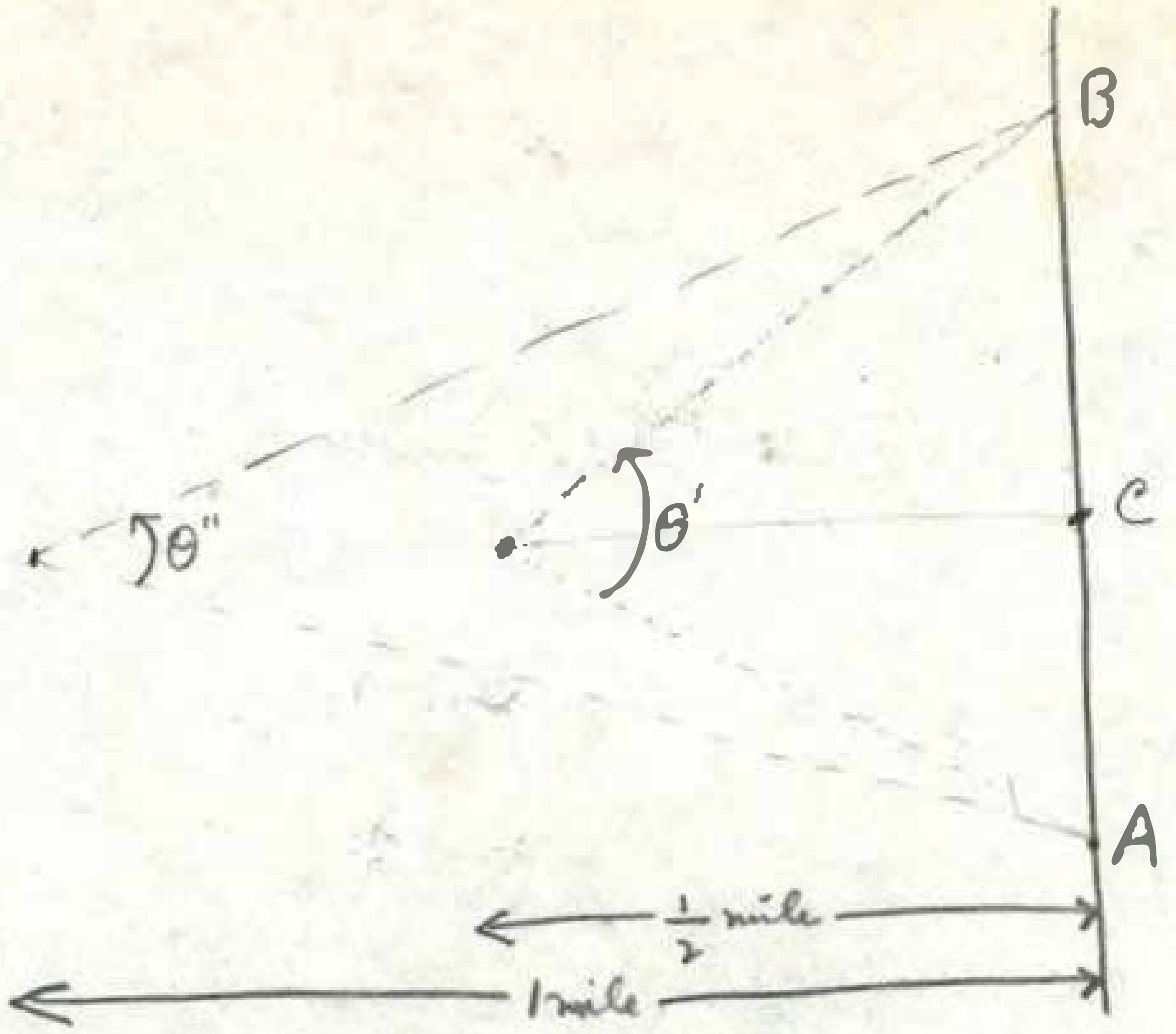
at 60°, cosine = .50

secant $= \frac{1}{\cos} = \frac{1}{.5} = 2$ ✓

$(\text{secant})^2 = \underline{4}$ or $\underline{8 \text{ times}}$ cosine.

$\therefore$ at 60°, $\frac{d}{dt}\tan x$ is 8 times $\frac{d}{dt}\sin x$, + tangent increases 8 times as rapidly as sine

Page 295) 12

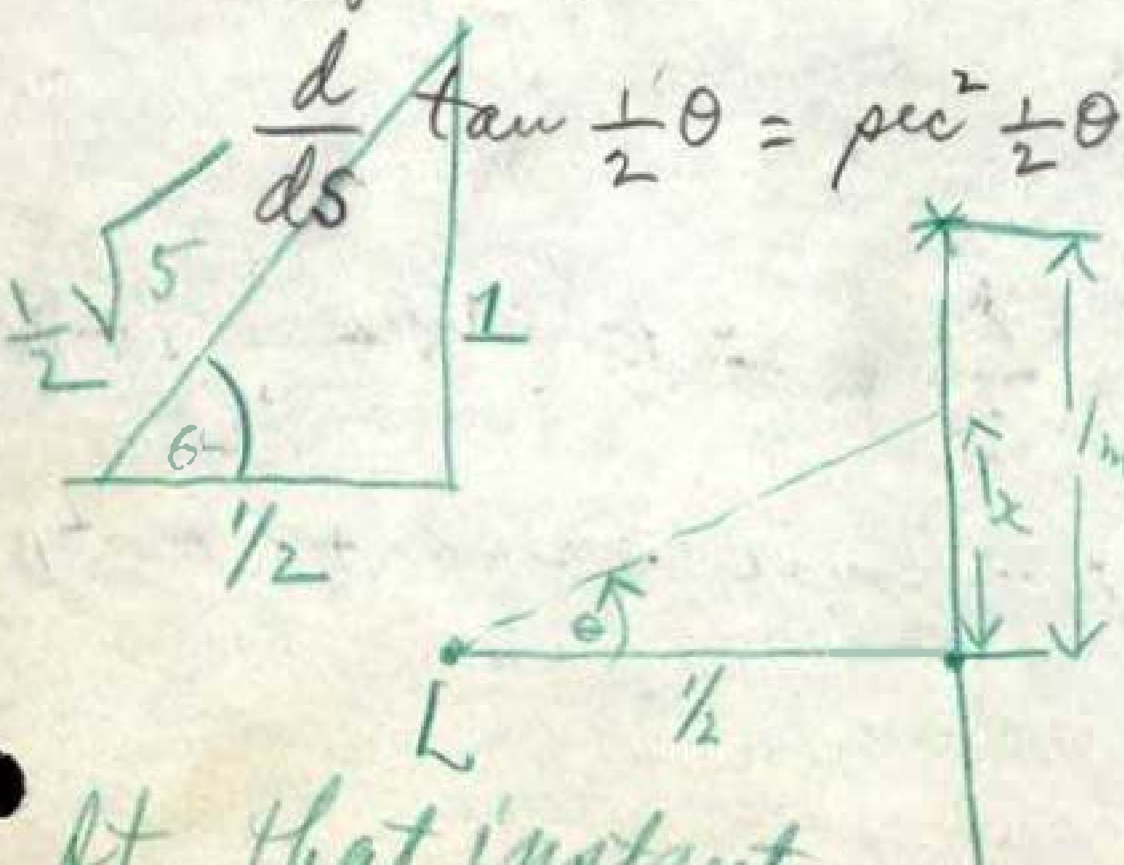


Let AB = distance traveled by light in 30 seconds along shore

BC = " " " " " 15 " " "

Thus speed of light is a function of angle θ, & specifically of tangent $\frac{1}{2}\theta$, which is in turn a function of s, distance from shore

$$\frac{d}{ds}\tan\frac{1}{2}\theta = \sec^2\frac{1}{2}\theta \cdot \frac{d}{d}$$



$$\left\{\begin{array}{l} \frac{d\theta}{dt} = 2\pi \\ \frac{dx}{dt} = ? \end{array}\right.$$

$\tan\theta = 2x$

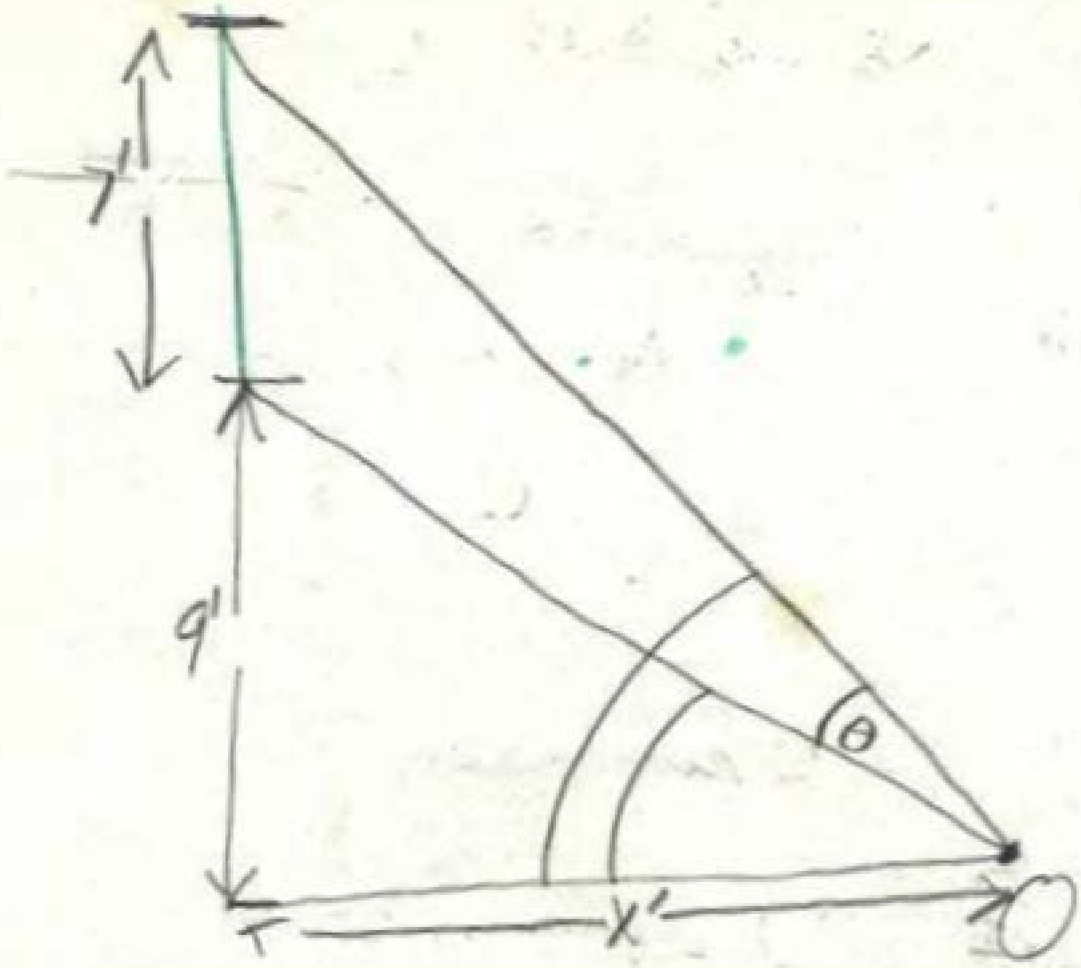
$x = \frac{1}{2}\tan\theta$

$\frac{dx}{dt} = \frac{1}{2}\sec^2\theta\,\frac{d\theta}{dt} = \underline{\pi\sec^2\theta}$

At that instant,

$dx/dt = \pi \cdot 5 = 5\pi$

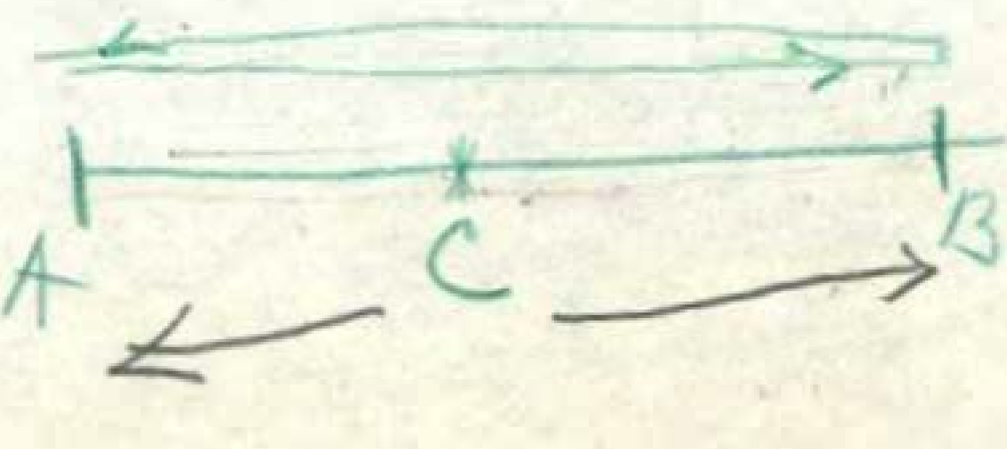
295/13



Find $\underline{X}$ so that θ be a maximum.

$$\theta = \arctan \frac{16}{x} - \arctan \frac{9}{x}$$

max. value $\approx \underline{S}$



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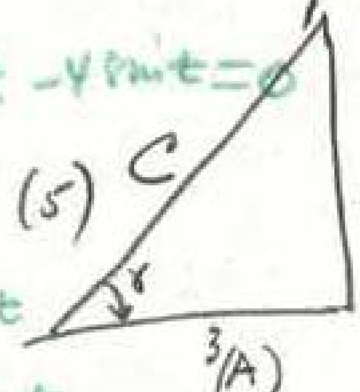
$S = 3\sin t + 4\cos t$ —— 0

The period is $\frac{2\pi}{1} = 2\pi$ for both terms

$\frac{ds}{dt} = 3\cos t - 4\sin t = 0$



$3\cos t = 4\sin t$

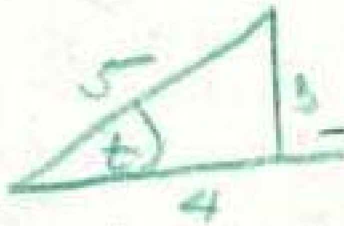
$3 = 4\tan t$

$\tan t = 3/4$

Hypotenuse $= \sqrt{9+16} = 5$

$B = C\sin\gamma = 4$

$A = C\cos\gamma = 3$



$\rightarrow S = 3 \cdot \frac{3}{5} + 4 \cdot \frac{4}{5} = \frac{9}{5} + \frac{16}{5} = 5$

Substituting,

$S = C\cos\gamma\sin t + C\sin\gamma\cos t$

~~Since~~ $\sin(A+B) = \sin A\cos B + \cos A\sin B$,

$S = C\sin(t+\gamma)$

$$* \begin{cases} S = A\sin 3t + B\cos 3t \\ v = 3A\cos 3t - 3B\sin 3t \\ a = -9A\sin 3t - 9B\cos 3t \end{cases}$$

This is formula for simple sine curve with

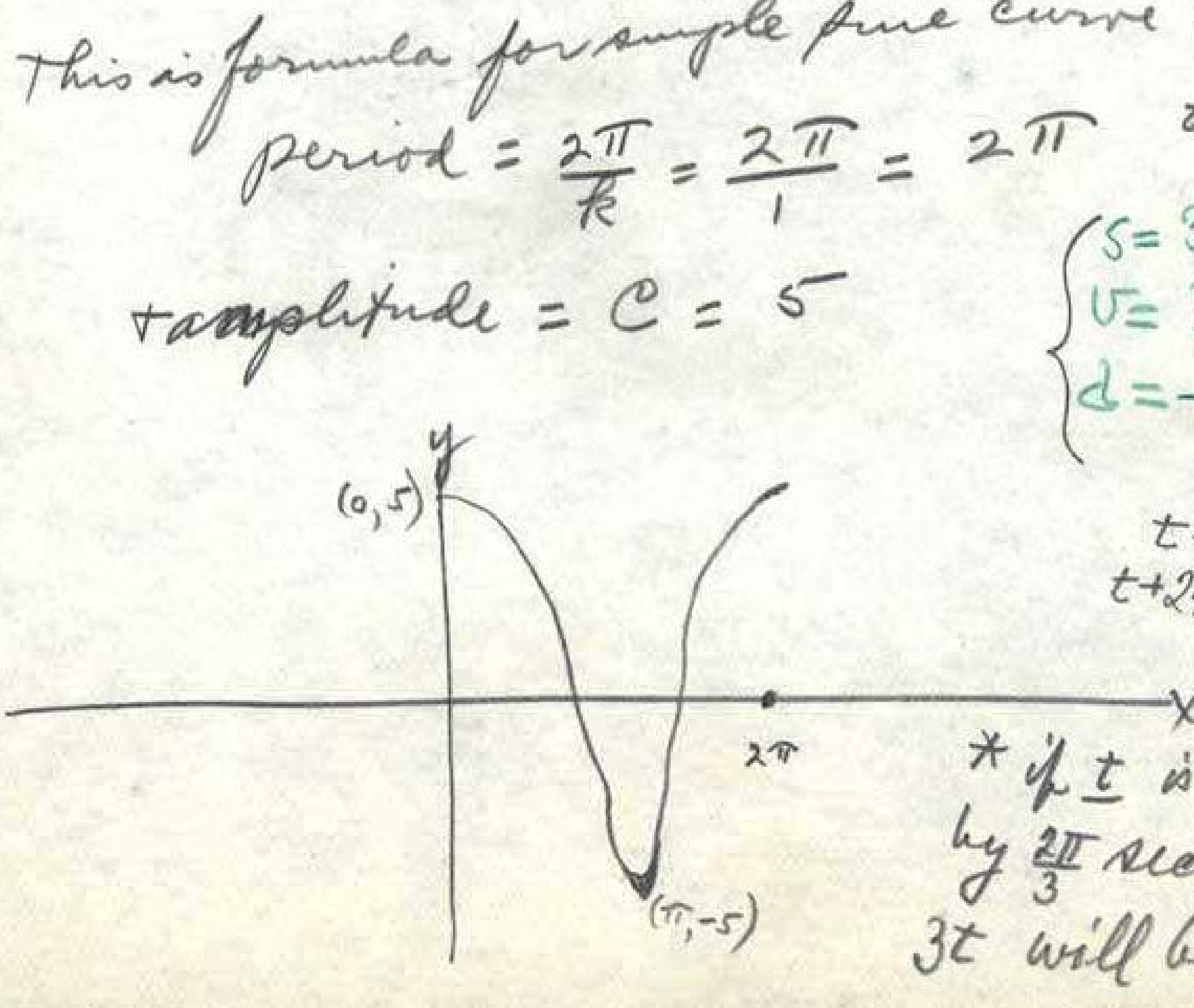
$t \rightarrow s, v, a$

$t + \frac{2\pi}{3} \rightarrow s, v, a$

period $= \frac{2\pi}{k} = \frac{2\pi}{1} = 2\pi$

+amplitude $= C = 5$

$$\begin{cases} S = 3\sin t + 4\cos t \\ v = 3\cos t - 4\sin t \\ a = -3\sin t - 4\cos t \end{cases}$$



$t \rightarrow s, v, a$

$t + 2\pi \rightarrow s, v, a$

* if t is augmented by $\frac{2\pi}{3}$ seconds, then $3t$ will be augm. by 2π

306) 2

$$s = 3e^{-\frac{1}{2}t} \sin 2t$$

$$v = \frac{ds}{dt} = \frac{d}{dt} 3e^{-\frac{1}{2}t} \sin 2t$$

$$= \left[3e^{-\frac{1}{2}t} \frac{d}{dt} \sin 2t\right] + \left[\sin 2t \frac{d}{dt} 3e^{-\frac{1}{2}t}\right]$$

$$= \left[3e^{-\frac{1}{2}t} (\cos 2t)(2)\right] + \left[\sin 2t \left(3e^{-\frac{1}{2}t}\right)\left(-\frac{1}{2}\right)\right]$$

$$= 6e^{-\frac{1}{2}t} \cos 2t - \frac{3}{2} e^{-\frac{1}{2}t} \sin 2t$$

$$= e^{-\frac{1}{2}t}\left(6 \cos 2t - \frac{3}{2} \sin 2t\right)$$

$$a = \frac{dv}{dt} = \frac{d}{dt} e^{-\frac{1}{2}t}\left(6 \cos 2t - \frac{3}{2} \sin 2t\right)$$

$$= e^{-\frac{1}{2}t} \frac{d}{dt}\left(6 \cos 2t - \frac{3}{2} \sin 2t\right) + \left(6 \cos 2t - \frac{3}{2} \sin 2t\right) \frac{d}{dt} e^{-\frac{1}{2}t}$$

$$= e^{-\frac{1}{2}t}\left(-12 \sin 2t - 3 \cos 2t\right) + \left(6 \cos 2t - \frac{3}{2} \sin 2t\right) e^{-\frac{1}{2}t}\left(-\frac{1}{2}\right)$$

$$= e^{-\frac{1}{2}t}\left(-12 \sin 2t - 3 \cos 2t - 3 \cos 2t + \frac{3}{4} \sin 2t\right)$$

$$= e^{-\frac{1}{2}t} \frac{\left(-45 \sin 2t - 24 \cos 2t\right)}{4}$$

(see next page)

30) 2 cont.

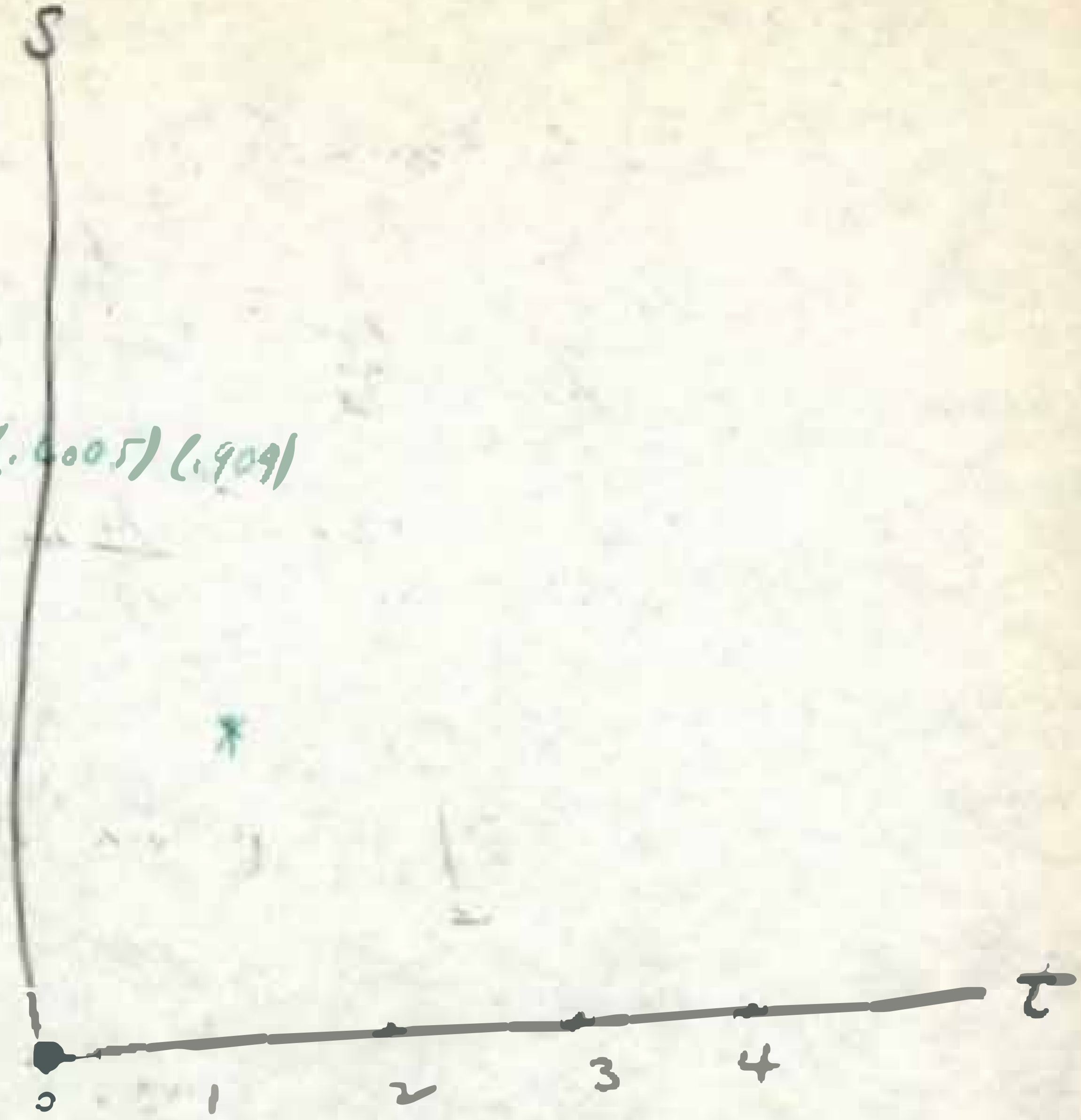
When $t=0$, $s=3e^{-\frac{1}{2}t}\sin 2t=0$

a) $t=1$, $s=3e^{-\frac{1}{2}}\underline{\sin 2}=3(.6005)(.9091)$ 2 radians

$t=2$, $s=3e^{-1}\sin 4=$

$t=3$, $s=3^{-\frac{3}{2}t}\sin 6=$

$t=4$, $s=3^{-2}\sin 8=$



b) When $v=0$, $0=e^{-\frac{1}{2}t}\left(6\cos 2t-\frac{3}{2}\sin 2t\right)$

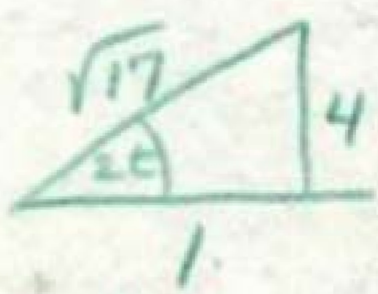
then $6\cos 2t-\frac{3}{2}\sin 2t=0$

$$\frac{6\cos 2t}{\cos 2t}=\frac{\frac{3}{2}\sin 2t}{\cos 2t}$$

$6=\frac{3}{2}\tan 2t$, $\tan 2t=4$

$2t=1.32$

$t=.66$



$S=3e^{-.33}\frac{4}{\sqrt{17}}$

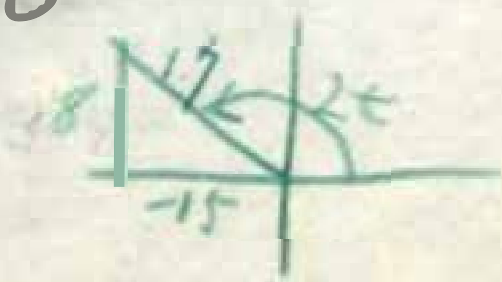
c) When $a=0$, $0=e^{-\frac{1}{2}t}\frac{\left(-45\sin 2t-24\cos 2t\right)}{4}$

$-45\sin 2t-24\cos 2t=0$

$$-15\frac{\sin 2t}{\cos 2t}-\frac{8\cos 2t}{\cos 2t}=0$$

$-15\tan 2t=8$

$\tan 2t=-8/15$



306) 5b

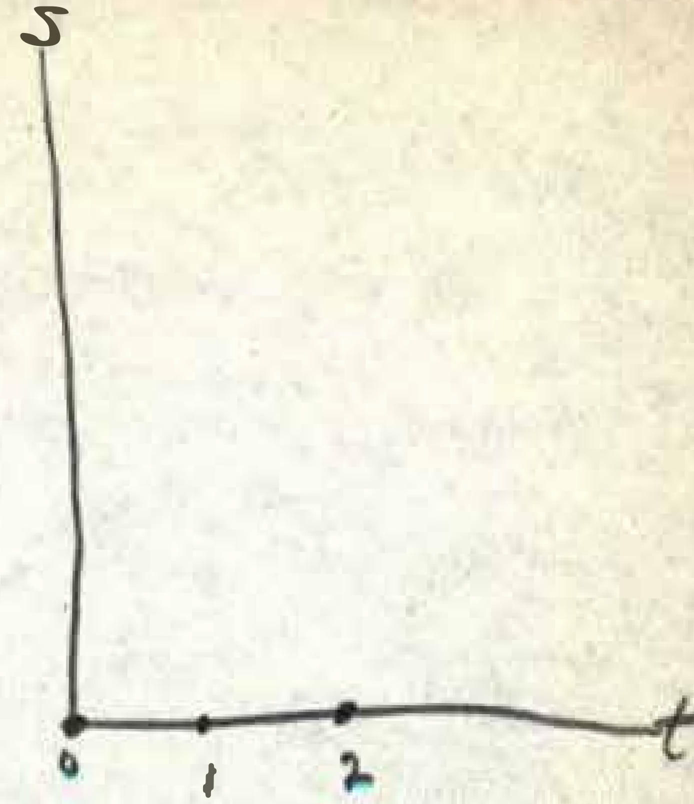
(radians)

$s = \frac{1}{2} t \sin t$

When $t=0$, $s = 0 \sin 0 = 0$

$t=1$, $s = \frac{1}{2} \sin 1 =$

$t=2$, $s = \sin 2 =$



$$v = \frac{ds}{dt} = \frac{d}{dt} \frac{1}{2} t \sin t = \frac{1}{2} t \cos t + \frac{\sin t}{2}$$

$$= \frac{1}{2} (t \cos t + \sin t)$$

When $t=0$, $v = \frac{0(1) + 0}{2} = 0$

$t=1$, $v = \frac{\cos 1 + \sin 1}{2} =$

$t=2$, $v = \frac{2 \cos 2 + \sin 2}{2} =$

$$a = \frac{dv}{dt} = \frac{d}{dt} \frac{1}{2} (t \cos t + \sin t)$$

$$= \frac{1}{2} \left[t \cdot (-\sin t \cdot (1)) + \cos t \cdot (1) \right] + \left[\frac{1}{2} \cos t (1) \right]$$

$$= \frac{-t \sin t + \cos t}{2} + \frac{\cos t}{2}$$

$$= \frac{-t \sin t + 2 \cos t}{2}$$

When $t=0$, $a = \frac{-0 \sin 0 + 2 \cos 0}{2} = 1$

$t=1$, $a = \frac{-1 \sin 1 + 2 \cos 1}{2} =$

When $t=2$,

$a = \frac{-2 \sin 2 + 2 \cdot \cos 2}{2}$

$=$

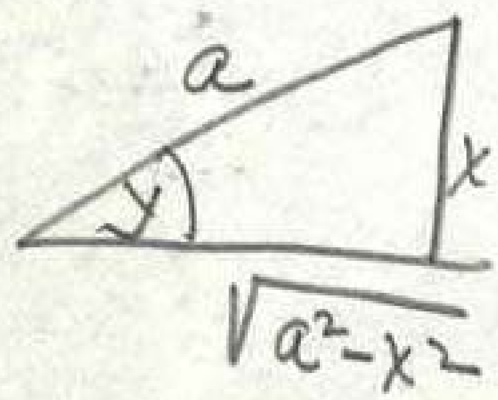
311) 1a

$$y = \arcsin \frac{x}{a}$$

$$\sin y = \frac{x}{a}$$

$$x = a \sin y$$

$$\frac{dx}{dy} = a \cos y$$



$$\frac{dy}{dx} = \frac{1}{a \cos y} = \frac{1}{\cancel{a} \cdot \frac{\sqrt{a^2-x^2}}{\cancel{a}}} = \frac{1}{\sqrt{a^2-x^2}}$$

311) 2a $\qquad y = \arcsin \frac{x}{a}$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}} = \frac{\frac{1}{a}}{\sqrt{1-\frac{x^2}{a}}} = \frac{1}{a\sqrt{1-\frac{x^2}{a^2}}} = \frac{1}{\sqrt{a^2-x^2}}$$

---

311) 2b $\qquad y = \arccos \frac{1}{2} x$

$$\frac{dy}{dx} = -\frac{\frac{1}{2}}{\sqrt{1-\frac{1}{4}x^2}} = -\frac{1}{2\sqrt{1-\frac{1}{4}x^2}} = \frac{1}{\sqrt{4-x^2}}$$

---

311) 2c $\qquad y = \arctan \frac{x}{a}$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}\left(\frac{x}{a}\right)}{1+\frac{x^2}{a^2}} = \frac{\frac{1}{a}}{1+\frac{x^2}{a^2}} = \frac{1}{a}\cdot\frac{a^2}{a^2+x^2} = \frac{a}{a^2+x^2}$$

---

311) 2d $\qquad y = \arcsin \frac{a}{x}$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}\frac{a}{x}}{\sqrt{1-\frac{a^2}{x^2}}} = \frac{\frac{-a}{x^2}}{\sqrt{1-\frac{a^2}{x^2}}} = -\frac{a}{x^2\sqrt{1-\frac{a^2}{x^2}}}$$

$$= \frac{-a}{\sqrt{x^4-\frac{x^4a^2}{x^2}}} = -\frac{a}{\sqrt{x^4-x^2a^2}}$$

311) 2f

$$y = \arctan \frac{2x}{1-x^2}$$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}\left(\frac{2x}{1-x^2}\right)}{1+\left(\frac{2x}{1-x^2}\right)^2}$$

$$= \frac{\frac{(1-x^2)(2) - \left[(2x)(-2x)\right]}{(1-x^2)^2}}{1+\left(\frac{2x}{1-x^2}\right)^2}$$

$$= \frac{2-2x^2+4x^2}{\left[1+\frac{4x^2}{1-2x^2+x^4}\right](1-x^2)^2}$$

$$= \frac{2+2x^2}{\frac{1-2x^2+x^4+4x^2(1-2x^2+x^4)}{(1-2x^2+x^4)}}$$

$$= \frac{2(1+x^2)}{(1+x^2)^2}$$

$$= \frac{2}{1+x^2}$$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$x = \tan A$

$A = \arctan x$

$y = \arctan\left(\frac{2\tan A}{1-\tan^2 A}\right)$

$= \arctan(\tan 2A)$

$= 2A$

$\frac{dy}{dx} = 2\frac{dA}{dx}$

$= 2 \cdot \frac{1}{1+x^2}$

311) 3 b

$$y = \operatorname{arc\,cos} \frac{1}{2} x$$

$$\cos y = \frac{1}{2} x$$

$$x = 2 \cos y$$

If $x = 1$,

$$2 \cos y = 1$$

$$\cos y = \frac{1}{2}, \quad y = \pi/3$$

$$y' = - \frac{\frac{d}{dx} \frac{1}{2} x}{\sqrt{1 - \frac{1}{4}x^2}} = - \frac{\frac{1}{2}}{\sqrt{1 - \frac{1}{4}x^2}}$$

$$= - \frac{1}{2\sqrt{1 - \frac{1}{4}x^2}} = - \frac{1}{\sqrt{4 - x^2}}$$

when $x = 1$,

$$y' = - \frac{1}{\sqrt{4 - \cancel{\pm}\, 1}} = - \frac{1}{\sqrt{3}} \cancel{= \infty}$$

3.11) 3 h

$$y = x \arcsin \tfrac{1}{4} x$$

$$x \sin y = \frac{1}{4} x$$

$\arcsin \frac{1}{4} x = y/x$

$x = 1,\ y = \arcsin 1/4$

$$\sin y = \frac{1}{4}$$

$\sin (y/x) = \frac{1}{4} x$

$$y' = x \frac{\frac{d}{dx} \frac{1}{4} x}{\sqrt{1 - \frac{x^2}{16}}} + \left(\arcsin \frac{1}{4} x \cdot 1\right)$$

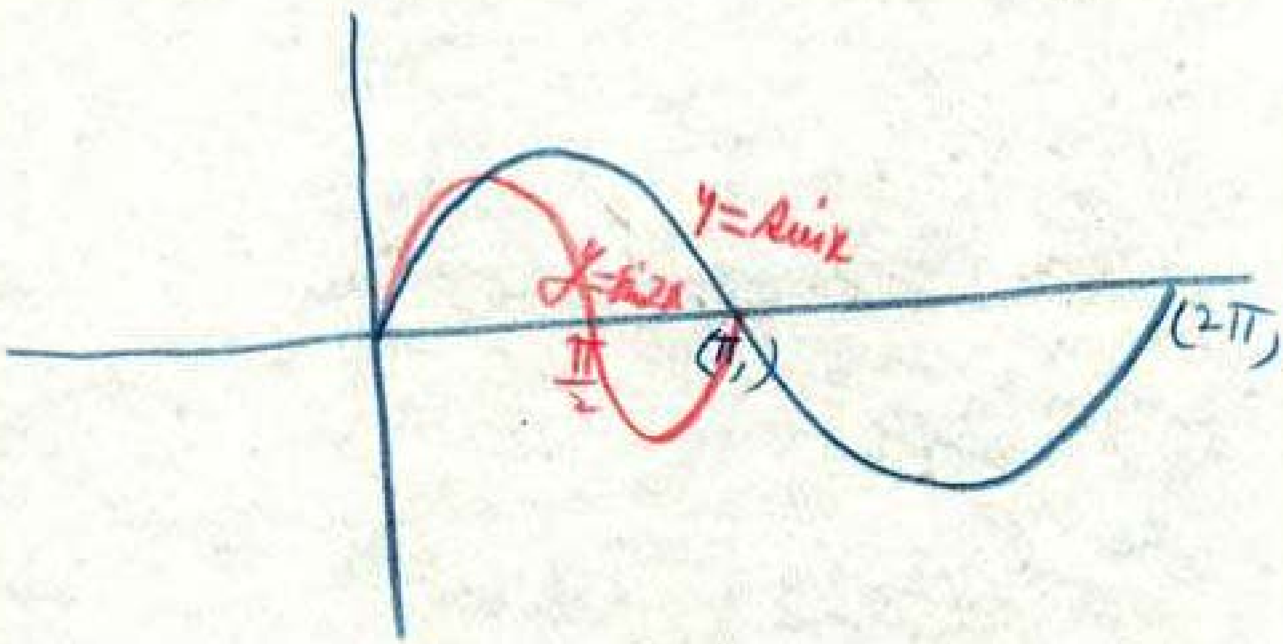
$$= \frac{\frac{1}{4} x}{\sqrt{1 - \frac{x^2}{16}}} + \arcsin \frac{1}{4} x$$

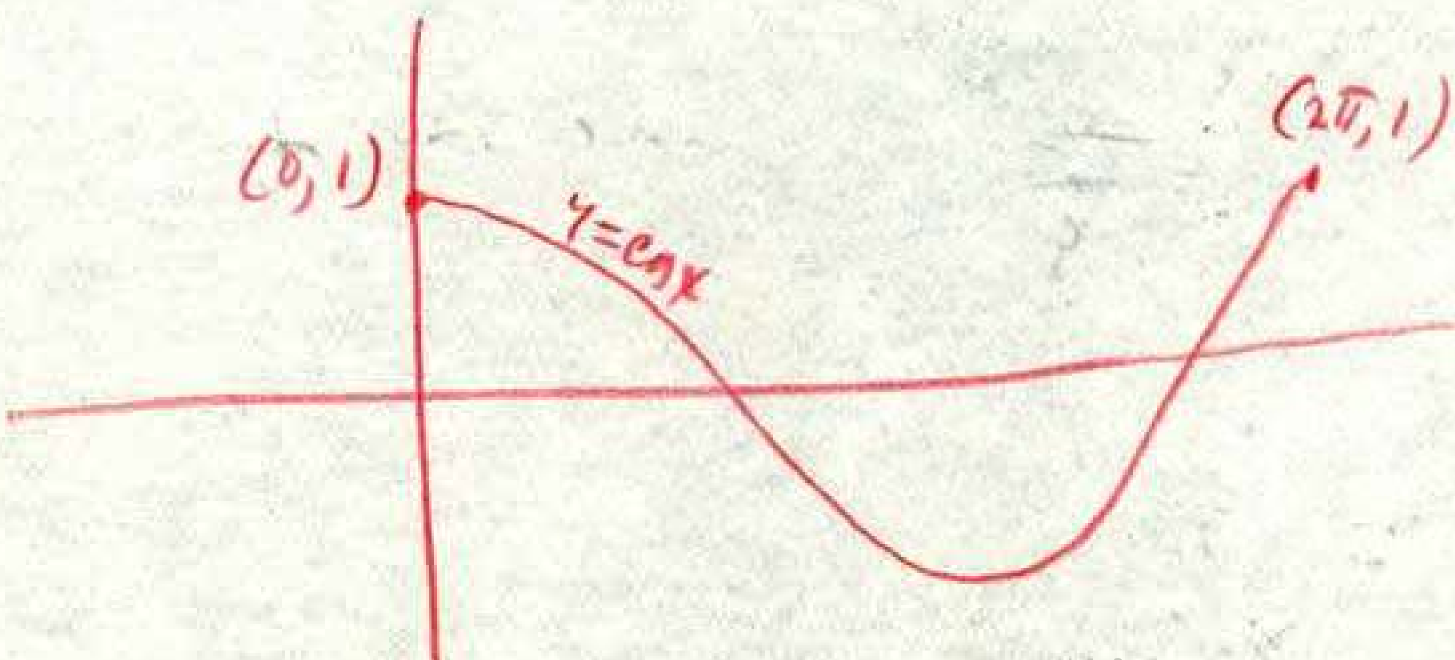
$$= \frac{x}{\sqrt{16 - x^2}} + \arcsin \frac{1}{4} x \ \checkmark$$

When $x = 1$

$$y' = \frac{1}{\sqrt{15}} + \arcsin \frac{1}{4}$$

$$\left.\begin{aligned} \sin(A+2\pi) &= \sin A \\ \cos(A+2\pi) &= \cos A \\ \tan(A+2\pi) &= \tan A \end{aligned}\right\} \text{Period} = 2\pi$$





$$\int \tan u \, du = \ln \sec u + C$$

~~$\frac{d}{du}(\quad) = \tan u$~~

$$\frac{d}{du}(\ln \sec u) = \frac{1 \cdot \cancel{\sec u} \tan u}{\cancel{\sec u}} = \tan u$$

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$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

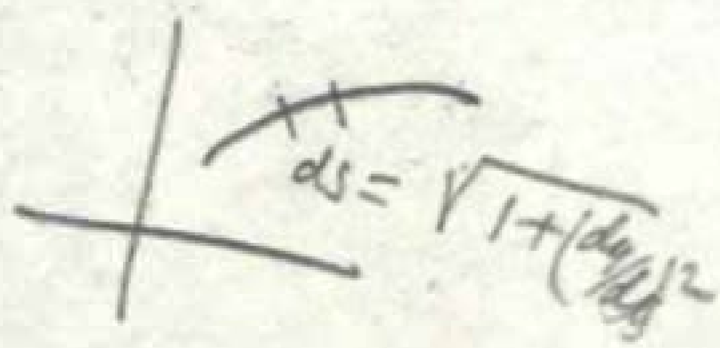
$$\frac{d}{dx}\left(\frac{1}{a}\sin ax\right) = \cancel{\frac{1}{a}}\, a \cdot \cos ax$$

$$\int \cos ax\,dx$$

$u = ax$

$du = a\,dx$

$$= \frac{1}{a}\int \underline{\cos u} \cdot \cancel{\frac{1}{a}}\,\underline{du}$$



$$= \frac{1}{a}\sin u + C$$

$$= \frac{1}{a}\sin ax + C$$

$$\int \underline{\sin^3 2x}\ \underline{\cos 2x\ dx} = \frac{1}{2}\int u^3\,du$$

$u = \sin 2x$

$du = 2\cos 2x\,dx$

$$= \frac{1}{2}\cdot\frac{u^4}{4} + C$$

$$= \frac{1}{8}\sin^4 2x + C$$

323/1b

$\int 2 \sin 3x\,dx = \int \frac{2}{3} \sin u\,du$

Let $u = 3x$
$du = 3\,dx$

$= \frac{2}{3} \int \sin u\,du = -\frac{2}{3} \cos 3x + C$

1f) $\int e^x \sin e^x\,dx = -\cancel{e^x} \cos e^x + C$

1g) $\int \frac{dx}{\sin^2 x} = \int \csc^2 x\,dx = -\operatorname{ctn} x + C$

1j) $\int \operatorname{ctn}^2 x\,dx = \int (\csc^2 x - 1)\,dx = -\operatorname{ctn} x - \overset{x}{\cancel{x}} + C$

$\operatorname{ctn}^2 = \csc^2 - 1$

325) 1 l

$$\int \frac{\sec^2 x\,dx}{2+3\tan x} = \int \frac{\tan^2 x + 1}{3\tan x + 2}(dx) = \int \left(\frac{\frac{\sin^2 x}{\cos^2 x} + 1}{3\frac{\sin x}{\cos x} + 2} \right) dx = \int \frac{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}}{\frac{3\sin x + 2\cos x}{\cos x}} (dx)$$

$u = 2 + 3\tan x$

$du = 3 \sec^2 x \; dx$

$\frac{1}{3}\int \frac{du}{u}$

$= \frac{1}{3} \ln u + c$

$= \frac{1}{3} \ln (2+3\tan x) + c$

$$= \int \frac{1}{\cos^2 x} \cdot \frac{\cos x}{3\sin x + 2\cos x} (dx)$$

$\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$

$$= \int \left(\frac{1}{\cos x} \cdot \frac{1}{3\sin x + 2\cos x} \right) dx$$

$\frac{1}{3\sin x \cos x + 2\cos^2 x}$

$$= \int \left(\frac{1}{3\cos x \sin x} + \frac{1}{2\cos^2 x} \right) dx$$

$$= \int \left(\frac{1}{\frac{3}{2}(\sin 2x)} + \frac{1}{2} \sec^2 x \right) dx$$

$$= \int \left(\frac{2}{3} \csc 2x + \frac{1}{2} \sec^2 x \right) dx$$

$$= \frac{2}{3} \ln (\csc 2x - \operatorname{ctn} 2x) + \frac{1}{2} \tan x + C$$

326) 1.

$$\int (\sec\theta - \tan\theta)^2 \, d\theta = \int (\sec^2\theta - 2\tan\theta\sec\theta + \tan^2\theta)\, d\theta$$

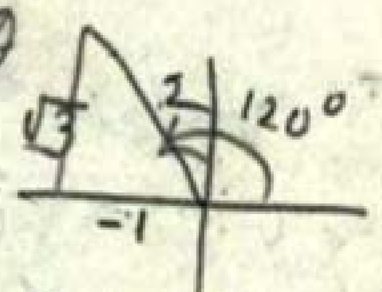
$$= \int (\sec^2\theta - 2\tan\theta\sec\theta + \sec^2\theta - 1)\, d\theta$$

$$= \int \left(2\sec^2\theta - 2\,\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} - 1\right) d\theta$$

$$= \int \left(2\sec^2\theta - 2\,\frac{\sin}{\cos^2\theta} - 1\right) d\theta$$

$$= \int \left(2\cdot\frac{1}{\cos^2\theta} - 2\,\frac{\sin\theta}{\cos^2\theta} - 1\right) d\theta$$

$$= \int \left[2\sec^2\theta\,(1 - \sin\theta) - 1\right] d\theta$$



$$\frac{d}{dx}\left(-\cos\tfrac{1}{3}\pi x\right) = -\left(\tfrac{1}{3}\pi\right)\sin\tfrac{1}{3}\pi x$$

$\cos 120^\circ = -1/2$

326) 1(o)

$$\int (\sec\theta - \tan\theta)^2 d\theta = \int (\sec^2\theta - 2\tan\theta\sec\theta + \tan^2\theta)\, d\theta$$

$$= \int (\sec^2\theta - 2\tan\theta\sec\theta + \sec^2\theta - 1)\, d\theta$$

$$= 2\tan\theta - 2\sec\theta - \theta + C$$

326) 2j $\int_0^2 \sin\left(\frac{1}{3}\pi x\right) dx = -\cos\left(\frac{1}{3}\pi x\right)\Big]_0^2 = \left[-\cos\frac{2\pi}{3}\text{radians} + \cos 0\,\text{radians}\right]$

$\frac{1}{3}\pi x = u$ $\quad \frac{du}{dx} = \frac{1}{3}\pi$

$\frac{1}{3}\pi\, dx = du$

$dx = du / \frac{\pi}{3} = \frac{3}{\pi} du$

$\frac{3}{\pi}\int \sin u\, du = \frac{3}{\pi}(-\cos u)\Big]$

$= -\frac{3}{\pi}\cos\frac{1}{3}\pi x\Big]_0^2$

$\frac{3}{\pi}(0.5 + 1) = \cancel{0.495}$ $\quad (1.5)\frac{3}{\pi}$

326) 2l $\int_1^2 ctn\left(\frac{1}{4}\pi x\right) dx = \frac{4}{\pi}\ln\sin\frac{1}{4}\pi x\Big]_1^2 = \frac{4}{\pi}\left[\ln\sin\frac{\pi}{2} - \ln\frac{\pi}{4}\right]$

$u = \frac{1}{4}\pi x$

$du = \frac{1}{4}\pi\, du$

$\frac{4}{\pi}\int ctn\, u\, du$

$= \frac{4}{\pi}\ln\sin u\Big]$

$(4/\pi)\ln\sin\frac{\pi x}{4}\Big]$

$\frac{4}{\pi}\left[\ln 1 - \ln 0.71\right]$

$= 4/\pi\left[0 - (9.658 - 10)\right]$

$= +(0.342)\, 4/\pi$

326) 2n $\int_0^2 (x + \cos x)\, dx = \frac{x^2}{2} + \sin x\Big]_0^2 = 2 + \sin \cancel{2\ \text{radians}}$

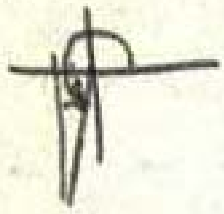
$= 2 + 0.909$

$= \underline{2.909}$

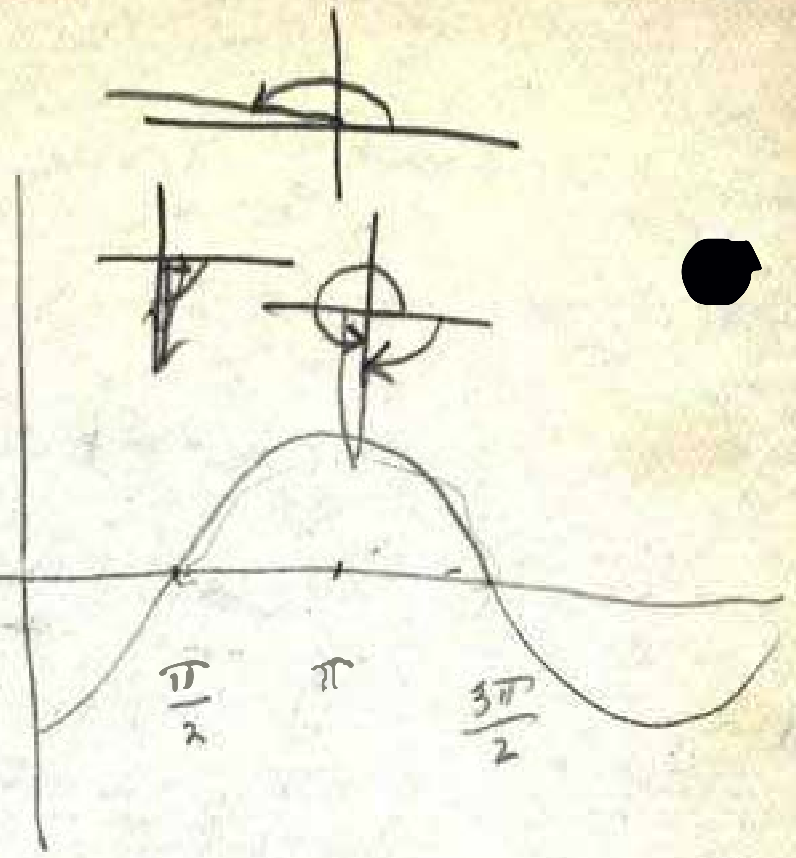
326) 3 b

$y = \cos 2x$

Period $= \frac{2\pi}{2} = \pi$



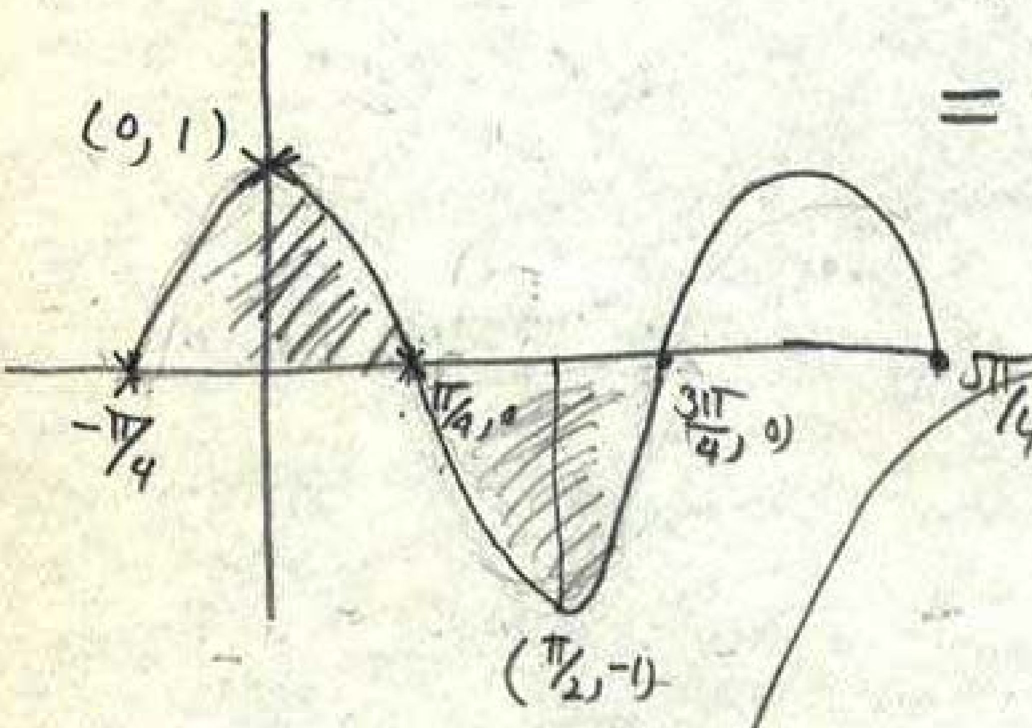
x	y
0	1
$\pi/4$	0
$\pi/2$	-1
$3\pi/4$	0
$-\pi/4$	0



Element of Area $= y\,dx$

Total Area $= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} y\,dx = 2\int_0^{\pi/4} \cos 2x\,dx$

$= 2\frac{1}{2}\sin 2x\Big]_0^{\pi/4}$



$= \cancel{\frac{\sin\frac{3}{2}\pi}{2} - \frac{\sin\frac{1}{2}\pi}{2}}$

$= \cancel{-.5 - .5 = -1.}$

Book gives 1.

$= 1$