Smith, Langley & Wilson

Page 8 No. 1 a) $x^2 - 6 > 3$ $\rightarrow$ $|x| > 3$ $\left\{ \begin{array}{l} x > 3 \checkmark \\ x < -3 \checkmark \end{array} \right.$

$x^2 > 9$ (add 6 to each side)

$\therefore$ if $x > 3$, $x^2 - 6 \geq 3$

~~$-3 < x < 3$~~ ~~$-3 > x > 3$~~

e)

$x^2 < x + 6$

$x^2 - x < 6$

$x^2 - x + \frac{1}{4} < 6\frac{1}{4}$

$(x - \frac{1}{2})^2 < 6\frac{1}{4}\ (\frac{25}{4})$

$(x - \frac{1}{2}) < \pm\frac{5}{2}$

then $x < 3$

$\therefore$ if $x < 3$, $x^2 < x + 6$

or if $x > -2$, $x^2 < x + 6$

$x - \frac{1}{2} < \frac{5}{2} \checkmark$ $x < 3$

$|x - \frac{1}{2}| < \frac{5}{2}$

$-x + \frac{1}{2} < \frac{5}{2} \checkmark$ $-2 < x$ $x > -2$

$(-2 < x < 3)$

c) $\underbrace{(x-4)(x+5)}$ > 0

Then x must be > 4

or x .... $(5 - 5 = 0)$ < -5

~~$x+5>0$~~ $x + 5 < 0$

$x - 4 > 0$ ~~$x-4<0$~~

Page 8 No. 2. a Find roots a) $|x - 3| = 1$

If x is positive (greater than 3), then $x = 1 + 3 = 4$

If x is negative (less than 3), x = ~~[illegible]~~ $= 3 - 1 = 2$

$4 - 3 = 1$

$2 - 3 = -1$

* Not completely clear (absolute values & roots)

2 c) $|x|^2 + 5|x| + 6 = 0$ (possible to factor)

$(|x| + 3)(|x| + 2) = 0$

If $|x| + 3 = 0$, $|x| = -3$ ✗

If $|x| + 2 = 0$, $|x| = -2$ ✗

If $x > 0$, this is impossible.

Consult Mr. Morris about this

Page 8 No. 3a

$-5 < 7$

$-5 > 7$

x

$|x-3| < 1$

~~Then $x < 4$~~

$x-3 < 1$ if + no.

& $3-x < 1$ if neg. no

$x-3$

$x-3 < 1$

$x < 4$

$3-x < 1$

$-x < -2$ ✓

$x > 2$

$2 < x < 4$

$4 > x > 2$

1) ~~$x-3 > 0$~~

~~then $x > 4$~~

1) $x-3 > 0$, $x-3 < 1$, $x < 4$

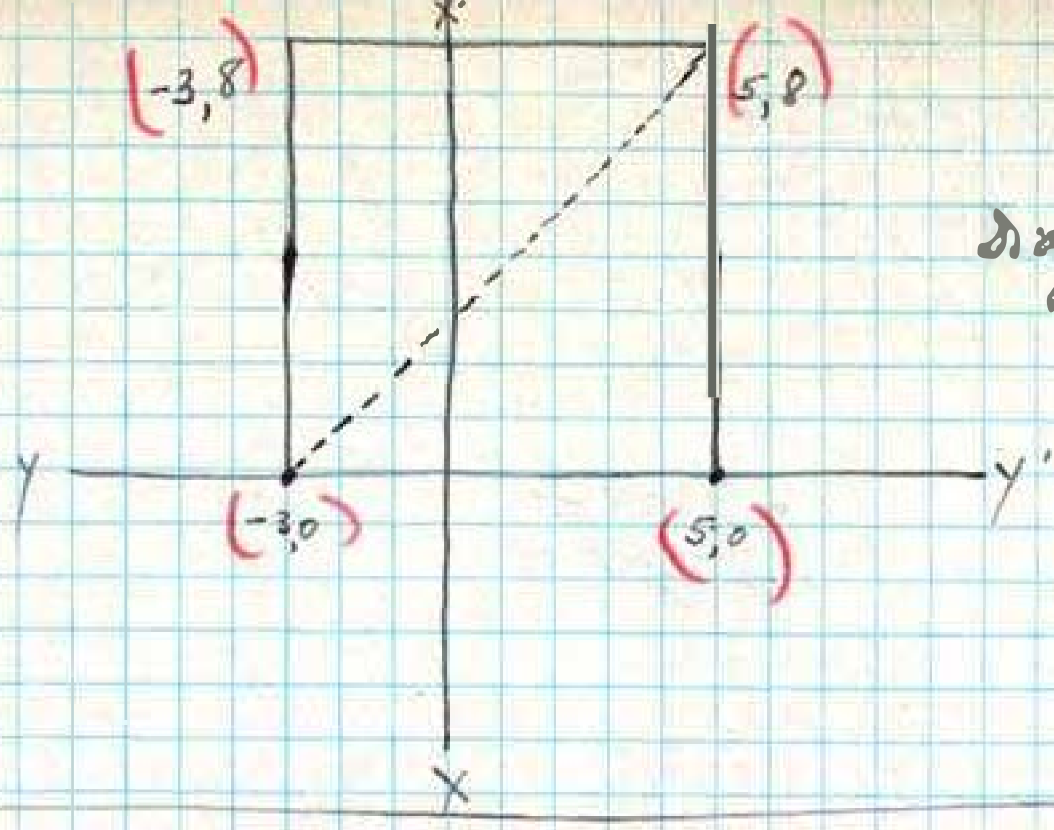
2) $x-3 < 0$ $\quad x > 2$

$(3 < x < 4)$

$(2 < x < 3)$

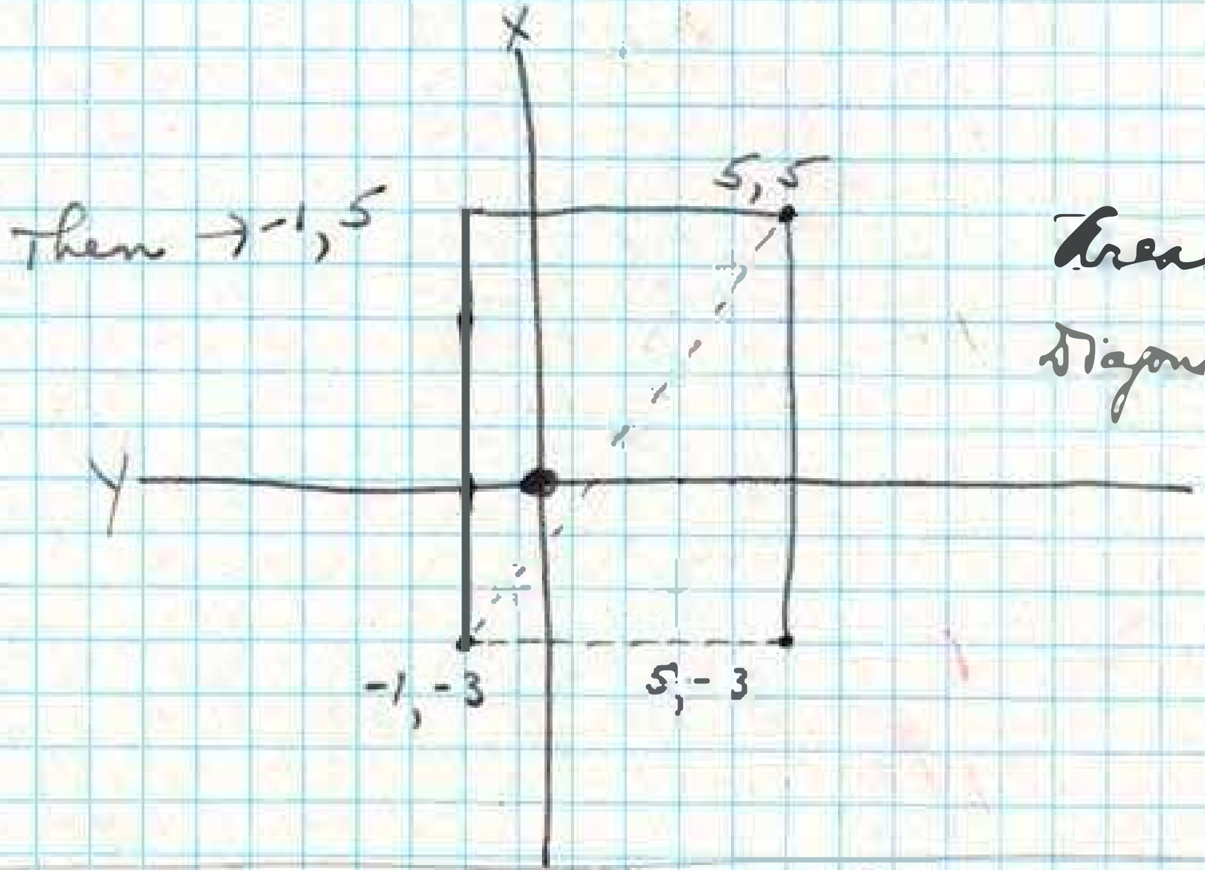
L. S. Wilson

Page 8 No. 7



Diagonal $= \sqrt{64+64}$
$= \sqrt{128}$
$= 8\sqrt{2}$

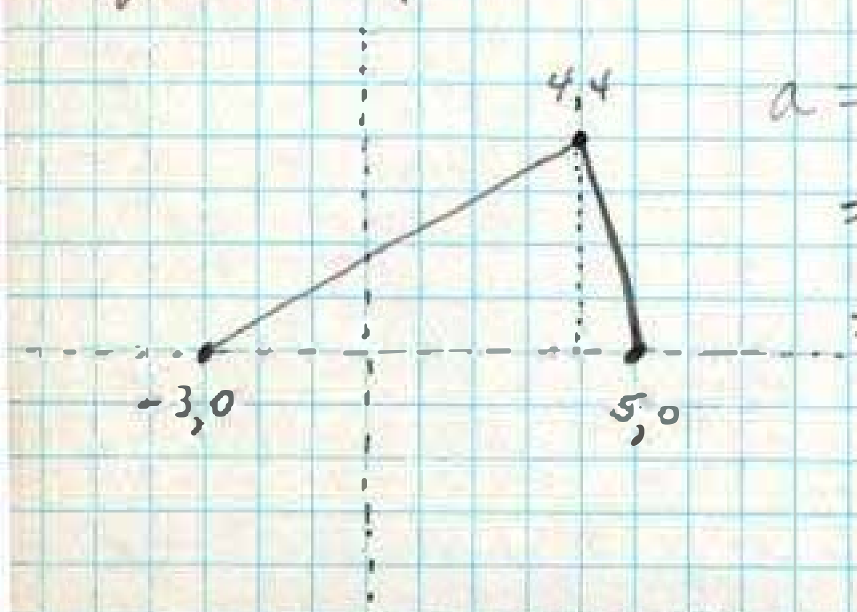
Page 8 No. 8



Area $= 6 \times 8 = 48$

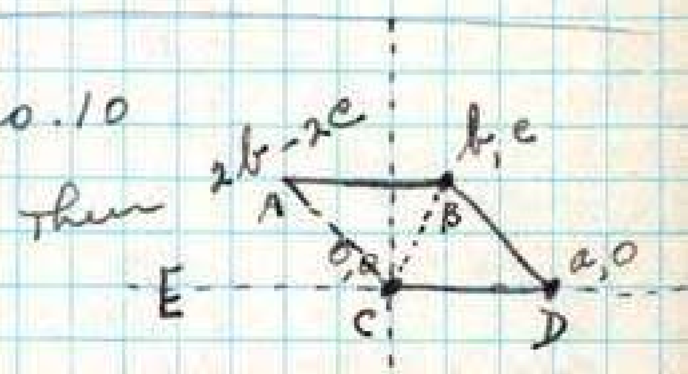
Diagonal $= \sqrt{36+64}$
$= \sqrt{100}$
$= 10$

Page 8 No 9 c



$a = \frac{1}{2} ba$
$= \frac{8 \times 4}{2}$
$= 16$

Page 8 No. 10



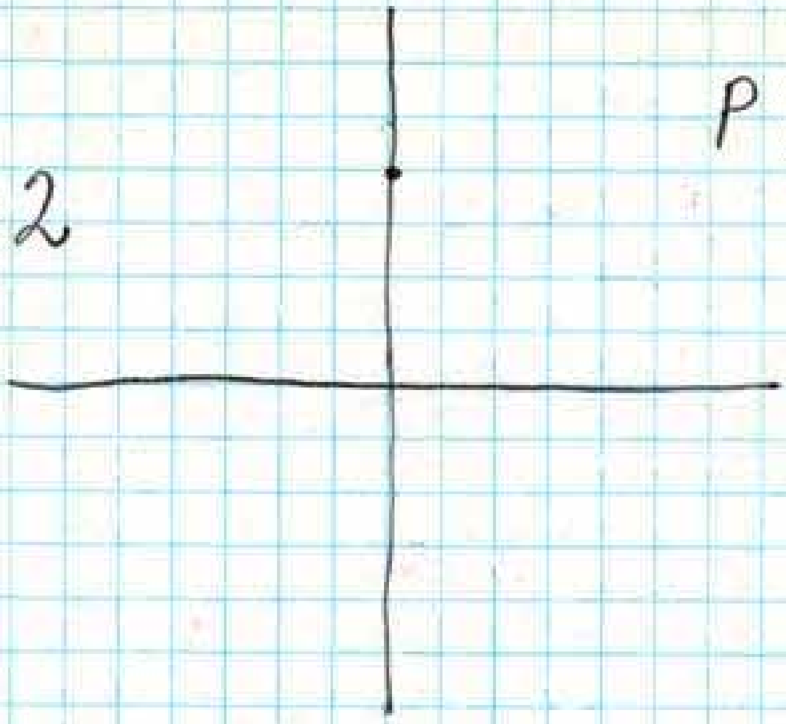
$\angle ABC = \angle BCD$
BC is common side
$\angle ACE = BDC$
$\cancel{\angle BDC} + \angle BCD + \angle CBD = 180° = \cancel{\angle ACE} + \angle ACB + \cancel{\angle BCD}$
$\therefore \angle CBD = \angle ACB$

2 triangles are equal if one side and
The adjacent angles are equal

Page 9 c, e, 8

c

$|x-2| \leqq 2$



$P(x, y) =$

L.S.W.

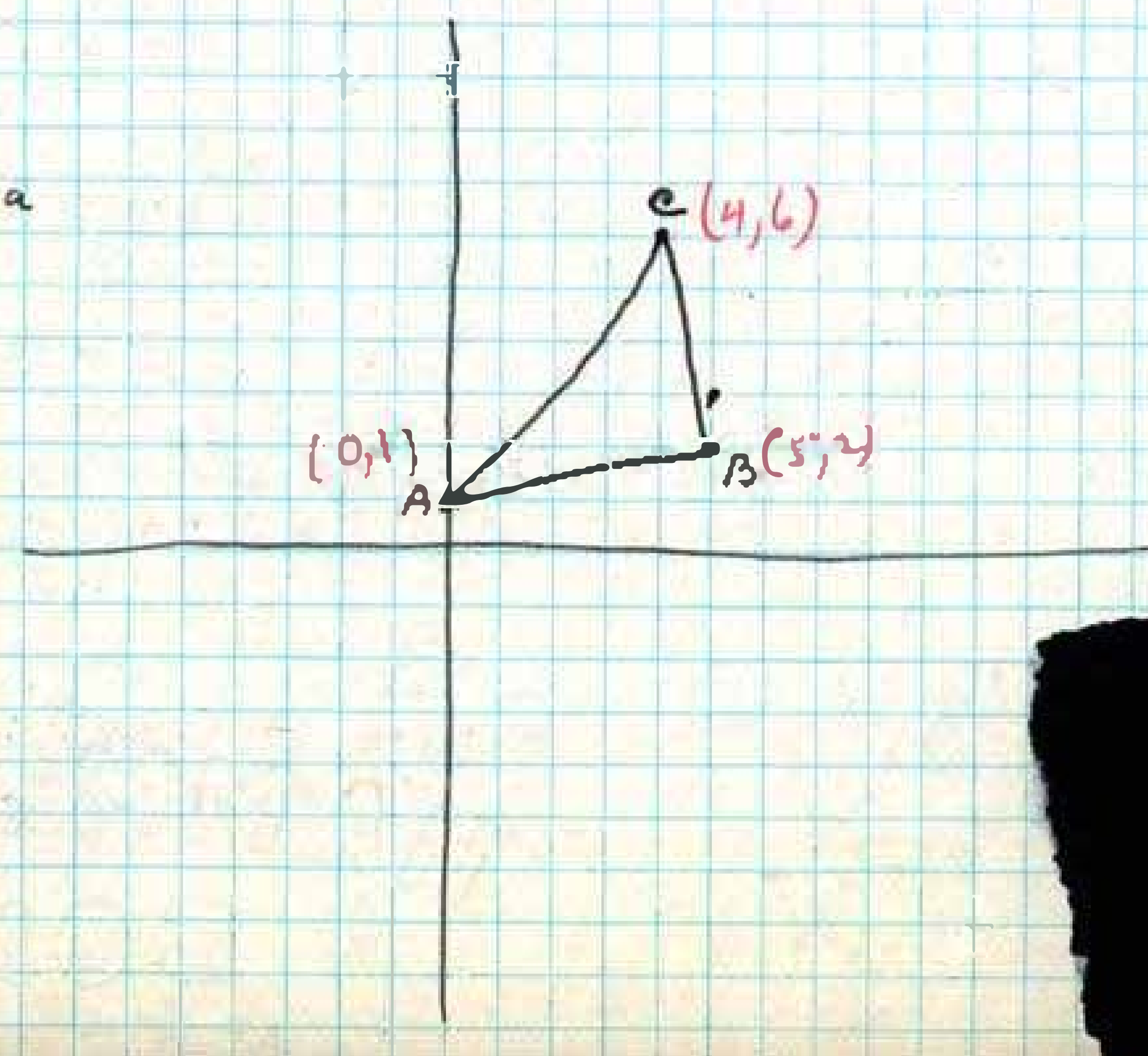
Page 16 1a, 3, 4

17 9, 10, 15, 16

Page 24 1a, 3c, 5, 8, 9, 10, 13a,

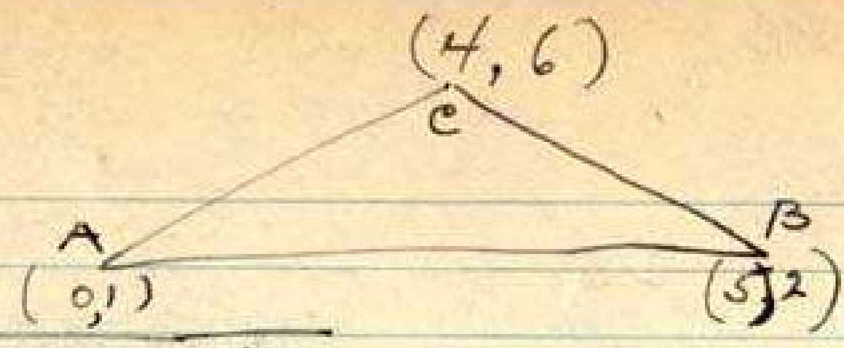
25 16, 18, 22

Page 16 1a



L.S., W

Page 16 (1a)



$$d = \sqrt{(x_2 - x_1)^2 + (y^2 - y^1)^2}$$

$$AB = \sqrt{(5-0)^2 + (2-1)^2} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

$$BC = \sqrt{(5-4)^2 + (2-6)^2} = \sqrt{1+16} = \sqrt{17}$$

Page 16 No. 4 (c)

Coordinates of Q

$$x = \frac{x_1 + x_2}{2} = \frac{12}{2} = 6$$

$$y = \frac{y_1 + y_2}{2} = \frac{5+1}{6} = 3$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Then $QC = \sqrt{(12-6)^2 + (5-3)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$\therefore QC = \frac{AC}{2}$

Then $QD = \sqrt{(6-5)^2 + (3-6)^2}$

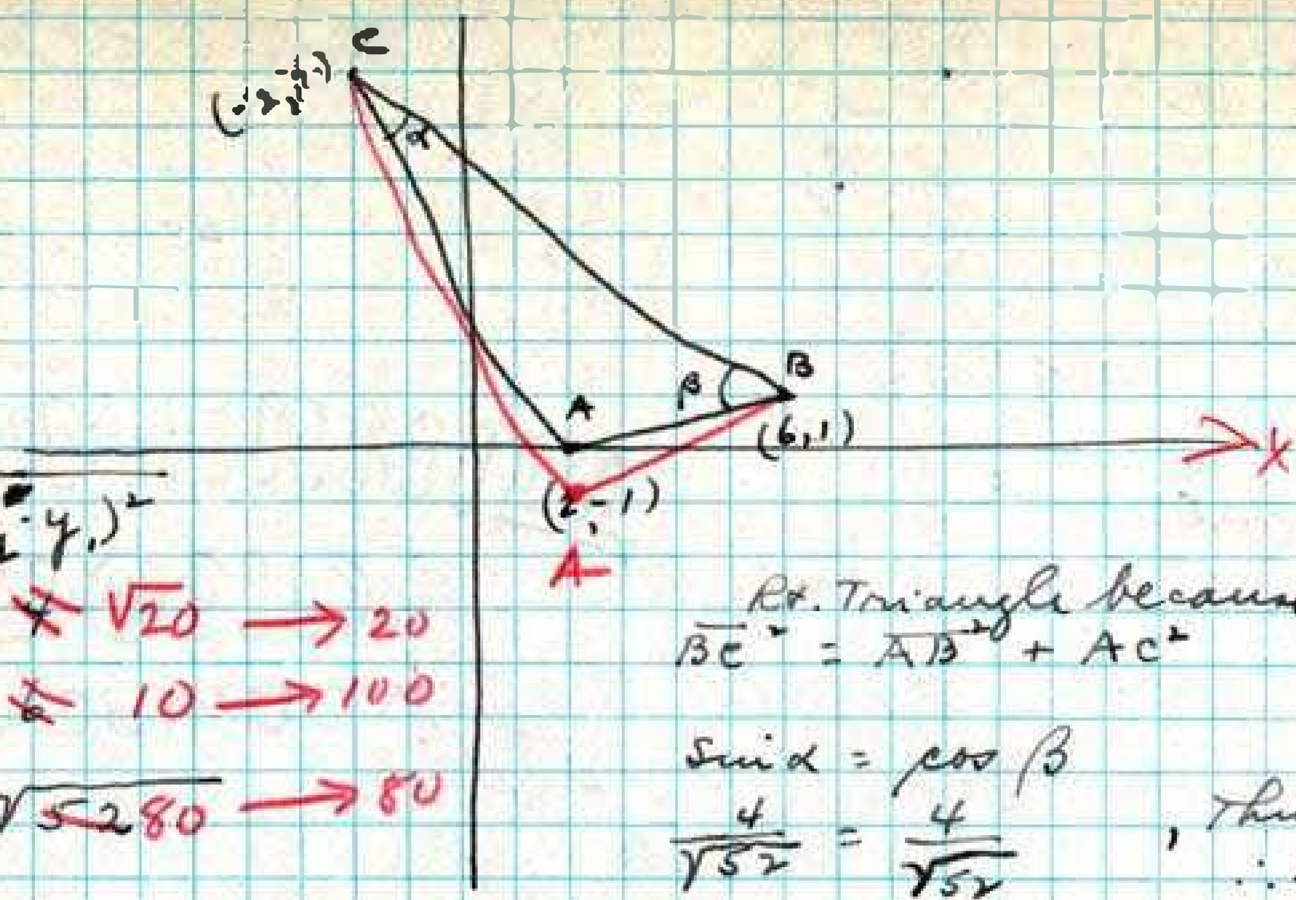
$= \sqrt{1+9}$

$= \sqrt{10}$

$\therefore QD = \frac{BD}{2}$

L, S, + W

Page 16 No. 3



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{16 + \cancel{9}4} = \cancel{7}\ \sqrt{20} \longrightarrow 20$

$\cancel{A}B C = \sqrt{\cancel{0}_{64} + 36} = \cancel{6}\ 10 \longrightarrow 100$

$\cancel{A}A C = \sqrt{16 + \cancel{36}_{64}} = \sqrt{\cancel{52}80} \longrightarrow 80$

Rt. Triangle because

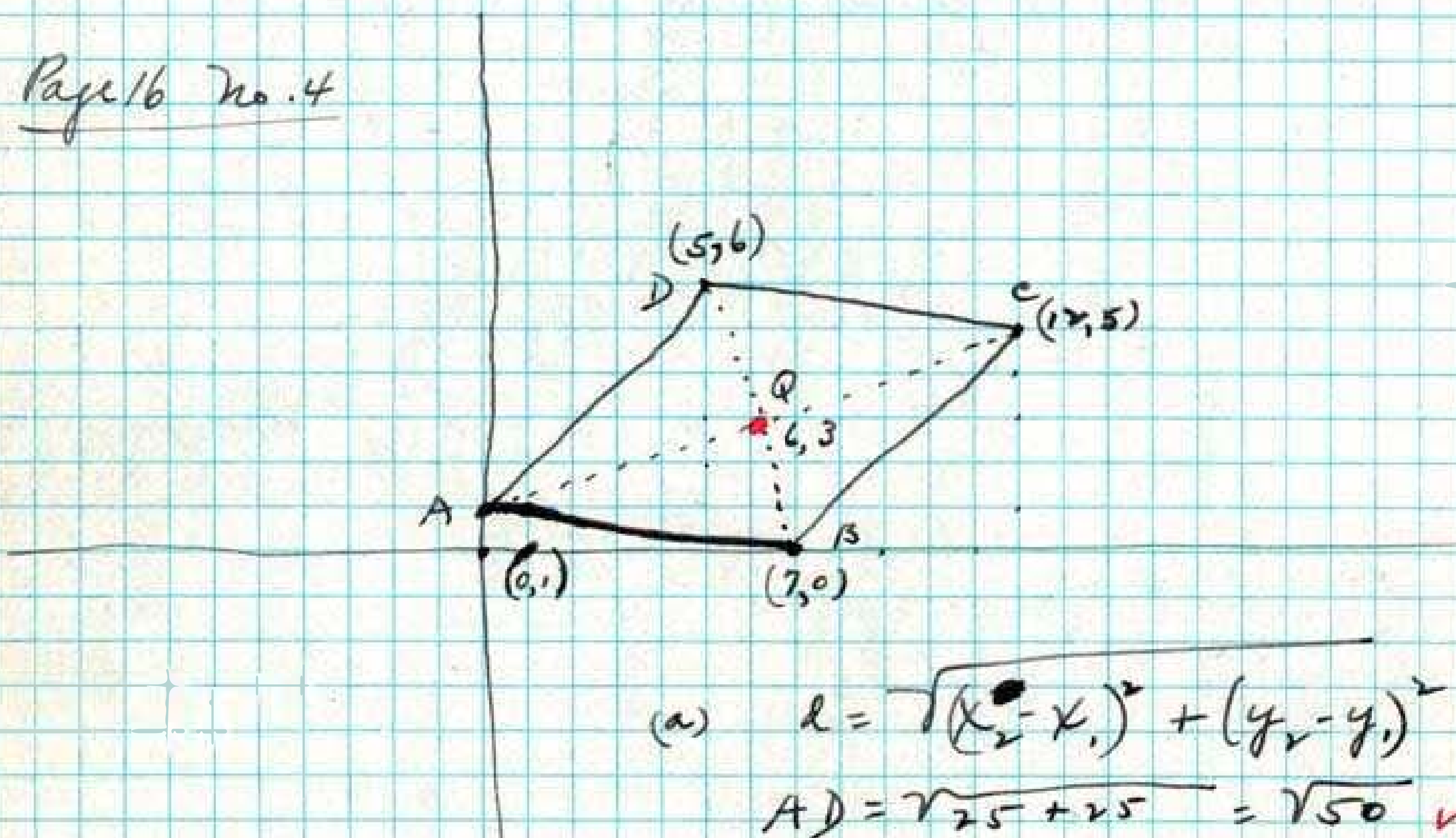
$\overline{BC}^2 = \overline{AB}^2 + AC^2$

$\sin \alpha = \cos \beta$

$\frac{4}{\sqrt{52}} = \frac{4}{\sqrt{52}}$, Thus $\alpha + \beta = 90°$

$\therefore \angle BAC = 180° - 90° = 90°$

Page 16 No. 4



(a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AD = \sqrt{25 + 25} = \sqrt{50}$ ✓

$AB = \sqrt{49 + 1} = \sqrt{50}$ ✓

$BC = \sqrt{25 + \sqrt{25}} = \sqrt{50}$ ✓

$CD = \sqrt{49 + 1} = \sqrt{50}$ ✓

$\therefore$ Figure is parallelogram

b) $AC = \sqrt{12^2 + 4^2} = \sqrt{160} = 4\sqrt{10}$ ✓

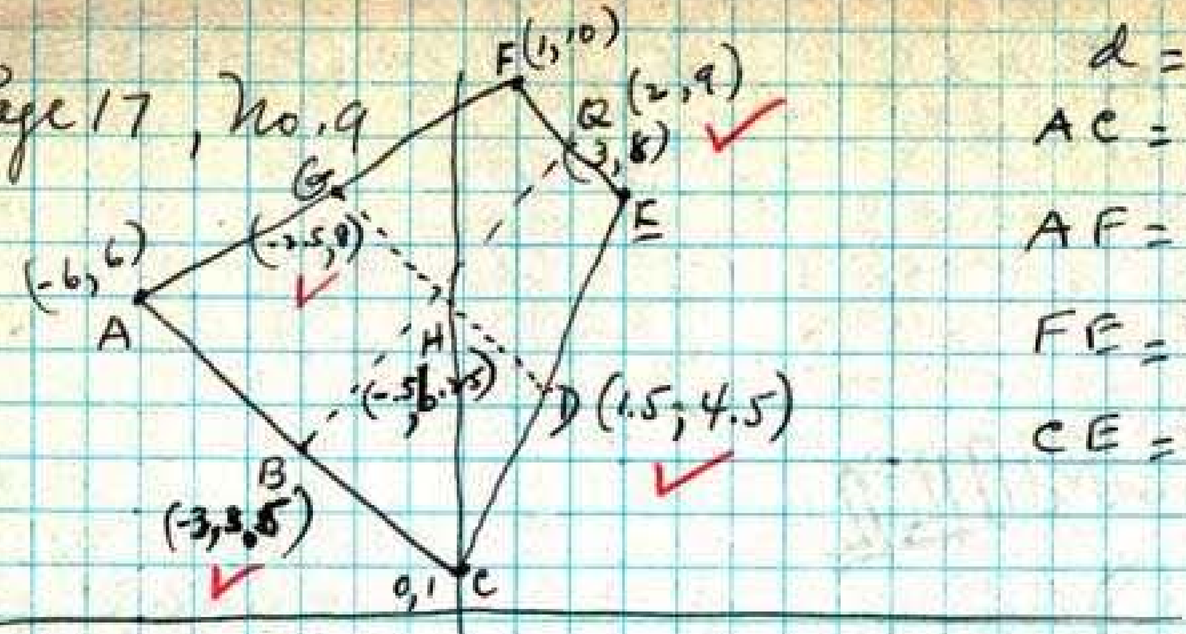
$BD = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$ ✓

Thus $AC = 2BD$

c) $AQ = CQ$
+ $DQ = BQ$ } because $\triangle ADQ = \triangle ABQ$ (the 3 ∠s are equal)

← see

Page 17, No. 9



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AC = \sqrt{36+25} = \sqrt{61}$

$AF = \sqrt{49+16} = \sqrt{65}$

$FE = \sqrt{4+4} = \sqrt{8}$

$CE = \sqrt{9+49} = \sqrt{58}$

not necessary for problem

Coordinates of B

$x = \frac{x_2 + x_1}{2}$ $\qquad y = \frac{y_2 + y_1}{2}$

$= \frac{-6}{2} = -3$ $\qquad y = \frac{6+1}{2} = 3.5$

Coordinates of G $\left\{ \begin{array}{l} x = -2.5 \\ y = 8 \end{array} \right.$

" $\quad$ Q $\{ x = 2, \ y = 9$

" $\quad$ D $\{ x = 1.5, \ y = 4.5$

Then Coordinates of H are

$x = -\frac{1}{2}, \ y = 6\frac{1}{4}$ (from BQ) | $x = -\frac{1}{2} \quad y = 6\frac{1}{4}$ from DG

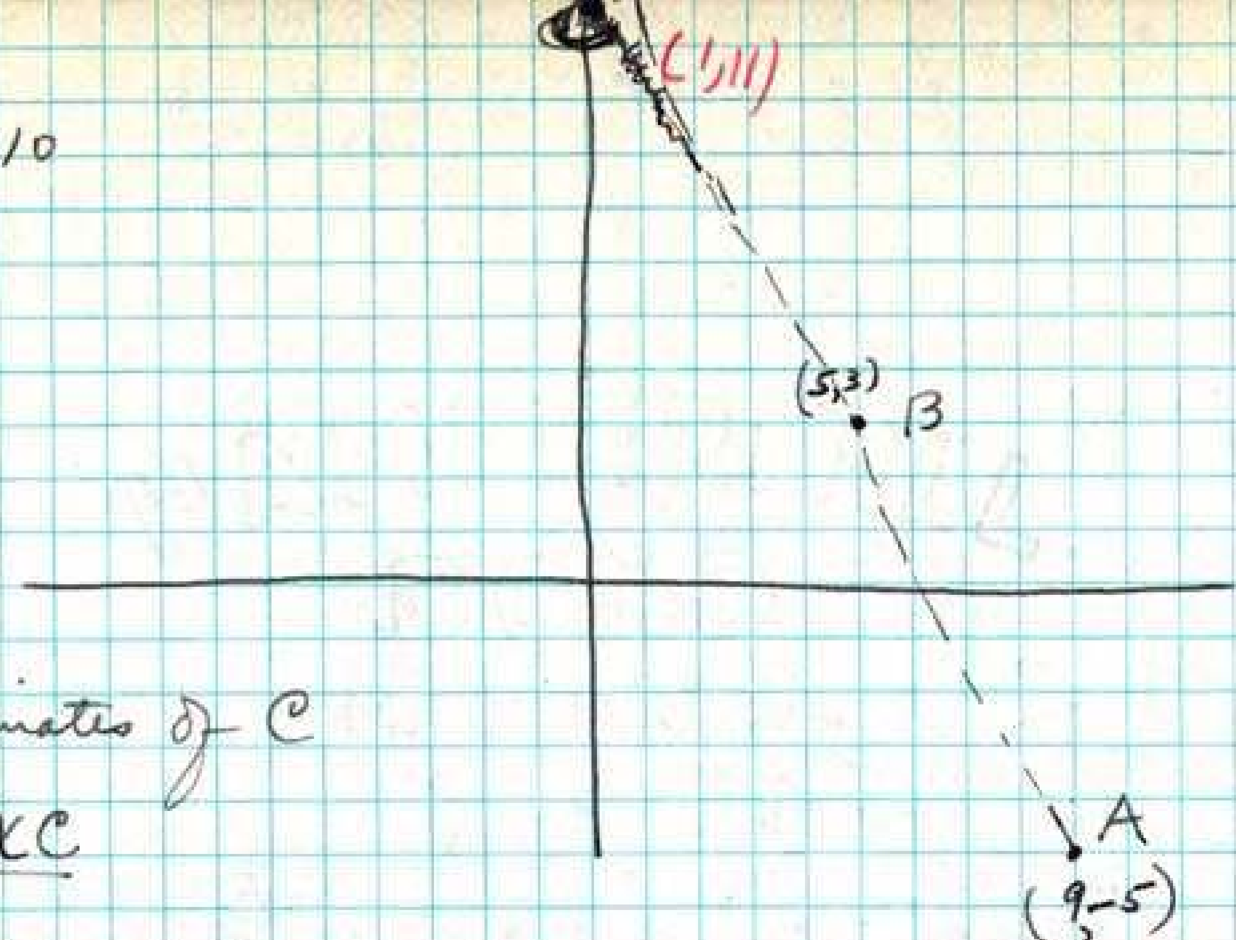
Then $BH = \sqrt{(-3 - -.5)^2 + (3.5 - 6.25)^2} = \sqrt{(-2.5)^2 + (2.75)^2} =$

$QH = \sqrt{(2-(-.5))^2 + (9-6.25)^2} = \sqrt{(2.5)^2 + (2.75^2)} =$ $\Big\}$ each other

$DH = \sqrt{(1.5-(-.5))^2 + (4.5-6.25)^2} = \sqrt{2^2 + (-1.75)^2} =$

$GH = \sqrt{[-2.5 - (-.5)]^2 + (8-6.25)^2} = \sqrt{(-2)^2 + (1.75)^2} =$ $\Big\}$ each other

Page 17 No. 10



To find coordinates of C

$$xB = \frac{xA + xC}{2}$$

or $xC = 2xB - xA$

$= 10 - 9 = 1$

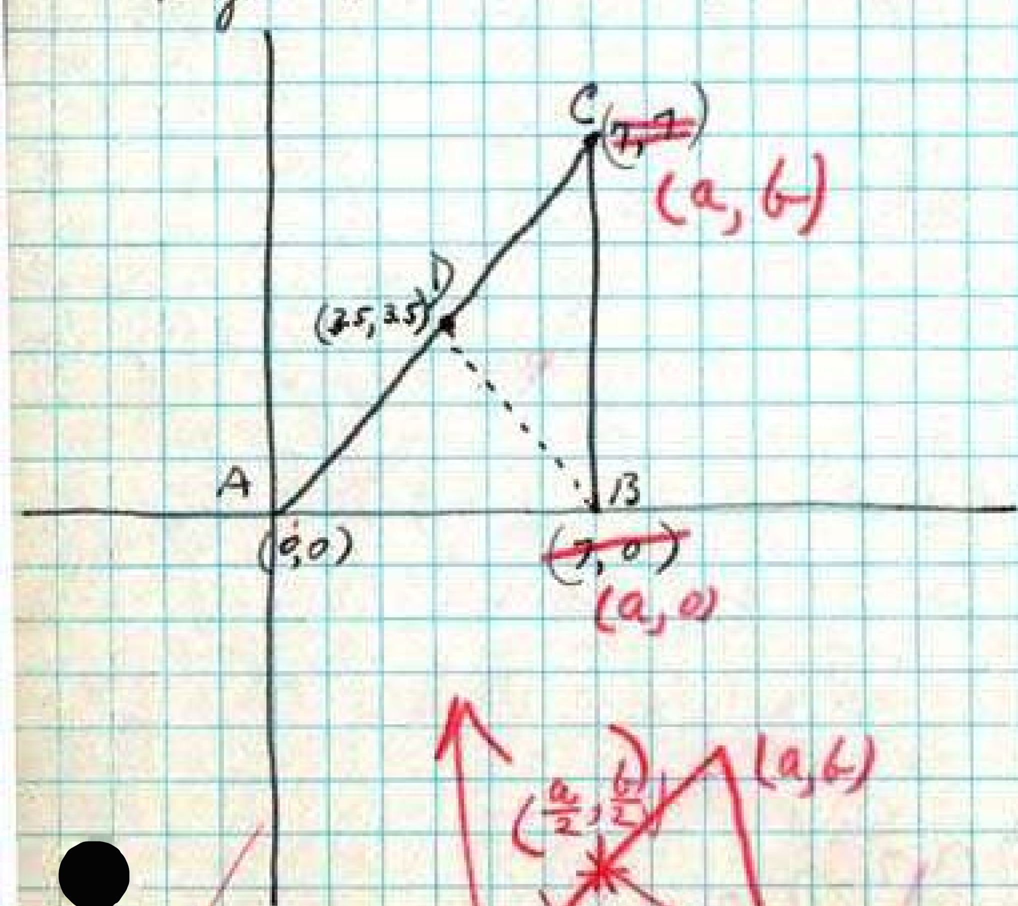
$\therefore$ Coordinates of $C = (1, 11)$

$$yB = \frac{y_A + y_C}{2}$$

$yC = 2yB - yA$

$= 6 - (-5) = 11$

Page 17 No. 15



To prove $AD = CD = BD$

Coordinates of $D \rightarrow x = 3.5 \quad y = 3.5$

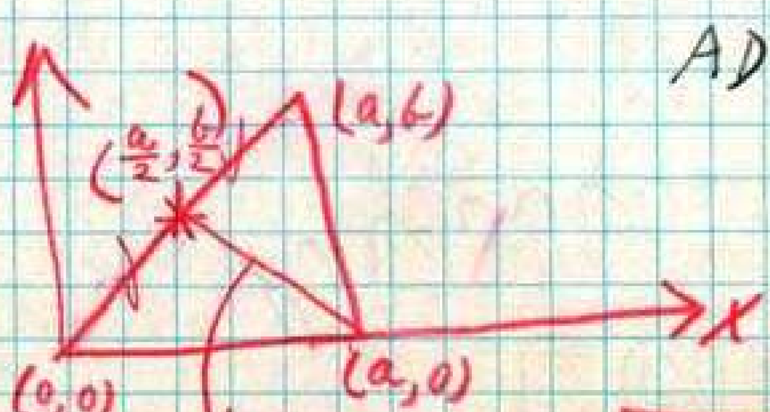
$AD = CD$ (given)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$BD = \sqrt{(7-3.5)^2 + (0-3.5)^2} = \sqrt{3.5^2 + 3.5^2}$

$CD = \sqrt{(7-3.5)^2 + (7-3.5)^2} = \sqrt{3.5^2 + 3.5^2}$

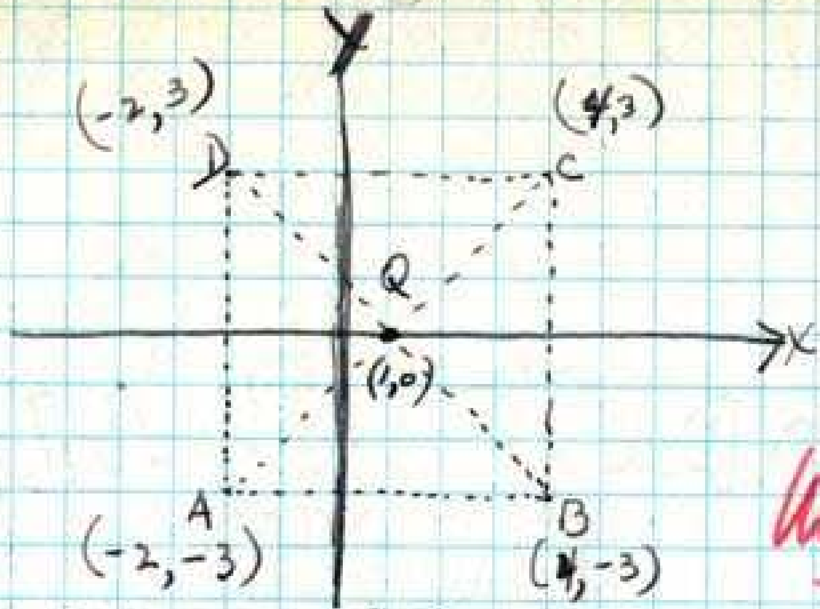
$AD = \sqrt{3.5^2 + 3.5^2}$



$\sqrt{a^2 + b^2}$

$\sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \sqrt{\frac{a^2 + b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2}$

Pg. 17, No. 16



Use letters.

To prove $DQ = BQ$ + $AQ = CQ$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{[4-(-2)]^2 + [3-(-3)]^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$BD = \sqrt{[4-(-2)]^2 + [-3-3]^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

Coordinates of Q are

$$\left(x = \frac{x_2 + x_1}{2}, \; y = \frac{y_2 + y_1}{2}\right)$$

$$x = \frac{4-2}{2} \qquad y = \frac{3-3}{2}$$

$$x = 1 \qquad y = 0$$

Then

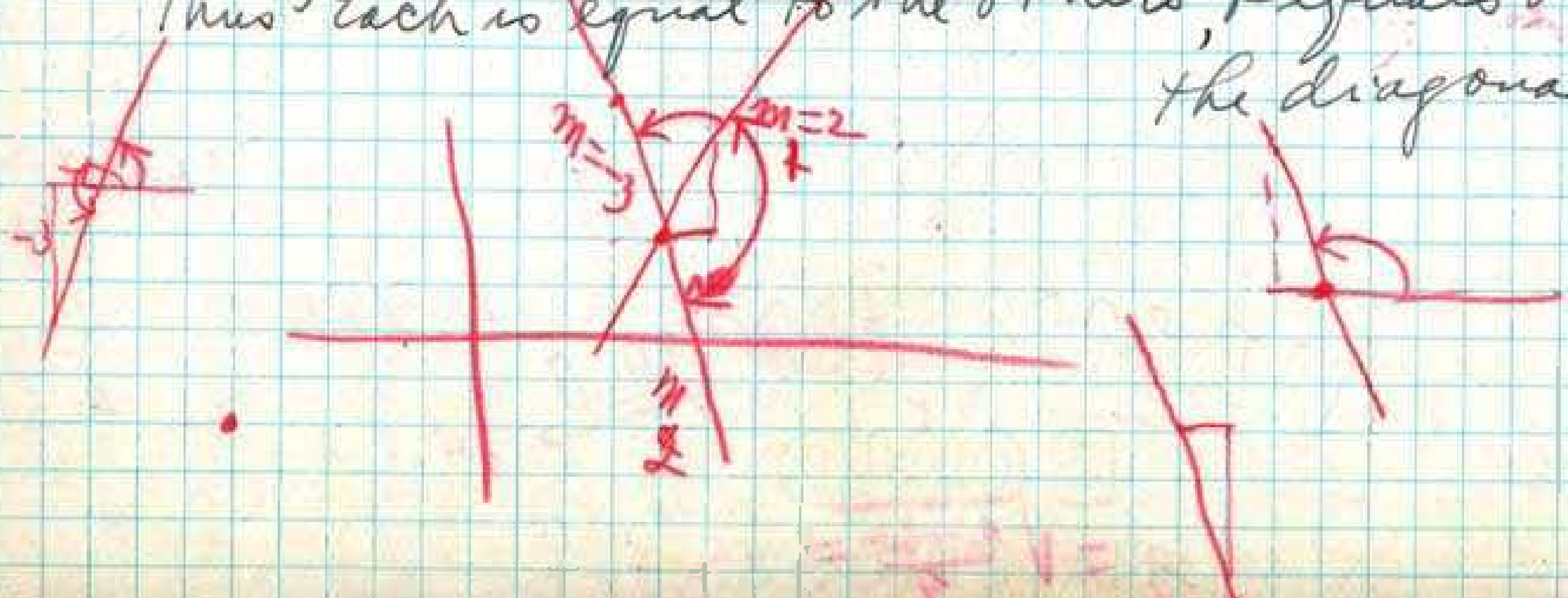
$$DQ = \sqrt{(-2-1)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BQ = \sqrt{9+9} = 3\sqrt{2}$$

$$CQ = \sqrt{9+9} = 3\sqrt{2}$$

$$AQ = \sqrt{9+9} = 3\sqrt{2}$$

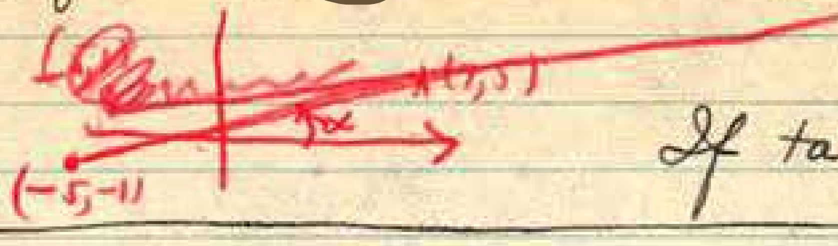
Thus Each is equal to the others, & equals one half of the diagonal.



S, L, r Wilson

Page 24 (1a)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(5-(-1))}{(7-(-5))} = \frac{6}{12} = \frac{1}{2}$$



If $\tan\alpha = 1/2$, $\alpha = 26°34'$

(3e)

$m_1 = 2$
$m_2 = -3$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{2-(-3)}{1 + 2\cdot(-3)}$$

ask about the (−1) →

$$= \frac{5}{-5} = -1$$

~~$\therefore \theta = 135°$~~

(No. 5)

$\theta = 45°$
$m_1 = 3$
To find m_2

$$\tan\theta(1 + m_1 m_2) = m_1 - m_2$$

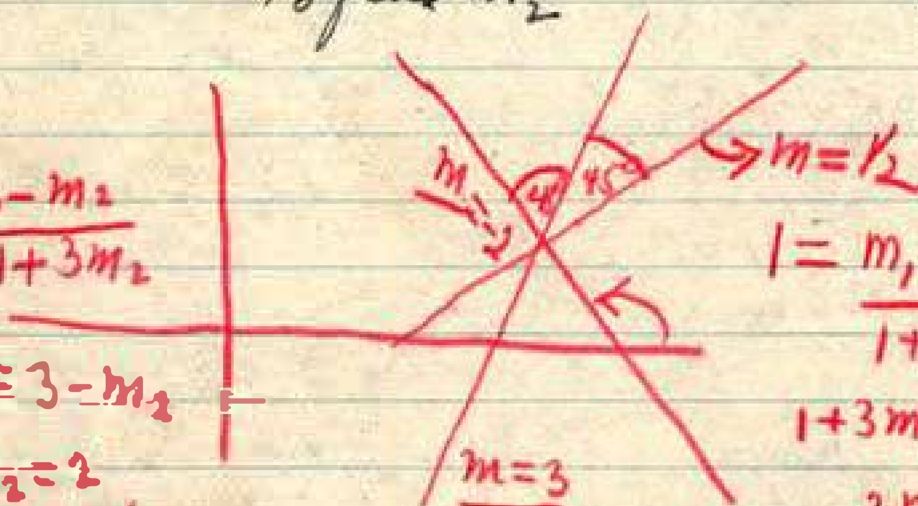
$$\tan\theta + \tan\theta\, m_1 m_2 = m_1 - m_2$$

$$\tan\theta\, m_1 m_2 + m_2 = m_1 - \tan\theta$$

$$m_2(\tan\theta\, m_1 + 1) = m_1 - \tan\theta$$

$$m_2 = \cancel{\frac{\tan\theta\, m_1 + 1}{m_1 - \tan\theta}}\ \frac{m_1 - \tan\theta}{m_1 \tan\theta + 1}$$

$$= \frac{1\cdot 3 + 1}{3 - 1} = \frac{3+1}{2} = \frac{4}{2} = 2$$



$$1 = \frac{3 - m_2}{1 + 3m_2}$$

$$1 + 3m_2 = 3 - m_2$$

$$4m_2 = 2$$

$$m_2 = 1/2$$

$$1 = \frac{m_1 - 3}{1 + 3m_1}$$

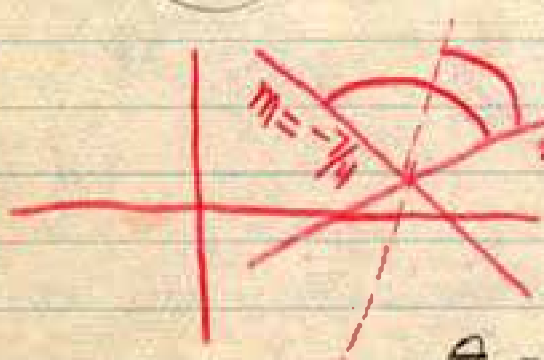
$$1 + 3m_1 = m_1 - 3$$

$$2m_1 = -4$$

$$m_1 = -2$$

(No. 8)

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-\frac{7}{4} - \frac{1}{8}}{1 + \left(-\frac{7}{4}\cdot\frac{1}{8}\right)} = \frac{\frac{-15}{8}}{1 - \frac{7}{32}} = \frac{\frac{-15}{8}}{\frac{25}{32}}$$



$$= -\frac{\cancel{15}^{3}}{\cancel{8}} \cdot \frac{\cancel{32}^{4}}{\cancel{25}_{5}} = \frac{-12}{5}$$

$\theta = 67°23'$

$\frac{\theta}{2} = 33°41'30''$ $\quad \tan\frac{\theta}{2} = \frac{2}{3}$

Book gave 2 without minus sign

Question: This not possible because my answer must be part of line!

$$m_3 = \frac{\tan\theta\, m_1 + 1}{m_1 - \tan\theta} = \frac{\left(\frac{2}{3}\cdot\frac{1}{8}\right) + 1}{\frac{1}{8} - \frac{2}{3}} = \frac{\frac{1}{12} + 1}{\frac{1}{8} - \frac{2}{3}} = \frac{13}{12} \cdot \frac{-24}{13} = -2$$

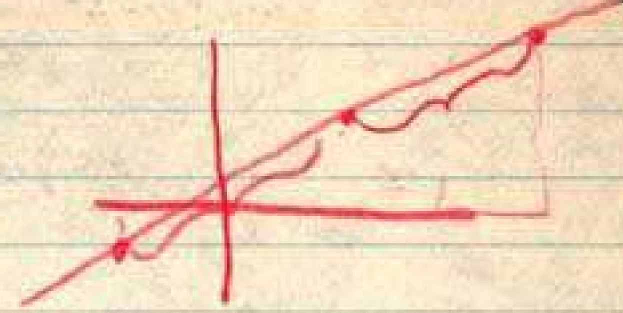
or [illegible]

Page 24 No. 9

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-(-1)}{3-(-3)} = \frac{2+1}{3+3} = \frac{1}{2} \checkmark$$

Points $(-3,-1)$
$(3,2)$
$(7,4)$

$$= \frac{4-2}{7-3} = \frac{1}{2}$$

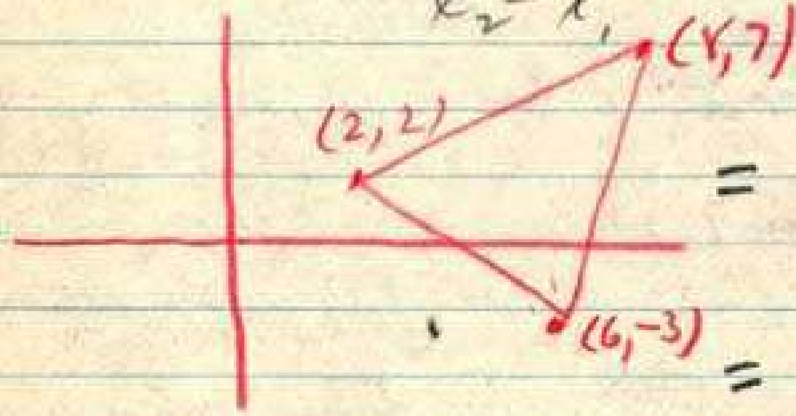


$$= \frac{4-(-1)}{7-(-3)} = \frac{1}{2}$$

Since slopes of lines joining any two of given three points are equal, the points must be on same straight line

Page 24 (No. 10) Points $(6,-3)(8,7)(2,2)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-(-3)}{8-6} = 5 \checkmark$$



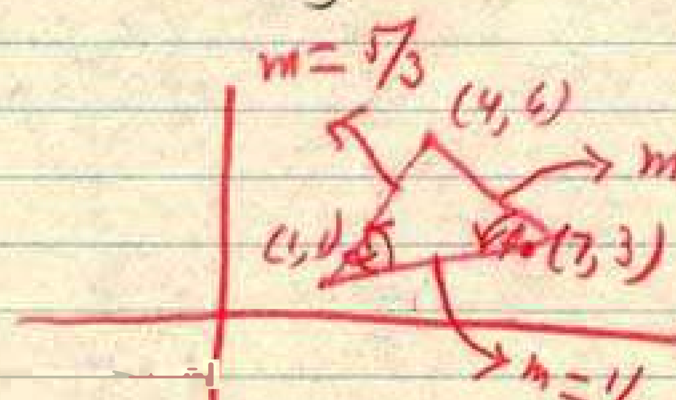
$$= \frac{2-7}{2-8} = \frac{5}{6} \checkmark$$

$$= \frac{2-(-3)}{2-6} = \frac{5}{-4} = -\frac{5}{4} \checkmark$$

The above points are not the vertices of a right triangle because no one line is perpendicular to either one of the other two (no slope of one is the negative reciprocal of either of the other two)

Page 24 No. 12(a)

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{6-3}{4-7} = \frac{3}{-3} = -1 \checkmark$$



$$= \frac{1-6}{1-4} = \frac{-5}{-3} = \frac{5}{3} \checkmark$$

$$\tan\alpha = \frac{m_2-m_1}{1+m_2m_1}$$

$$= \frac{1-3}{1-7} = \frac{-2}{-6} = \frac{1}{3} \checkmark$$

$$\tan\alpha = \frac{5/3 - 1/3}{1+5/9}$$

$$\tan\beta = \frac{1/3+1}{1+(\frac{-1}{3})}$$

Page 25 No. 13a Prove that points $(-6,0)$ $(0,-6)$ $(8,6)$ $(2,12)$ are vertices of parallelogram.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-0}{0-(-6)} = -1$$

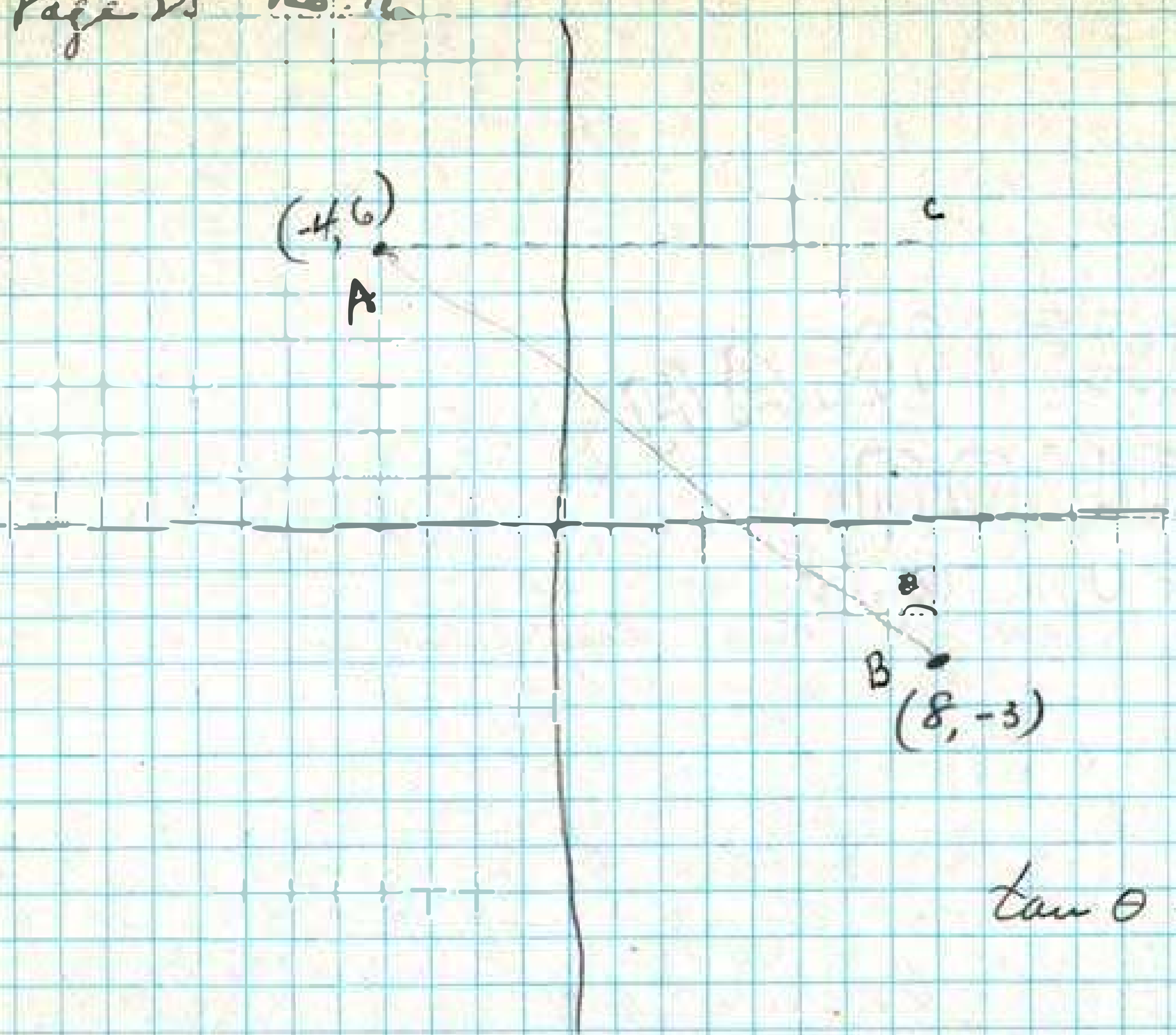
$$= \frac{6+6}{8-0} = \frac{3}{2}$$

$$= \frac{12-6}{2-8} = -1$$

$$= \frac{12-0}{2-(-6)} = \frac{12}{8} = \frac{3}{2}$$

these two lines must be parallel because their slopes are equal.

Page 25 No. 16



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$mAB = \frac{6-(-3)}{-4-8} = \frac{9}{-12} = -\frac{3}{4}$$

$\angle ACB = 90°$

$AC = BC$

$\angle ABC = \angle CAB = 45°$
$= \theta$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$m_2 = \frac{\tan\theta\, m_1 + 1}{m_1 - \tan\theta}$$
(mBC)

$$= \frac{1\cdot\left(-\frac{3}{4}\right)+1}{-\frac{3}{4}-1} = \frac{\frac{1}{4}}{-\frac{7}{4}}$$

$$= \frac{1}{4}\cdot -\frac{4}{7} = -\frac{1}{7}$$

Then $mAC = 7$ (AC is perpendicular to BC, hence their slopes are neg. recip.)

<u>To find Coordinates of C</u>

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let coordinates of $C = x, y$

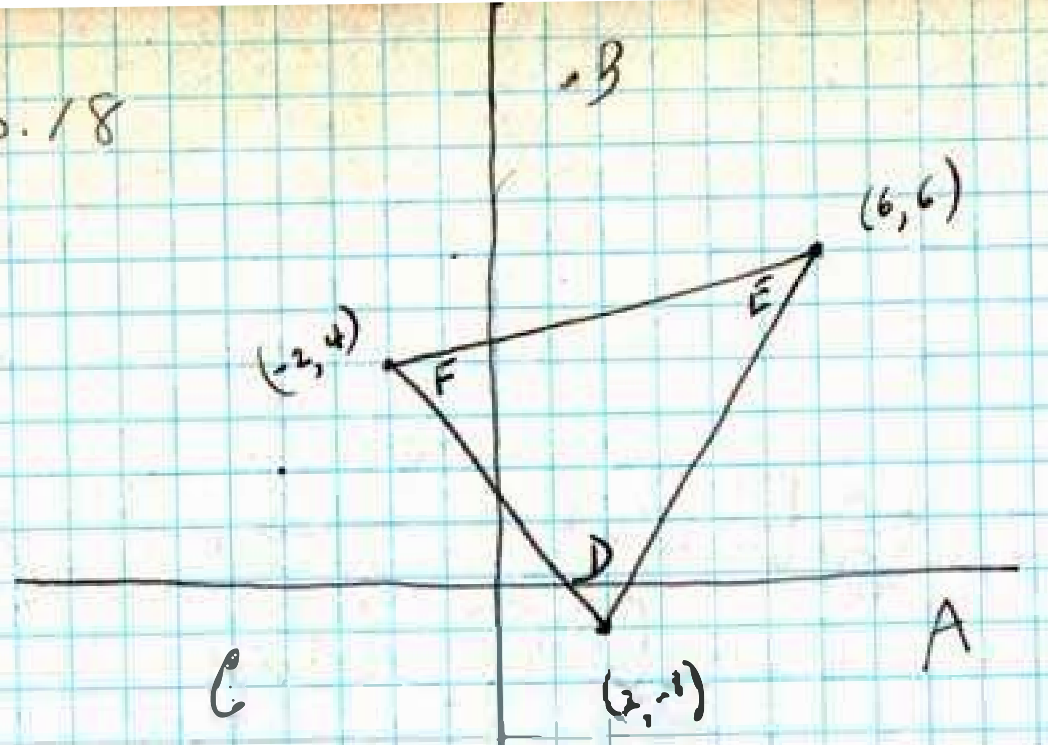
Then $mAC = \dfrac{y-6}{x-(-4)} = 7$

$mBC = \dfrac{y-(-3)}{x-8} = -\dfrac{1}{7}$ (How to multiply)

$y - 6 = 7x + 28$ or $7x - y = 34$

$-7y + 21 = -x + 8$ or $x - 7y = -29$

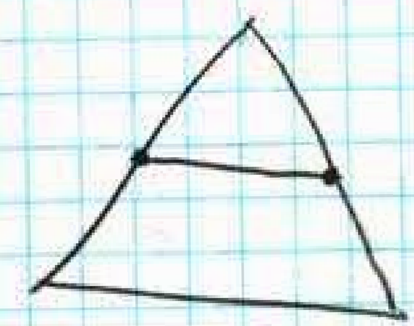
Page 25 No. 18



Let A, B, C be the vertices (coordinates unknown)

$m\,BC = m\,DE = \frac{6-(-1)}{6-2} = \frac{7}{4}$

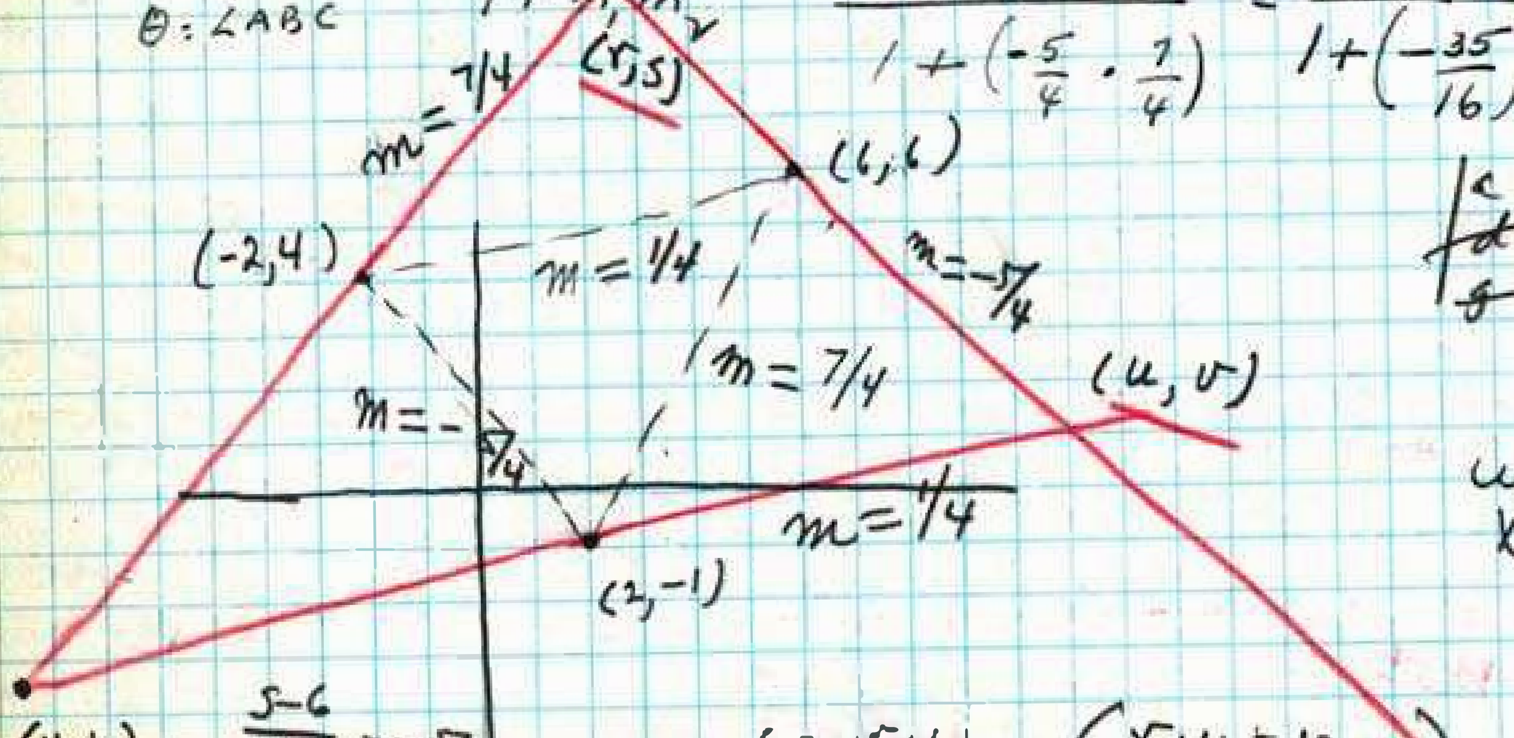
$m\,AB = m\,DF = \frac{4-(-1)}{-2-2} = -\frac{5}{4}$



$m\,AC = m\,EF = \frac{6-4}{6+2} = \frac{1}{4}$

$\tan\theta = \frac{m_1 - m_2}{1+m_1 m_2} = \frac{-\frac{5}{4} - \frac{7}{4}}{1+\left(-\frac{5}{4}\cdot\frac{7}{4}\right)} = \frac{-3}{1+\left(-\frac{35}{16}\right)} = \frac{-3}{\frac{-19}{6}} = 3\cdot\frac{6}{19} = \frac{18}{19}$

$\theta = \angle ABC$



~~c b e~~
~~d e f~~
~~g h i~~

u = 10
x = -6

(x,y)

$\frac{s-6}{r-6} = -\frac{5}{4}$

$\frac{v-6}{u-6} = -\frac{5}{4}$

$2 = \frac{x+u}{2}$

$-1 = \frac{y+v}{2}$

$6 = \frac{r+u}{2}$

$6 = \frac{s+v}{2}$

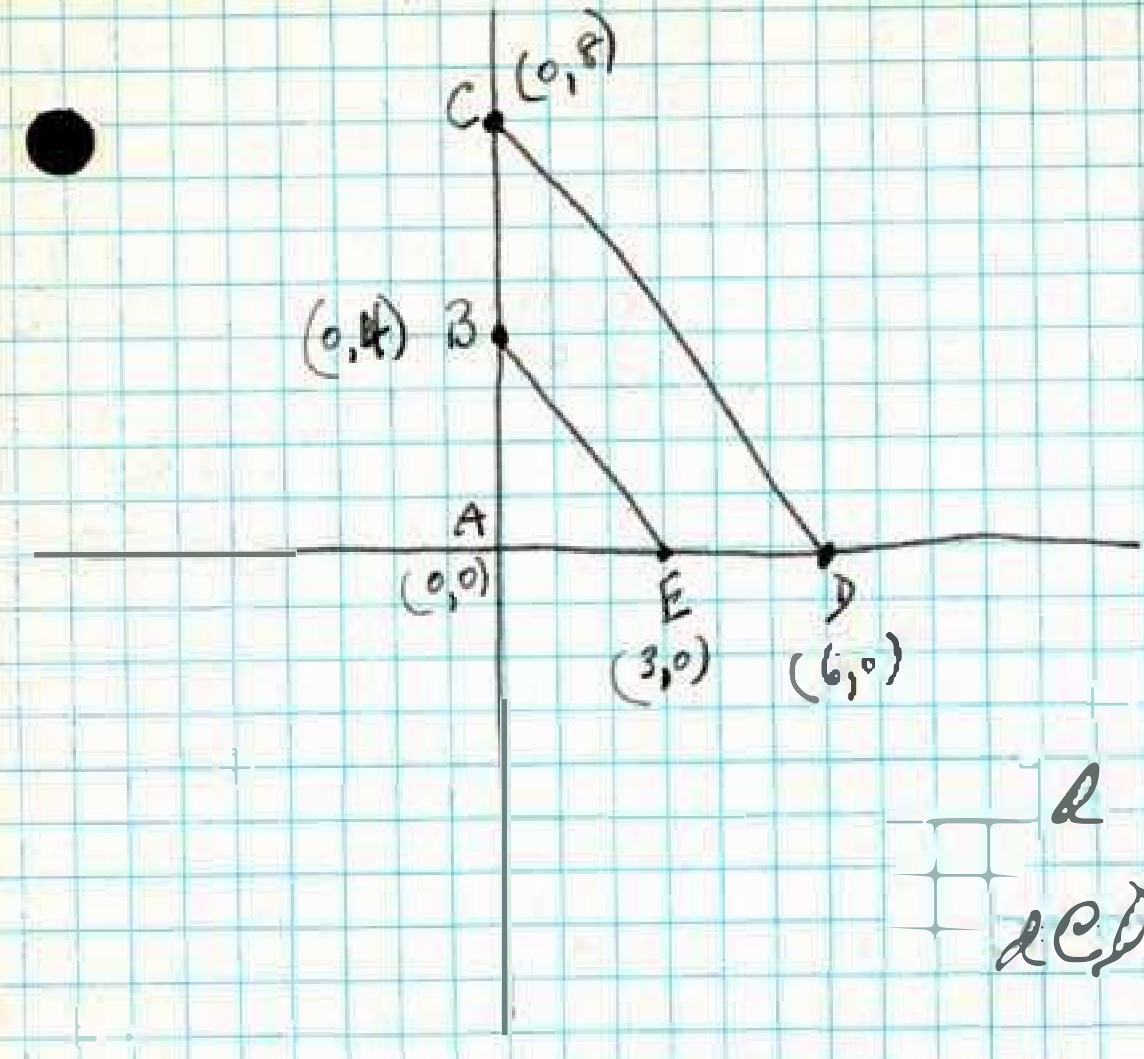
$-2 = \frac{r+x}{2}$

$4 = \frac{s+y}{2}$

$$\begin{cases} r+u = 12 \\ s+v = 12 \\ r+x = -4 \\ s+y = 8 \\ x+u = 4 \\ y+v = -2 \end{cases}$$

u - x = 16
u + x = 4

Page 25 No. 22



$m = \dfrac{y_2 - y_1}{x_2 - x_1} =$

$mBE = \dfrac{4-0}{0-3} = \dfrac{-4}{3}$

$mCD = \dfrac{8-0}{0-6} = -\dfrac{4}{3}$

$\therefore$ BE + CD are parallel, because their slopes are equal

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$dCD = \sqrt{(0-6)^2 + (8-0)^2}$

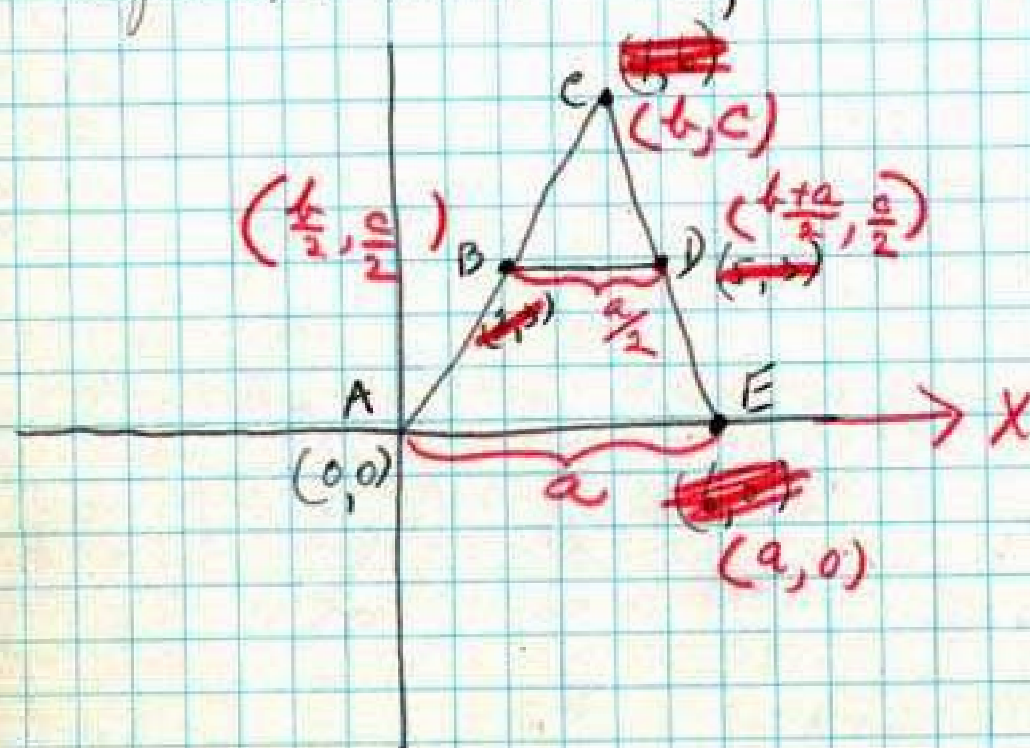
$= \sqrt{36+64} = \sqrt{100} = 10$

$dBE = \sqrt{(0-3)^2 + (4-0)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$

$\therefore BE = \frac{1}{2}$ of CD

If $\triangle$ is not rt. $\triangle$,



Coordinates of B $\begin{cases} x = \dfrac{4+0}{2} = 2 \\ y = \dfrac{6+0}{2} = 3 \end{cases}$

" D $\begin{cases} x = \dfrac{4+6}{2} = 5 \\ y = \dfrac{6+0}{2} = 3 \end{cases}$

$mBD = \dfrac{3-3}{5-2} = 0$

$mAC = \dfrac{0-0}{6-0} = 0$

$\therefore$ BD + AC are parallel

$dBD = \sqrt{(5-2)^2 + (3-3)^2} = \sqrt{9} = 3$

$dAE = \sqrt{(0-0)^2 + (6-0)^2} = \sqrt{36} = 6$

$\therefore BD = \frac{1}{2} AE$

Page 29 No. 1, 4, 7, 8, 9, 12, 13, 14

30 No. 18

31 No. 1(c), 8, 10, 11

32 No. 14

33 No. 2a, 3a

34 No. 6, 7, 10

Page 29 No. 1

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = 2/3$

$y - 10 = \frac{2}{3}(x - 6)$

Proof: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{10-4}{6+3} = \frac{2}{3}$

$2x - 3y + 18 = 0$

$-3y = -2x - 18$

$3y = 2x + 18$

$y = \frac{2}{3}x - 6$

m

(6,10)

$= \frac{10 - y}{6 - x} = \cancel{\frac{-4 - y}{-3 - x}}$ $\frac{4-y}{-3-x}$

$-30 + 3y - 10x + \cancel{xy} = 24 - 6y - 4x + \cancel{xy}$

$-10x + 4x + 3y + 6y - 30 - 24 = 0$

$-6x + 9y - 54 = 0$

$6x - 9y + 54 = 0$

$2x - 3y + 18 = 0$

No. 4 $\quad y - y_1 = m(x - x_1)$

$y = -2x + 1$ (thus $m = -2$)

$y + 3 = -2(x - 2)$

$= -2x + 4$

$2x + y - 1 = 0$ Proof

(0,1)

$(2, -3) \rightarrow 4 - 3 - 1 = 0$ ✓

No. 7 $\quad \alpha = 135°$, $\therefore m = -1$ ✓

$y - y_1 = m(x - x_1)$

$y + 6 = -1(x - 3)$

$= -x + 3$

$x + y + 3 = 0$ ✓

Proof: $y = -x - 3$ $(m = -1)$ ✓

No. 8 $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

(6,3) $\quad m = 0$

$y - 3 = 0(x - 6)$

$y - 3 = 0$

$= \frac{3 - y}{6 - x} = \frac{3 - y}{0 - x}$

$-3x + xy = 18 - 6y - 3x + xy$

(zero) $0x + 6y - 18 = 0$

m of x axis or line parallel to x axis is 0

$6y = -0x + 18$ $\quad (m = 0)$

$y = 3$ (checks $\bar{c}$ y above)

Page 29 No. 9

$$m = \frac{-2-y}{4-x} = \frac{0-y}{4-x}$$

$$-8-4y+2x+xy = 0-4y-0+xy$$

$$2x+0y-8=0$$

$$m = \frac{-2-0}{4-4} = \frac{-2}{0} = 0$$

$$x+0y-4=0$$

$\underline{x=4}$ (checks)

$$0y = -x+4$$

$$y = -\frac{x}{0} + \frac{4}{0} = 0$$

m = coeff. of $x = 0$

No. 12

$$\frac{x}{a} + \frac{y}{b} = 1 \quad ✓$$

$$m = \frac{6-0}{0-3} = -2$$

$$\frac{x}{3} + \frac{y}{-6} = 1 \quad ✓$$

$$-y = -2x+6$$

$$-6x+3y=-18$$

$$6x-3y=18$$

$$2x-y=6 \quad ✓$$

No. 13

$$m = \frac{-3+y}{1-x} = \frac{0-y}{5-x}$$

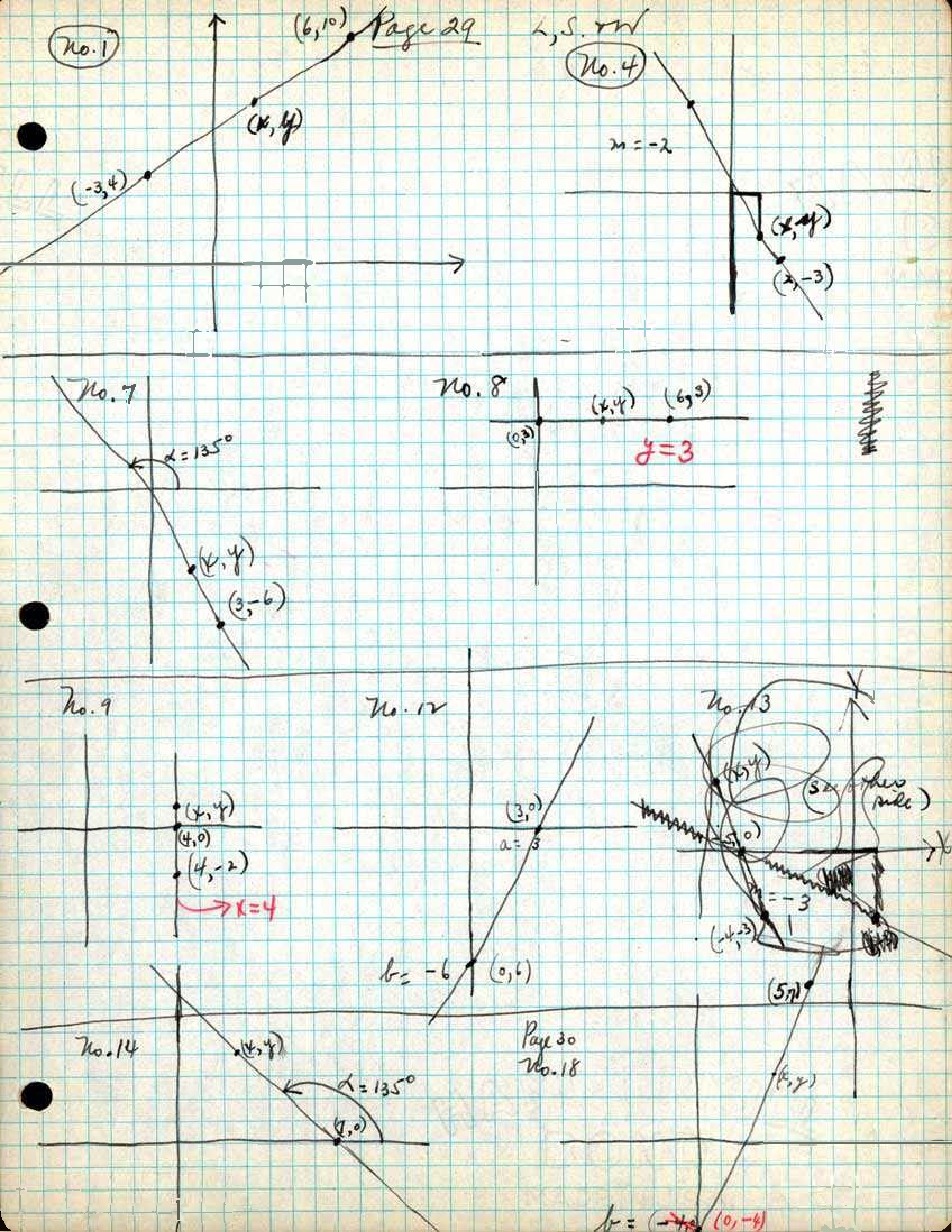
$$15+5y+3x+xy = -y+xy$$

$$3x+6y+15=0$$

$$x+2y+5=0$$

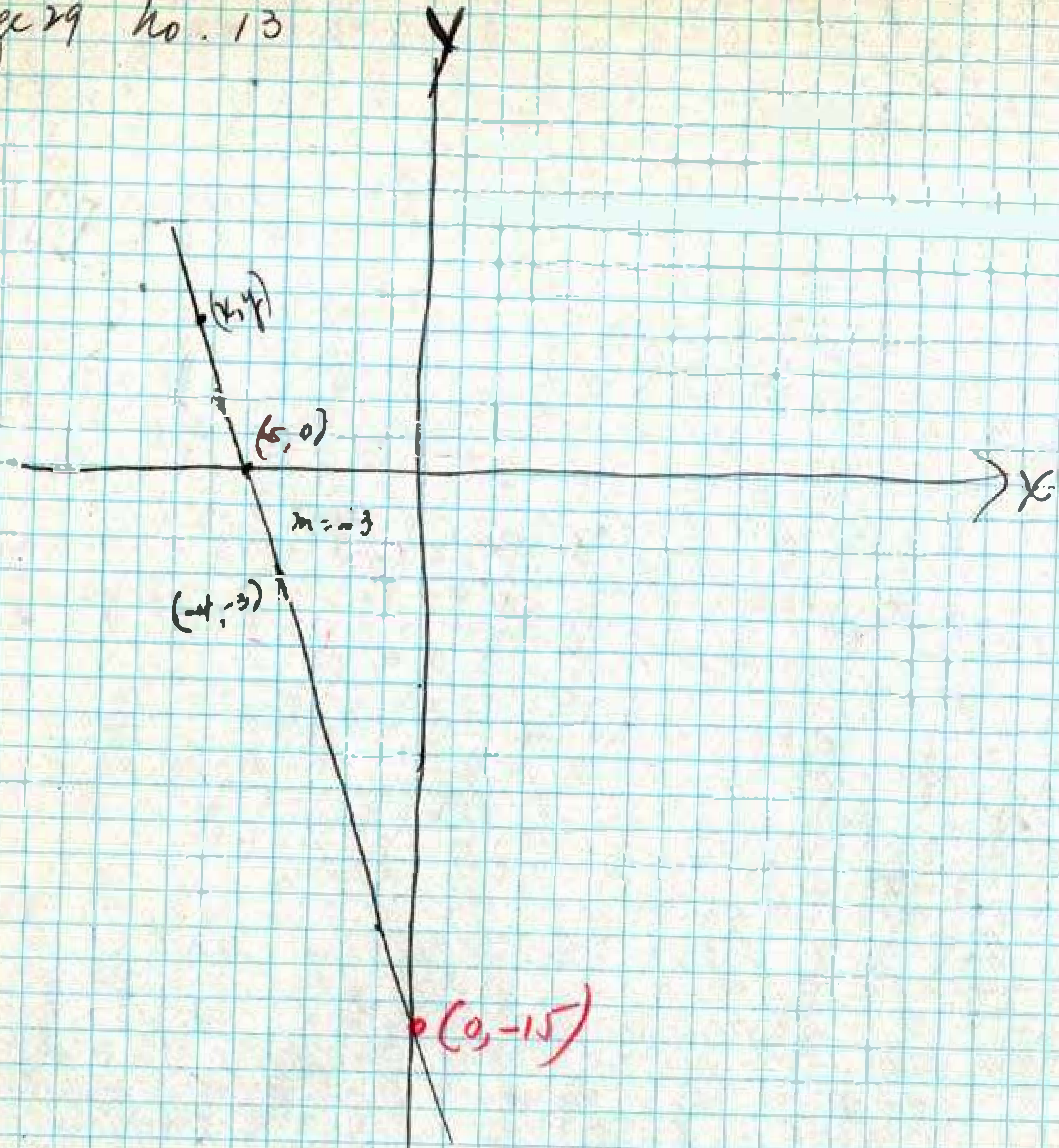
Proof: Then $2y = -x-5$

$y =$



Page 29
L.S. rW
No. 1
(6,10)
(x,y)
(-3,4)
No. 4
m = -2
(x,y)
(x,-3)
No. 7
α = 135°
(x,y)
(3,-6)
No. 8
(x,y)
(6,3)
(0,3)
y=3
No. 9
(x,y)
(4,0)
(4,-2)
x=4
No. 12
(3,0)
a = 3
b = -6
(0,6)
No. 13
(x,y)
(5,0)
m = -3
1
(-4,3)
(5,7)
No. 14
(x,y)
α = 135°
(7,0)
Page 30
No. 18
(x,y)
b = (0,-4)

Page 29 No. 13



Page 29 No. 13

$$m = \frac{0-y}{-5-x} = \frac{-3-y}{-4-x}$$

$y - y_1 = m(x - x_1)$

$4y + xy = 15 + 5y + 3x + xy$

$m = \frac{-3-0}{-4+5} = \frac{-3}{+1} = -3$

$y = -3(x+5)$

$-3x - y - 15 = 0$

$y = -3x - 15$

$3x + y + 15 = 0$

$y = -3x - 15$

$m = -3$

[illegible] - y intercept is 15. [illegible]

Page 29 No. 14

$\alpha = 135 \quad \therefore \quad m = -1$

$(7,0)$

$y - y_1 = m(x - x_1)$

$y = mx + b$

$y - 0 = -1(x-7)$

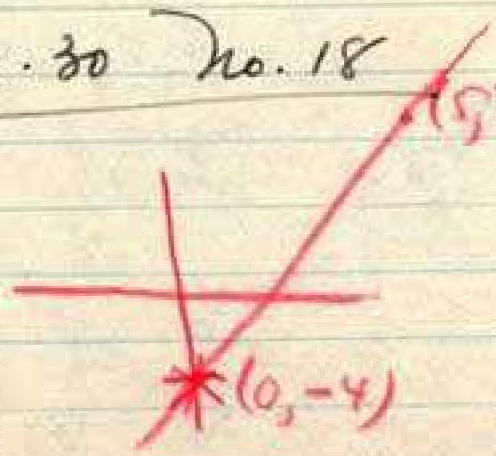
Check $\quad y = -x + 7$

$= -x + 7$

$x + y - 7 = 0$

$m = -1$

Page 30 No. 18



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \cancel{\frac{7-0}{5-(-4)} = \frac{7}{9}} \quad \frac{11}{5}$$

$$\frac{y-7}{x-5} = \frac{y-0}{x+4}$$

$y = \frac{11}{5}x - 4$

$5y = 11x - 20$

$xy - 7x + 4y - 28 = xy - 5y$

$-7x + 9y - 28 = 0$

$9y = 7x + 28$

$7x - 9y + 28 = 0$

$y = \frac{7}{9}x + \frac{28}{9}$

$35 - 63 + 28 = 0$

$y - 7 = \frac{11}{5}(x-5)$

$m = \frac{7}{9}$

$5y - 35 = 11x - 55, \quad 5y = 11x - 20$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Then in problem 1c

$$\frac{x}{\frac{5}{2}} + \frac{y}{-5} = 1$$

$$\frac{x}{\frac{5}{2}} - \frac{y}{5} = 1$$

$$2x - y = 5$$

?

Page 31 No. 1(c)

Equation $2x - y - 5 = 0$

If $x = 0$, $y = -5$ $(y\text{-int} = b = -5)$

If $y = 0$, $x = \frac{5}{2}$ $(x\text{-int.} = a = 5/2)$

Then coordinates of points where the line cuts the axis are $(0, -5)$ + $(\frac{5}{2}, 0)$

$y = 2x - 5$ $(0, b)$

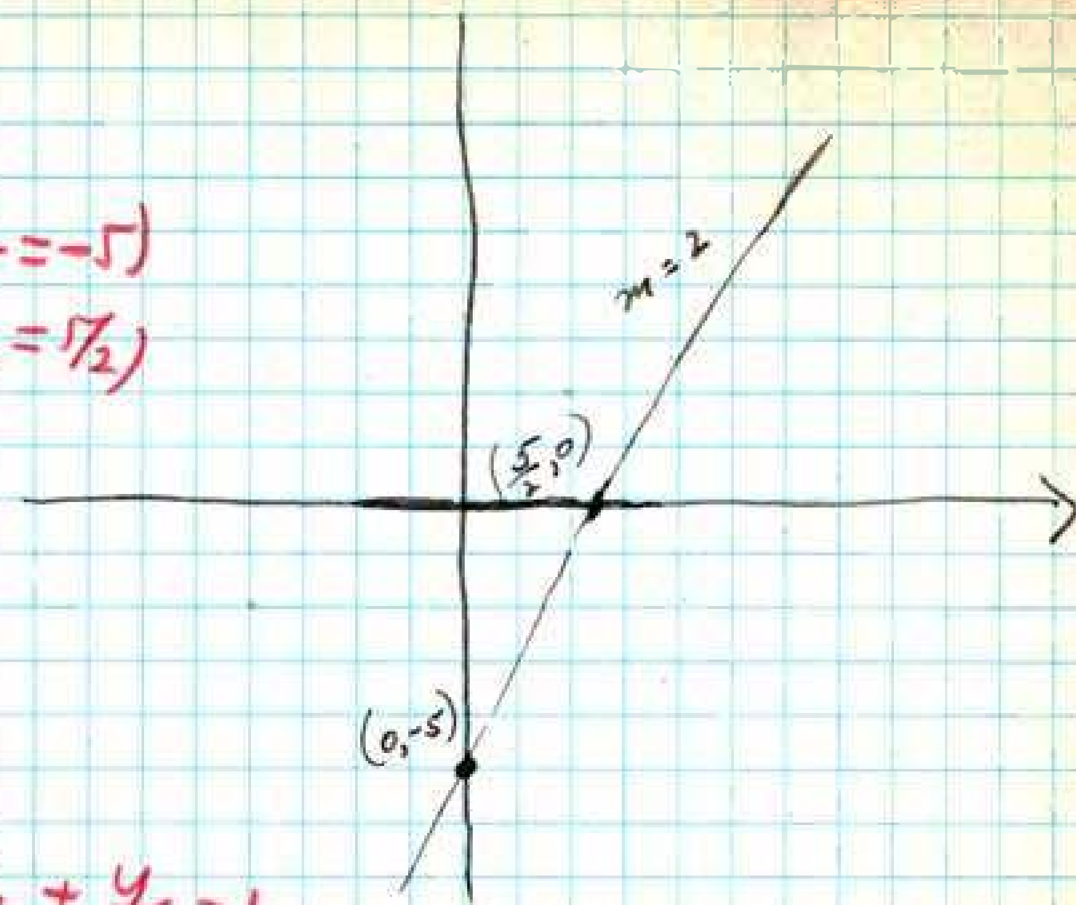
$(0, 2a)$

$\therefore$ Slope $= 2$

$\tan\alpha$ ($\angle$ of inclination) $= 2$

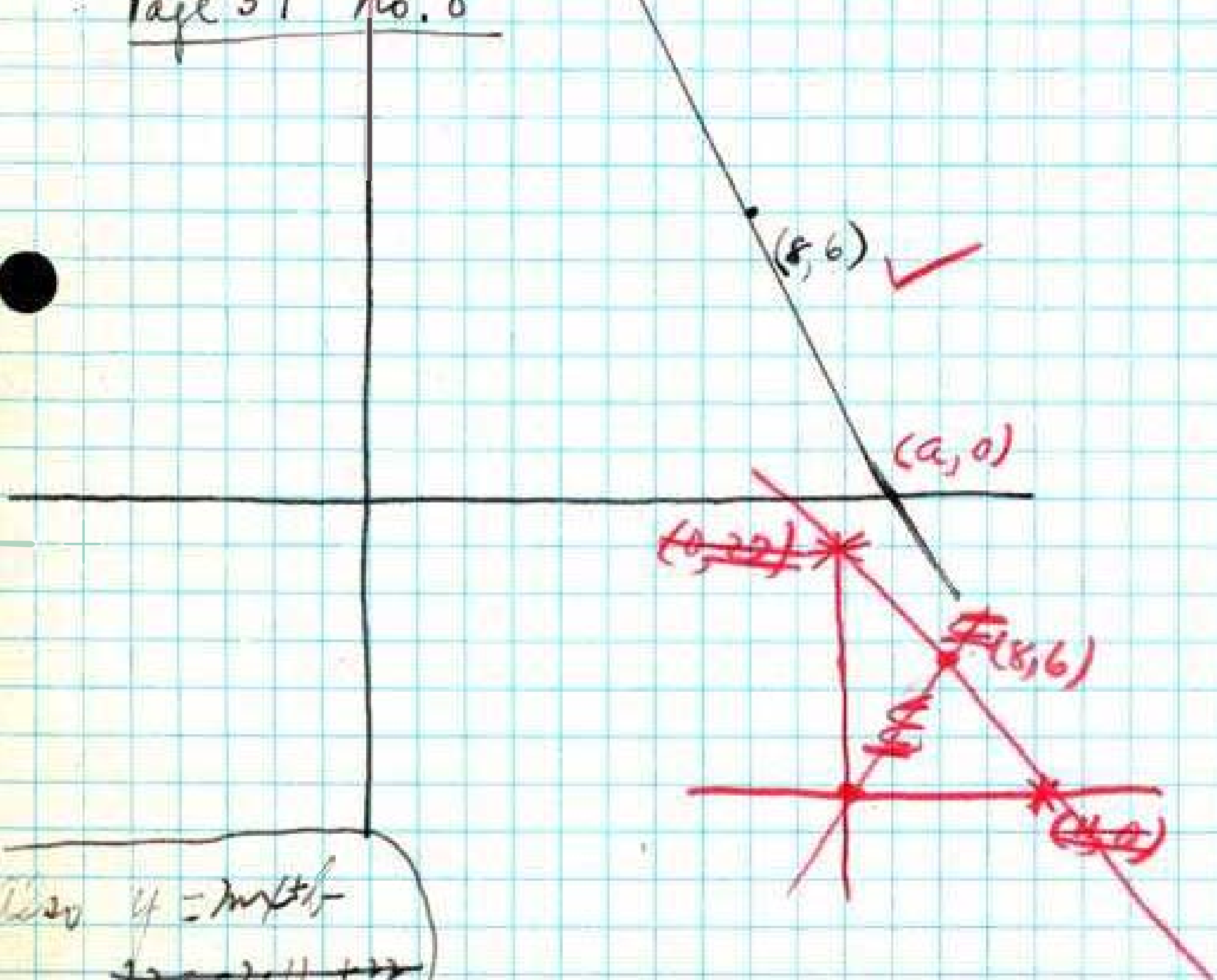
$\therefore \alpha = \tan^{-1}(2)$

$\alpha = \text{arc}\tan 2$



$$\frac{x}{a} + \frac{y}{b} = 1$$

Page 31 No. 8



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{6 - 0}{8 - \cancel{x}a} = \frac{6 - 2\cancel{a}}{8 - 0} \quad (\text{since } y = 2x)$$

$48 = 48 - 16 - 6x + 2x^2$

$2x^2 - 22x = 0$

$x^2 - 11x = 0$ ~~x = 0~~, $x = 11$

$x(x - 11) = 0$

$x^2 - 11x + \left(\frac{11}{2}\right)^2 = \left(\frac{11}{2}\right)^2$

$x - \frac{11}{2} = \pm\frac{11}{2}$

$a = \cancel{x_{int}} = \frac{22}{2} = 11$

b: $\cancel{y_{int}} = 22$

~~x int~~, $y = 0$, $x = 11$

~~y int~~, $x = 0$, $y = 22$

$y = mx + b$

~~$22 = -2 \cdot 11 + 22$~~

$y = -2x + 22$

$2x + y - 22 = 0$

?

Proof
$$\frac{22 - 6}{0 - 8} = \frac{0 - 22}{11 - 0}$$

$-2 = -2$

$$\frac{x}{11} + \frac{y}{22} = 1$$

$2x + y = 22$

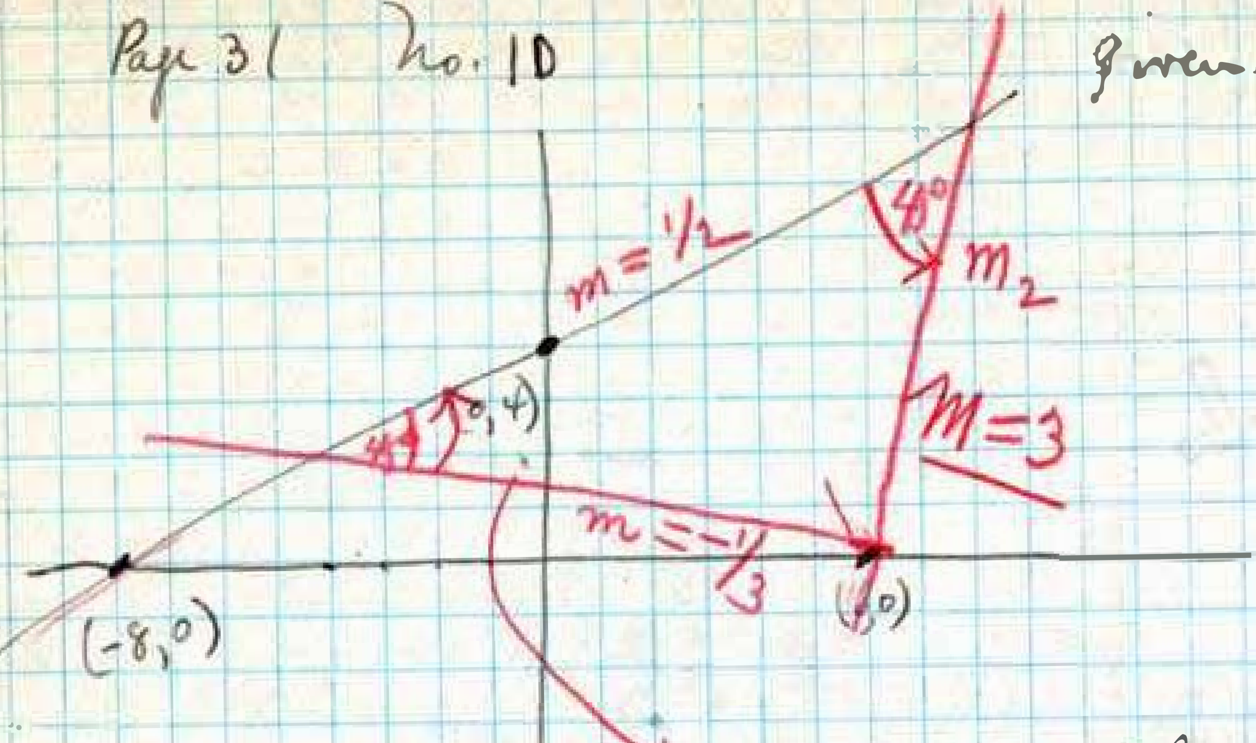
?

$y = -2x + 22$

$m = -2$

Question $\rightarrow$?

Page 31 No. 10



$$y = -\frac{1}{3}(x-6)$$

$$1 = \frac{\frac{1}{2} - m}{1 + \frac{1}{2}m}$$

$$1 + \frac{1}{2}m = \frac{1}{2} - m$$

$$2 + m = 1 - 2m$$

$$3m = -1$$

$$m = -\tfrac{1}{3}$$

Given line $x - 2y + 8 = 0$

If $x = 0$, $y = 4$

$y = 0$, $x = -8$

Then two points on line are

$(0,4)$ + $(-8,0)$

$\left(m = \frac{1}{2}\right)$

Unknown line makes angle of 45° with given line.

Then $\tan\theta = \dfrac{m_2 - m_1}{1 + m_2 m_1}$

$$\tan 45^\circ = \frac{m_2 - \frac{1}{2}}{1 + m_2 \cdot \frac{1}{2}}$$

$$1 = \frac{m_2 - \frac{1}{2}}{1 + \frac{m_2}{2}}$$

$$1 + \frac{m_2}{2} = m_2 - \frac{1}{2}$$

$$2 + m_2 = 2m_2 - 1$$

$$2m_2 - m_2 = 2 + 1$$

$$m_2 = 3$$

Unknown line passes through $(6,0)$ + has slope of 3.

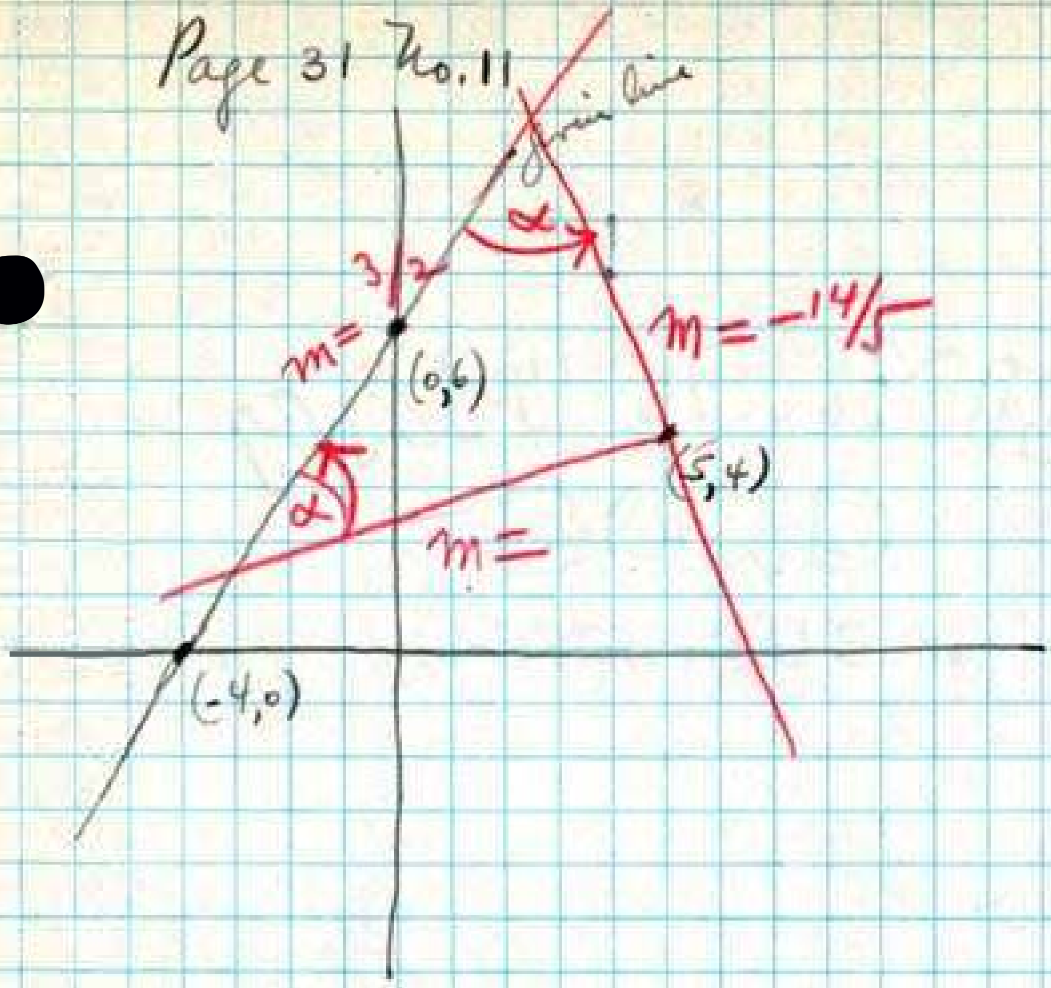
$$y - y_1 = m(x - x_1)$$

$$y - 0 = 3(x - 6)$$

$$y = 3x - 18$$

$3x - y - 18 = 0$ is Equation of unknown line

Page 31 No. 11



Given line $3x - 2y + 12 = 0$

If $x = 0$, $y = 6$

$y = 0$, $x = -4$

Then two points on line are $(0,6)$ + $(-4,0)$

$$\tan\theta = \frac{1}{2}$$

Slope of given line $= \frac{3}{2}$

$$-3x + 2y - 12 = 0$$

$$2y = 3x + 12$$

$$y = \frac{3}{2}x + 6$$

$$\frac{1}{2} = \frac{\frac{3}{2} - m}{1 + \frac{3}{2}m}$$

$m =$ ______

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$\frac{1}{2} = \frac{m_2 - \frac{3}{2}}{1 + m_2 \cdot \frac{3}{2}}$$

$$\frac{1}{2} + \frac{3}{4}m_2 = 2m_2 - \frac{6}{2}$$

$$2 + 3m_2 = 8m_2 - 12$$

$$8m_2 - 3m_2 = -12 - 2$$

$$5m_2 = -14$$

$$m_2 = -\frac{14}{5}$$

Unknown line passes through $(5,4)$ + has slope of $-\frac{14}{5}$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{14}{5}(x - 5)$$ ✓

$$y - 4 = -\frac{14}{5}x + 14$$

$$5y - 20 = -14x + 70$$

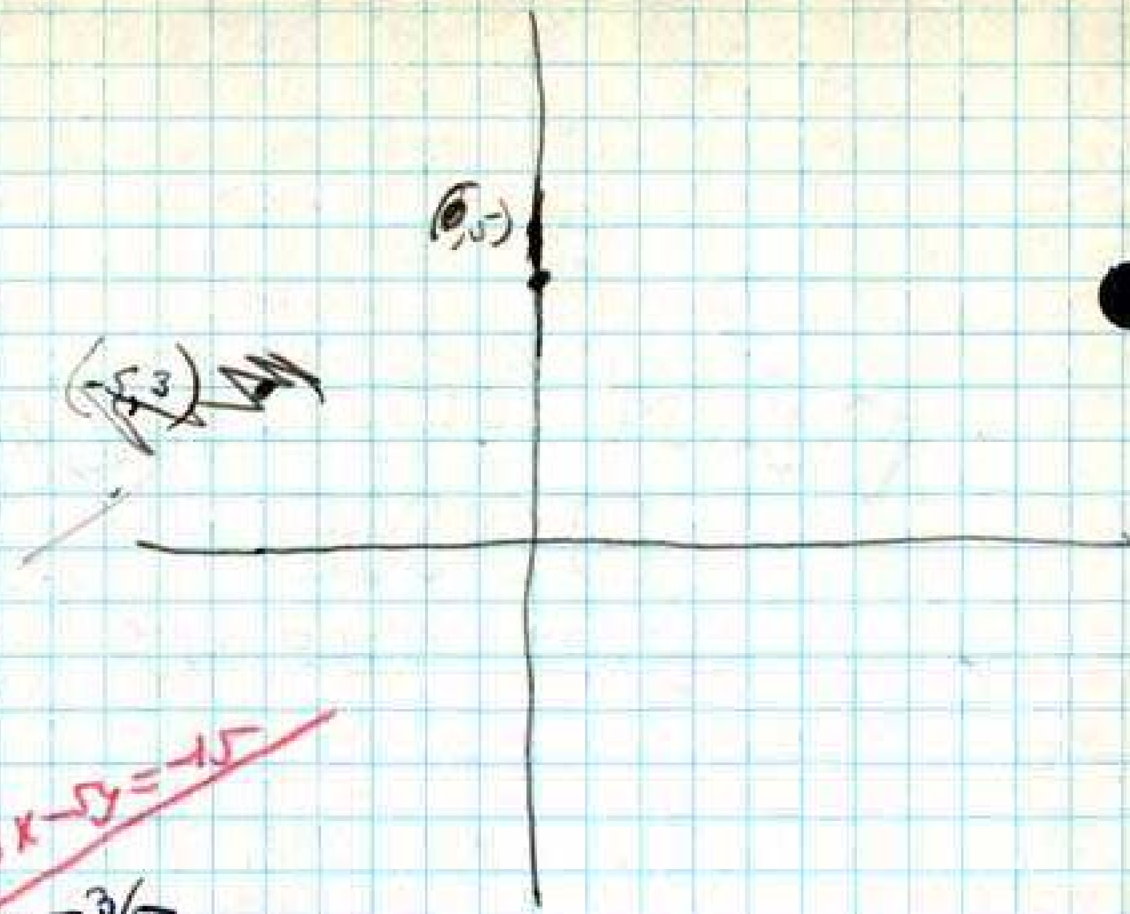
$$14x + 5y - 90 = 0$$ (equation of unknown line)

Page 32 No. 14

$3x - 5y + 15 = 0$

$\left.\begin{array}{l} \text{If } x = 0, \; y = 3 \\ y = 0, \; x = -5 \end{array}\right\}$

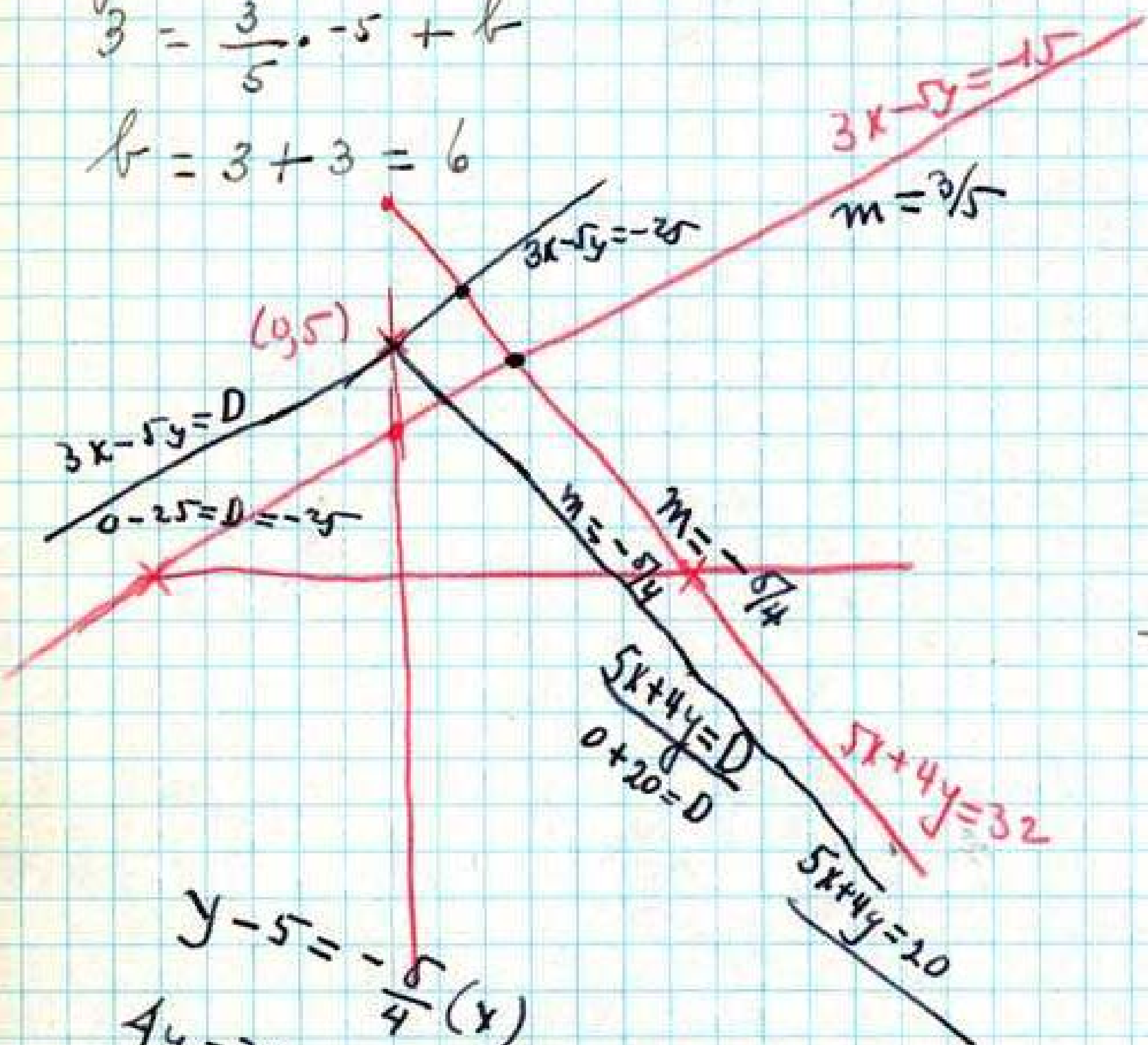
$m = 3/5$



$y = mx + b$

$3 = \frac{3}{5} \cdot -5 + b$

$b = 3 + 3 = 6$



$y - 5 = -\frac{5}{4}(x)$

$4y - 20 = -5x$

$3x - 5y + 15 = 0$

$\underline{5x + 4y - 32 = 0}$

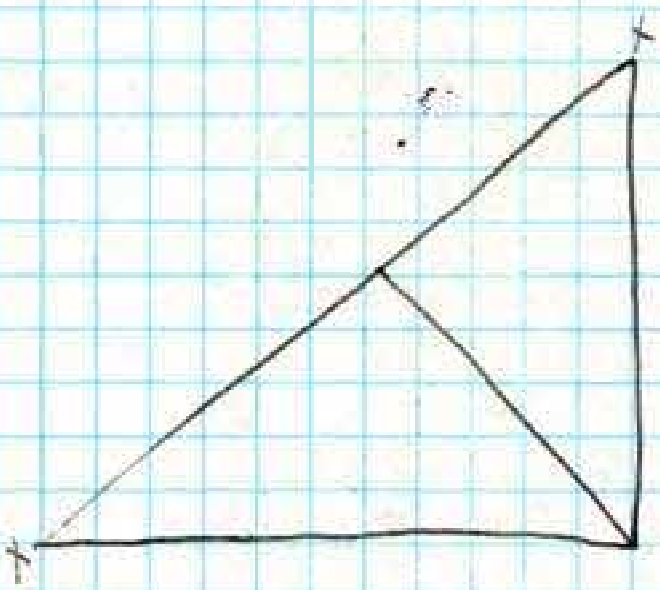
$15x - 25y + 75 = 0$

$\underline{15x + 12y - 96 = 0}$

$-37y + 171 = 0$

Slopes of lines ē given equations are not equal, ∴ they are not parallel

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{3}{5}(x -$$

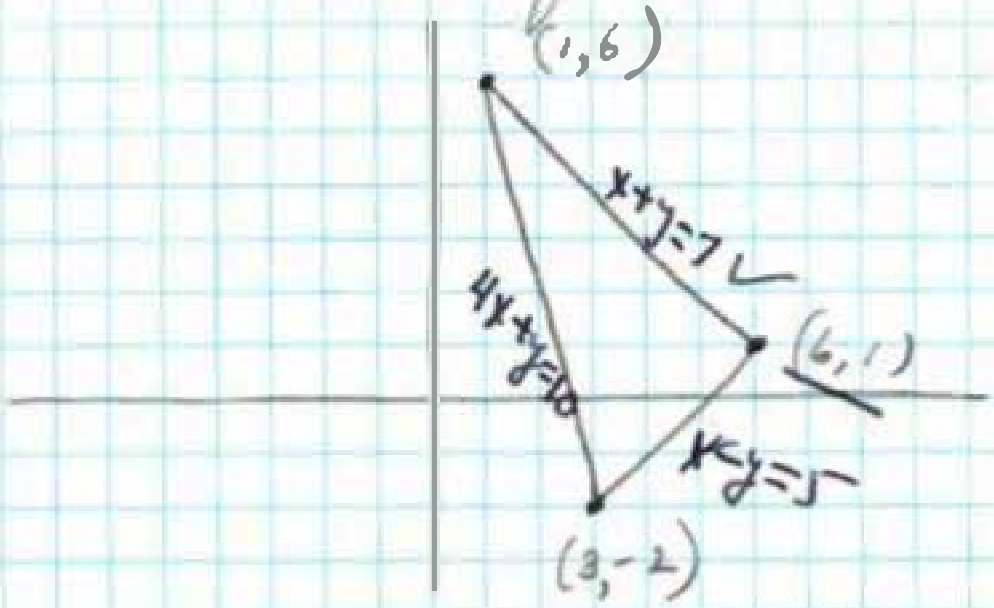


Page 33 No. 2a

Given

$$\begin{array}{r} x+y=7 \\ x-y=5 \\ \hline 2x=12 \\ x=6 \\ y=1 \end{array} \qquad \begin{array}{r} x+y=7 \\ 4x+y=10 \\ \hline -3x=-3 \\ x=1 \\ y=6 \end{array} \qquad \begin{array}{r} x-y=5 \\ 4x+y=10 \\ \hline 5x=15 \\ x=3 \\ y=-2 \end{array}$$

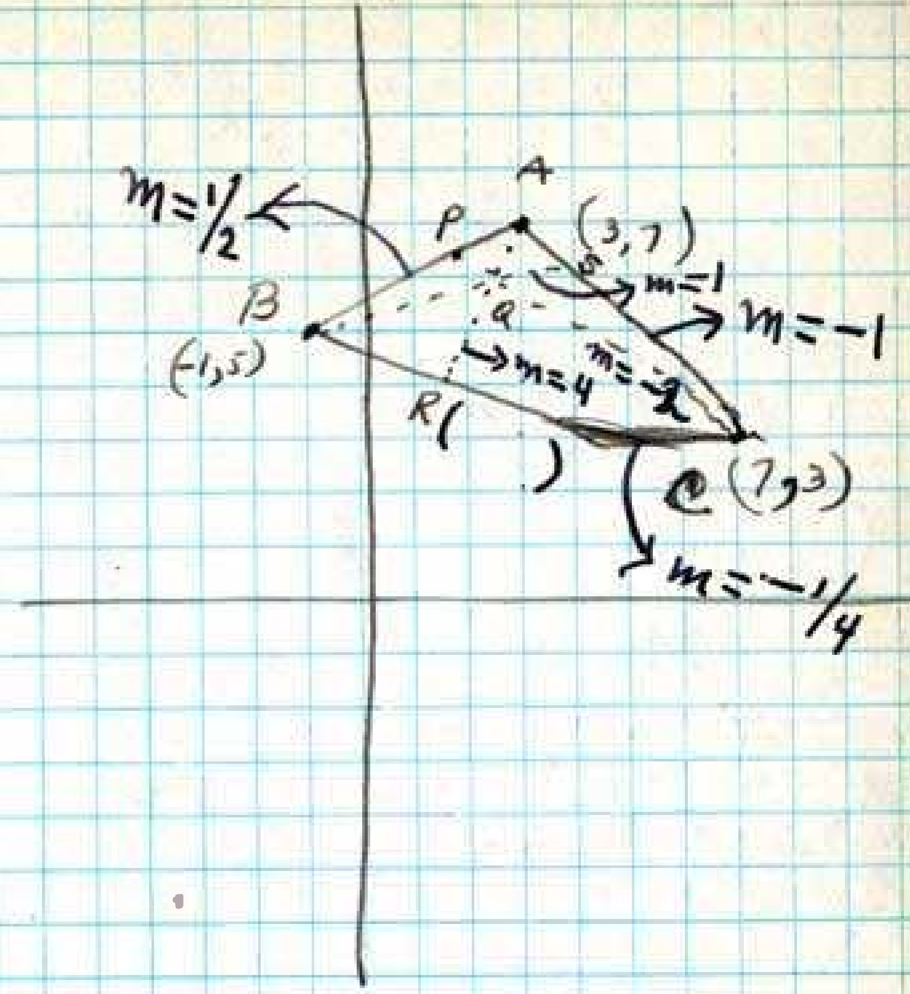
Thus, the coordinates of the vertices are (6,1) (1,6) (3,-2)



Page 33 No 3a

$m\,AB = \frac{7-5}{3+1} = \frac{2}{4}$

Then $m\,CP = \frac{4}{2} = -2$



$m\,AC = \frac{7-3}{3-7} = \frac{4}{-4} = -1$

Then $m\,BS = 1$

$m\,BC = \frac{3-5}{7+1} = \frac{-2}{8} = -\frac{1}{4}$

Then $m\,AR = 4$

To find equations of PC, AR, & BS

$y - y_1 = m(x - x_1)$

For PC
$$y - 3 = -2(x-7)$$
$$y - 3 = -2x + 14$$
$$2x + y - 17 = 0 \checkmark$$

For AR
$$y - 7 = 4(x-3)$$
$$= 4x - 12$$
$$4x - y - 5 = 0 \checkmark$$

For BS
$$y - 5 = 1(x+1)$$
$$= x + 1$$
$$x - y + 6 = 0 \checkmark$$
$$\cancel{1 - 5 - 4 = 0}$$

$2x + y - 17 = 0 \checkmark$

$4x - y - 5 = 0 \checkmark$

$6x - 22 = 0$

$x = \frac{22}{6} = \frac{11}{3}$

$\frac{22}{3} + y - 17 = 0$

$y = 17 - \frac{22}{3} = 29/3$

$y = \frac{29}{3}$

Coordinates of Q

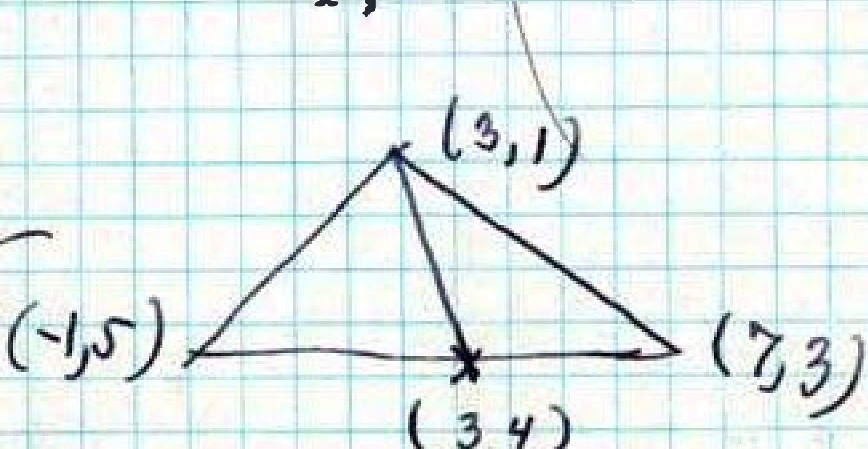
$4x - y - 5 = 0$

$x - y + 6 = 0$

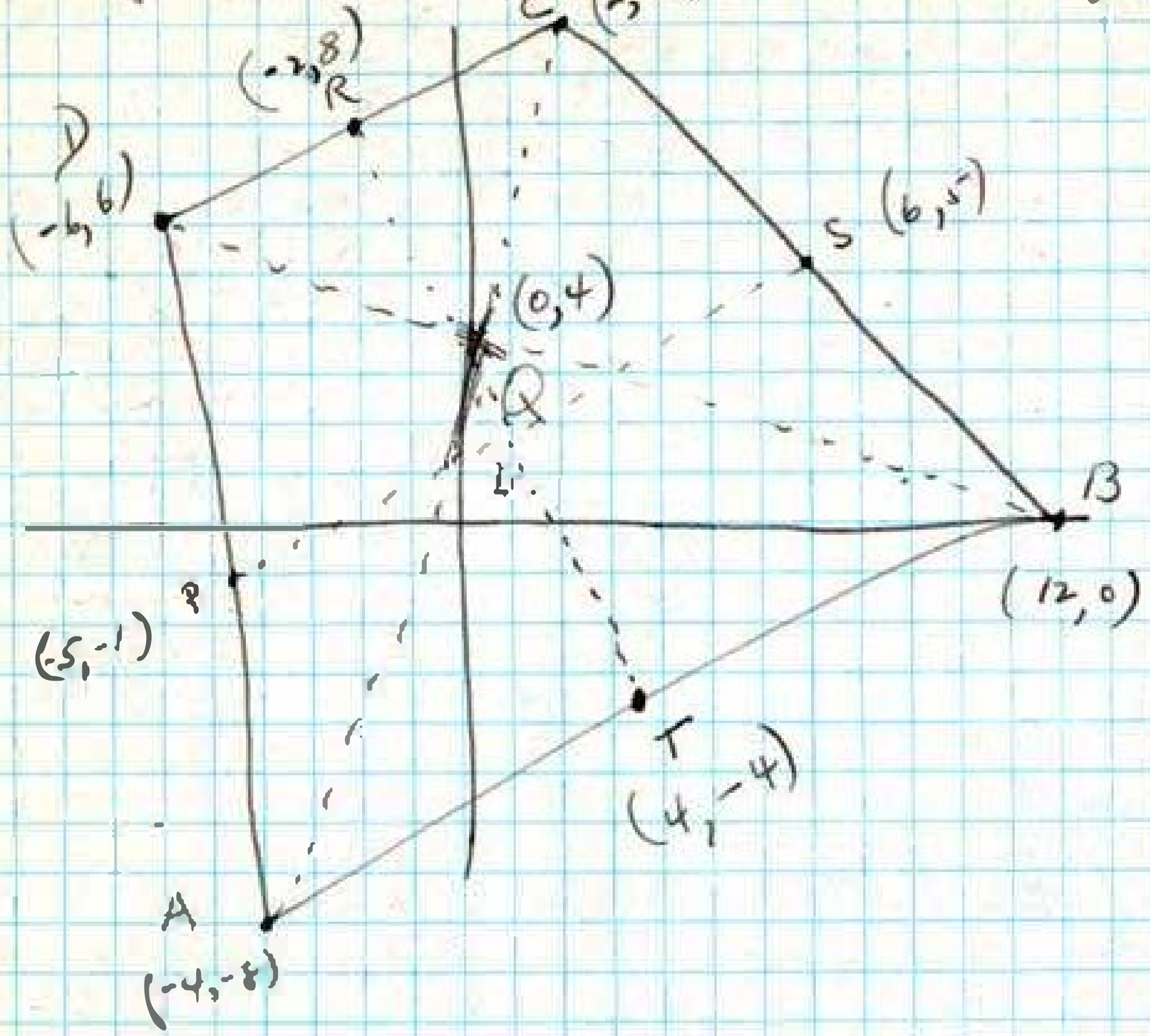
$3x - 11 = 0$

$x = 11/3$

2y



Page 34 No. 6



Eq. of BD

$$\frac{0-y}{12-x} = \frac{6-y}{-6-x}$$

$$6y + xy = 72 - 12y - 6x + xy$$

$$6x + 18y - 72 = 0$$

$$x + 3y - 12 = 0$$

Eq. AC

$$\frac{10-y}{2-x} = \frac{-8-y}{-4-x}$$

$$-40 + 4y - 10x + \cancel{xy} = -16 - 2y + 8x + xy$$

$$-18x + 6y - 24 = 0$$

$$3x - y + 4 = 0$$

$$3x - y + 4 = 0$$

$$\left\{ \begin{array}{l} x + 3y - 12 = 0 \\ 9x - 3y + 12 = 0 \end{array} \right.$$

$$10x = 0$$

$$\left. \begin{array}{l} x = 0 \\ \text{Then } y = 4 \end{array} \right\}$$

(*) Coordinates of Q = (0,4)

~~Then Coordinates of L~~

$$x - 11y + 14 = 0$$

$$4x + 5y - 12 = 0$$

$$4x - 44y + 56 = 0$$

$$49y - 68 = 0$$

$$y = \frac{68}{49}$$

$$x =$$

Coordinates of P, R, S, T (points bisecting respective lines) are P (-5,-1)

R (-2,8)

S (6,5)

T (4,-4)

Eq. RT $\frac{y+4}{x-4} = \frac{y-8}{x+2}$

$$xy + 4y + 2y + 8 = xy - 8x - 4y + 32$$

$$6y + 8 = -8x - 4y + 32$$

$$8x + 10y - 24 = 0$$

$$4x + 5y - 12 = 0$$

Eq. PS $\frac{y-5}{x-6} = \frac{y+1}{x+5}$

$$xy - 5 + 5y - 25 = xy + x - 6y - 6$$

$$-5 + 5y - 25 = x - 6y - 6$$

$$-x + 11y - 14 = 0$$

~~x + 11y~~

$$x - 11y + 14 = 0$$

Page 42 1a

2a

Page 43 4a (a of 3)

48 1a

1c

1e

1g

1k

1m

1o

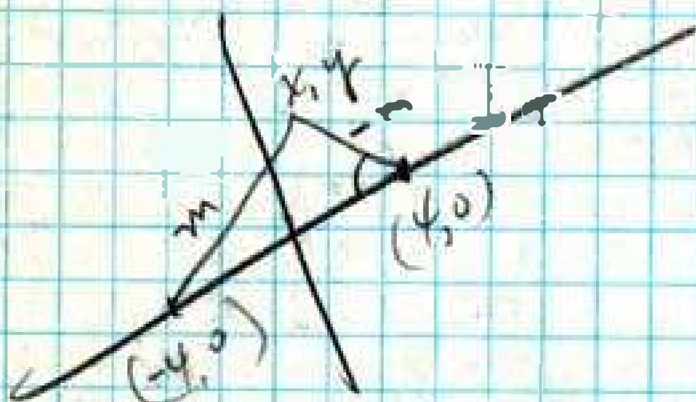
1u

1t

1w

Page 56 2a

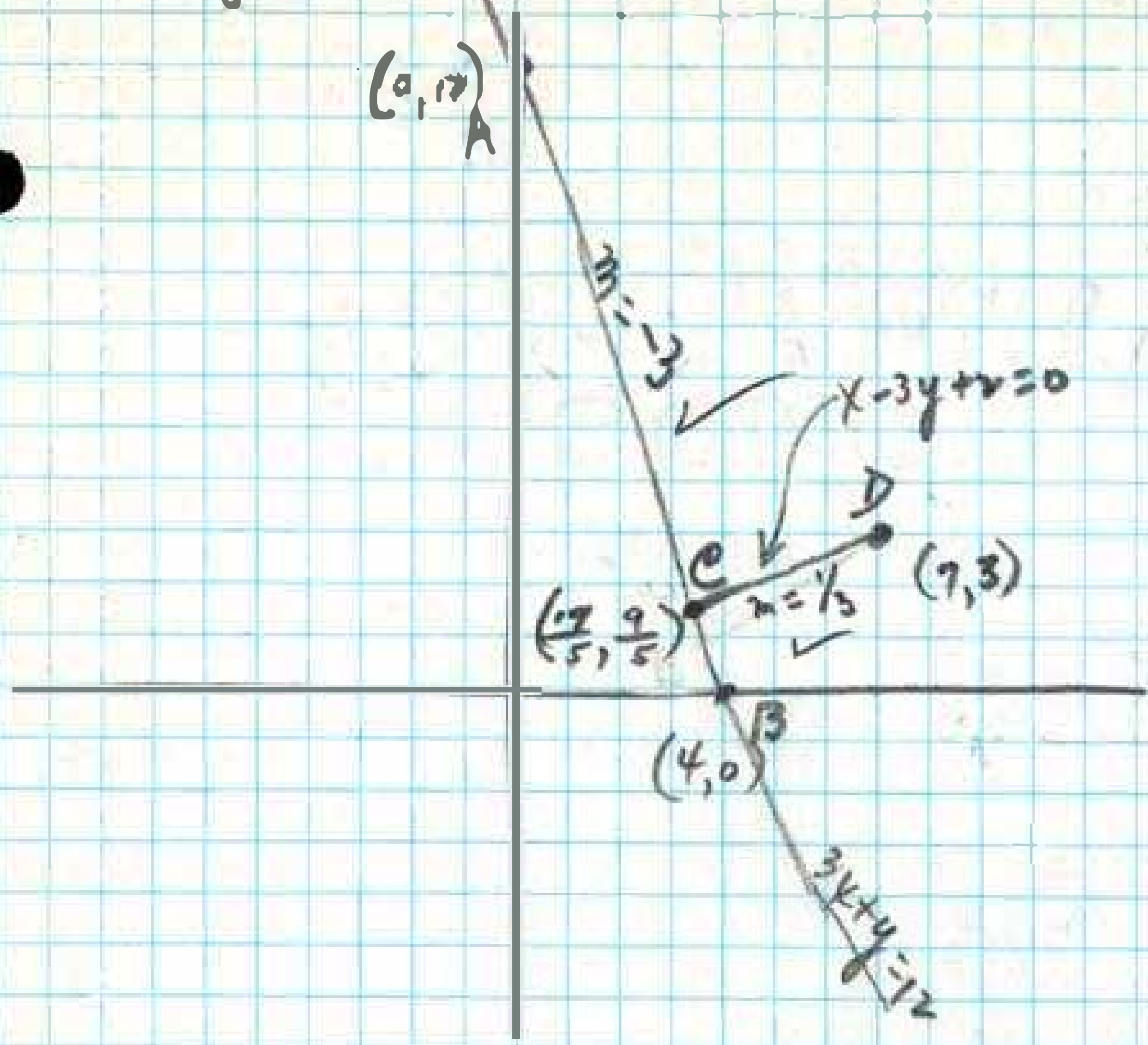
5a



$$y = m(x+4) + b$$

$$y = -m(x-4) + b$$

Page 34 No. 7



(AB) given line $3x+y=12$
Then two points on line are $(4,0)$ + $(0,12)$
(D) given point is $(7,3)$

$$3x+y=12$$
$$y=-3x+12$$
$$mAB=-3$$
Then $mCD=\frac{1}{3}$

$$y-y_1=m(x-x_1)$$
$$y-3=\frac{1}{3}(x-7)$$
$$3y-9=x-7$$
$$x-7=3y-9$$
$$x-3y+2=0$$

Equation of CD

$$\begin{cases}3x+y-12=0\\3x-9y+6=0\end{cases}$$
$$10y-18=0$$
$$5y-9=0$$

Coordinates of C (point of intersection of lines AB + CD) $\rightarrow \begin{cases}y=\frac{9}{5}\\x=\frac{17}{5}\end{cases}$

$\left(x-\frac{27}{5}+2=0\right.$
$x=\frac{17}{5}$

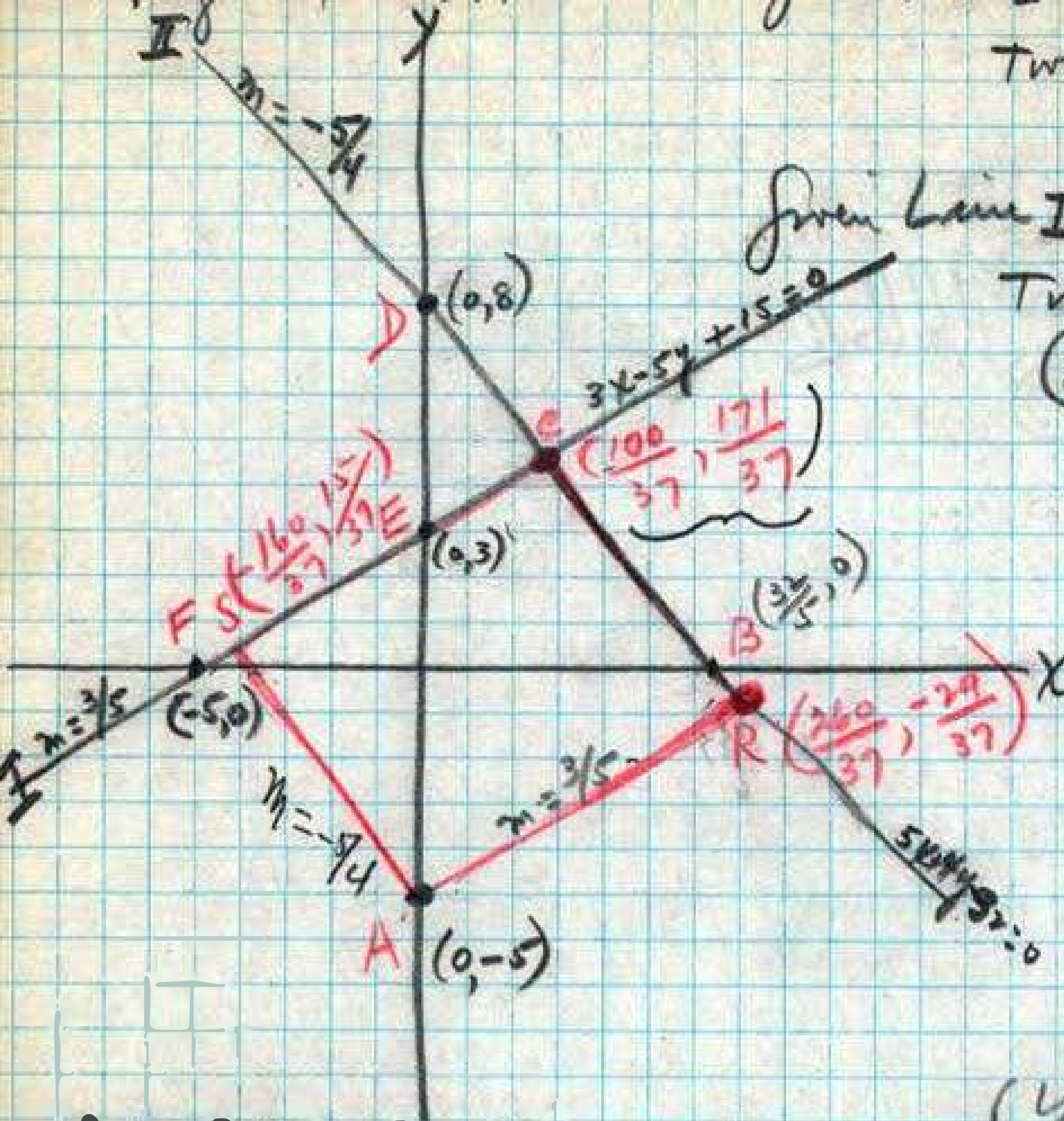
$$d(CD)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(7-\frac{17}{5}\right)^2+\left(3-\frac{9}{5}\right)^2}$$
$$=\sqrt{\left(\frac{18}{5}\right)^2+\left(\frac{6}{5}\right)^2}$$
$$=\sqrt{\frac{324}{25}+\frac{36}{25}}=\sqrt{\frac{360}{25}}$$
$$=\frac{6}{5}\sqrt{10}$$

Checks

$$3x+y=12$$
$$3\cdot\frac{17}{5}+\frac{9}{5}=12$$
$$\frac{51}{5}+\frac{9}{5}=12$$

$$3x+y=12$$
$$y=-3x+12$$
$$d_{(CD)}=\frac{y_1-mx_1-b}{\sqrt{m^2+1}}$$
$$=\frac{3-(-3\cdot7)-12}{\sqrt{(-3)^2+1}}$$
$$=\frac{3+21-12}{\sqrt{10}}=\frac{12}{\sqrt{10}}$$
$$=\frac{12\sqrt{10}}{10}$$
$$=\frac{6}{5}\sqrt{10} \longleftrightarrow =\frac{6}{5}\sqrt{10}$$

Page 34 No. 10



Given line I $3x-5y+15=0$

Two points then are

$(-5,0)$ + $(0,3)$

Given Line II $5x+4y-32=0$

Two points then are

$\left(\frac{32}{5},0\right)$ + $(0,8)$

slope I = $\frac{3}{5}$

slope II = $-\frac{5}{4}$

A is given vertex $(0,-5)$

C must be vertex because given lines intersect here

$$3x-5y+15=0$$
$$5x+4y-32=0$$
$$15x-25y+75=0$$
$$15x+12y-96=0$$
$$-37y+171=0$$

$$\left.\begin{aligned} y &= \frac{171}{37} \\ x &= \frac{100}{37} \end{aligned}\right\} \text{coordinates of } C$$

Eq. of line parallel to Line I

$$y-y_1 = m(x-x_1)$$
$$y+5 = \frac{3}{5}(x-0)$$
$$= \frac{3}{5}x$$
$$5y+25 = 3x$$
$$3x-5y-25=0$$
$$5x+4y-32=0$$
$$15x-25y-125=0$$
$$15x+12y-96=0$$
$$-37y-29=0$$

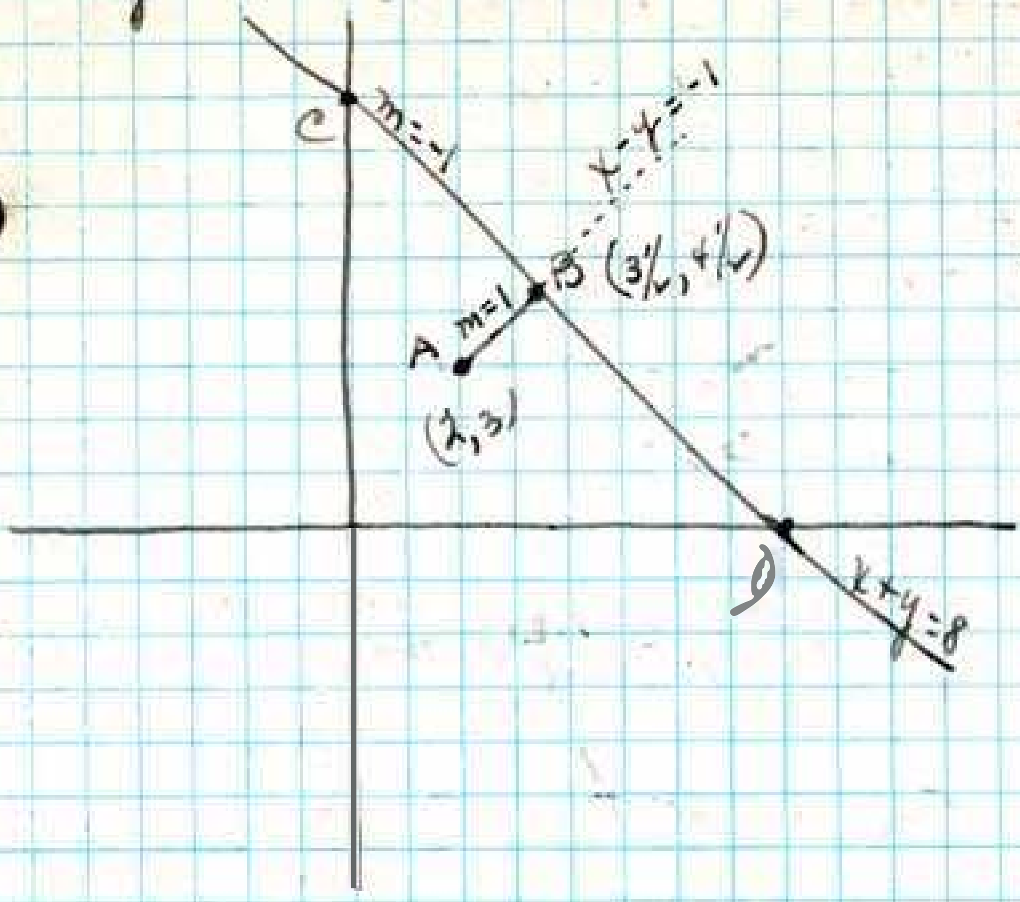
$$\left.\begin{aligned} y &= -\frac{29}{37} \\ x &= \frac{260}{37} \end{aligned}\right\} \text{coordinates of } R$$

Eq. of line parallel to II

$$y+5 = -\frac{5}{4}(x-0)$$
$$= -\frac{5}{4}x$$
$$\frac{5}{4}x+y+5=0$$
$$5x+4y+20=0$$
$$3x-5y+15=0$$
$$15x+12y+60=0$$
$$15x-25y+75=0$$
$$37y-15=0$$

$y = \frac{15}{37}$, + $x = -\frac{160}{37}$ (coordinates of S)

Page 42 1(a)



Given line $x + y = 8$
CD

$y = -x + 8$

$m = -1$

To find AB ($\perp$ to CD)

$\therefore m = 1$

$y - y_1 = m(x - x_1)$

$y - 3 = (1)(x - 2)$

$= x - 2$

$x - y = -1$ (Equation AB ✓

$\underline{x + y = 8}$

$2x = 7$

$x = 3\tfrac{1}{2}$
$y = 4\tfrac{1}{2}$ } coordinates of A

(AB) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3\tfrac{1}{2} - 2)^2 + (4\tfrac{1}{2} - 3)^2}$

$= \sqrt{\frac{9}{4} + \frac{9}{4}}$

$= \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$ ✓

Check

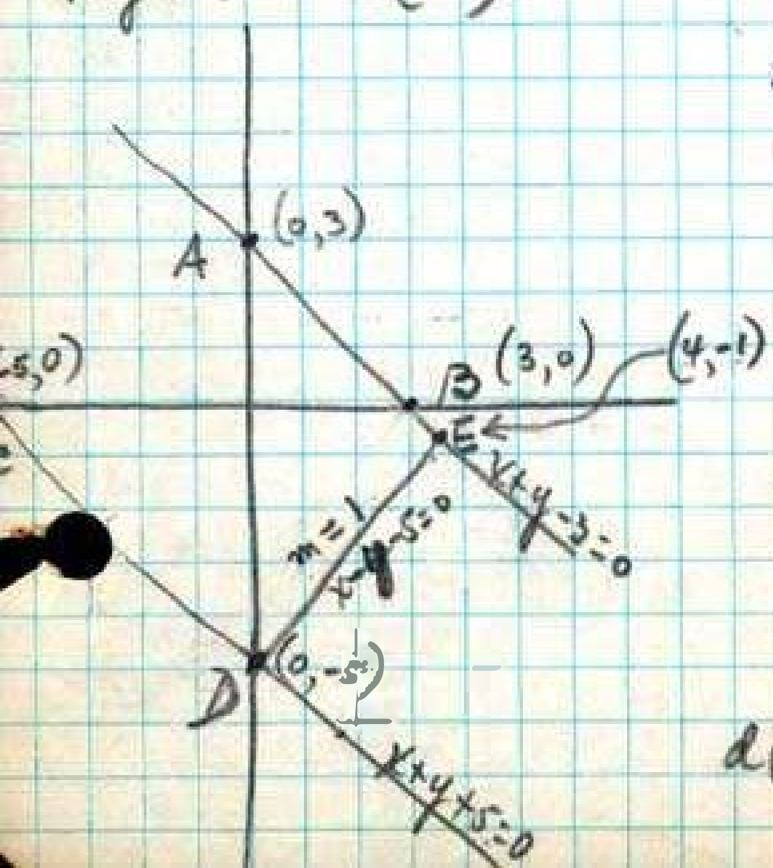
$$d = \frac{y_1 - mx_1 - b}{\sqrt{m^2 + 1}}$$

$$= \frac{3 - (1 \cdot 2) - 8}{\sqrt{(-1)^2 + 1}}$$

$$= \frac{3 + 2 - 8}{\sqrt{1 + 1}} = \frac{-3}{\sqrt{2}} = -\frac{3}{2}\sqrt{2}$$

minus & plus signs are reconciled ? by $\sqrt{\ } = \pm$?

Page 42 2(a)



To find DE

slope of each line $= -1$

$\therefore$ slope DE $= 1$

$y = mx + b$

$y = 1 \cdot x + (-5)$

$x - y - 5 = 0$ (equation DE)

$\underline{x + y - 3 = 0}$

$2x - 8 = 0$

$x = 4$
$y = -1$ } coordinates of E

$d(DE) = \sqrt{(4-0)^2 + (-1-(-5))^2}$

$= \sqrt{16 + 16} = 4\sqrt{2}$ ✓

Check

$$d = \frac{y_1 - mx_1 - b}{\sqrt{m^2 + 1}}$$

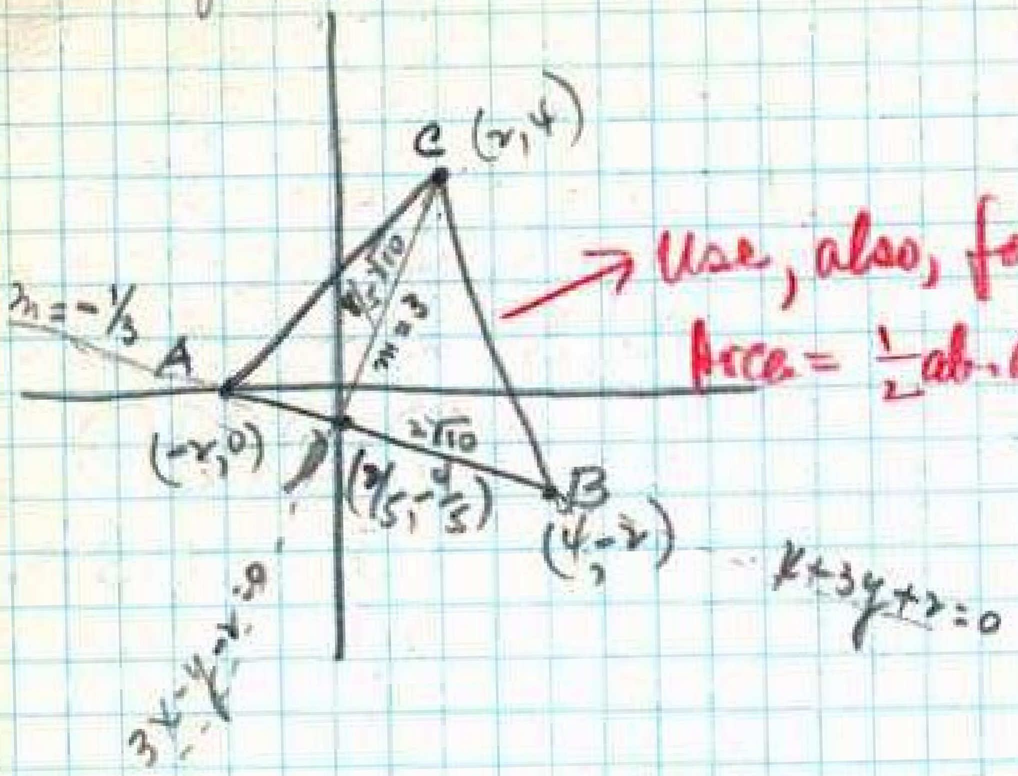
$$= \frac{-1 - (1 \cdot 4) - (-5)}{\sqrt{1 + 1}}$$

$$= \frac{-1 + 4 + 5}{\sqrt{2}}$$

$$= \frac{8}{\sqrt{2}} = \frac{8}{2}\sqrt{2}$$

$$= 4\sqrt{2}$$

Page 43 No. 4



Use, also, formula Area $=\frac{1}{2}ab\sin C$

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x+2}=\frac{-2-0}{4+2}$$

$$4y+2y=-2x-4$$

$$2x+6y+4=0$$

$\checkmark$ $x+3y+2=0$ (Equation AB)

$$m\,AB=-\frac{1}{3}\ \checkmark$$

$$d\,AB=\sqrt{(4+2)^2+(-2)^2}$$

$$=\sqrt{36+4}=\sqrt{40}=2\sqrt{10}$$

slope CD $=3$

$$y-y_1=m(x-x_1)$$

$$y-4=3(x-2)$$

$$=3x-6$$

$$3x-6=y-4$$

$3x-y-2=0$ (Equation CD)

$$x+3y+2=0$$

$$9x-3y-6=0$$

$$10x-4=0$$

$$5x-2=0$$

$x=\frac{2}{5}$, $y=-\frac{4}{5}$ coordinates of D $\checkmark$

$$d\,(CD)=\sqrt{(2-\tfrac{2}{5})^2+(4-(-\tfrac{4}{5}))^2}$$

$$=\sqrt{\left(\frac{8}{5}\right)^2+\left(\frac{24}{5}\right)^2}$$

$$=\sqrt{\frac{64}{25}+\frac{576}{25}}=\sqrt{\frac{640}{25}}$$

$$=\frac{8}{5}\sqrt{10}$$

Area $\triangle=\frac{1}{2}ba$

$$=\frac{1}{2}\cdot 2\sqrt{10}\cdot\frac{8}{5}\sqrt{10}$$

$$=\frac{1\cdot 2\sqrt{10}\cdot 8\sqrt{10}}{10}$$

$$=\frac{160}{10}=16$$

$x+3y+2=0$ $y=-\frac{1}{3}x-\frac{2}{3}$

check

$$d_{(CD)}=\frac{y_1-mx_1-b}{\sqrt{m^2+1}}$$

$$=\frac{4-(-\frac{1}{3}\cdot 2)-(-\frac{2}{3})}{\sqrt{\frac{1}{9}+1}}$$

$$=\frac{4+\frac{2}{3}+\frac{2}{3}}{\sqrt{\frac{10}{9}}}$$

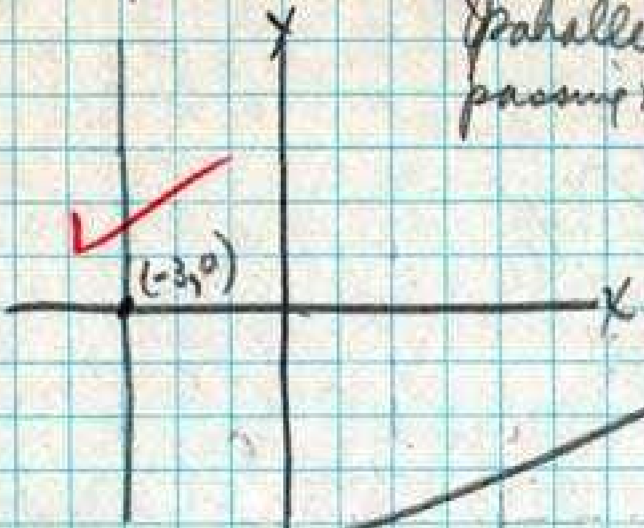
$$=\frac{\frac{16}{3}}{\frac{\sqrt{10}}{3}}=\frac{16}{3}\cdot\frac{3}{\sqrt{10}}=\frac{16}{\sqrt{10}}$$

$$=\frac{16\sqrt{10}}{10}=\frac{8}{5}\sqrt{10}$$

Page 48 1(a)

$x = -3$

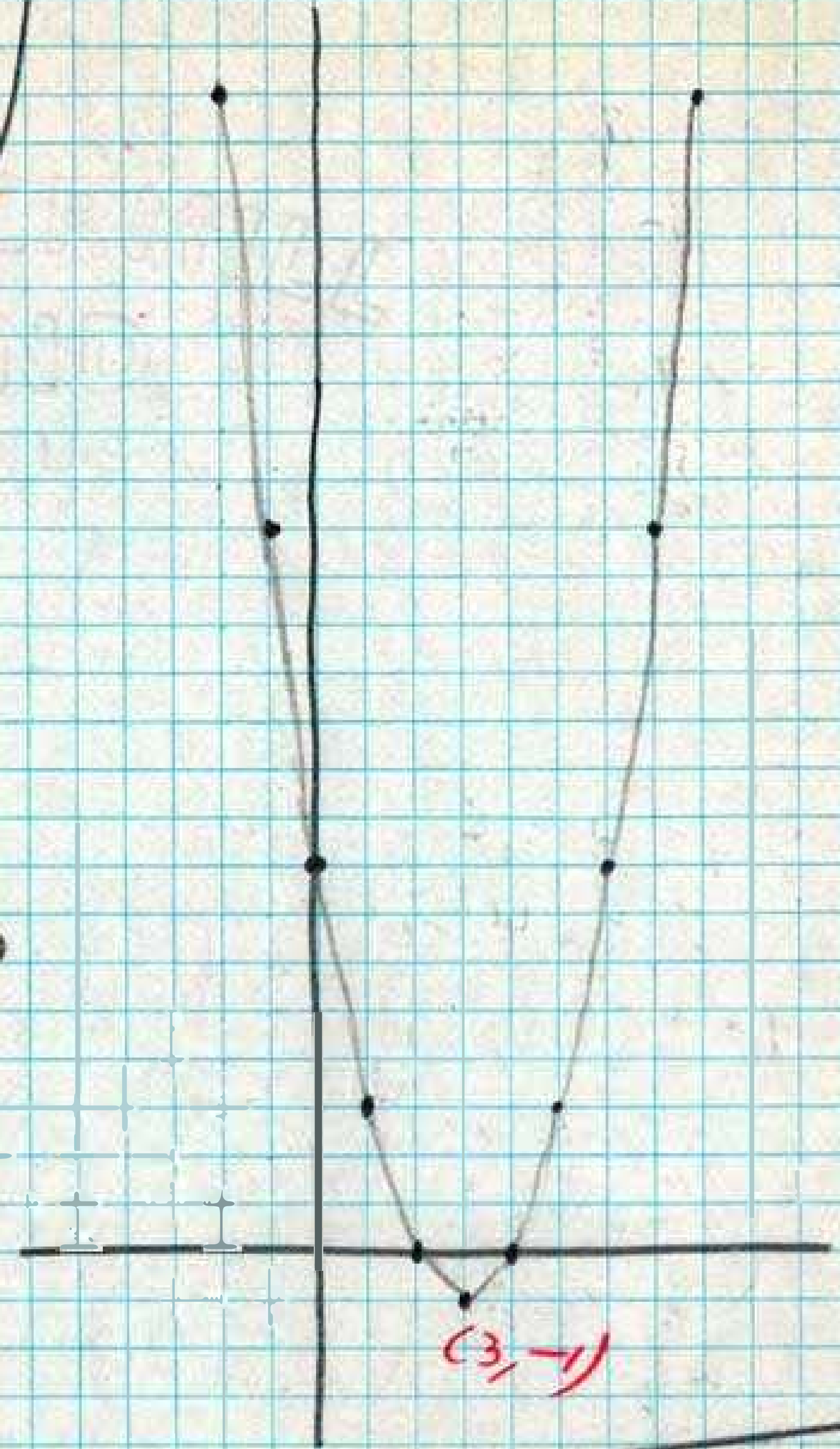
(Graph is line parallel to y axis passing thru $(-3,0)$)



1e

$y = x^2 - 6x + 8$

x	y
-2	24
-1	15
0	8
1	3
2	0
3	-1
4	0
5	3
6	8
7	15
8	24



1e

$x^2 + y^2 = 32$

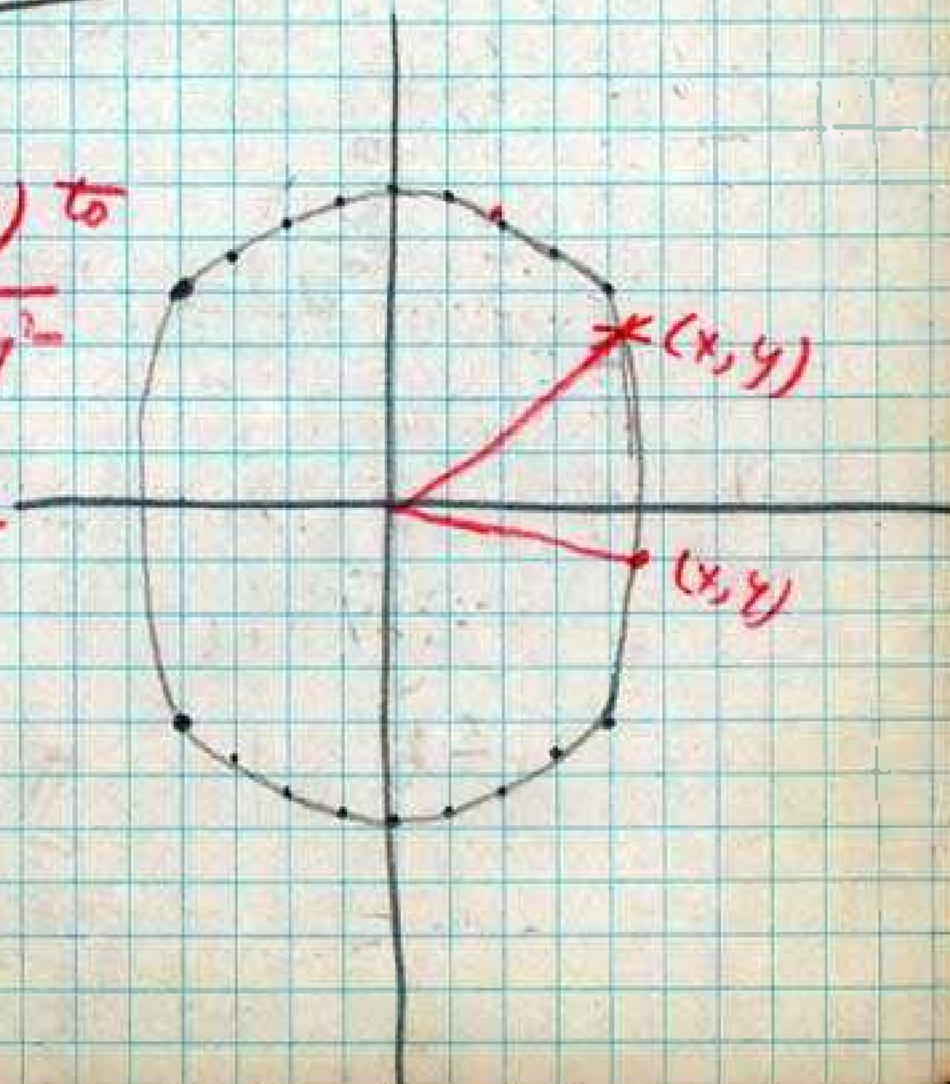
$y = \pm\sqrt{32 - x^2}$

x	y	
-4	4	
-3	$\sqrt{23}$	4.78
-2	$\sqrt{28}$	5.29
-1	$\sqrt{31}$	5.57
0	$4\sqrt{2}$	5.65
1	$\sqrt{31}$	
2	$\sqrt{28}$	
3	$\sqrt{23}$	
4	4	

$\pm$

dist. from $(0,0)$ to $(x,y) = \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = \sqrt{32}$



Page 48

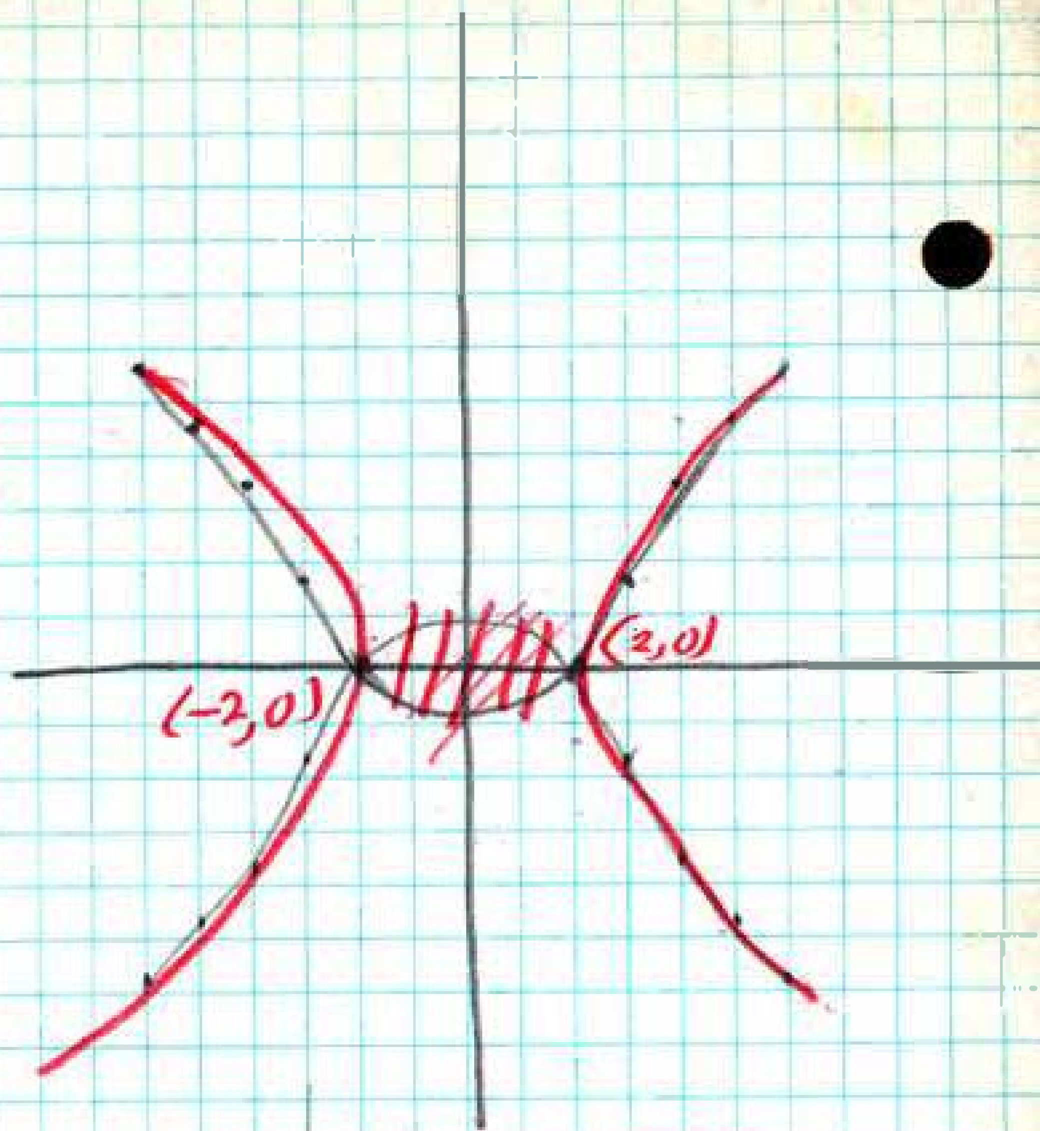
1g

$$x^2 - y^2 = 4$$

$$-y^2 = 4 - x^2$$

$$y^2 = x^2 - 4 \quad \therefore y = \pm\sqrt{x^2-4}$$

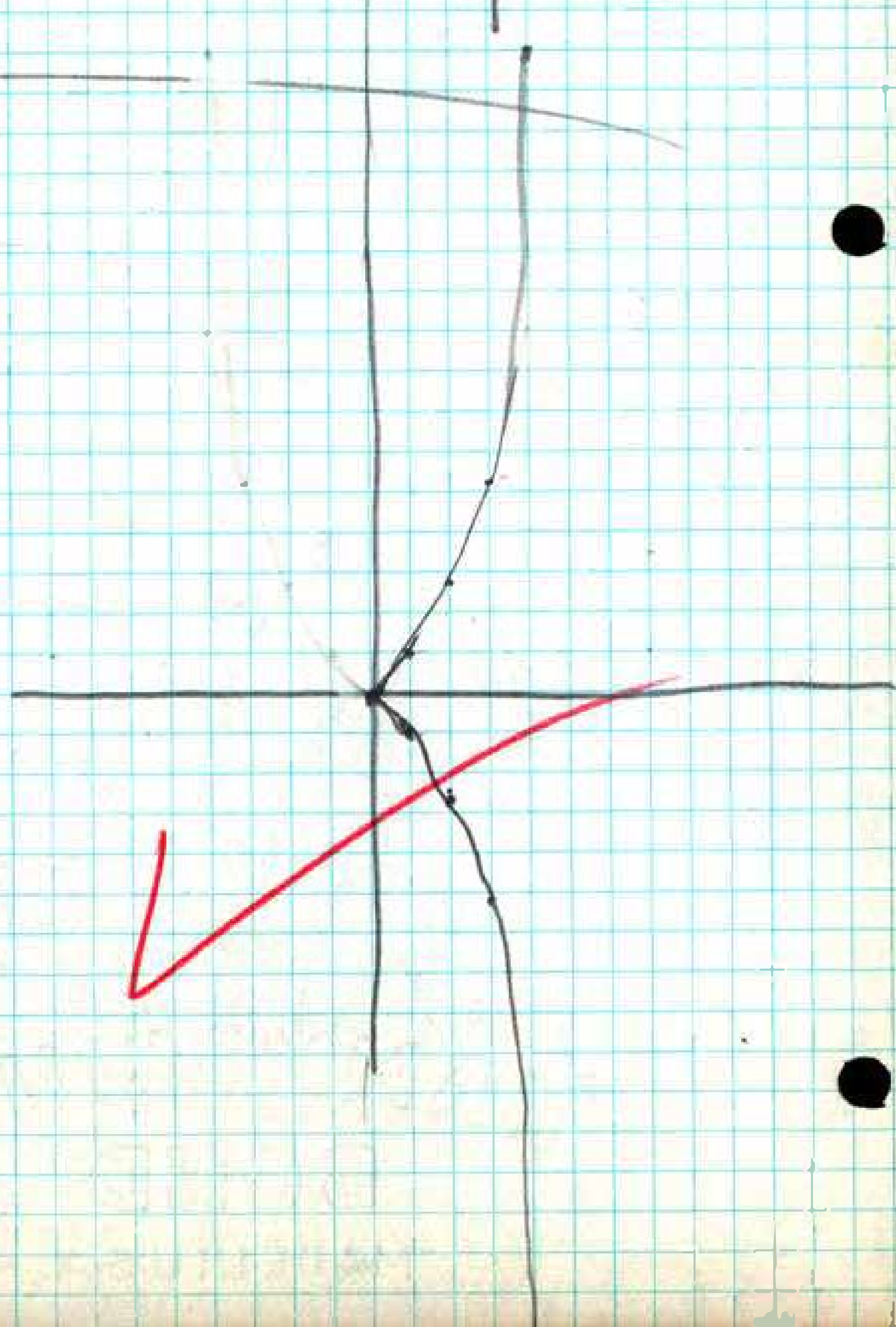
x	y	
−6	$\pm\sqrt{32}$	(5.66)
−5	$\pm\sqrt{21}$	(4.58)
−4	$\pm\sqrt{12}$	(3.46)
−3	$\pm\sqrt{5}$	(1.73)
−2	0	
1		
2	0	
3	$\pm\sqrt{5}$	
4	$\pm\sqrt{12}$	
5	$\pm\sqrt{21}$	
6	$\pm\sqrt{32}$	



1k

$$y^2 = x^3$$

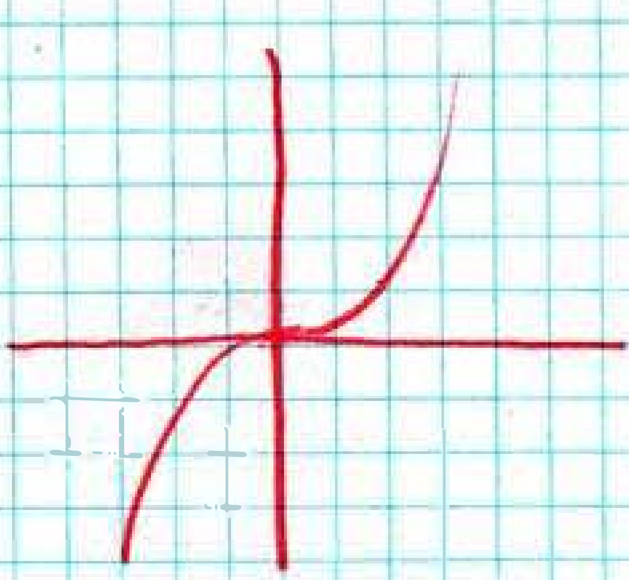
x	y	
0	0	
1	± 1	
2	$\pm\sqrt{8}$	(2.83)
3	$\pm\sqrt{27}$	(5.2)
4	± 16	

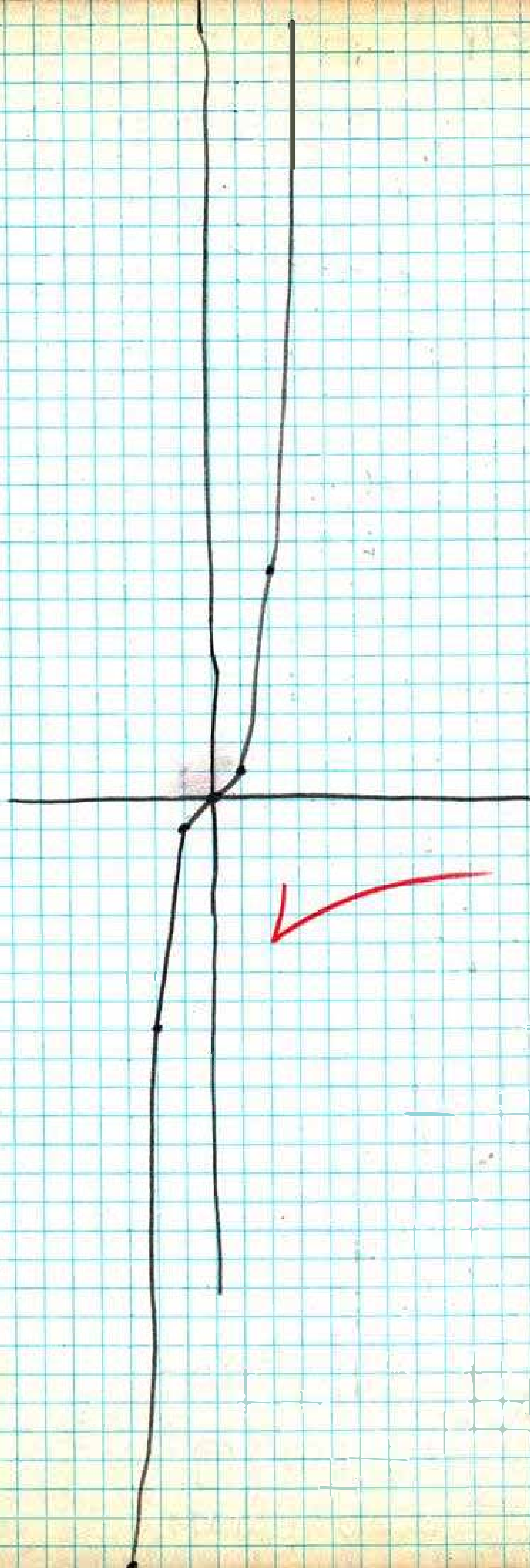


Page 48 1m

$y = x^3$

x	y
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27



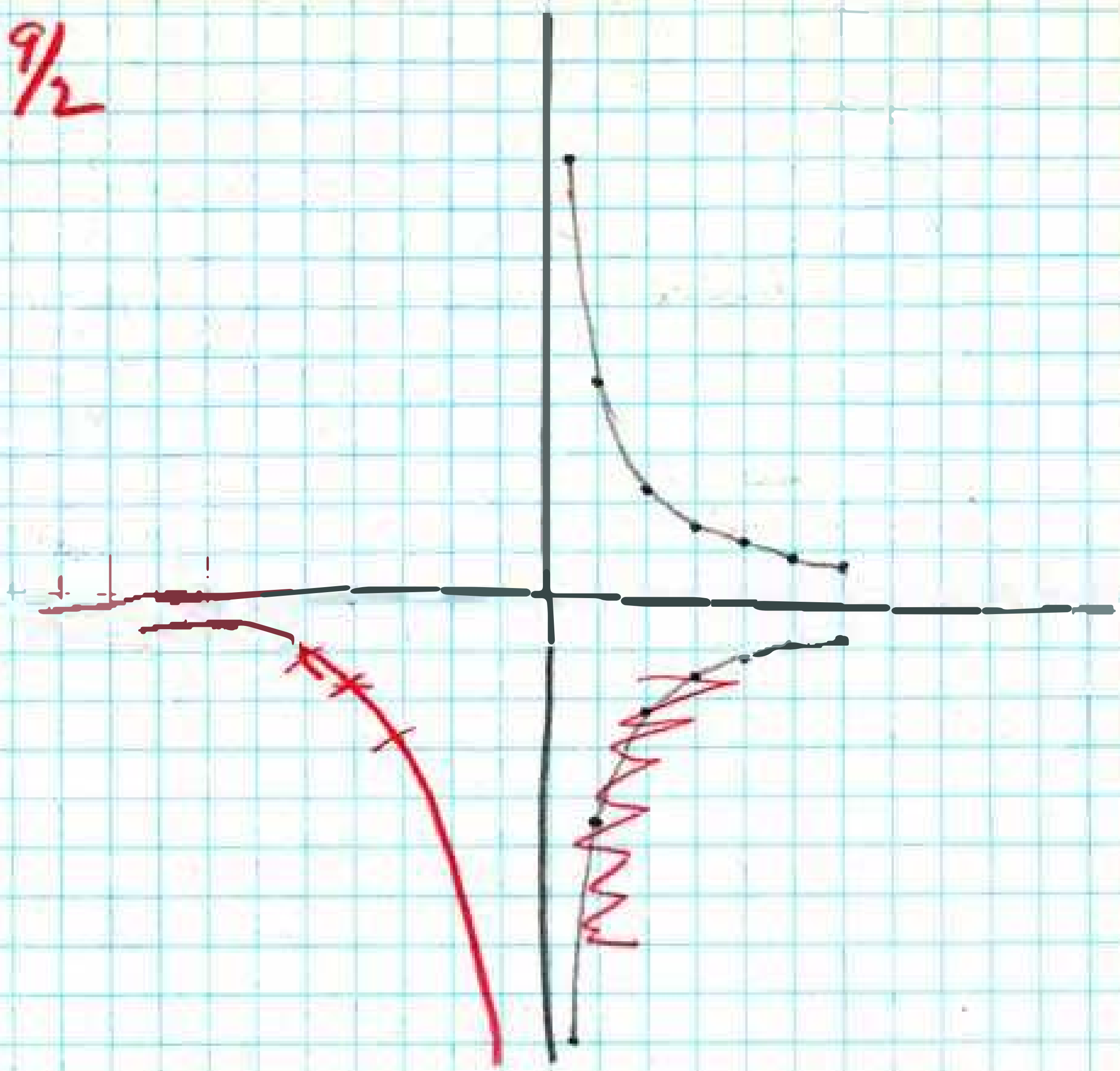


Page 48 1(o)

$2\underline{xy} = 9$, $xy = 9/2$

$y = \frac{9}{2x}$

x	y	x	y
-5	-0.9		
-4	-1.125		
-3	-1.5		
-2	-2.25		
-1	-4.5	$-\frac{1}{2}$	-9
1	4.5	$\frac{1}{2}$	9
2	2.25		
3	1.5		
4	1.125		
5	0.9		
6	0.75		



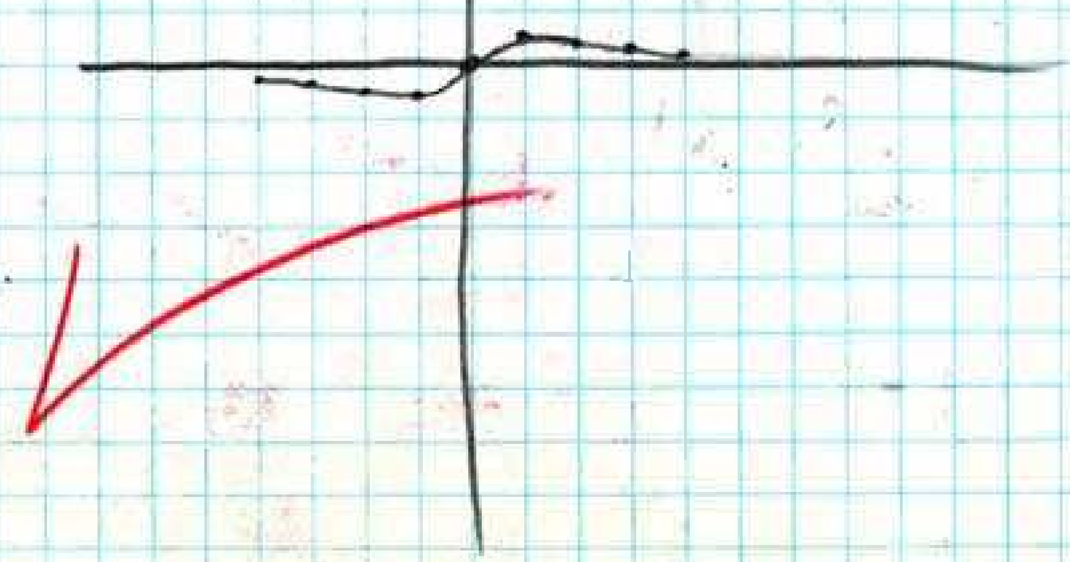
1 m

$x^2y + y = x$

$y(x^2+1) = x$

$y = \frac{x}{x^2+1}$ ✓ $y = \frac{1}{x+\frac{1}{x}}$

x	y
-4	-4/17
-3	-.3
-2	-.4
-1	-.5
0	0
1	.5
2	.4
3	.3
4	4/17



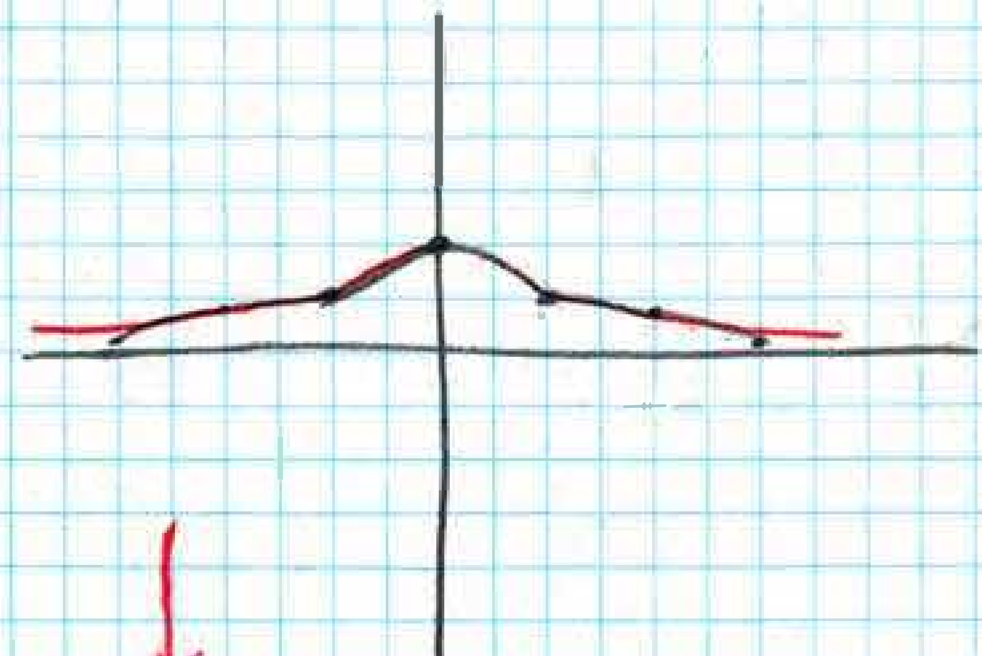
Page 48 No. 1 (t)

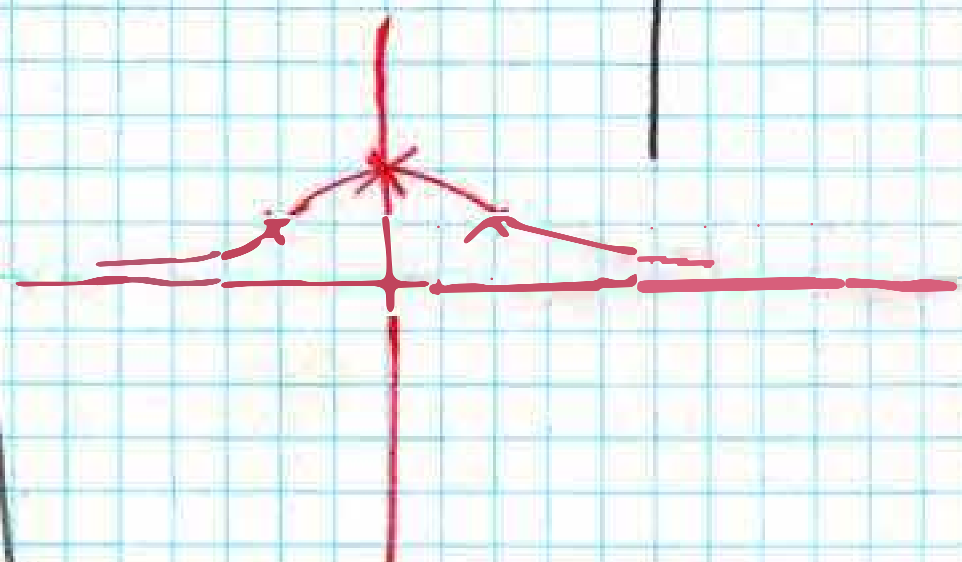
$$x^2y + 4y = 8$$

$$y(x^2+4) = 8$$

$$y = \frac{8}{x^2+4}$$

x	y
-6	$-.2$
-4	$.8$
-2	1
0	2
2	1
4	$.8$
6	$.2$





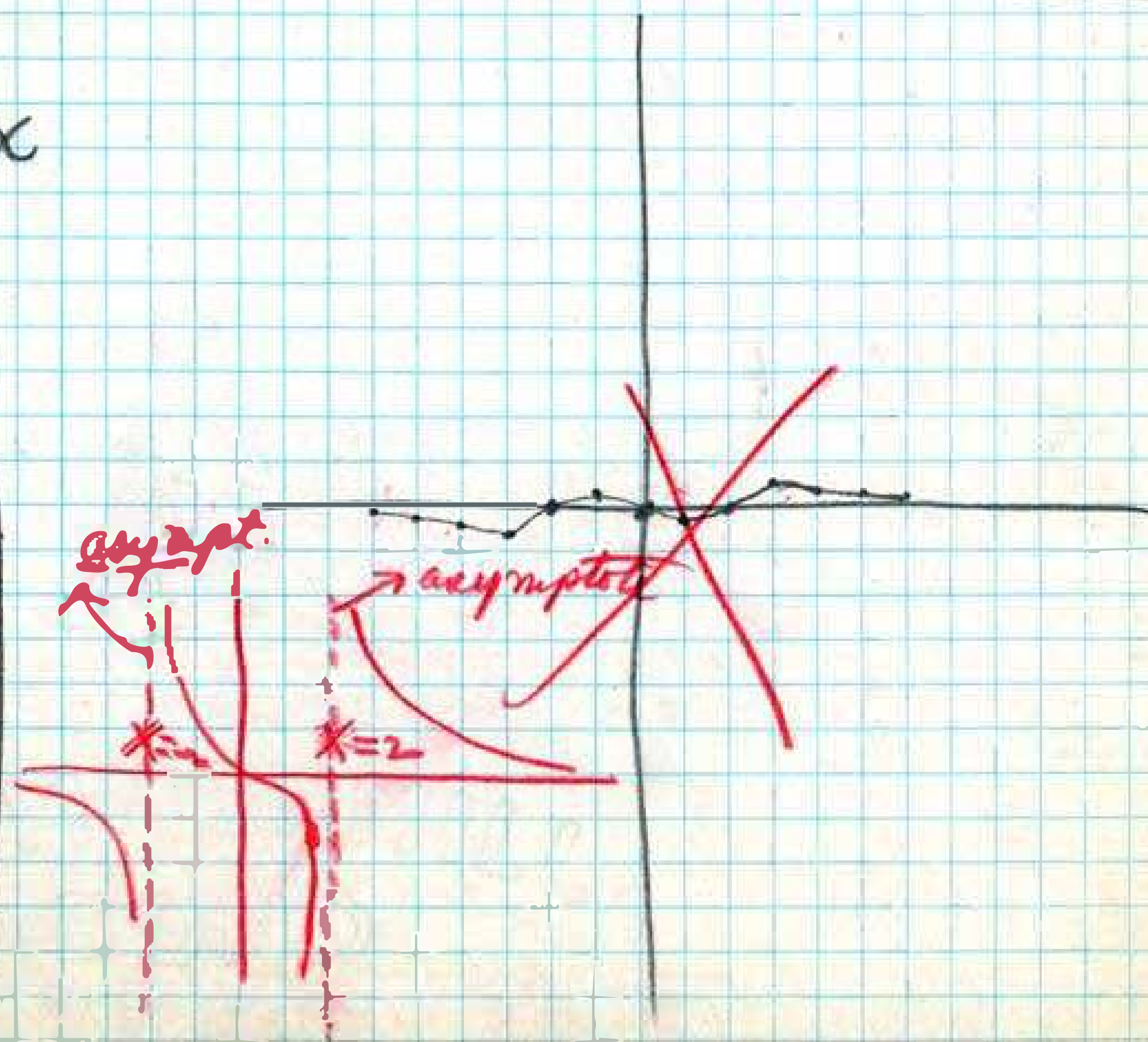
(1 v)

$$x^2y - 4y = x$$

$$y(x^2-4) = x$$

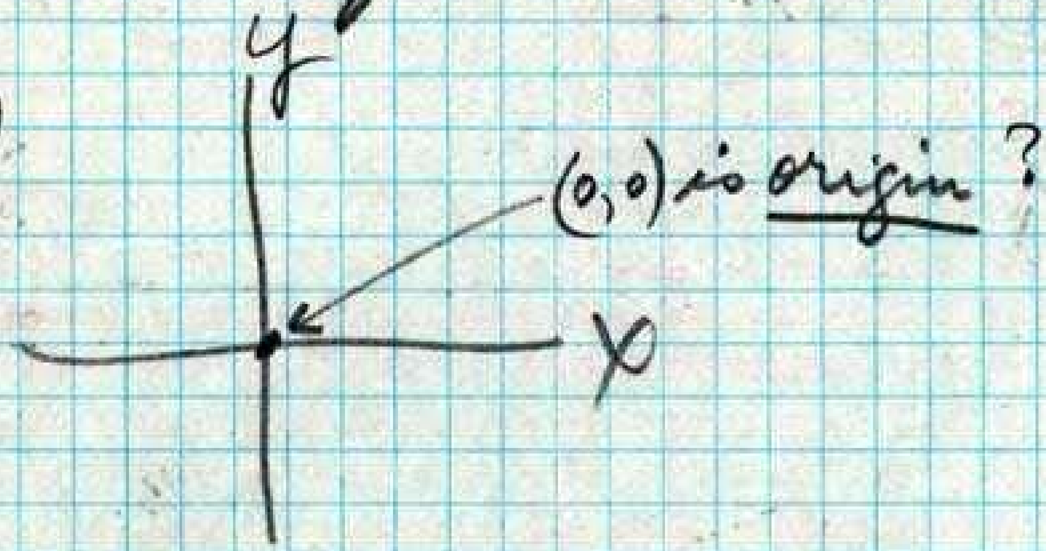
$$y = \frac{x}{x^2-4}$$

x	y
-6	$-3/16$
-5	$-5/21$
-4	$-4/3$
-3	$-3/5$
-2	8
-1	$1/3$
0	0
1	$-1/3$
2	0
3	$3/5$
4	$1/3$
5	$5/24$
6	$3/16$



Questions — Just to make sure

(1)



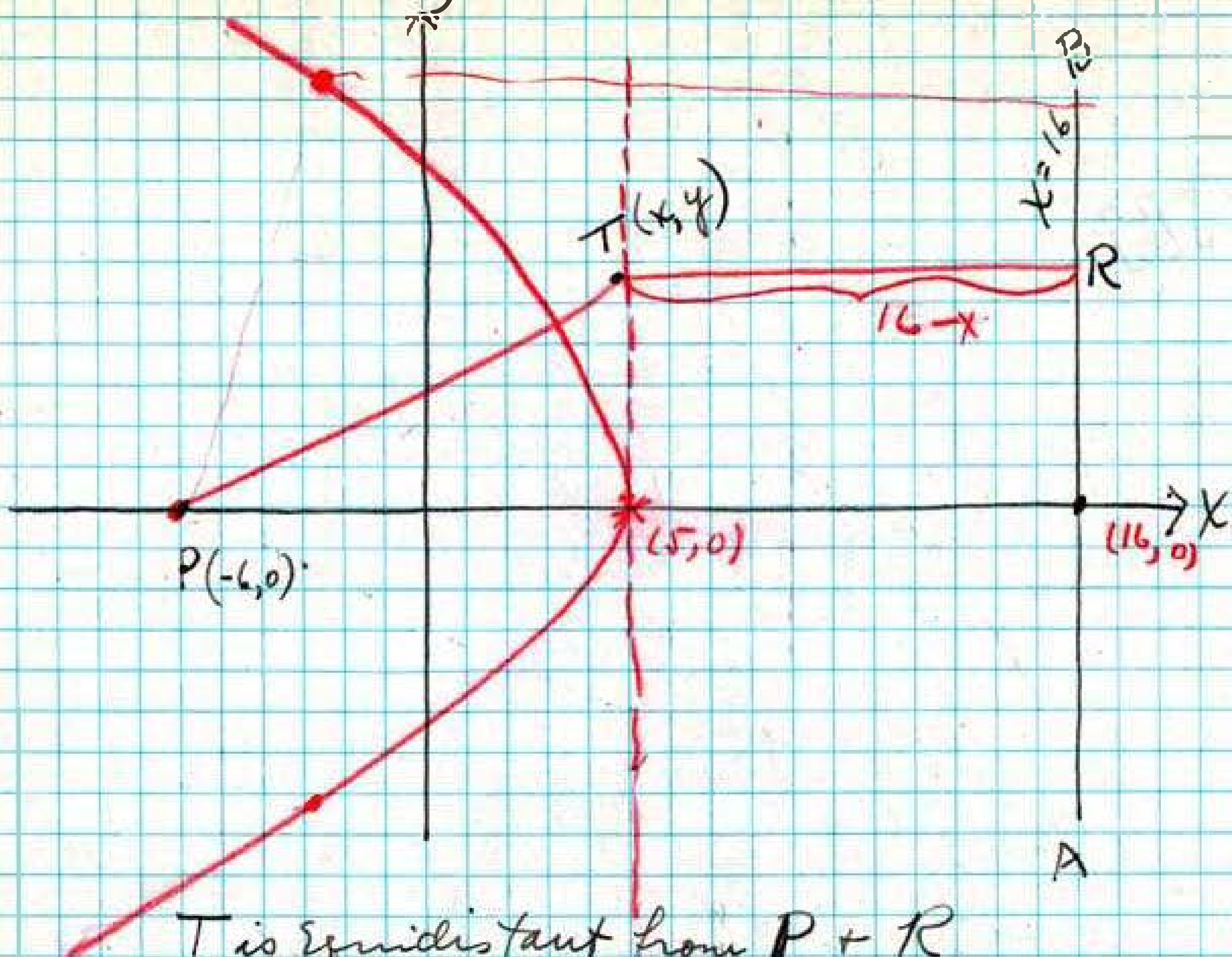
(2) Equation of line is $x + 3y + 2 = 0$

If I write it in $y = mx + b$ form

$y = mx + b$

$y = -\frac{1}{3}x - \frac{2}{3}$, does this mean the 2 in the equation by being transformed to $-\frac{2}{3}$ now represents or rather is, the Y intercept of the line?

Page 56 (20)



T is Equidistant from P + R

$\therefore \quad PT = TR$

$PT = \sqrt{(x+6)^2+y^2}$

$TR = x-16$

$\therefore \sqrt{(x+6)^2+y^2} = \cancel{x-16}\ 16-x$

$(x+6)^2+y^2 = (x-16)^2$

$x^2+12x+36+y^2 = x^2-32x+256$

$12x+y^2+32x = 256-36$

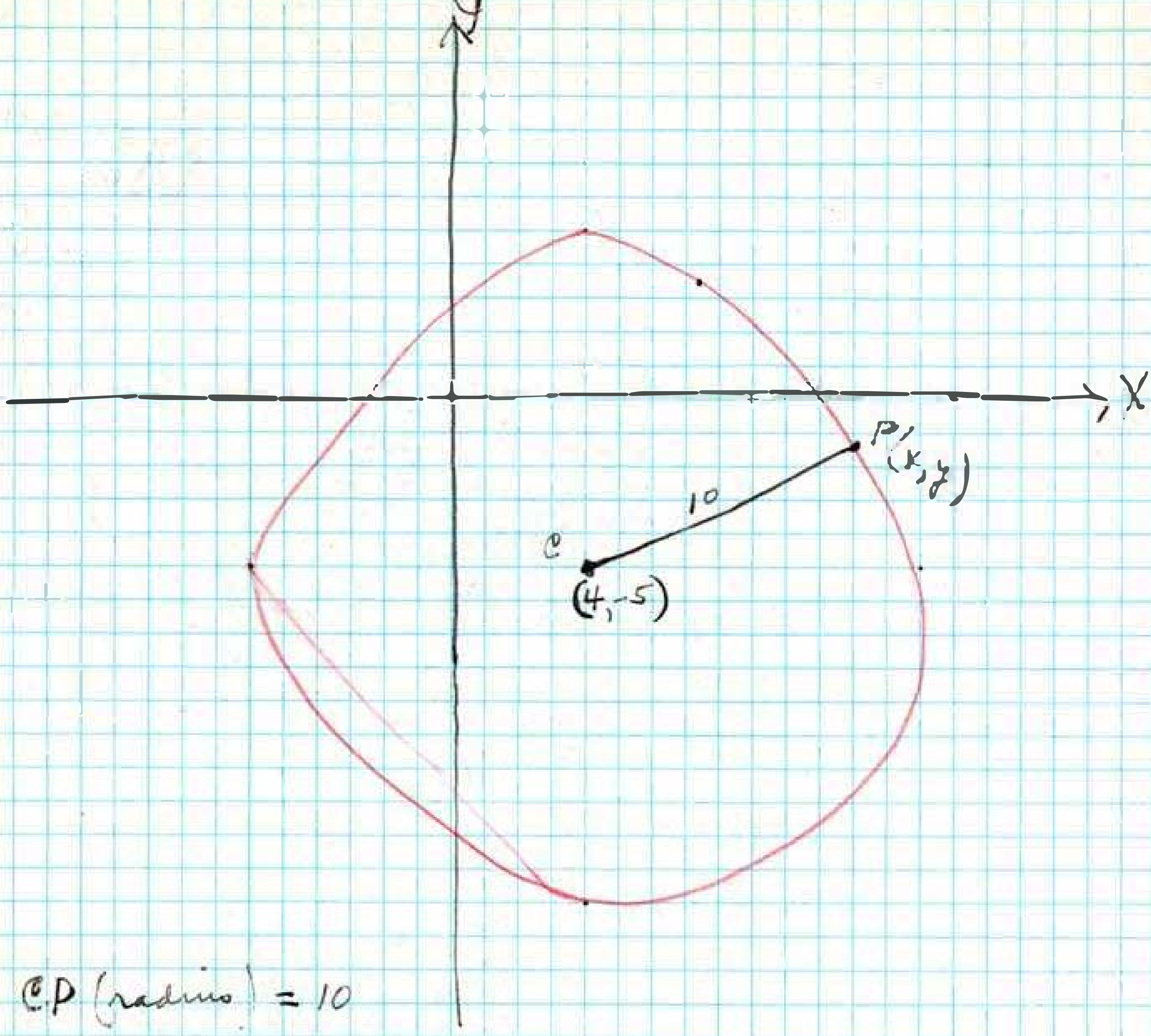
$y^2+44x = 220$ ✓

$y^2 = 220-4x$

$y = \pm\sqrt{220-4x}$

(Equation of locus of point T equidistant from P(-6,0) + line (x=16)

x y



CP (radius) = 10

$$CP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 10$$

$$= \sqrt{(x-4)^2 + (y+5)^2} = 10$$

Squaring $(x-4)^2 + (y+5)^2 = 100$

$$x^2 - 8x + 16 + y^2 + 10y + 25 = 100$$

$$x^2 - 8x + y^2 + 10y = 100 - 25 - 16$$

Equation of Circle $x^2 - 8x + 10y + y^2 = 59$

249) 1c

Question: To reduce this equation quickly to $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

$6x^2-2y^2+25=0$

$6x^2-2y^2=-25$

$2y^2-6x^2=25$

Real Intercepts = $y=\pm\frac{5\sqrt{2}}{2}$

Imaginary Intercepts = $x=\pm\frac{5\sqrt{6}}{6}$

$a=\pm\frac{5\sqrt{2}}{2}$, $b=\pm\frac{5\sqrt{6}}{6}$

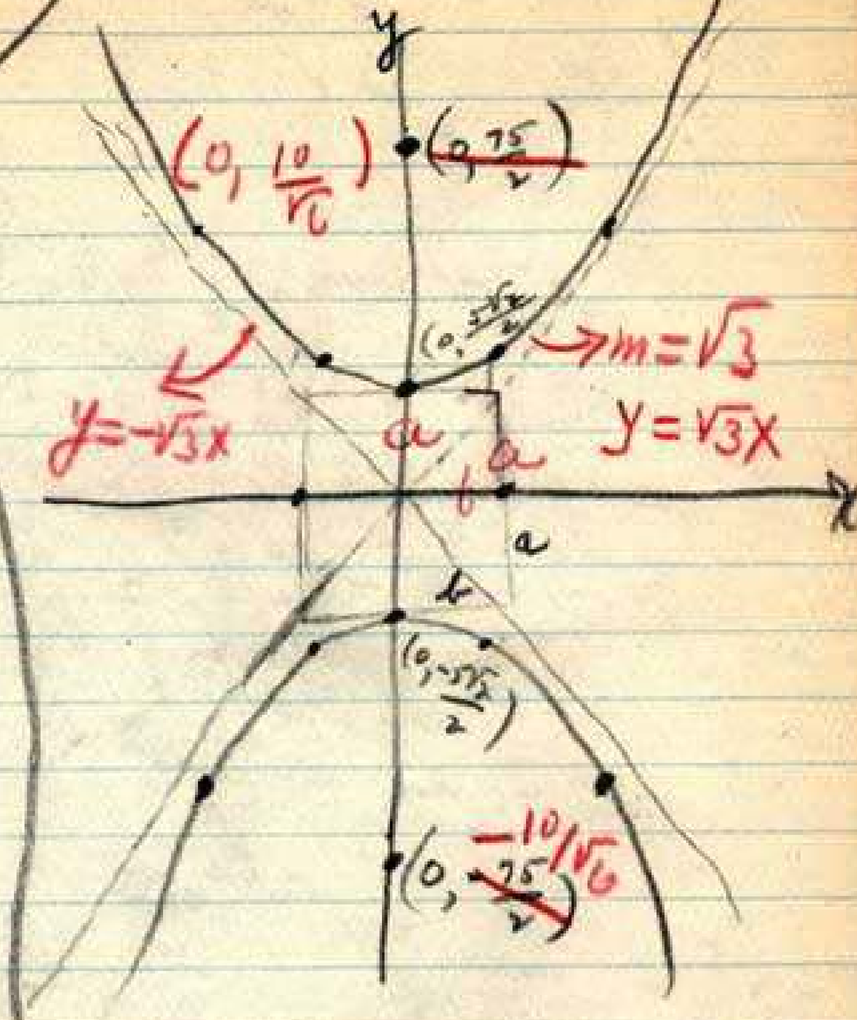
$c^2=a^2+b^2=\frac{50}{4}+\frac{25}{6}=\frac{75}{2}$ $\frac{200}{12}=\frac{100}{6}$

$c=\sqrt{\frac{75}{2}}=\frac{5\sqrt{3}}{\sqrt{2}}=\pm\frac{5\sqrt{6}}{2}$

$c=\frac{10}{\sqrt{6}}$

If $x=\pm1$, $y=\pm\sqrt{\frac{31}{2}}=\pm\frac{\sqrt{62}}{2}$

If $x=\pm2$, $y=\frac{7\sqrt{2}}{2}$



$2y^2-6x^2=25$

$\frac{2y^2}{25}-\frac{6x^2}{25}=1$

$\frac{y^2}{\frac{25}{2}}-\frac{x^2}{\frac{25}{6}}=1$

Thus $a=\pm\sqrt{\frac{25}{2}}$

$+b^2=\pm\sqrt{\frac{25}{6}}$

Asymptote:

slope = $\frac{a}{b}$ $\sqrt{\frac{25}{2}}\div\sqrt{\frac{25}{6}}$

$=\pm\frac{\frac{5\sqrt{2}}{2}}{\frac{5\sqrt{6}}{6}}=\pm\left(\frac{5\sqrt{2}}{2}\cdot\frac{6}{5\sqrt{6}}\right)=\pm\frac{3\sqrt{2}}{\sqrt{6}}=\pm\frac{3\sqrt{12}}{6}$

$=\pm\frac{\sqrt{12}}{2}=\pm\frac{2\sqrt{3}}{2}$

$=\pm\sqrt{3}$

$\frac{5}{\sqrt{2}}\cdot\frac{\sqrt{6}}{5}=\sqrt{3}$

244) 1 f

$$9x^2 - 16y^2 + 144 = 0$$

$$9x^2 - 16y^2 = -144$$

$$16y^2 - 9x^2 = 144$$

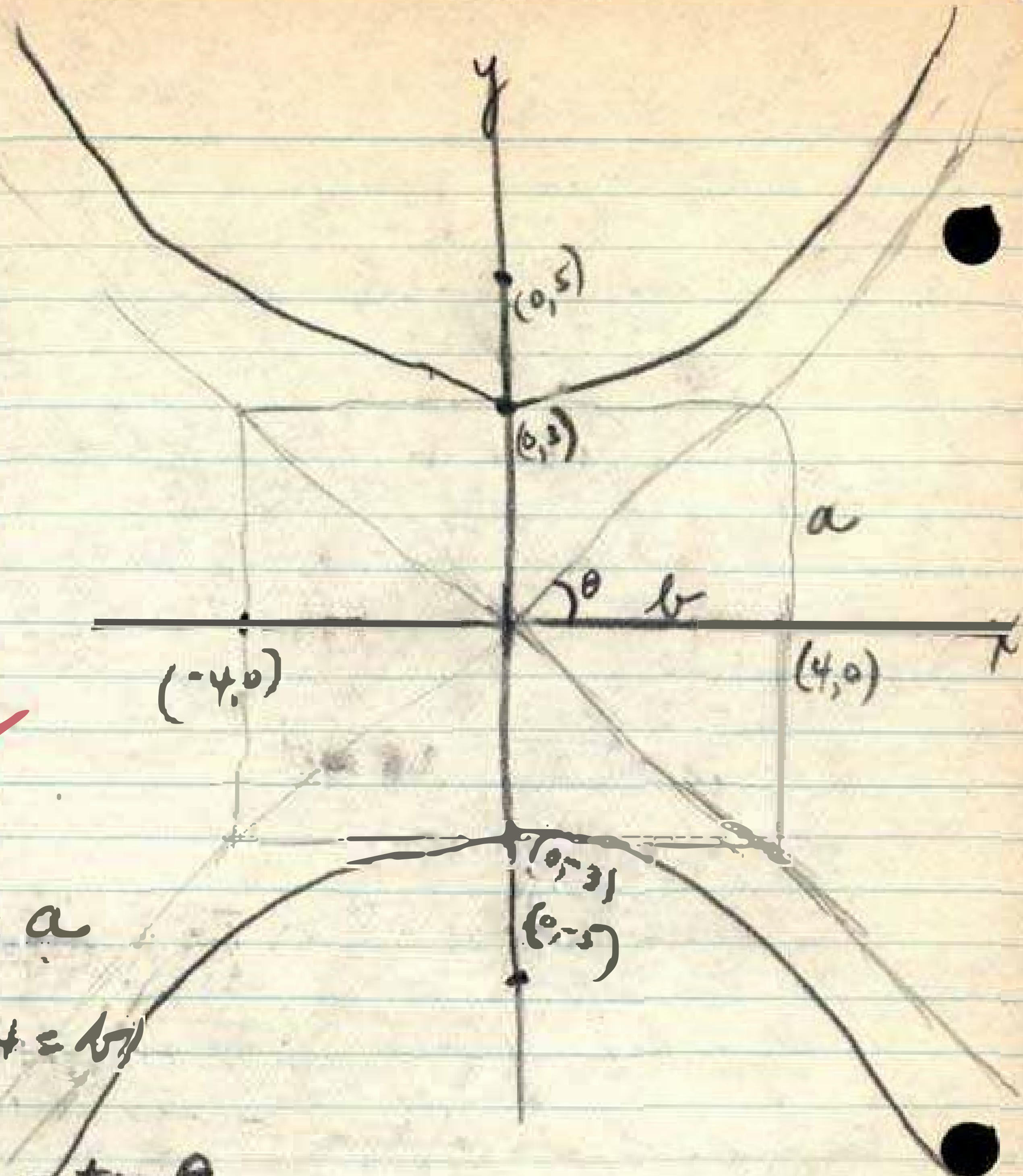
$$\frac{16y^2}{144} - \frac{9x^2}{144} = 1$$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

Real intercepts $= y = \pm 3 = a$

Imaginary intercepts $= x = \pm 4 = b$

Asymptote $= \pm \frac{a}{b} = \pm \frac{3}{4} = \tan\theta$



$$e^2 = a^2 + b^2$$

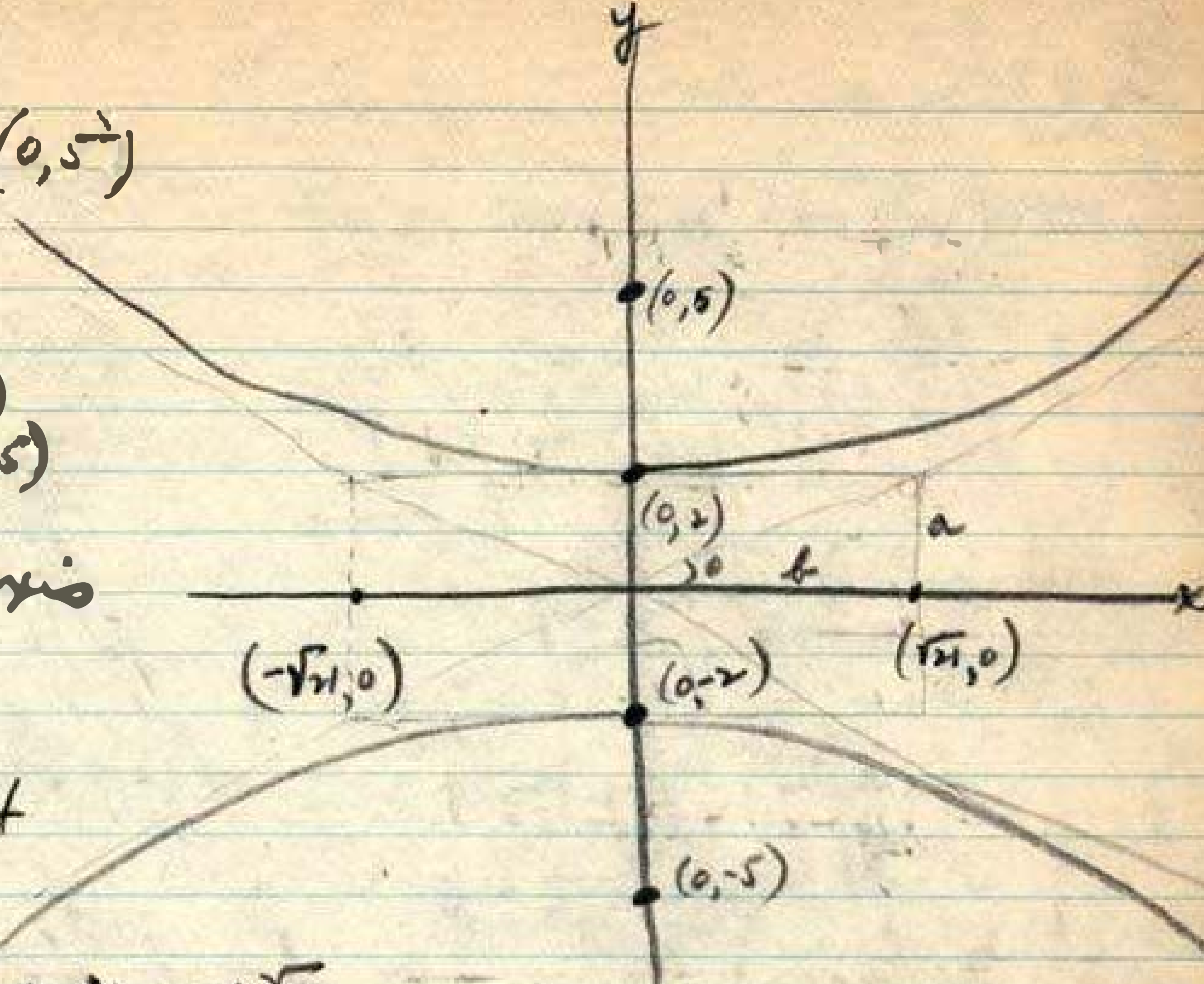
$$= 9 + 16 = 25$$

$$e = \pm 5$$

2.49) 2a

Vertex $(0,2)$, focus $(0,5)$

Center at origin



Then other vertex is $(0,-2)$

" focus is $(0,-5)$

Principal axis on y-axis

$a = 2$

$c = 5$

$b^2 = c^2 - a^2 = 25 - 4$

$b = \pm\sqrt{21}$

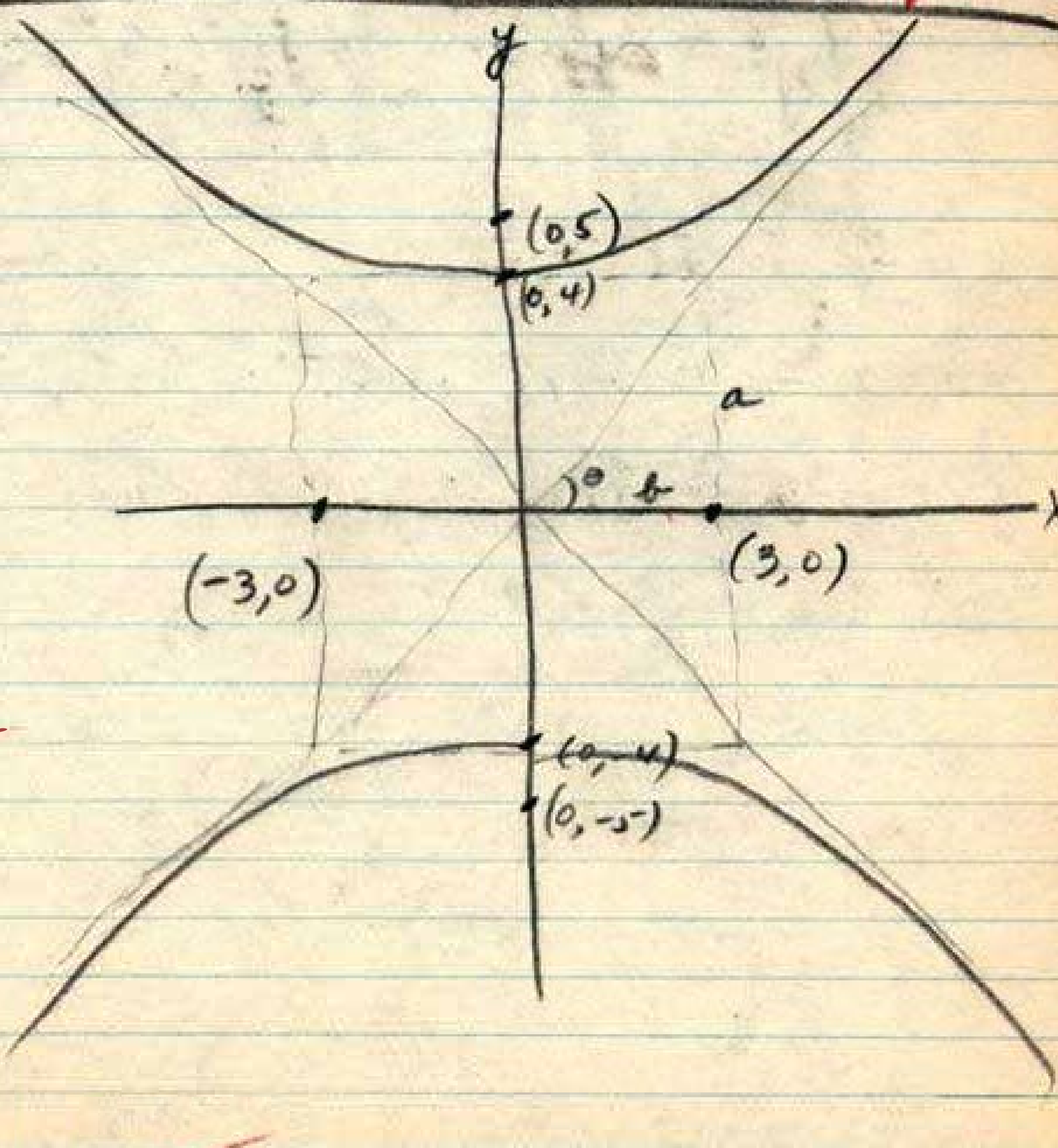
Asymptote = $\tan\theta = \pm\frac{a}{b} = \pm\frac{2}{\sqrt{21}} = \pm\frac{2\sqrt{21}}{21}$

Equation of Hyperbola = $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, or $\frac{y^2}{4} - \frac{x^2}{21} = 1$

2.49) 2b

Focus $(0,5)$, Conj. axis 6

Center at origin



$c = 5$

$b = 3$

$a^2 = c^2 - b^2 = 25 - 9 = 16$

$a = \pm 4$

Vertices at $(0,4)$ + $(0,-4)$

Asymptote = $\tan\theta$

$= \pm\frac{a}{b} = \pm\frac{4}{3}$

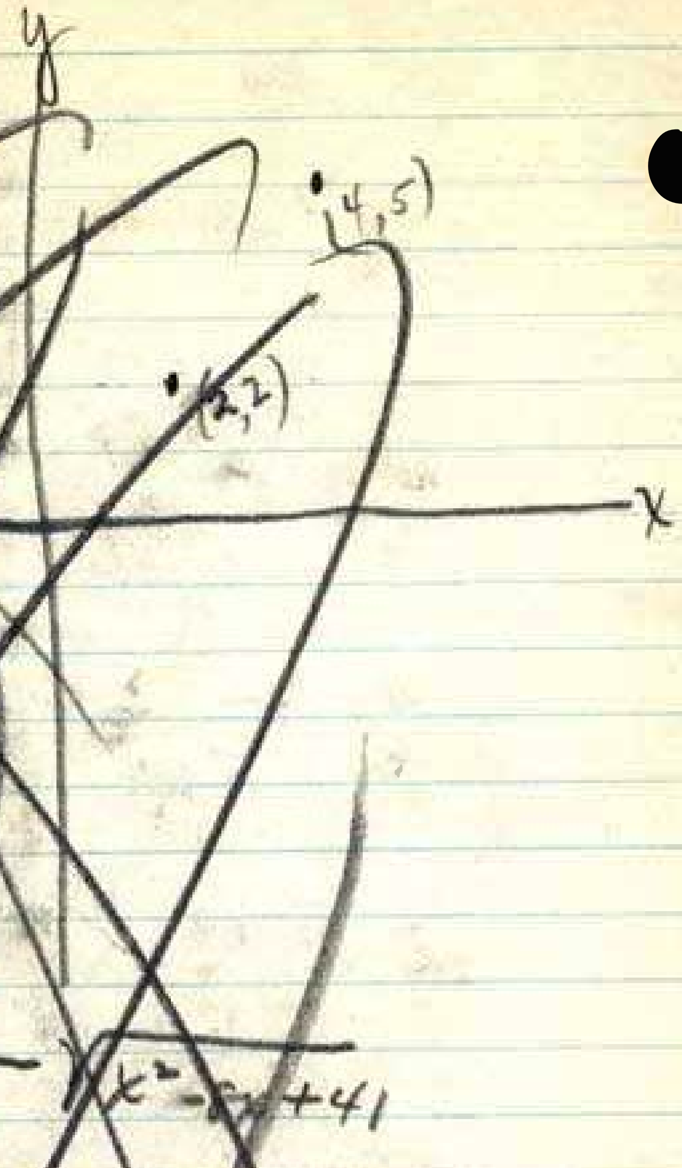
Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{y^2}{16} - \frac{x^2}{9} = 1$

249) 3 b

Let (x, y) be coordinates of one focus

$(-x, y)$ other "



Then $\sqrt{(2+x)^2+(2-y)^2} - \sqrt{(2-x)^2+(2-y)^2}$

$= \sqrt{(4+x)^2+(5-y)^2} - \sqrt{(4-x)^2+(5-y)^2}$

Since $y = 0$

$\sqrt{4+4x+x^2+4} - \sqrt{4-4x+x^2+4}$

$= \sqrt{16+8x+x^2+25} - \sqrt{16-8x+x^2+25}$

$\sqrt{x^2+4x+8} - \sqrt{x^2-4x+8} = \sqrt{x^2+8x+41} - \sqrt{x^2-8x+41}$

Squaring

$x^2+4x+8 - 2\sqrt{(x^2+4x+8)(x^2-4x+8)} - x^2+4x-8$

$= x^2+8x+41 - 2\sqrt{(x^2+8x+41)(x^2-8x+41)} - x^2+8x-41$

$8x - 2\sqrt{(x^2+4x+8)(x^2-4x+8)} = 16x - 2\sqrt{(x^2+8x+41)(x^2-8x+41)}$

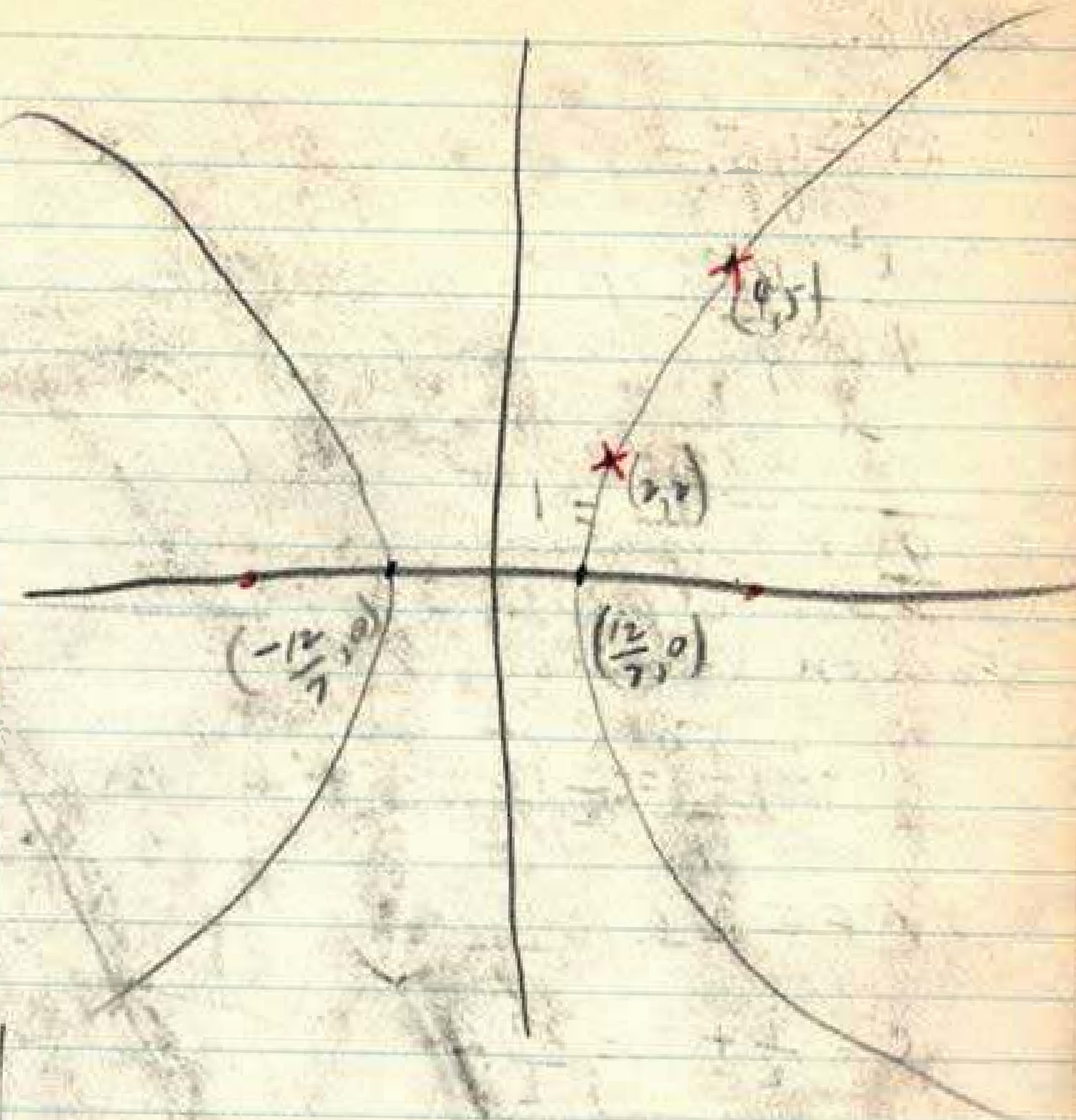
$-8x = -2\sqrt{x^4-18x^2+1681} + 2\sqrt{x^4+64}$

$64x = -4x^4 - 32x^2 - 6724 - 8\sqrt{(x^4+18x^2+1681)(x^4+64)} + 4x^4 + 296$

$32x^2+64x+6428 = -8\sqrt{x^8+18x^6+1745x^4+1152x^2+17584}$

249) 3 b

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{4}{a^2} - \frac{4}{b^2} = 1$$

$$\frac{16}{a^2} - \frac{25}{b^2} = 1$$

$$\frac{4}{a^2} - \frac{4}{b^2} = \frac{16}{a^2} - \frac{25}{b^2}$$

$$4b^2 - 4a^2 = 16b^2 - 25a^2$$

$$-12b^2 = -21a^2$$

$$12b^2 = 21a^2$$

$$4b^2 = 7a^2$$

$$a^2 = \frac{4}{7}b^2$$

Then $\frac{4}{\frac{4b^2}{7}} - \frac{4}{b^2} = 1$

$$\frac{7}{b^2} - \frac{4}{b^2} = 1$$

$$b^2 = 3$$

$$a^2 = \frac{12}{7}$$

$$\boxed{c^2 = a^2 + b^2 = \frac{12}{7} + 3 = \frac{33}{7}}$$

Equation:

$$\frac{x^2}{\frac{12}{7}} - \frac{y^2}{3} = 1$$

or

$$3x^2 - \frac{12}{7}y^2 = \frac{36}{7}$$

$$21x^2 - 12y^2 = 36$$

$$7x^2 - 4y^2 = 12$$

249) 8

$$2x^2 - y^2 = 9$$

$$\frac{2x^2}{9} - \frac{y^2}{9} = 1$$

$$\frac{x^2}{\frac{9}{2}} - \frac{y^2}{9} = 1$$

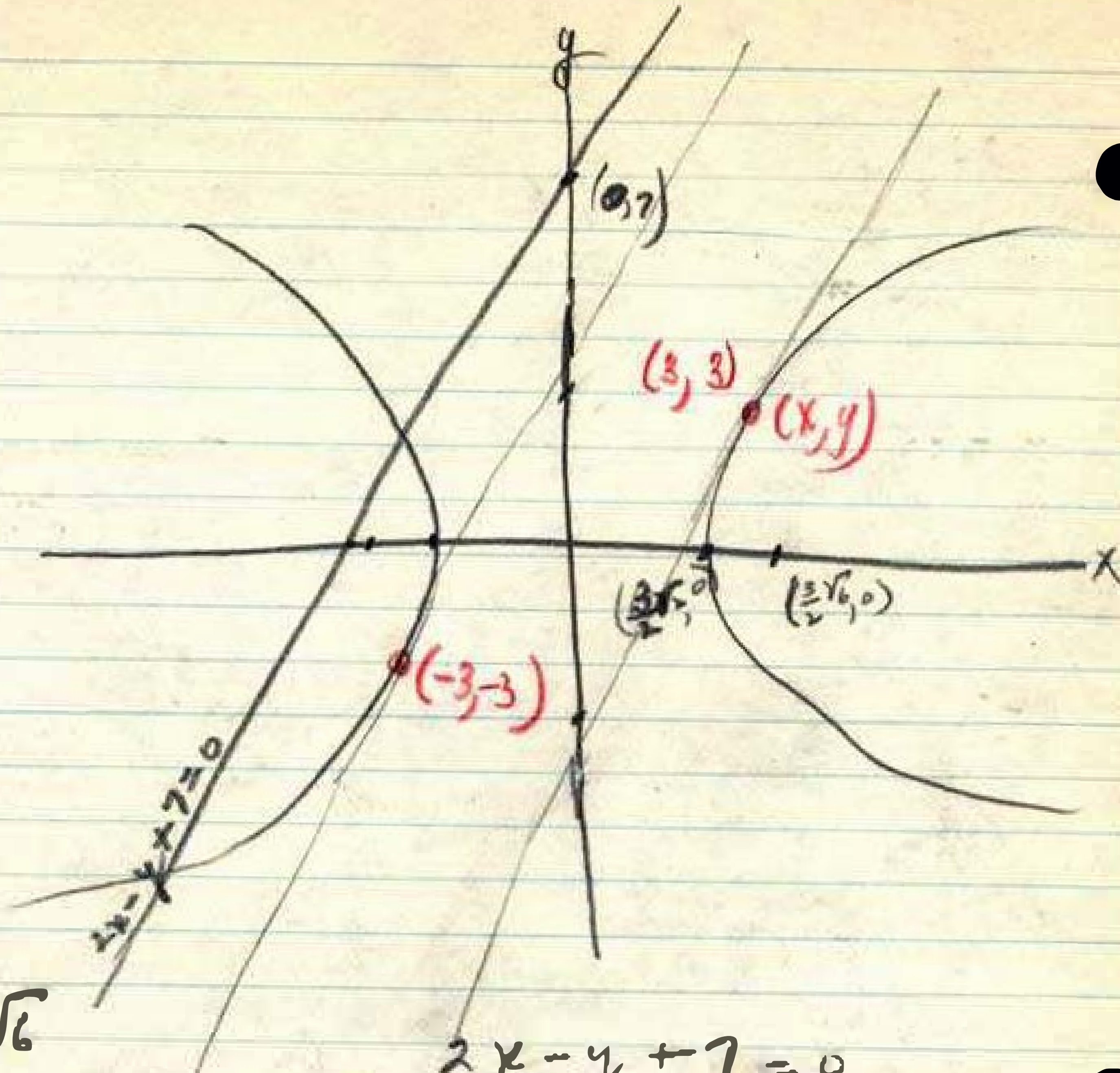
Foci on x-axis

$$a = \pm\sqrt{\frac{9}{2}} = \pm\frac{3}{2}\sqrt{2}$$

$$b = \pm 3$$

$$c^2 = \frac{9}{2} + 9 = \frac{27}{2}$$

$$c = \pm\sqrt{\frac{27}{2}} = \pm 3\sqrt{\frac{3}{2}} = \pm\frac{3}{2}\sqrt{6}$$



$$2x - y + 7 = 0$$

if $x = 0$, $y = 7$

$y = 0$, $x = -\frac{7}{2}$

$$-y = -2x - 7$$

$$y = 2x + 7$$

slope of line $= 2$ ✓

$$2x^2 - y^2 = 9$$

differentiating,

$$4x - 2y\frac{dy}{dx} = 0$$

$$-2y\frac{dy}{dx} = -4x$$

$$\frac{dy}{dx} = \frac{4x}{2y} = \frac{2x}{y}$$ ✓ $\longrightarrow$ $$2 = \frac{2x}{\pm\sqrt{2x^2-9}}$$

$$\boxed{\begin{aligned} 2x^2 - y^2 &= 9 \\ -y^2 &= 9 - 2x^2 \\ y^2 &= 2x^2 - 9 \end{aligned}}$$

$$\pm 2\sqrt{2x^2-9} = 2x$$

$$8x^2 - 36 = 4x^2$$

$$4x^2 = 36$$

$$x^2 = 9$$

$$x = \pm 3$$

$$\frac{2x}{y} = 2$$

$$x = y = \pm 3$$

Equation: $2 = \dfrac{y-3}{x-3}$

$$2x - 6 = y - 3$$

$$2x - y = 3$$

or ✓ $2 = \dfrac{y+3}{x+3}$

$$2x + 6 = y + 3$$

$$2x - y = -3$$

250) 11

$x^2 - y^2 = 5$

$\frac{x^2}{5} - \frac{y^2}{5} = 1$

Foci on x-axis

$a = \sqrt{5}$

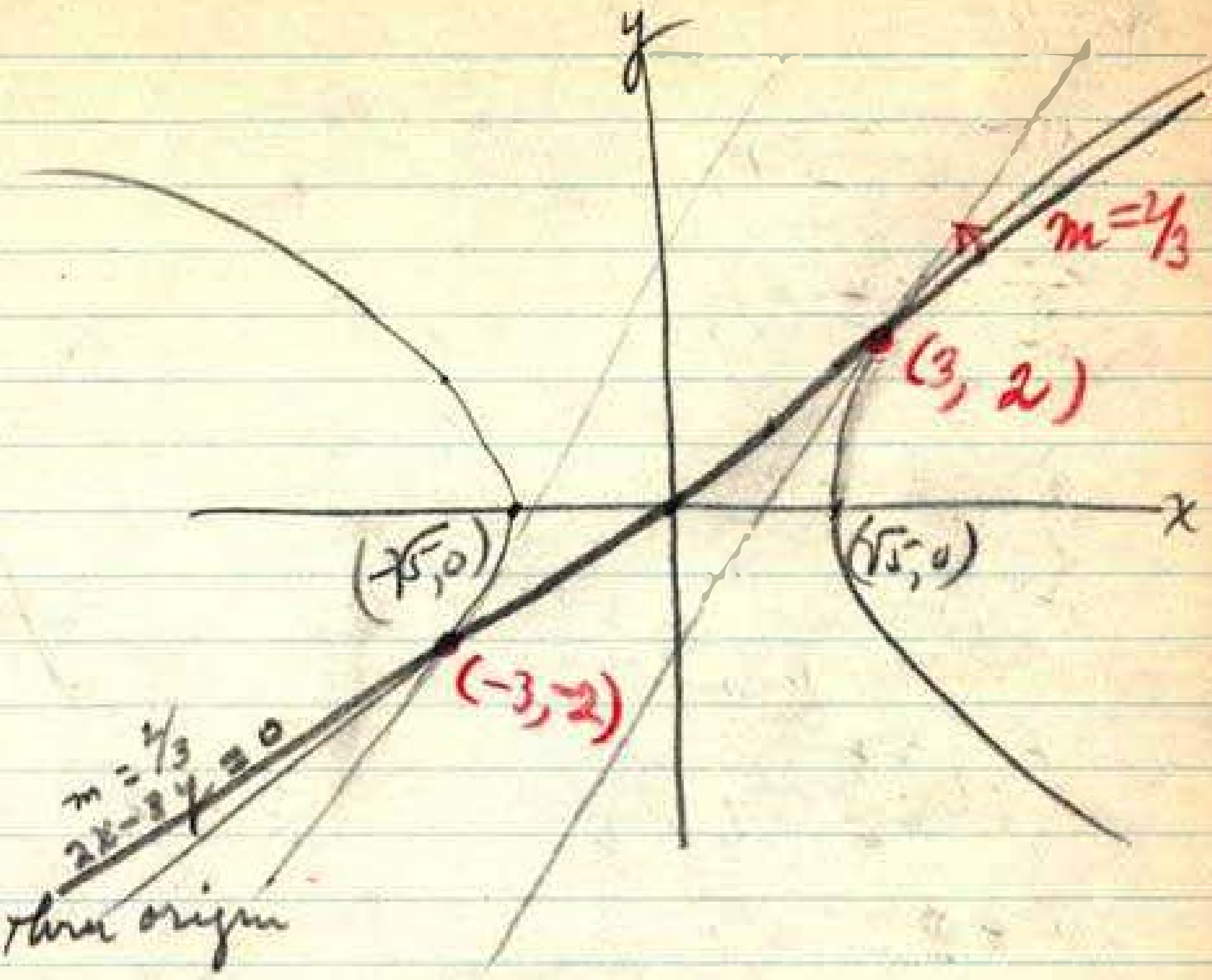
$b = \sqrt{5}$

when $y = \pm x$, $x = \pm 3$

Line: $2x - 3y = 0$

when $\left.\begin{array}{l} y = 0,\ x = 0 \\ x = 0,\ y = 0 \end{array}\right\}$ ∴ line passes thru origin

When $x = \sqrt{5}$, $y = \frac{2}{3}\sqrt{5}$



$-3y = -2x$

$y = \frac{2}{3}x$

slope $= 2/3$

$x^2 - \left(\frac{2}{3}x\right)^2 = 5$

$x^2 - \frac{4}{9}x^2 = 5$

$9x^2 - 4x^2 = 45$

$5x^2 = 45$

$x^2 = 9$

$x = \pm 3$

$9 - y^2 = 5$

$y^2 = 4$

$y = \pm 2$

$x^2 - y^2 = 5$

$2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{x}{y}$

Intersection between line + hyperbola occurs at $(3, 2)$ + $(-3, -2)$

Slope of tangent $= \frac{x}{y} = \frac{3}{2}$

$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{\frac{3}{2} - \frac{2}{3}}{1 + \left(\frac{3}{2} \cdot \frac{2}{3}\right)}$

$= \frac{\frac{9}{6} - \frac{4}{6}}{2} = \frac{5}{6} \cdot \frac{1}{2} = \frac{5}{12}$

Then $\theta = 22°38'$

250) 14

$$x^2 - 4y^2 = 20$$

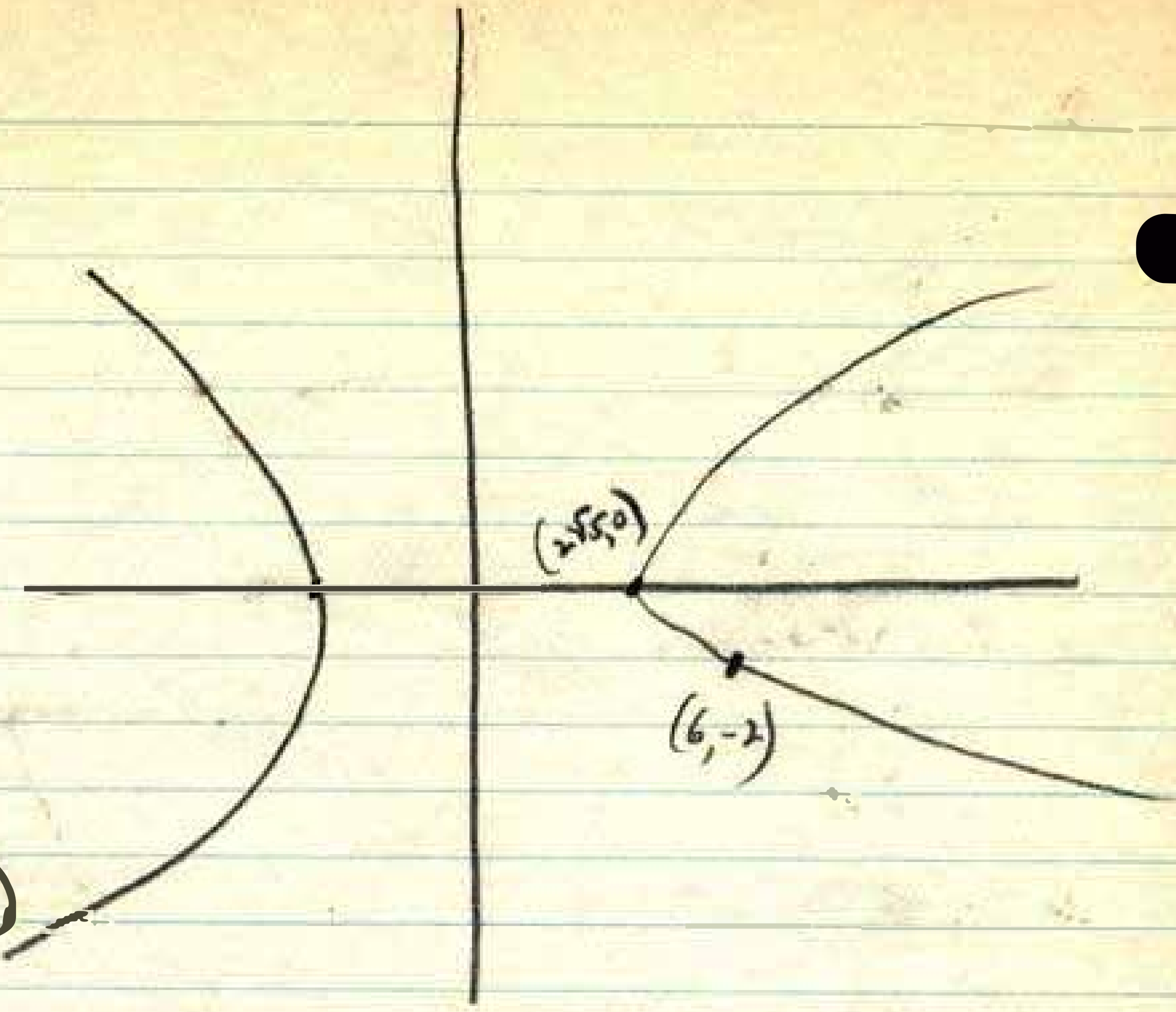
$$\frac{x^2}{20} - \frac{y^2}{5} = 1$$

Foci on x-axis

$a = \pm 2\sqrt{5}$

$b = \pm\sqrt{5}$

Center at origin



$x^2 - 4y^2 = 20$

Differentiating with respect to time (t)

$$2x\frac{dx}{dt} - 8y\frac{dy}{dt} = 0$$

$$2x\frac{dx}{dt} = 8y\frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{8y}{2x}\frac{dy}{dt} = \frac{4y}{x}\frac{dy}{dt}$$

But $\frac{dx}{dt} = 3$

$$\therefore\ 3 = \frac{4y}{x}\frac{dy}{dt}$$

Given point (6, -2)

$$\frac{dy}{dt} = \frac{3x}{4y} = \frac{18}{-8} = -\frac{9}{4}$$

$\therefore$ at given point y is decreasing at rate of $\frac{9}{4}$ units/sec

250) 15

$$x^2 - y^2 = 16$$

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

Foci on x-axis

Center at origin

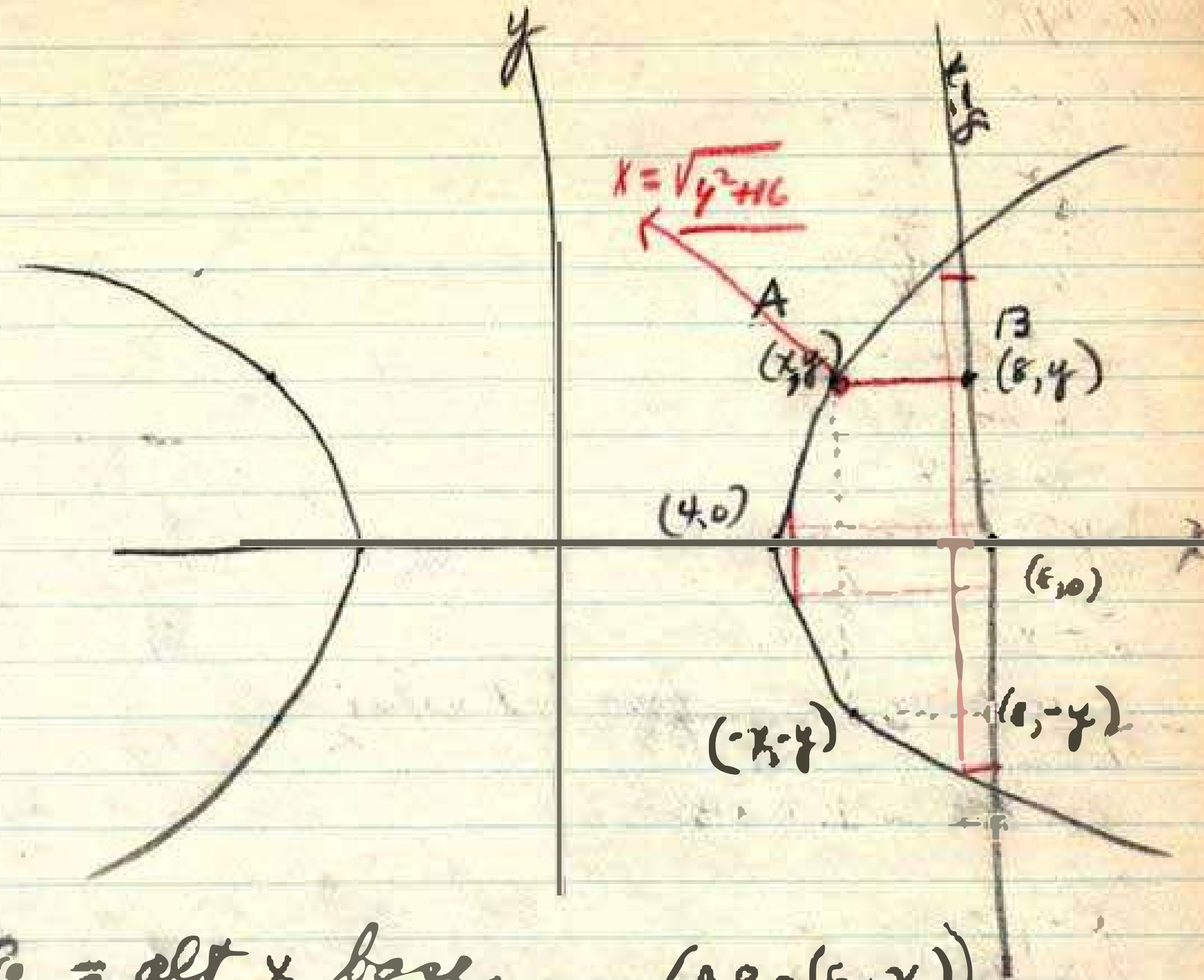
$a = \pm 4$

$b = \pm 4$

When $y = \pm 3$, $x = \pm 5$

$$x^2 = y^2 + 16$$

$$x = \pm\sqrt{y^2+16}$$



Area of rectangle = alt × base $\quad (AB = (8-x))$

$$A = 2y \cdot (8 - x)$$

$$= 2y \cdot (8 \pm \sqrt{y^2+16})$$

$$= 16y \pm 2y\sqrt{y^2+16}$$

$$= 16y \pm \sqrt{4y^4 + 64y^2}$$

Answer:

Dimensions

$2y = 7.44$

$8 - x = 2.54$

$$\frac{dA}{dy} = 16 \pm \tfrac{1}{2}(4y^4+64y^2)^{-\frac{1}{2}}(16y^3+128y)$$

$$= 16 \pm \frac{16y^3 + 128y}{2\sqrt{4y^4+64y^2}} = 16 \pm \frac{8y^3+64y}{\sqrt{4y^4+64y^2}}$$

Setting $\frac{dA}{dy} = 0$, $\quad = \frac{16\sqrt{4y^4+64y^2} \pm 8y^3 \pm 64y}{\sqrt{4y^4+64y^2}}$

$$16\sqrt{4y^4+64y^2} \pm 8y^3 \pm 64y = 0$$

$$2\sqrt{4y^4+64y^2} \pm y^3 \pm 8y = 0$$

$$y^3 + 8y = \pm 2\sqrt{4y^4+64y^2}$$

$$y^6 + \cancel{16y^4} + 64y^2 = \cancel{16y^4} + 256y^2$$

$$y^6 - 192y^2 = 0$$

$y^2(y^4 - 192) = 0$

$y = 0$

$y^4 = 192$, $y^2 = 13.86$

$y = 3.72$, $2y = 7.44$

$x^2 = 16 + 13.86 = 29.86$

$x = 5.46$ $\quad \boxed{8 - x = 2.54}$

250) 16

$x^2 - y^2 - 16 = 0$

$\frac{x^2}{16} - \frac{y^2}{16} = 1$

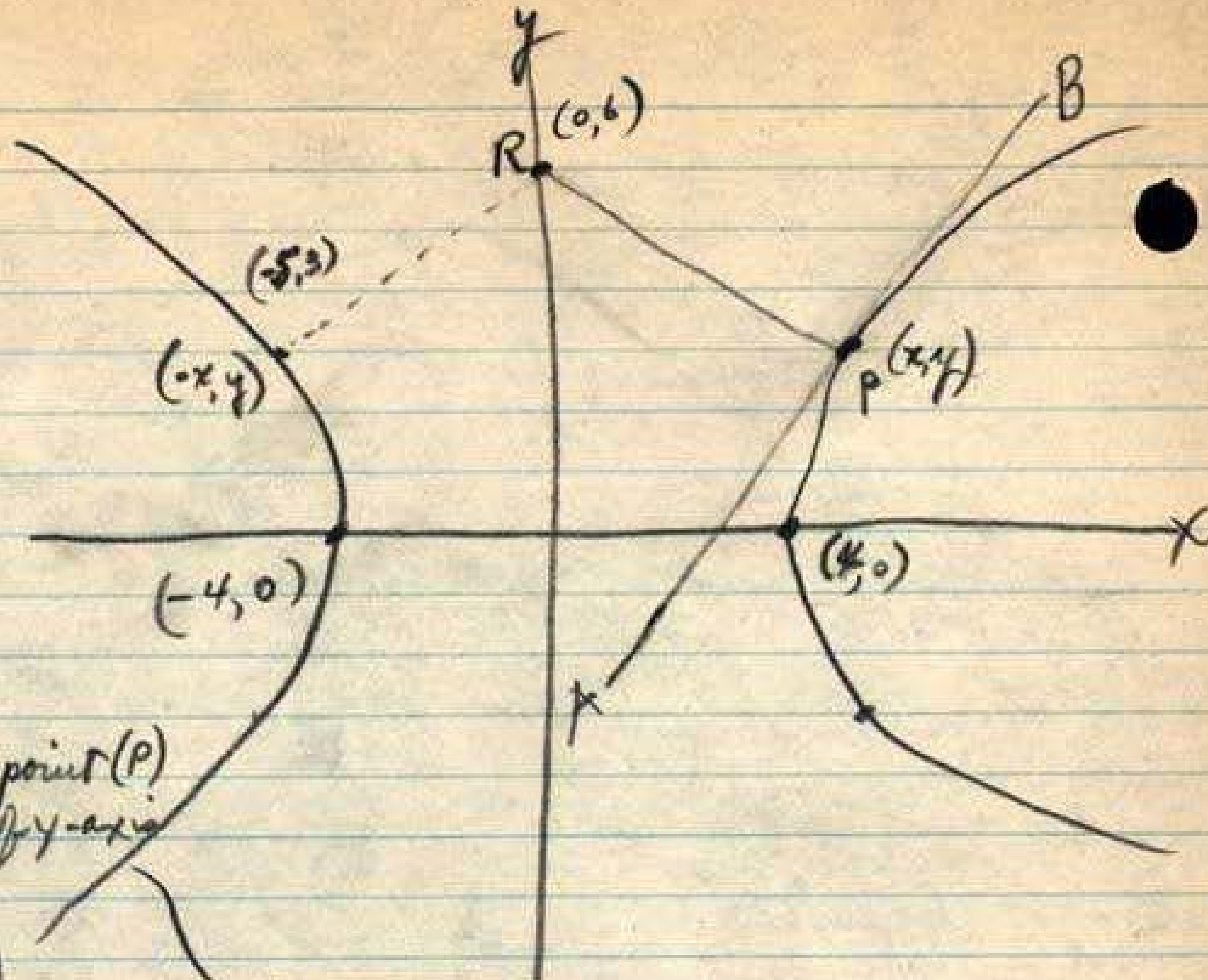
Foci on x-axis

Center at origin

$a = \pm 4$

$x^2 = y^2 + 16 \quad x = \pm\sqrt{y^2+16}$

Let (x, y) or $(-x, y)$ be the coordinates of desired point (P) on portion of hyperbola to rt. of y-axis



AB is tangent at P

& PR is $\perp$ to AB at P

$x^2 - y^2 - 16 = 0$

Differentiating, $2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

slope AB $= \frac{x}{y}$

Is this correct?

Slope PR $= -\frac{y}{x} = \frac{y-6}{x-0}$ (1)

$-xy = xy - 6$

$2xy = 6$

$xy = 3$

$x = \frac{3}{y}$

Substituting in (1)

$\frac{-y}{\frac{3}{y}} = \frac{y-6}{\frac{3}{y}}$

$2y = 6$

$y = 3$

Then $x = \frac{3}{3} = 1$

Slope of PR $= -\frac{3}{1}$

$S = PR = \sqrt{x^2 + (y-6)^2}$

$= \sqrt{x^2 + y^2 - 12y + 36}$

$= \sqrt{y^2+16+y^2-12y+36} = \sqrt{2y^2-12y+56}$ ✓

Differentiating,

$\frac{\cancel{dy}\ dS}{dy} = \frac{4y-12}{2\sqrt{2y^2-12y+56}}$

Setting derivative at 0,

$4y - 12 = 0$

$4y = 12 \quad , y = 3$ ✓

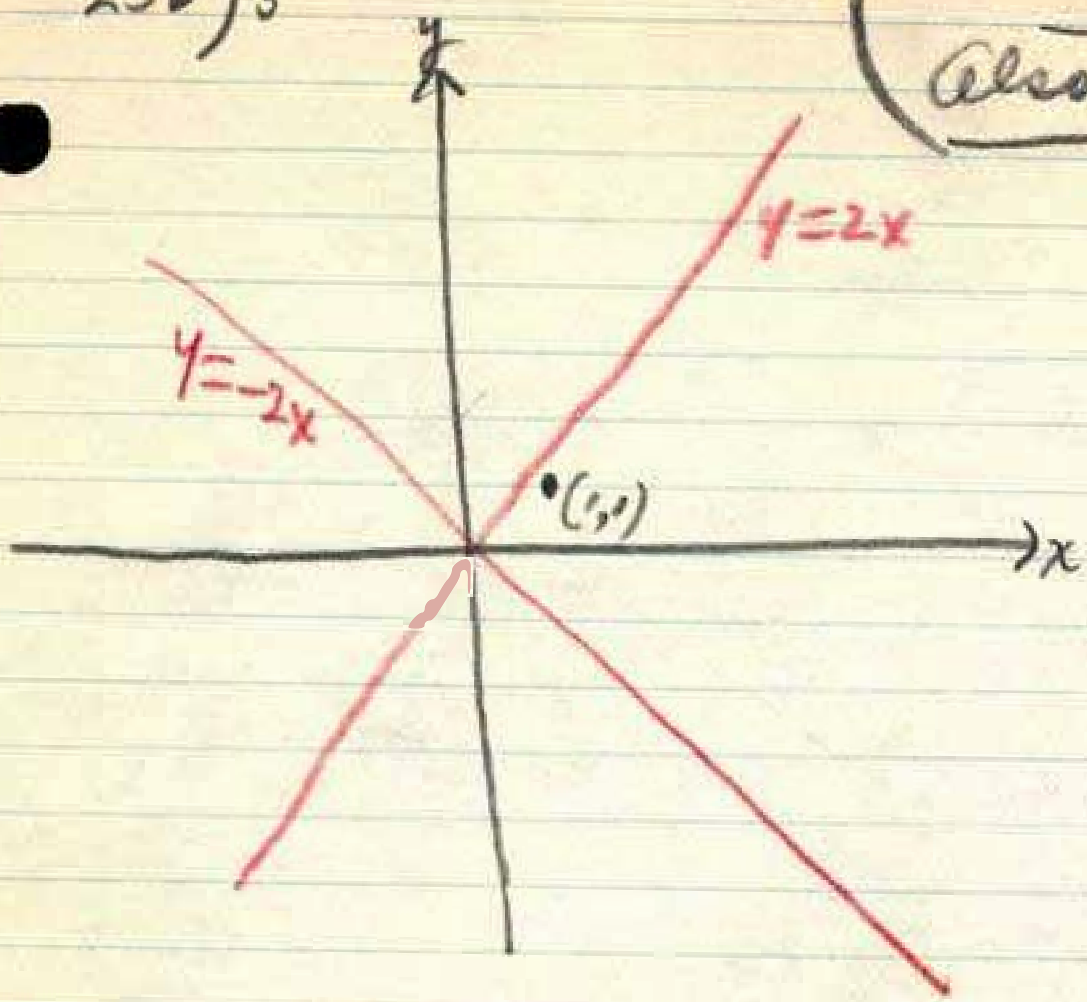
$x^2 = y^2 + 16 = 25$

$x = \pm 5$

Coordinates of P are $(5,3)$ $(-5,3)$ ✓

(*) Discuss Section 150 page 250
Also Equilateral Hyperbola + or Asymptotes
Also $xy = c$

252) 3



Asymptotes $y = \pm 2x$

$$\text{slope} = \frac{b}{a} = \frac{y}{x}$$

$$y = \frac{bx}{a} = \pm 2x$$

$$b = 2a$$

$$c^2 = a^2 + b^2$$

$$= a^2 + 4a^2 = 5a^2$$

$$c = \pm a\sqrt{5}$$

[illegible]

[illegible]

[illegible]

See attached paper

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$c = \pm .34$$

$$a\sqrt{5} = .34$$

$$a = \frac{.34}{\sqrt{5}} = \frac{.34}{2.4} = .14$$

$$b = .28$$

(14) $x^2 - y^2 = .28$

$$.0196\, x^2 - y^2 = .28$$

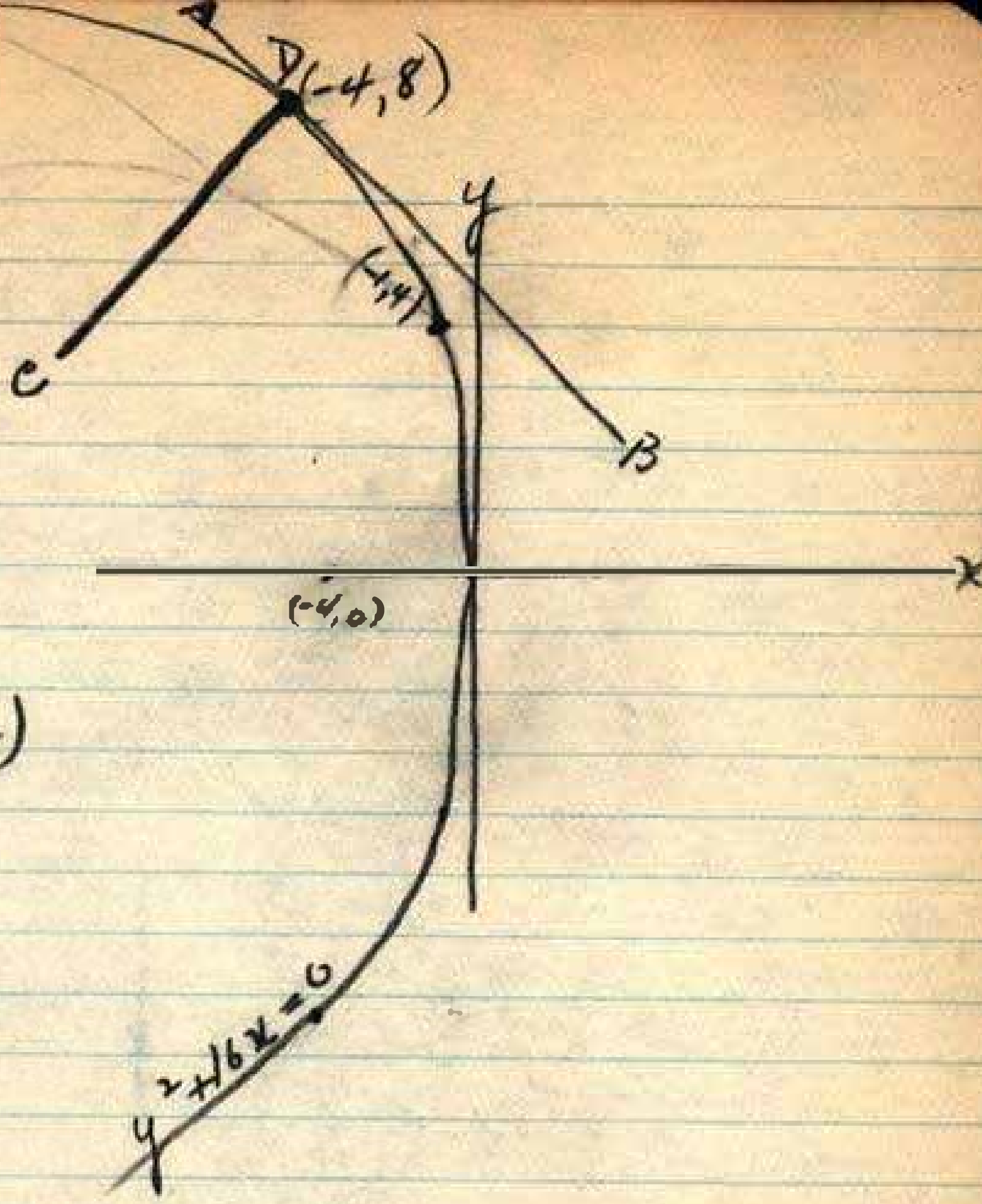
$$.02\, x^2 - y^2 = .28$$

$$2x^2 - 100y^2 = 28$$

$$x^2 - 50y^2 = 14$$

$$\frac{x^2}{(.14)^2} - \frac{y^2}{(.28)^2} = 1$$

252) 5a



$y^2+16x=0$

$y^2=-16x$

$2p=-16$

$p=-8$

AB is tangent to Parabola at $(-4,8)$
CD is $\perp$ to tangent at $(-4,8)$

$y^2+16x=0$

Differentiating,

$$2y\frac{dy}{dx}+16=0$$

$$\frac{dy}{dx}=\frac{-16}{2y}=\frac{-8}{y}$$

at $(-4,8)$, slope $=\frac{-8}{8}=-1$

$$\left.\begin{aligned} -1&=\frac{y-8}{x+4}\\ -x-4&=y-8\\ -x-4-y+8&=0\\ x+y-4&=0\end{aligned}\right\}\text{Equation of tangent}$$

$$\left.\begin{aligned} \text{Slope of normal}&=1\\ 1&=\frac{y-8}{x+4}\\ x+4&=y-8\\ x-y+12&=0\end{aligned}\right\}\text{Equation of normal}$$

252) 8a

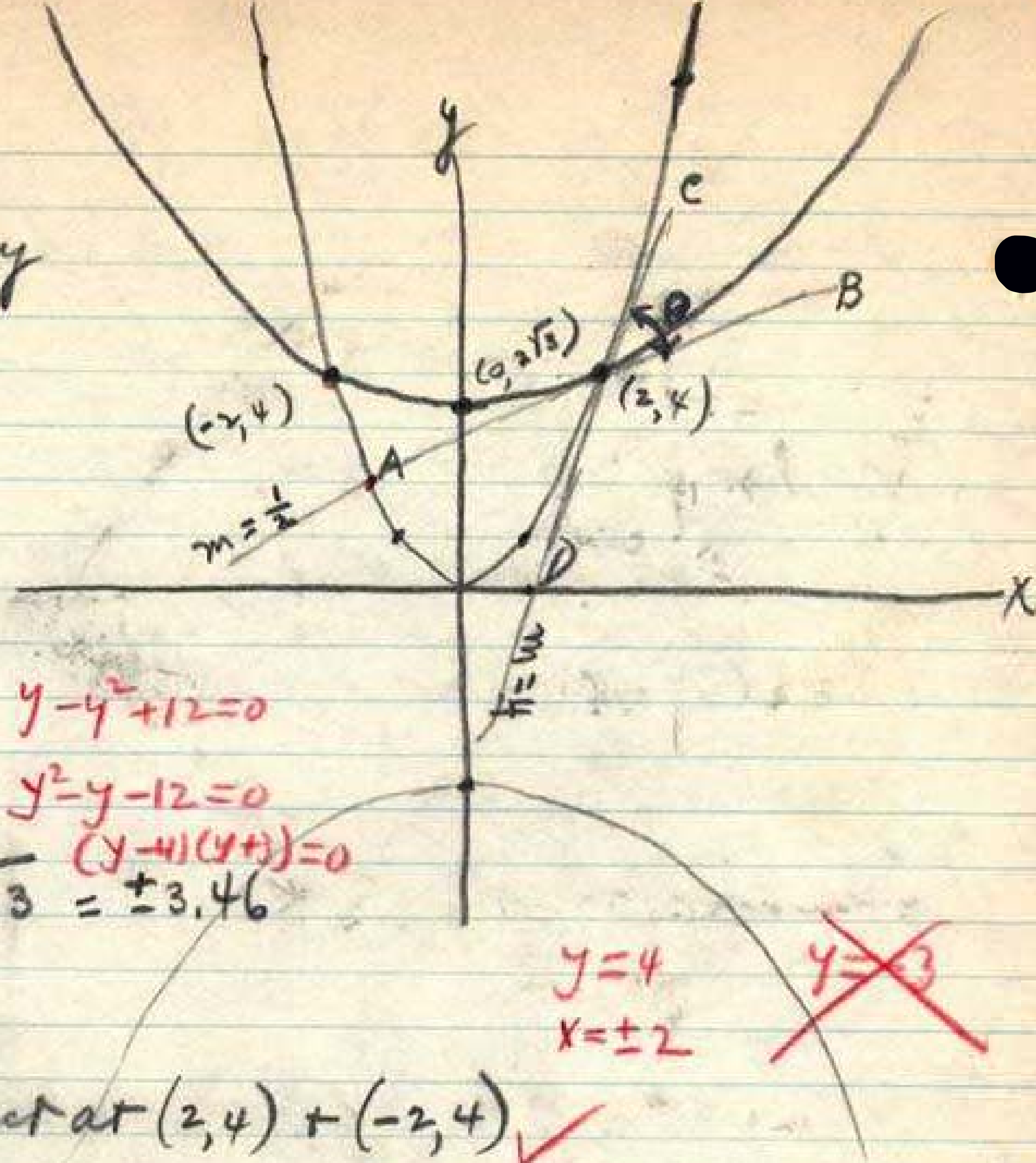
$y = x^2$

This is a parabola of type $x^2 = -2py$

When $x = \pm 1$, $y = 1$

$x = \pm 2$, $y = 4$

$x = \pm 3$, $y = 9$



$x^2 - y^2 + 12 = 0$

$x^2 - y^2 = -12$

$y^2 - x^2 = 12$

$\frac{y^2}{12} - \frac{x^2}{12} = 1$ Hyperbola

$y - y^2 + 12 = 0$

$y^2 - y - 12 = 0$

$(y-4)(y+3) = 0$

center at origin, $a = \pm 2\sqrt{3} = \pm 3.46$

foci on y axis

When $x = \pm 2$, $y = 4$

$y = 4$

$x = \pm 2$

$\cancel{y = -3}$

Parabola + Hyperbola intersect at $(2,4)$ + $(-2,4)$

Let AB be tangent to Hyperbola at $(2,4)$

CD " " " Parabola "

$y = x^2$

$\frac{dy}{dx} = 2x$

at $(2,4)$, $\frac{dy}{dx} = 4$ = slope of CD

$x^2 - y^2 + 12 = 0$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

at $(2,4)$, $\frac{dy}{dx} = \frac{2}{4} = \frac{1}{2}$

$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$

$= \frac{4 - \frac{1}{2}}{1 + 4\cdot\frac{1}{2}} = \frac{7/2}{3}$

$= \frac{7}{6}$

$\theta = 49^\circ 24'$