380) re cont.

$v = \sqrt{v_y^2 + v_x^2} = \sqrt{(-3\sin t)^2 + (-4\sin 2t)^2}$

$v = \sqrt{9\sin^2 t + 16\sin^2 2t}$ ✓

$$\frac{dv}{dt} = \frac{18\sin t\,[\cos t] + \cancel{32}\sin 2t(\cos 2t)(2)}{2\sqrt{9\sin^2 t + 64\sin^2\cos^2 t}}$$

$$= \frac{18\sin t\cos t + 64\sin 2t\cos 2t}{2\sqrt{9\sin^2 t + 64\sin^2\cos^2 t}}$$

Setting $\frac{dv}{dt} = 0$,

$18\sin t\cos t + 64\sin 2t\cos 2t = 0$

~~18 sin t cos t + 64~~

$9\sin 2t + 64\sin 2t(1 - 2\sin^2 t) = 0$

$9\underline{\sin 2t} + 64\underline{\sin 2t} - 128\sin^2 t\,\underline{\sin 2t} = 0$

~~73 sin 2t = 128 sin² t = 0~~

~~73~~

$\sin 2t(9 + 64 - 128\sin^2 t) = 0$

$\sin 2t = 0$, $2t = 0, \pi, 2\pi, ---$ $\qquad \sin 2t = 0$, $t = 0$

$t = 0, \pi/2, \pi, ---$ $\qquad$ or $128\sin^2 t = -73$

| | $dv/dt$ |
|---|---|
| $\pi/2 > t > 0$ | (+) |
| $\pi > t > \pi/2$ | − |
| $3\pi/2 > t > \pi$ | + |

$\sin^2 t = \frac{-73}{128}$ discarded

Therefore, speed is at maximum when ~~t = 0~~ $t = \pi/2$ ✓

When $t = 0$, $\tan\alpha = \frac{v_y}{v_x} = \frac{-3\sin t}{-4\sin 2t} = 0$

$\boxed{\text{What is velocity? at } t=0}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{0+0}$ (?)

$t = 0$, $v = 0$

$t = \pi/2$, $v = \sqrt{9} = 3$

8.5

73.

649.00

6

381) 6b

$$\boxed{\begin{matrix} a_x = 6t \\ a_y = 2 \end{matrix}} \quad \left.\begin{matrix} \boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}} \\ v_x = 1 \\ v_y = 2 \end{matrix}\right\} t=0$$

$$\frac{dv_x}{dt} = 6t \qquad , \qquad \frac{dv_y}{dt} = 2$$

$$v_x = \int 6t\,dt \qquad\qquad v_y = \int 2\,dt$$

$$= 3t^2 + C_1 \checkmark \qquad\qquad = 2t + C_2 \checkmark$$

when $t=0$, $1 = 0 + C_1$ $\qquad\qquad 2 = 0 + C_2$

$$C_1 = 1 \checkmark \qquad\qquad C_2 = 2 \checkmark$$

Then component velocities are $\begin{cases} v_x = 3t^2 + 1 \\ v_y = 2t + 2 \end{cases}$

$$\frac{dx}{dt} = 3t^2 + 1 \checkmark \qquad\qquad \frac{dy}{dt} = 2t + 2 \checkmark$$

$$x = \int (3t^2 + 1)\,dx \qquad\qquad y = \int (2t + 2)\,dt$$

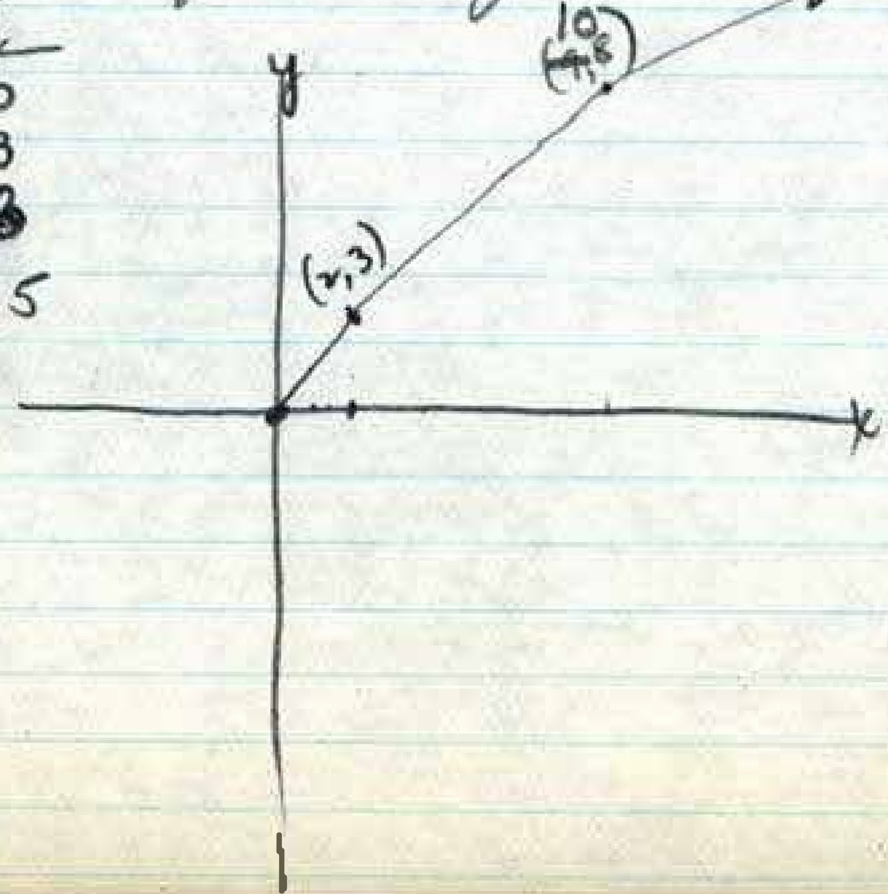
$$x = t^3 + t + C_3 \checkmark \qquad\qquad y = t^2 + 2t + C_4 \checkmark$$

Using initial conditions, $0 = 0 + 0 + C_3$ $\qquad\qquad 0 = 0 + 0 + C_4$

$$C_3 = 0 \checkmark \qquad\qquad C_4 = 0 \checkmark$$

Therefore, equations of motion equal $\left.\begin{matrix} x = t^3 + t \\ y = t^2 + 2t \end{matrix}\right\} \checkmark$

| t | x | y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 2 | 3 |
| 2 | 9 | 8 |
| 3 | 28 | 15 |



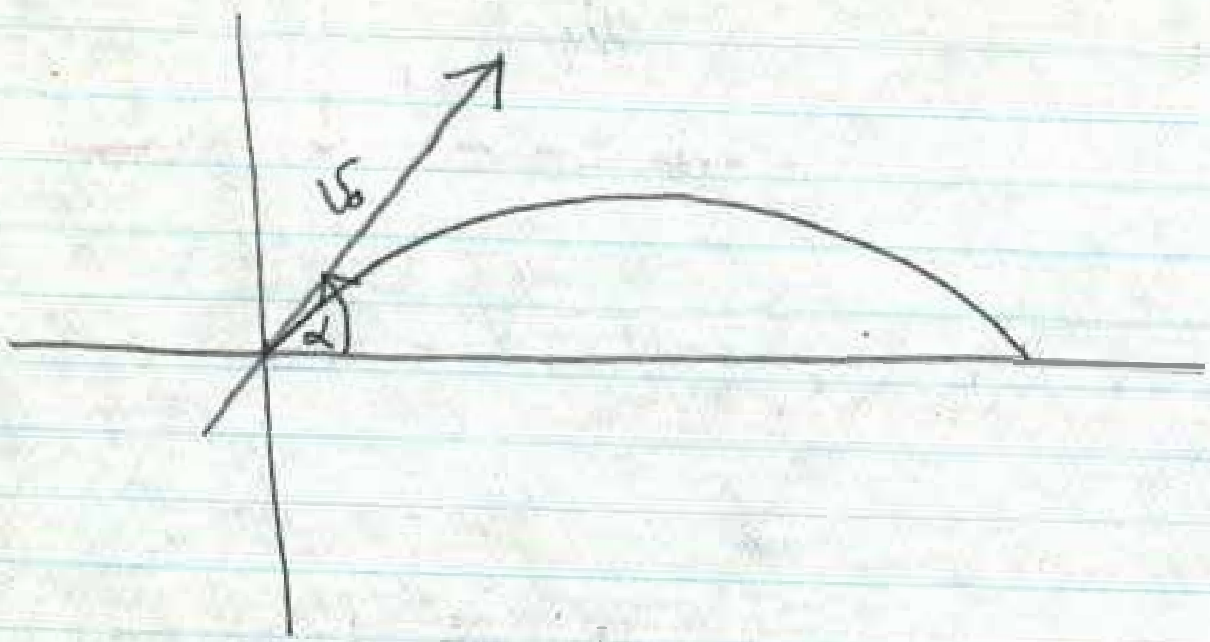

When $t = 2$, position is $(10, 8)$

and speed (velocity) $= \sqrt{v_x^2 + v_y^2}$

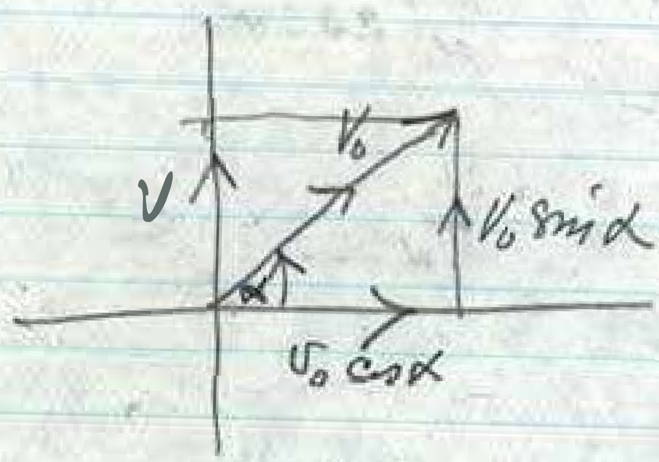
$$= \sqrt{(3t^2+1)^2 + (2t+2)^2}$$

$$= \sqrt{169 + 36}$$

$$= \sqrt{205} \checkmark \; \cancel{= 14.3 \text{ units/sec.}}$$




$(v_x)_0 =$




$$t=0, \begin{cases} V_x = V_0 \cos\alpha \\ V_y = V_0 \sin\alpha \end{cases}$$

381) 7

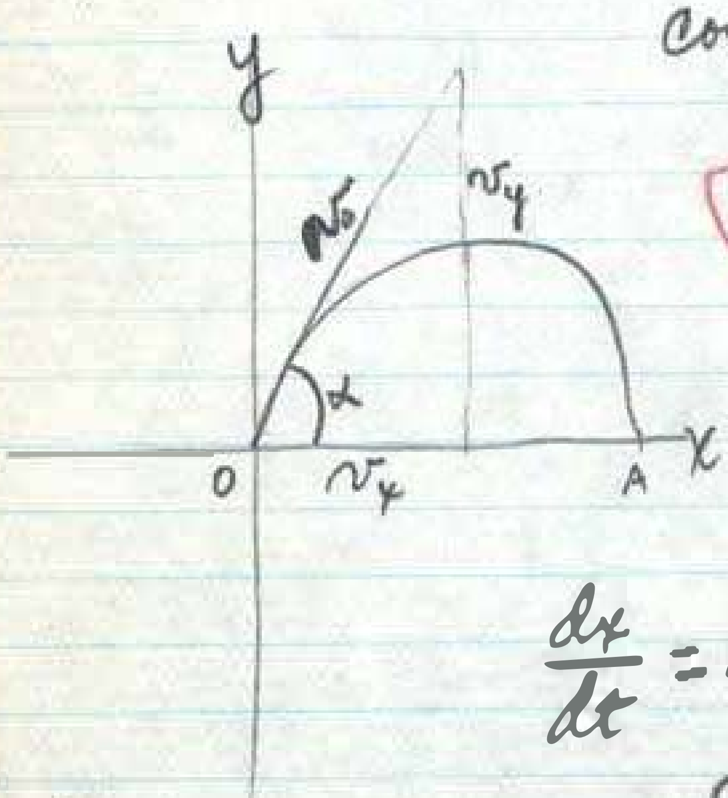
$$\left.\begin{array}{l} a_x = 0 \\ a_y = -32 \end{array}\right\}$$

$$\frac{dv_x}{dt} = 0$$

$$v_x = \cancel{0}\ C$$

$$\cos\alpha = \frac{v_x}{v_0}$$

$$\boxed{v_x = v_0 \cos\alpha}$$




$$\frac{dx}{dt} = v_0 \cos\alpha$$

$$x = \int (v_0 \cos\alpha)\, dt$$

$$x = v_0 t \cos\alpha + C_1$$

When $t = 0$; $x = 0$, $C_1 = 0$

$$\therefore x = v_0 t \cos\alpha$$

$$\frac{dv_y}{dt} = -32$$

$$v_y = \int -32\, dt = -32t + C_2$$

~~When $t = 0$, $C_2 = v_y$~~

$$V_0 \sin\alpha = C_2$$

$$\sin\alpha = \frac{v_y}{v_0}$$

$$v_y = v_0 \sin\alpha - 32t$$

$$\cancel{\frac{dy}{dt} = (C_2 - 32t)}$$

$$\frac{dy}{dt} = (v_0 \sin\alpha - 32t)$$

$$y = \int (v_0 \sin\alpha - 32t)\, dt$$

$$y = v_0 t \sin\alpha - 16t^2 + C_3$$

When $\left.\begin{array}{l} t = 0 \\ y = 0 \end{array}\right\}$, $C_3 = 0$

$$\therefore y = v_0 t \sin\alpha - 16t^2$$

381) 8

$$y = v_0 t \sin\alpha - 16t^2$$

$$x = v_0 t \cos\alpha$$

$$t = \frac{x}{v_0 \frac{v_x}{v_0}} = \frac{x}{v_x}$$

$$\tan\alpha = \frac{v_y}{v_x}$$

$$1 + \tan^2\alpha = \sec^2\alpha = \left(\frac{v_0}{v_x}\right)^2$$

$$\sin\alpha = \frac{v_y}{v_0}$$

$$\cos\alpha = \frac{v_x}{v_0}$$

$$y = v_0 \cdot \frac{x}{v_x} \cdot \frac{v_y}{v_0} - \frac{16x^2}{v_x^2}$$

$$= x \cdot \frac{v_y}{v_x} - \frac{16x^2}{v_x^2}$$

$$\boxed{v_x^2 = \frac{v_0^2}{\sec^2\alpha} = \frac{v_0^2}{1+\tan^2\alpha}}$$

$$= x\tan\alpha - \frac{16x^2(1+\tan^2\alpha)}{v_0^2}$$

$$= x\tan\alpha - \frac{16}{v_0^2}(1+\tan^2\alpha)x^2$$

$$t = \frac{x}{V_0\cos\alpha}$$

$$y = \frac{\cancel{V_0}\sin\alpha \cdot x}{\cancel{V_0}\cos\alpha} - \frac{16x^2}{V_0^2\cos^2\alpha}$$

$$y = x \cdot \tan\alpha - \frac{16x^2 \cdot \sec^2\alpha}{V_0^2} = x\tan\alpha - \frac{16x^2(1+\tan^2\alpha)}{V_0^2}$$


$$(ds)^2 = (dx)^2 + (dy)^2$$

$$= \sqrt{(dt)^2 + 4t^2(dt)^2}$$

$$= \sqrt{\cancel{(dt)^2}\left[1+4t^2\right]} \quad dt$$

383) 1a

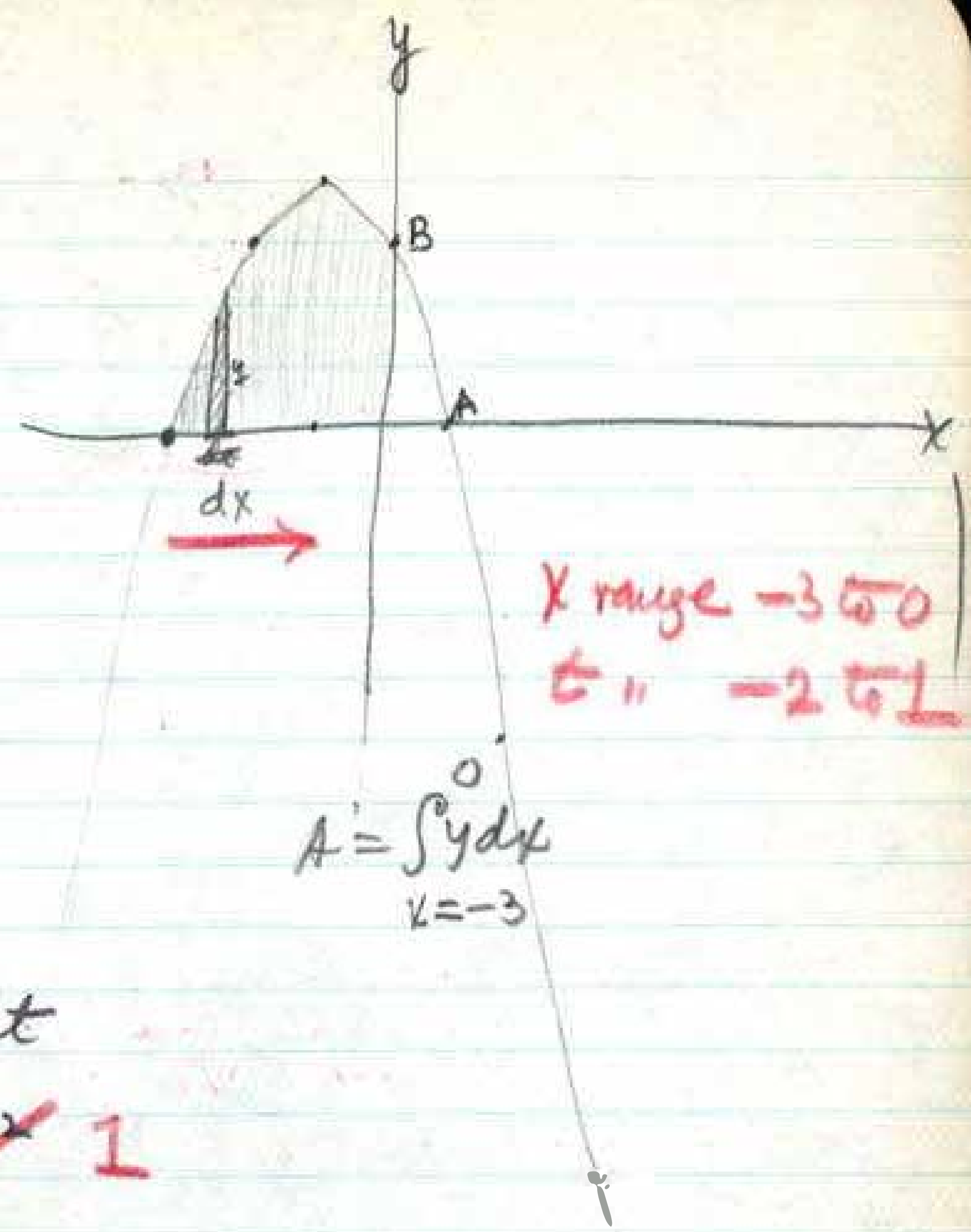
$x = t - 1 \quad (t = x+1)$

$y = 4 - t^2 \quad y = 4 - (x^2 + 2x + 1)$

$y = 4 - x^2 - 2x - 1$

$\boxed{y = 3 - x^2 - 2x}$

| t | x | y |
|---|---|---|
| 0 | -1 | 4 |
| 1 | 0 | 3 |
| 2 | 1 | 0 |
| 3 | 2 | -5 |
| 4 | 3 | -12 |
| -1 | -2 | 3 |
| -2 | -3 | 0 |




x range -3 to 0

t " -2 to 1

$A = \int_{x=-3}^{0} y\,dx$

$y = 4 - t^2$

$dx = \frac{d}{dt}(t-1) = dt$

El. area = y dt

$\text{Area} = \int_{1}^{-2} y\,dt = \int_{-2}^{1} (4 - t^2)\,dt$

$= 4t - \frac{t^3}{3}\Big]_{-2}^{1}$

$-\left(-8 + \frac{8}{3}\right) + \left(4 - \frac{1}{3}\right) = -8 + 8/3 - 4 + \frac{1}{3}$

$+ \quad - \quad + \quad -$

$= -12 + 3 = -9$ +

Area = 9 (to left of y axis)

(1b)

To find length of AB

$dx = dt$

$\frac{dy}{\cancel{dt}} = -2t\, dt$

$\frac{dy}{dx} = -2t$

$\boxed{\int \sqrt{u^2 + a^2}\,du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left(u + \sqrt{u^2+a^2}\right) + C}$

$S = \int_{-3}^{0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$S = \int_{-2}^{1} \sqrt{1 + (-2t)^2}\, dt$

$S = \int_{1}^{2} \sqrt{1 + 4t^2}\, dt$

$S = 2\int \sqrt{\frac{1}{4} + t^2}\, dt = \frac{2t}{2}\sqrt{1+4t^2} + \frac{1}{2}\ln\left(2t + \sqrt{1+4t^2}\right)\Big]_1^2$

$= \left[2\sqrt{9} + \frac{1}{2}\ln\left(4 + \sqrt{9}\right)\right] - \left[\sqrt{5} + \frac{1}{2}\ln\left(2 + \sqrt{5}\right)\right]$

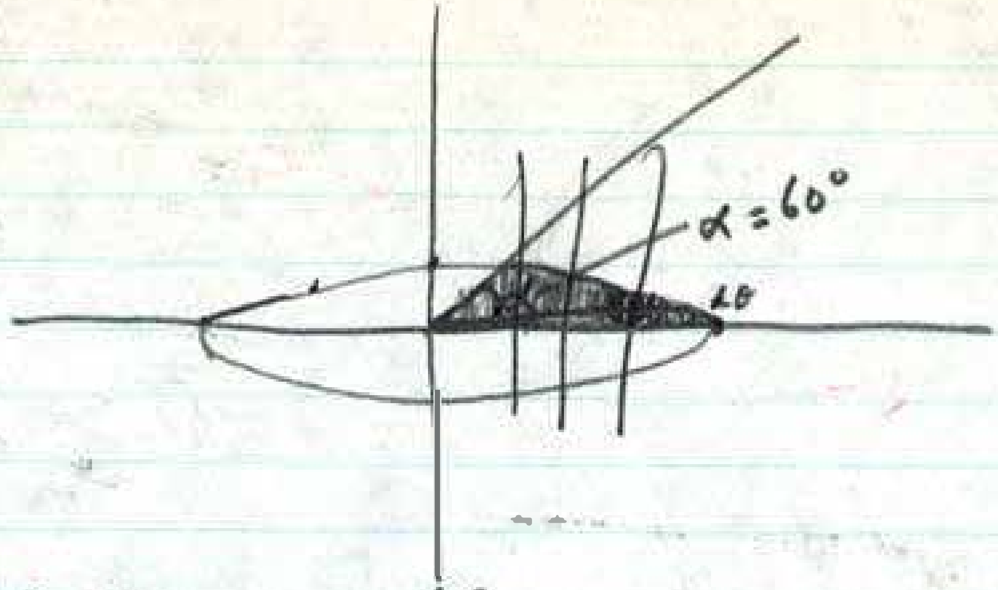
$= (6 + .9729) - (2.24 + .7223)$

$= 6.9729 - 2.9623 = 4.0106$

383) 2a  $x = 3\cos\theta = 3\cos t$

$y = \sin\theta = \sin t$

| $\theta$ | $x$ | $y$ |
|---|---|---|
| 0 | 3 | 0 |
| 60° | 1.5 | $\sqrt{3}/2$ |
| 90° | 0 | 1 |
| 120° | -1.5 | $\sqrt{3}/2$ |
| 180° | -3 | 0 |

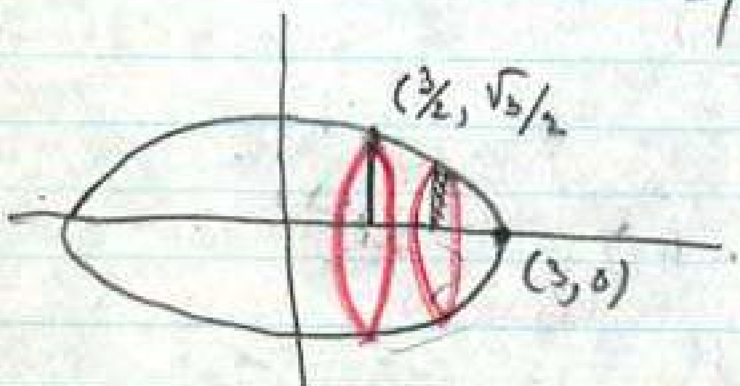



El. of area $= y\, d\theta$

Total Area $= \int_0^{\pi/3} y \,\cancel{d\theta}\, dx = \int_0^{\pi/3} (\sin\theta)\,\cancel{d\theta}\,(-3\sin\theta)\,d\theta$

$= -\cos\theta\Big]_0^{\pi/3} = (-.5) - (1.) = -1.5$

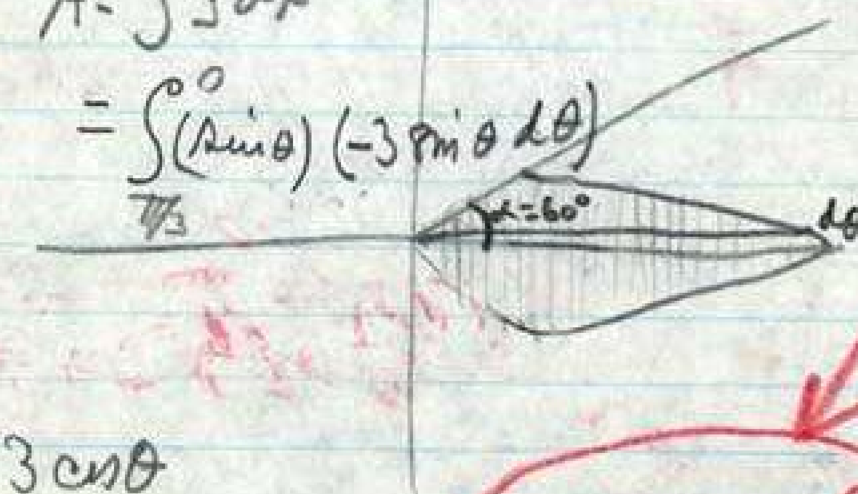
Ans. = 1.5 unit




$A = \int y\,dx$

$A = -3\int_{\pi/3}^{0} \sin^2\theta\, d\theta$  Corr.

2b $= \int_{\pi/3}^{0} (\sin\theta)(-3\sin\theta\, d\theta)$




El. of Volume $= \pi r^2 dx = \pi y^2 dx$

$= \pi y^2 \,\cancel{d\theta}\, dx$

$x = 3\cos\theta$

$dx = -3\sin\theta\ d\theta$

Total Volume $= \int_0^{\pi/3} (\pi \sin^2\theta)\,\cancel{d\theta}\,(-3\sin\theta\, d\theta)$

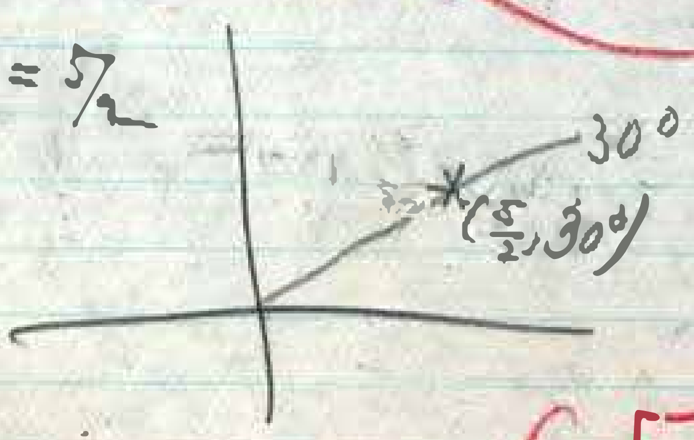
$V = -3\pi \int_{\pi/3}^{0} \sin^3\theta\, d\theta$

$= \left[\pi\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\right]_0^{\pi/3}$

$\rho = 5\sin\theta$

$= \pi\left(\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3}\right) - \left[\pi(0 - 0)\right]$

$\theta = 30°,\ \rho = 5/2$




$= \frac{\pi^2}{6} - \frac{\sqrt{3}}{8} = \frac{4\pi^2 - 3\sqrt{3}}{24}$

$t^2 = u$

$2t\,dt = du$

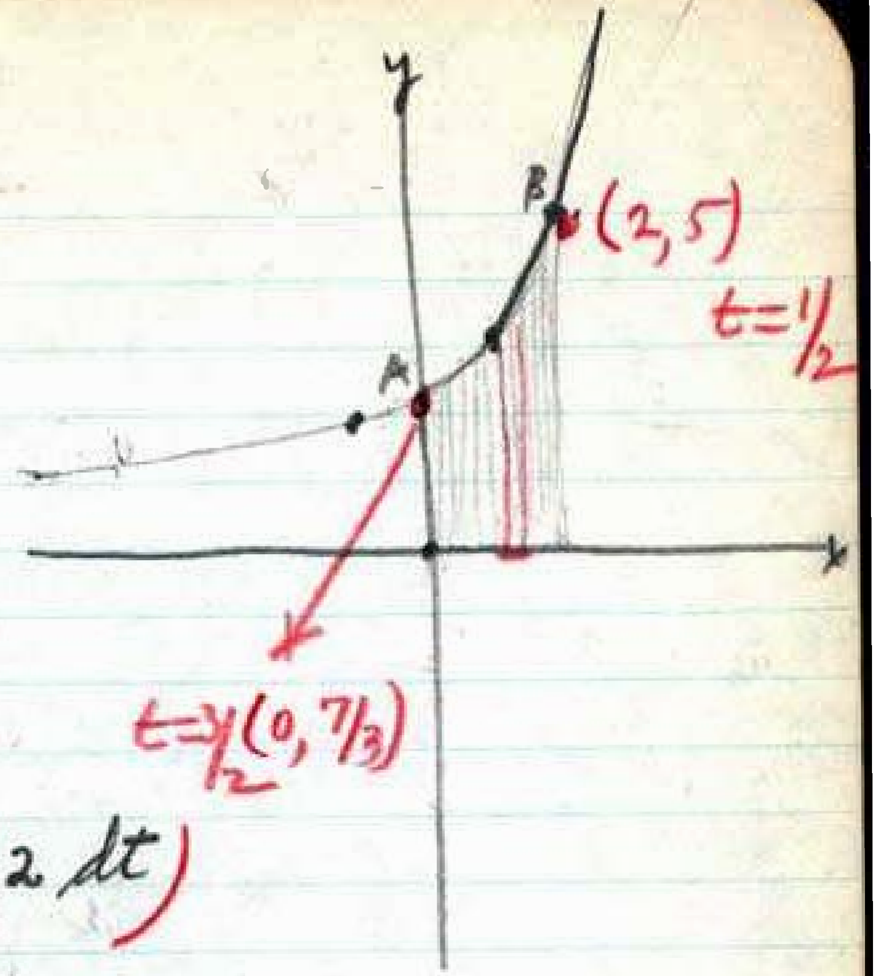
$S = 2\int \sqrt{1 + \frac{1}{t^4}}\ dt = 2\int \sqrt{\frac{t^4+1}{t^4}}\ dt$

$\int \frac{\sqrt{u^2+1}}{u}\ \frac{du}{2\sqrt{u}} = \int \frac{\sqrt{u^2+1}\,du}{2u^{3/2}}$    $S = 2\int \frac{\sqrt{t^4+1}}{t^2}\ dt$

383) 3a $x = 3-2t$

$y = 1+\frac{2}{t} = 1+2t^{-1}$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 3 | $\cancel{\infty}$ |
| $\frac{1}{2}$ | 2 | 5 |
| 1 | 1 | 3 |
| $\frac{3}{2}$ | 0 | $2\frac{1}{3}$ |
| 2 | -1 | 2 |
| 5 | -7 | 7/5 |



$\left\{ \begin{array}{l} dy = -2t^{-2}\,dt \\ dx = -2\,dt \end{array} \right.$

El. of area $= y\,dt$

Total area $= \int_{\frac{1}{2}}^{\frac{3}{2}} y \frac{dx}{dt}\,dt = \int_{\cancel{1}\;3/2}^{\cancel{1}\;1/2} (1+\frac{2}{t})(-2\,dt)$

$= -2\int_{\frac{1}{2}}^{\frac{3}{2}} (1+2t^{-1})\,dt = -2(t+2\ln t)\Big]_{\frac{1}{2}}^{\frac{3}{2}}$

$= -2t - 4\ln t\Big]$

$= \left[-3 - 4\ln\frac{3}{2}\right] - \left[-1 - \cancel{2}\,4\ln\frac{1}{2}\right]$

$= \left[-3 - .8110\right] - \left[-1 - .09788\right] = -0.7131$

$\int \frac{1}{t}\,dt = \ln t$

3b To find length of AB

$S = \int \sqrt{(dx)^2+(dy)^2} = \sqrt{4dt^2 + 4t^{-4}dt^2}$

$S = \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{1+\cancel{\left(\frac{-2}{t^2}\right)^2}}\,dt = \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\cancel{\left(1+\frac{4}{t^4}\right)}}\,dt$

$\frac{dx}{dt} = -2$

$\frac{dy}{dt} = \frac{-2}{t^2}$

$\frac{dy}{dx} = \cancel{\frac{-2}{t}}\ \frac{1}{t^2}$

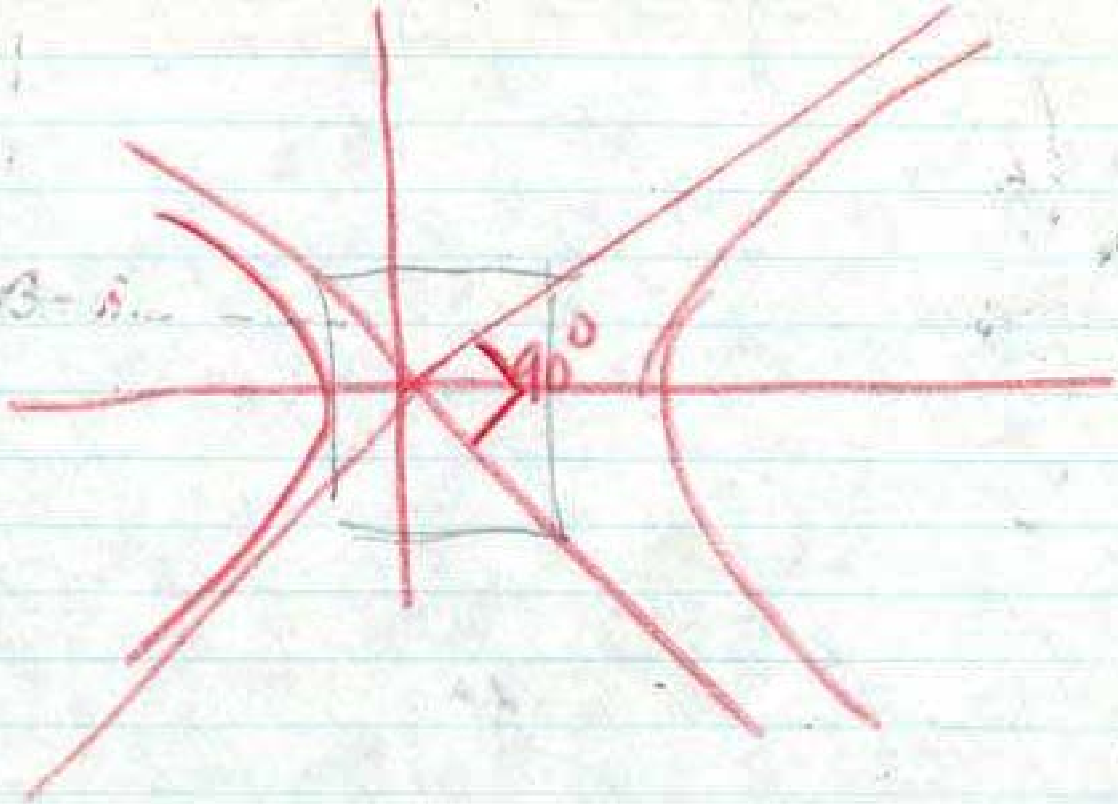
$= \cancel{\frac{1}{t^2}\sqrt{1+\frac{4}{t^4}} + \frac{1}{2}\ln\left(\frac{2}{t^2}+\sqrt{1+\frac{4}{t^4}}\right)}\Big]_{\frac{1}{2}}^{\frac{3}{2}}$

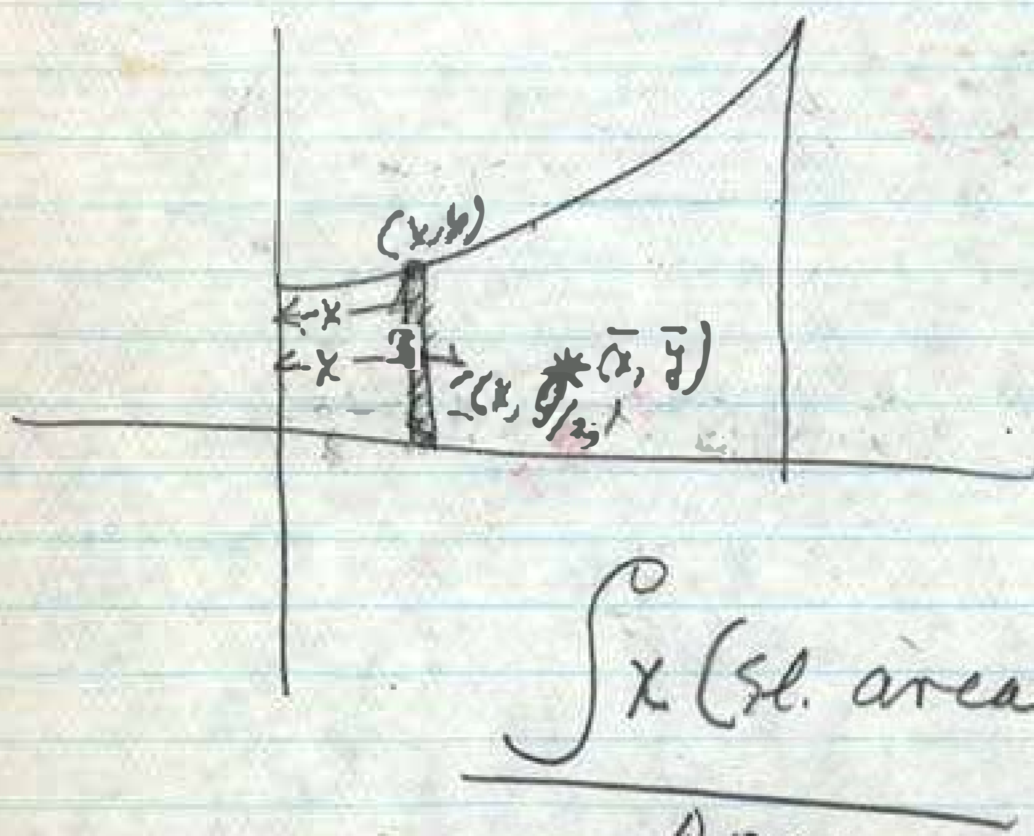
$= \left[\frac{4}{9}\left(\sqrt{1+\frac{4}{(\frac{3}{2})^4}}\right) + \frac{1}{2}\ln\left(\frac{8}{9}+\sqrt{1+\frac{4}{(\frac{3}{2})^4}}\right)\right]$

$-\left[4\sqrt{1+64} + \frac{1}{2}\ln\left(8+\sqrt{1+64}\right)\right]$

$S = 2\int\sqrt{1+t^{-4}}\,dt = \frac{4}{9}\left(\sqrt{1+\frac{64}{81}}\right) + \frac{1}{2}\ln\left(\frac{8}{9}+\sqrt{1+\frac{64}{81}}\right) - \left[4\sqrt{65}+\frac{1}{2}\ln\left(8+\sqrt{65}\right)\right]$

$= \left[\frac{4}{81}\sqrt{145} + \frac{1}{2}\ln\left(\frac{8}{9}+\frac{1}{9}\sqrt{145}\right)\right] - \left[4\sqrt{65}+\frac{1}{2}\ln\left(8+\sqrt{65}\right)\right]$




$$\frac{\int x(\text{El. area})}{\text{Area}} = \bar{x}$$

of the element

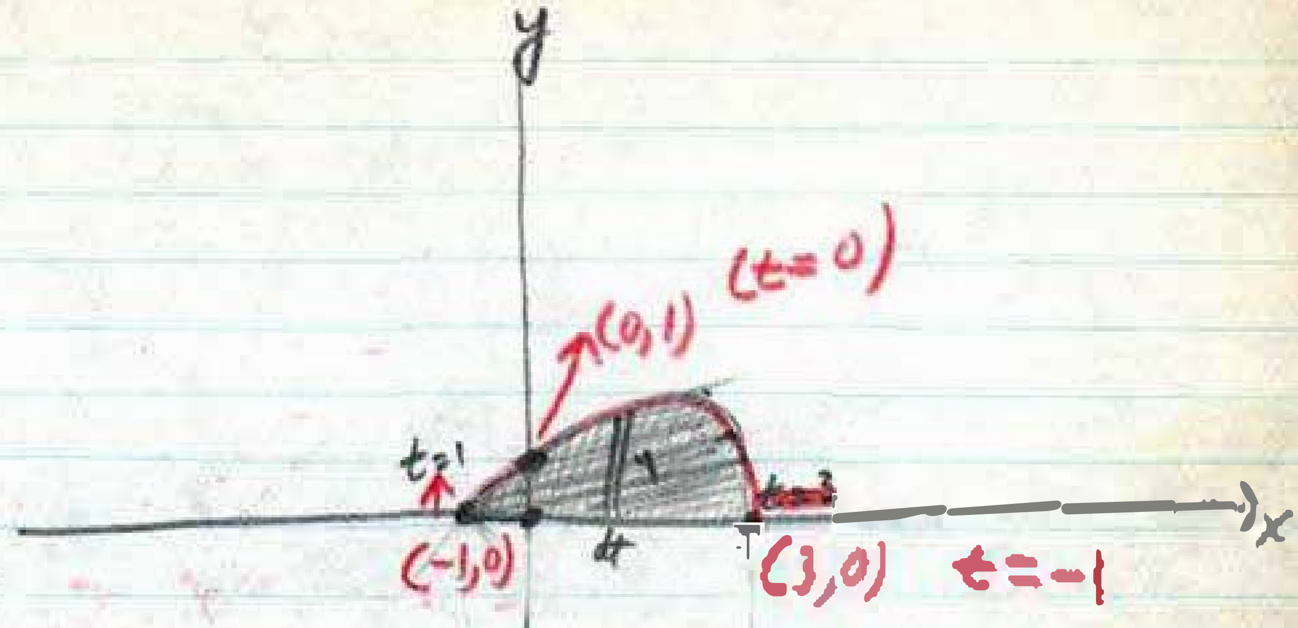
$$\frac{\int y(\text{El. area})}{\text{Area}} = \bar{y} \qquad \frac{\int y_{\text{mid-pt.}}(\text{El. area})}{\text{Area}}$$

3(3)4

$x = t^2 - 2t$

$y = 1 - t^2$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | -1 | 0 |
| 2 | 0 | -3 |
| 3 | 3 | -8 |
| 4 | 8 | -15 |
| -1 | 3 | 0 |
| -2 | 8 | -3 |




$A = \int_{-1}^{3} y\,dx$

-2

$= \int_{1}^{0} (1-t^2)(2t-2)\,dt$

El. of area = y dt

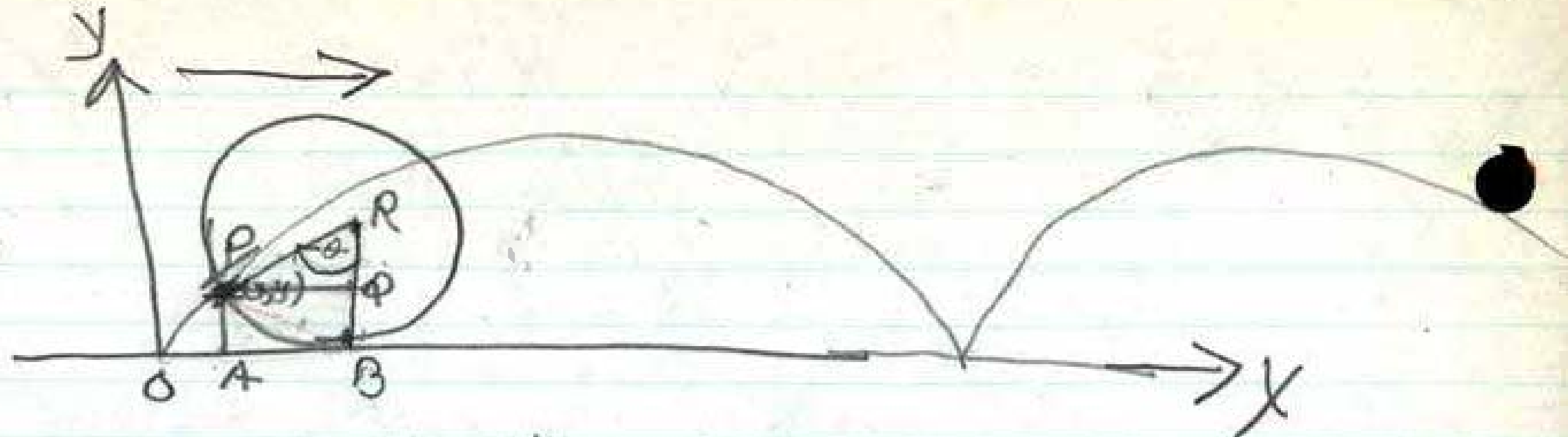
Total area $= \int_1^3 (1-t^2)\,dt$

$\cancel{= t - \frac{t^3}{3}\Big]_1^3 = (3-9) - \left(1 - \frac{1}{3}\right)}$

$\cancel{= -6 - 1 + \frac{1}{3}}$

$= -6\frac{2}{3}$

rad. $= a$

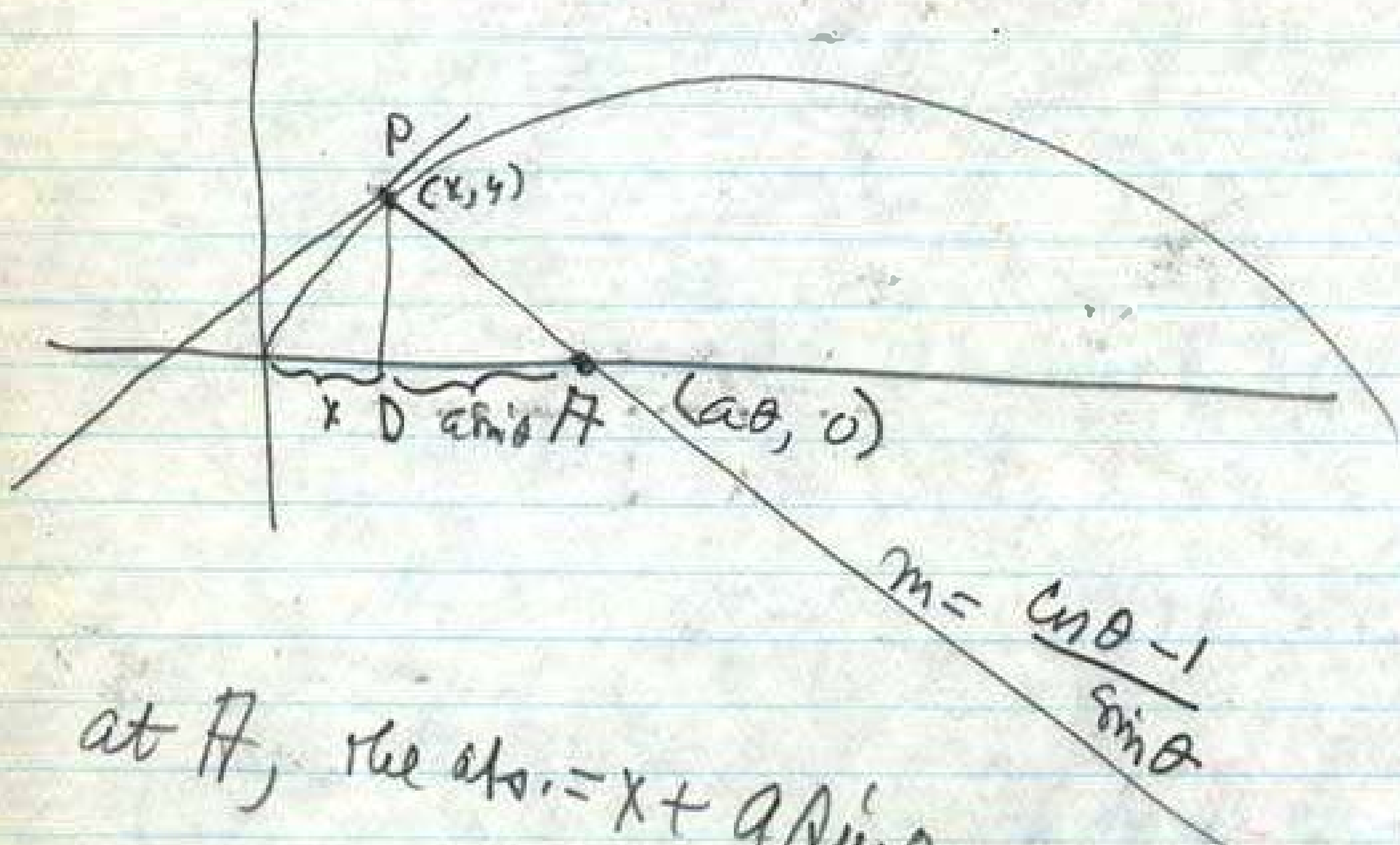



$$x = OA = OB - AB = \overset{\frown}{PB} - PQ = a\theta - a.\sin\theta = a(\theta - \sin\theta)$$

$$y = AP = BQ = BR - QR = a - a\cos\theta$$
$$= a(1-\cos\theta)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{a}.\sin\theta}{\cancel{a}(1-\cos\theta)}$$

$\therefore$ slope of Nor. $= -\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\cos\theta - 1}{\sin\theta}$




at A, the abs. $= x + a\sin\theta = a\theta - a\sin\theta + a\sin\theta$

$$\frac{y-0}{x-a\theta} = \frac{a(1-\cos\theta)}{\cancel{a\theta} - a\sin\theta - \cancel{a\theta}}$$

$$= \frac{\cancel{a}(1-\cos\theta)}{-\cancel{a}\sin\theta} = -\frac{1-\cos\theta}{\sin\theta} = \frac{\cos\theta - 1}{\sin\theta}$$

390) 1a

$x = a\cos\theta \qquad y = b\sin\theta$

$$\frac{dx}{d\theta} = -a\sin\theta \qquad \frac{dy}{d\theta} = b\cos\theta$$

$$\frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta} = y' = -\frac{b}{a}\cdot\cot\theta$$

$$\frac{dy'}{d\theta} = \frac{(-a\sin\theta)(-b\sin\theta) - (b\cos\theta)(-a\cos\theta)}{a^2\sin^2\theta}$$

$$= \frac{ab(\sin^2\theta + \cos^2\theta)}{a^2\sin^2\theta} = \frac{b}{a\sin^2\theta}$$

$$y'' = \frac{\frac{dy'}{dt}}{\frac{dx}{\cancel{dt}\ d\theta}} = \frac{\left(\frac{b}{a\sin^2\theta}\right)}{-a\sin\theta}$$

$$y'' = \frac{d}{dx}\left(\frac{b\cos\theta}{-a\sin\theta}\right)$$

$$= \frac{d}{d\theta}(\quad)\cdot\frac{d\theta}{dx}$$

$$= -\frac{b}{a^2\sin^3\theta} = \frac{-b}{a^2}\csc^3\theta$$

$$\left(\frac{1}{\sin\theta} = \csc\theta\right)$$

390) 1 b

$x = t+2$ $\qquad$ $y = 2t^2-3$

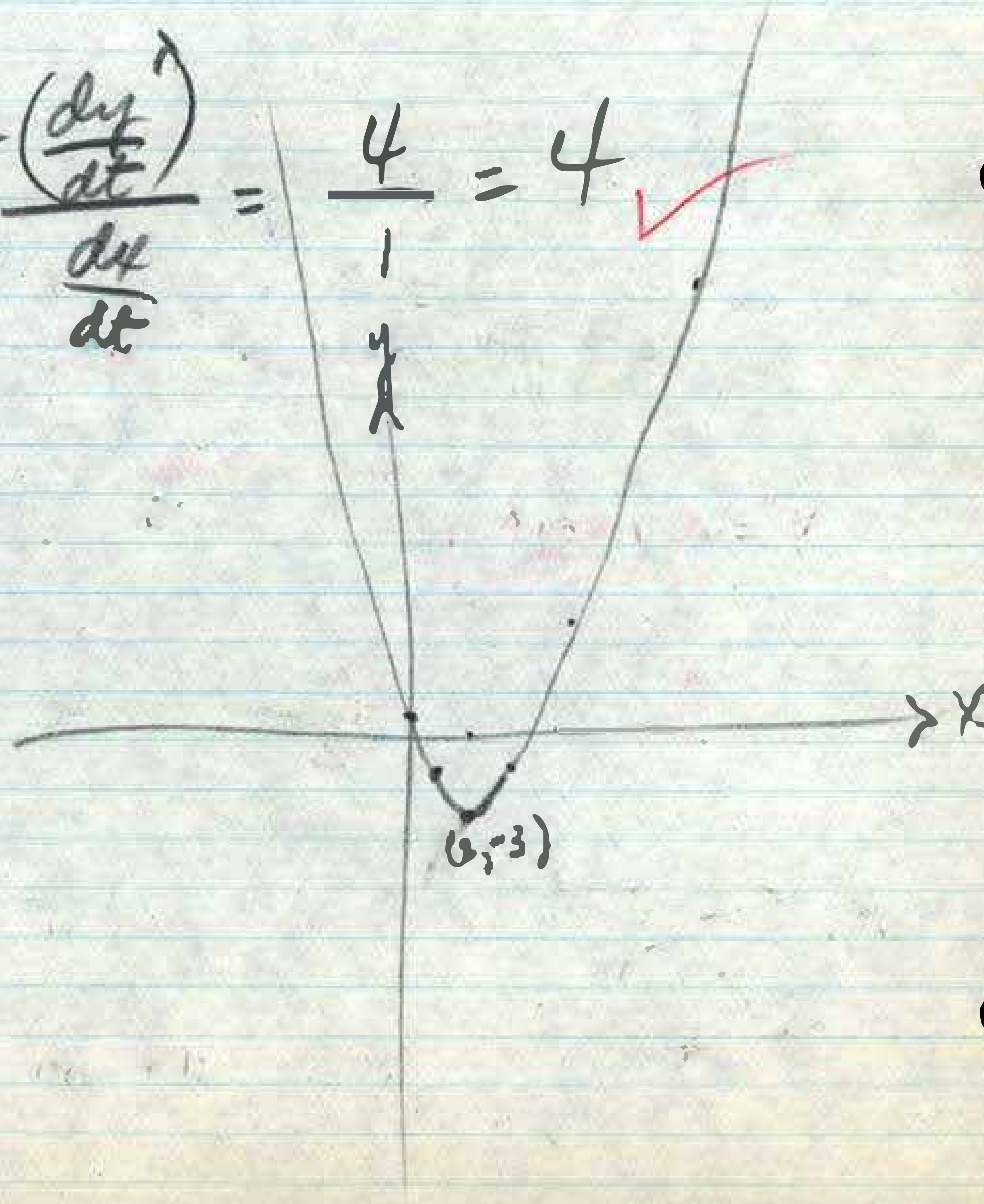
$\frac{dx}{dt} = 1$ $\qquad$ $\frac{dy}{dt} = 4t$

$$\frac{dy}{dx} = y' = 4t$$

$$\frac{dy'}{dt} = 4$$

$$y'' = \frac{\left(\frac{dy'}{dt}\right)}{\frac{dx}{dt}} = \frac{4}{1} = 4$$

| t | x | y |
|---|---|---|
| 0 | 2 | -3 |
| 1 | 3 | -1 |
| 2 | 4 | 5 |
| 3 | 5 | 15 |
| -1 | 1 | -1 |
| -2 | 0 | 1 |
| -3 | -1 | 15 |



Page 400) 1a

$y = x^2$ (parabola)

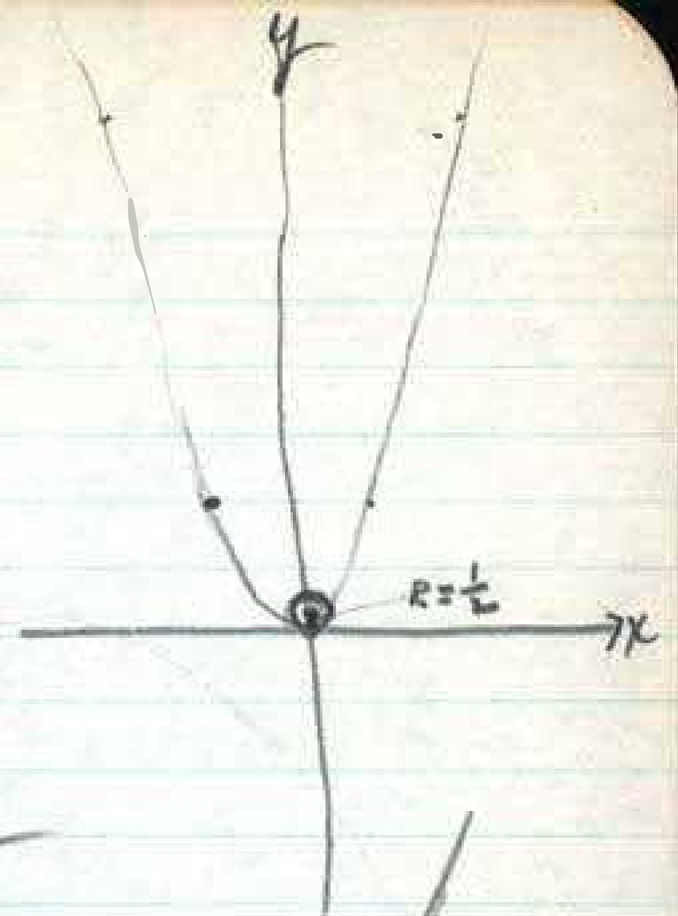
| y | x |
| --- | --- |
| 0 | 0 |
| 4 | ±2 |
| 16 | ±4 |

$y' = 2x$

$y'' = 2$

$R = \frac{1}{K} = \frac{\left[1+(y')^2\right]^{3/2}}{y''} = \frac{(1+4x^2)^{3/2}}{2}$

at $(0,0)$ $R = \frac{(1+0)^{3/2}}{2} = \frac{1}{2}$ ✓

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1c) $y^2 = x^3$ (semi-cubical parabola)

| y | x |
| --- | --- |
| 0 | 0 |
| ±1 | 1 |
| ±8 | 4 |

$y = \pm x^{3/2}$ ✓

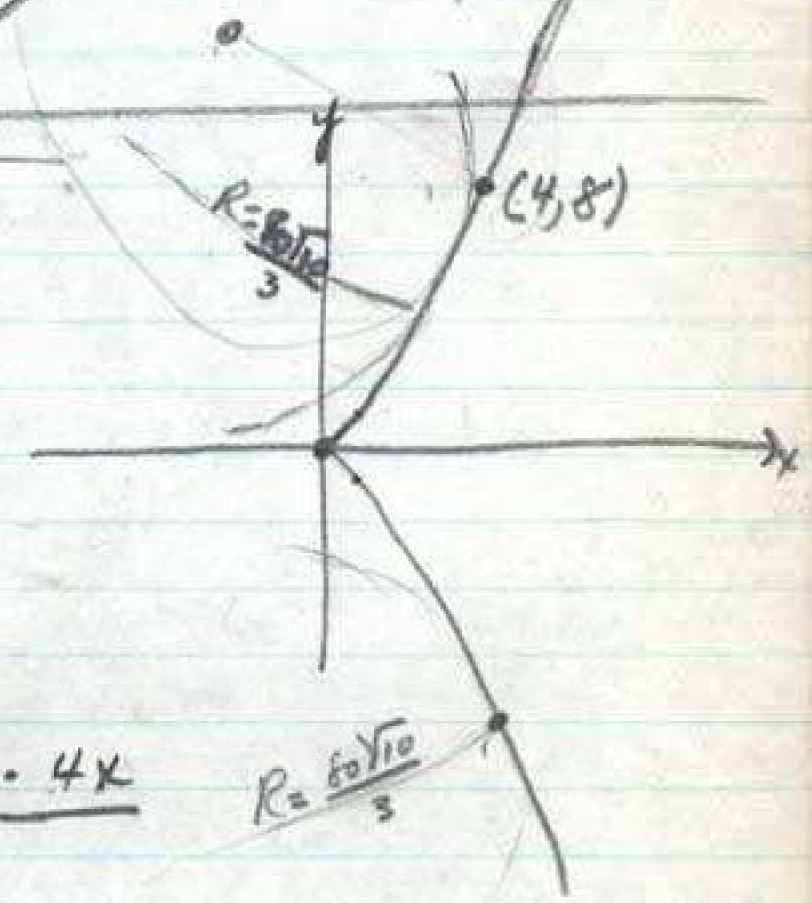
Taking x as pos. ($R=+$, $R=-$)

$y' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$ ✓

$y'' = \frac{3}{4}x^{-\frac{1}{2}} = \frac{3}{4\sqrt{x}} = \frac{3\sqrt{x}}{4x}$ ✓

$R = \left[\frac{1+\left(\frac{3\sqrt{x}}{2}\right)^2}{\frac{3\sqrt{x}}{4x}}\right]^{3/2} = \frac{\left(1+\frac{9x}{4}\right)^{3/2} \cdot 4x}{3\sqrt{x}}$

at $(4,8)$ $R = \frac{(10)^{3/2} \cdot 16}{6} = \frac{10\sqrt{10} \cdot 8}{3} = \frac{80\sqrt{10}}{3}$ ✓

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1g) $y = \sin x$

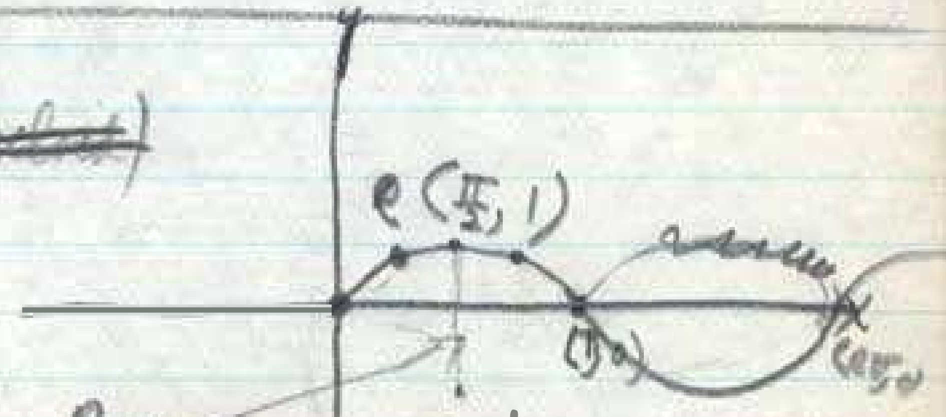
$y' = \cos x$

$y'' = -\sin x$ ✓

$R = \frac{\left[1+\cos^2 x\right]^{3/2}}{-\sin x}$

| y | x |
| --- | --- |
| 0 | 0 |
| $\frac{\sqrt{2}}{2}$ | $\frac{1}{4}\pi$ |
| 1 | $\frac{\pi}{2}$ |
| $\frac{\sqrt{2}}{2}$ | $\frac{3}{4}\pi$ |
| 0 | $\pi$ |

(~~explicit~~)




$R = \left|-\frac{3}{2}\sqrt{3}\right| = \frac{3}{2}\sqrt{3}$

Given $(\frac{1}{4}\pi, \frac{\sqrt{2}}{2})$, $\cos\frac{1}{4}\pi = \frac{\sqrt{2}}{2}$, $\sin\frac{1}{4}\pi = \frac{\sqrt{2}}{2}$

$R = \frac{(1+\frac{1}{2})^{3/2}}{-\frac{\sqrt{2}}{2}} = \frac{(\frac{3}{2})^{3/2}}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}}\sqrt{\frac{27}{8}} = -\sqrt{2}\cdot\sqrt{\frac{27}{8}} = -\sqrt{\frac{54}{8}}$

$= -\sqrt{\frac{27}{4}} = -\frac{3}{2}\sqrt{3}$ ✓

400) 1 i

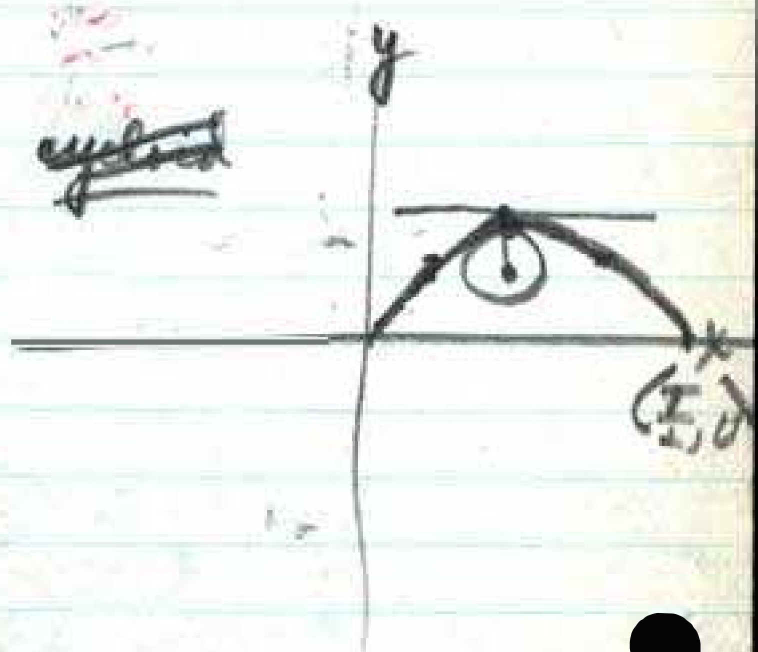
$$y = 2\sin 2x$$

$$y' = 4\cos 2x$$

$$y'' = -8\sin 2x$$

$$R = \frac{\left[1 + (4\cos 2x)^2\right]^{3/2}}{-8\sin 2x}$$

| $x$ | $y$ |
|---|---|
| $0$ | $0$ |
| $\frac{\pi}{6}$ | $\frac{2\sqrt{3}}{2} = \sqrt{3}$ |
| $\frac{\pi}{4}$ | $2$ |
| $\frac{\pi}{3}$ | $\sqrt{3}$ |
| $\frac{\pi}{2}$ | $0$ |




$$R = \frac{(1 + 16\cos^2 2x)^{3/2}}{-8\sin 2x}$$

at $(\frac{1}{4}\pi, 2)$, $2x = \frac{\pi}{2}$ $\begin{pmatrix}\cos 90° = 0 \\ \sin 90° = 1\end{pmatrix}$

$$\therefore R(\tfrac{1}{4}\pi, 2) = -\frac{1}{8}$$

$$R = \left|-\tfrac{1}{8}\right| = \tfrac{1}{8}$$

401) 6a

$$x = 3t \quad , \quad y = 2t^2 - 1$$

$$\frac{dx}{dt} = 3 \quad , \quad \frac{dy}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{4t}{3} = y' \checkmark$$

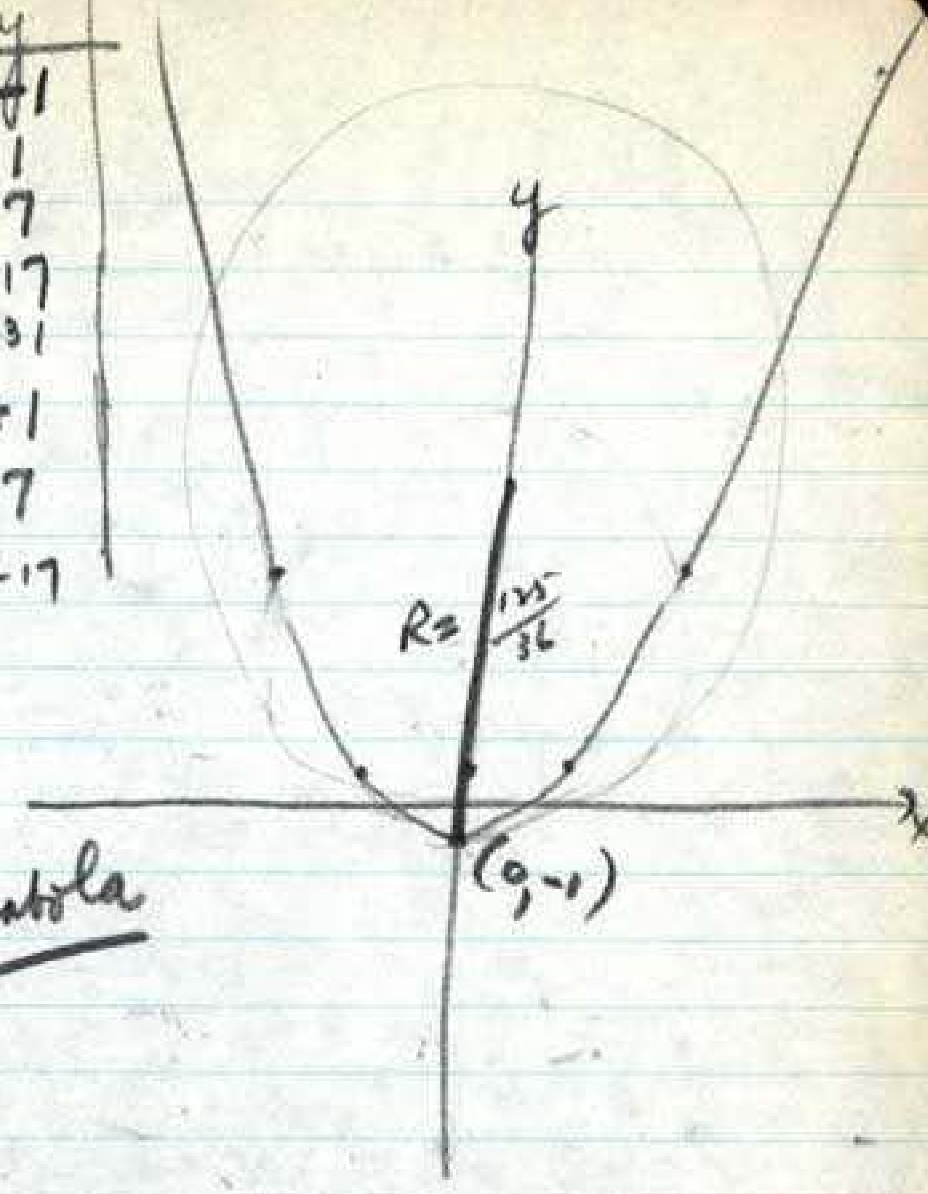
$$y'' = \cancel{\frac{4}{3}} \ 4/9$$

$$R = \frac{1}{K} = \left[\frac{1 + (y')^2}{y''}\right]^{3/2}$$

at $t = 1$, $R = \dfrac{\left(1 + \frac{16}{\cancel{4}\,9}\right)^{3/2}}{\frac{4}{\cancel{3}\,9}}$

$$R = \left(\frac{25}{9}\right)^{3/2} \cdot \frac{3}{4} = \frac{125 \cdot 3}{27 \cdot 4} = \frac{125}{36} \times$$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 0 | -1 |
| 1 | 3 | 1 |
| 2 | 6 | 7 |
| 3 | 9 | 17 |
| 4 | 12 | 31 |
| -1 | -3 | +1 |
| -2 | -6 | +7 |
| -3 | -9 | +17 |




Parabola

Book gives 125/12

$$\frac{dy}{dx} = \frac{4}{3}t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{4t}{3}\right)$$

$$= \frac{4}{3} \cdot \frac{dt}{dx}$$

$$= \frac{4}{3} \cdot \frac{1}{3} = \frac{4}{9}$$

$$\frac{dx}{dt} = \frac{4}{3}$$

$y''$

$$\frac{4/3}{3}$$

401) 6 b

$x = 4t \qquad y = \frac{2}{t}$

| t | x | y |
|---|---|---|
| ~~0~~ | ~~0~~ | ~~∞~~ |
| +1 | +4 | +2 |
| −1 | −4 | −2 |
| +2 | +8 | 1 |
| −2 | −8 | −1 |
| +3 | 12 | 4/3 |
| −3 | −12 | −2/3 |

$\frac{dx}{dt} = 4 \qquad \frac{dy}{dt} = -\frac{2}{t^2}$

$\frac{dy}{dx} = -\frac{1}{2t^2} = y'$ ✓

$\frac{d}{dx}\left(-\frac{1}{2t^2}\right) \frac{d}{dx}\left(\frac{t^{-2}}{2}\right)$

$= \cancel{2}\frac{t^{-3}}{\cancel{4}} \frac{dt}{dx}$

$\frac{1}{4}$

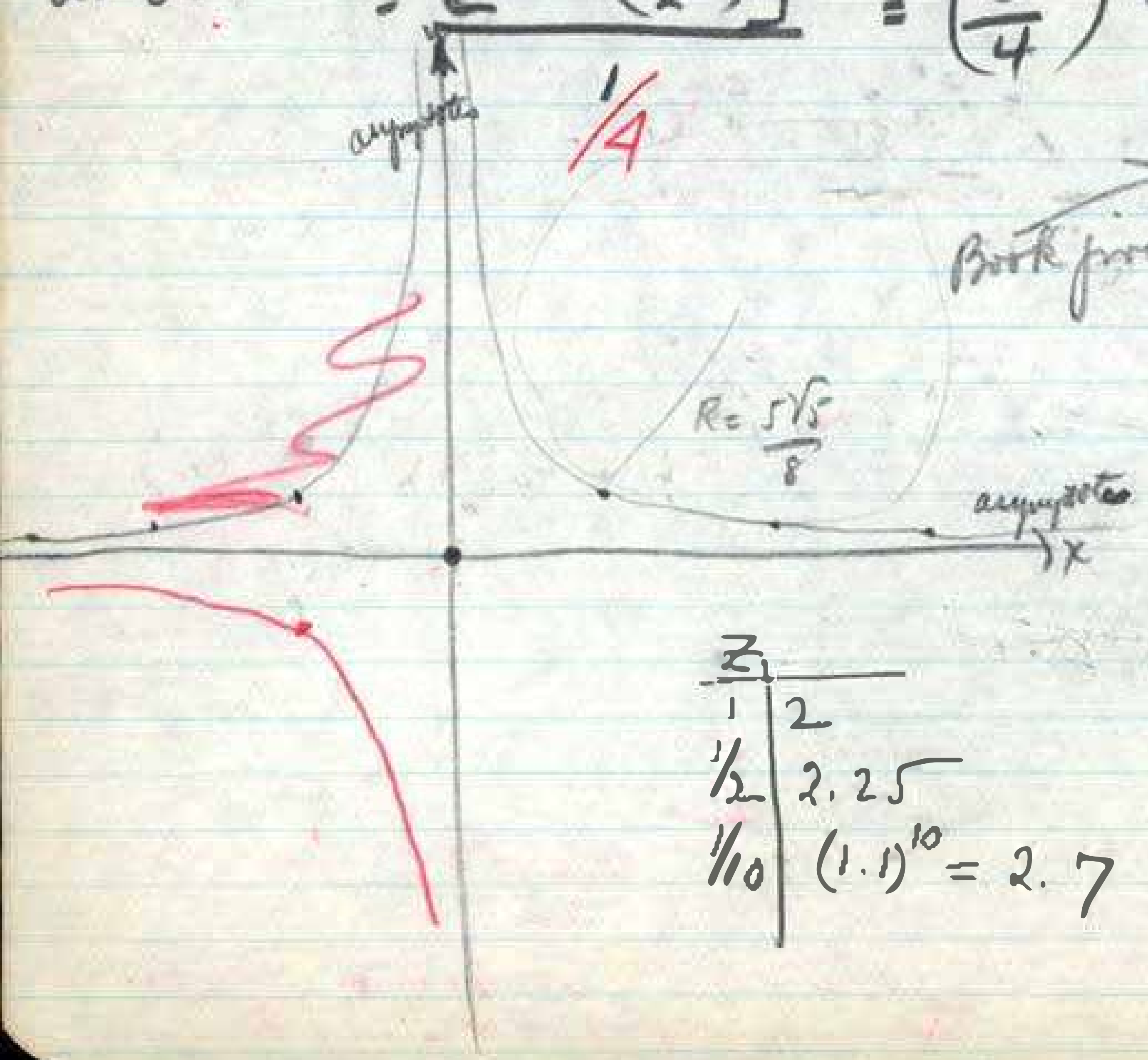
$= \frac{1}{4t^3}$

$y'' = \frac{1}{4t^3}$

$R = \frac{\left[1 + (y')^2\right]^{3/2}}{y''}$ ✓

at t = 1 $= \frac{\left[1 + \left(\frac{-1}{2}\right)^2\right]^{3/2}}{1/4} = \left(\frac{5}{4}\right)^{3/2} = \frac{5\sqrt{5}}{8}$ ✗

Book gives $\frac{5\sqrt{5}}{2}$ ✗ ✓




| z | |
|---|---|
| 1 | 2 |
| 1/2 | 2.25 |
| 1/10 | $(1.1)^{10} = 2.7$ |

~~$e = \lim (z+1$~~

$e = \lim_{z \to 0} \left((z+1)^{1/z}\right)$

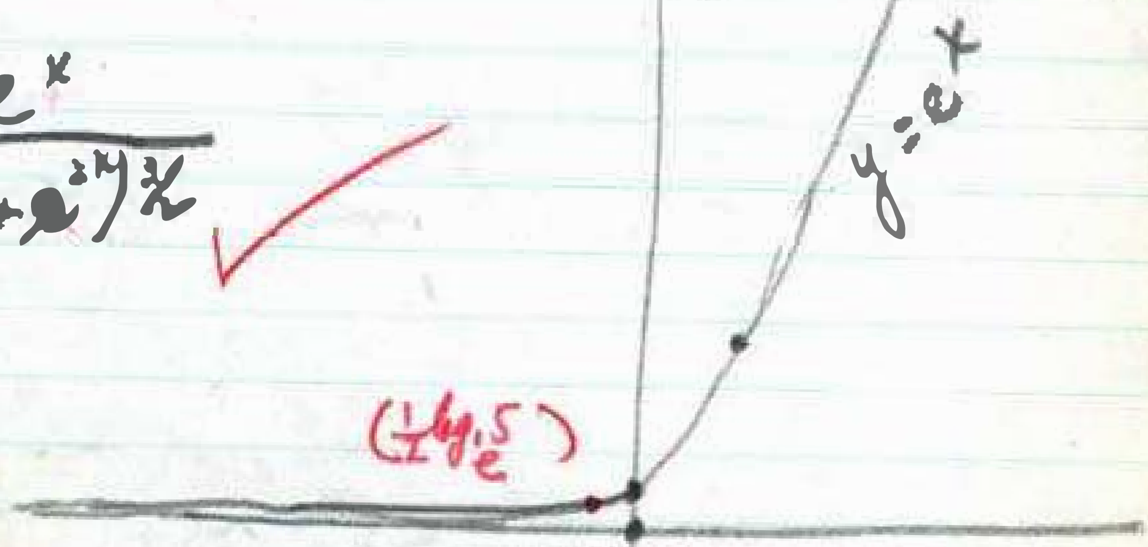
401) 9

$$y = e^x$$
$$y' = e^x$$
$$y'' = e^x$$

| x | y |
|---|---|
| 1 | 2.718 |
| 2 | 7.389 |
| 3 | 20.09 |
| 0 | 1. |
| −1 | 0.3679 |
| −2 | 0.1353 |
| −10 | 0.00005 |

$$R = \frac{\left[1 + (e^x)^2\right]^{3/2}}{e^x}$$

$$= \frac{(1 + e^{2x})^{3/2}}{e^x} \qquad K = \frac{e^x}{(1+e^{2x})^{3/2}}$$




$$\frac{dK}{dx} = \frac{(1+e^{2x})^{3/2}\underline{e^x} - \underline{e^x}\left[\frac{3}{\cancel{2}}(1+e^{2x})^{\frac{1}{2}}(\cancel{2}e^{2x})\right]}{(1+e^{2x})^3}$$

$\log(u^m) = m \cdot \log u$

$\log_e e = 1$

$$= \frac{e^x\left[(1+e^{2x})^{3/2} - 3e^{2x}(1+e^{2x})^{\frac{1}{2}}\right]}{(1+e^{2x})^3} = \frac{e^x(1+e^{2x})^{\frac{1}{2}}\left[\underline{(1+e^{2x})} \underline{-3e^{2x}}\right]}{(1+e^{2x})^3}$$

Setting $\frac{dK}{dx}$ at 0, $e^x(1+e^{2x})^{\frac{1}{2}}(1-2e^{2x}) = 0$

Then $e^x = 0$ (impossible)

or $1 + e^{2x} = 0$, $e^{2x} = -1$ Impos.

or $1 - 2e^{2x} = 0$, $e^{2x} = .5$,

$\log_e e^{2x} = \log_e (.5)$

$2x \cdot \log_e e = \log_e (.5)$

$2x = \log_e (.5)$, $x = \frac{1}{2}\log_e(.5)$

4·1) 10

$$3y = x^3 - 2x$$
$$y = \frac{x^3}{3} - \frac{2x}{3} \checkmark$$
$$y' = x^2 - \frac{2}{3} = \frac{3x^2-2}{3}$$
$$y'' = 2x \checkmark$$

$$9^{3/2} = 27$$

$$K = \frac{y''}{(1+(y')^2)^{3/2}} = \frac{2x}{\left[1 + \left(\frac{3x^2-2}{3}\right)^2\right]^{3/2}}$$

$$K = \frac{2x}{\left(1 + \frac{9x^4 - \cancel{6}^{12}x^2 + 4}{9}\right)^{3/2}} = \frac{2x}{\left(\frac{9x^4 - \cancel{2x^2}^{12} + 13}{9}\right)^{3/2}} = \frac{2x \cdot 27}{(9x^4 - 12x^2 + 13)^{3/2}} = \frac{54x}{(\quad)^{3/2}}$$

$$\frac{dK}{dx} = \frac{\left[\frac{(9x^4 - 6x^2 + 13)}{9}\right]^{3/2} \cdot 2 - 2x \cdot \frac{3}{2}\left(\frac{9x^4-6x^2+13}{9}\right)^{\frac{1}{2}}\left(4x^3 - \frac{4}{3}x\right)}{\left(\frac{9x^4-6x^2+13}{9}\right)^3}$$

$$\frac{dK}{dx} = \frac{2\left(\frac{9x^4-6x^2+13}{9}\right)^{3/2} - \left[(12x^4 - 4x^2)\left(\frac{9x^4-6x^2+13}{9}\right)^{\frac{1}{2}}\right]}{\left(\frac{9x^4-6x^2+13}{9}\right)^3}$$

Setting $\frac{dK}{dx} = 0$, $\left(\frac{9x^4-6x^2+13}{9}\right)^{\frac{1}{2}}\left[2\frac{(9x^4-6x^2+13)}{9} - 12x^4 - 4x^2\right] = 0$

Then either $\frac{9x^4 - \cancel{6}^{12}x^2 + 13}{9} = 0$, $9x^4 - 6x^2 + 13 = 0$

$a = 9$, $b = -6$, $c = 13$

$$x = \frac{6 \pm \sqrt{36 - 4(9 \times 13)}}{18}$$

$x^2 = -$ ~~X~~ discard

$x^2 = +$

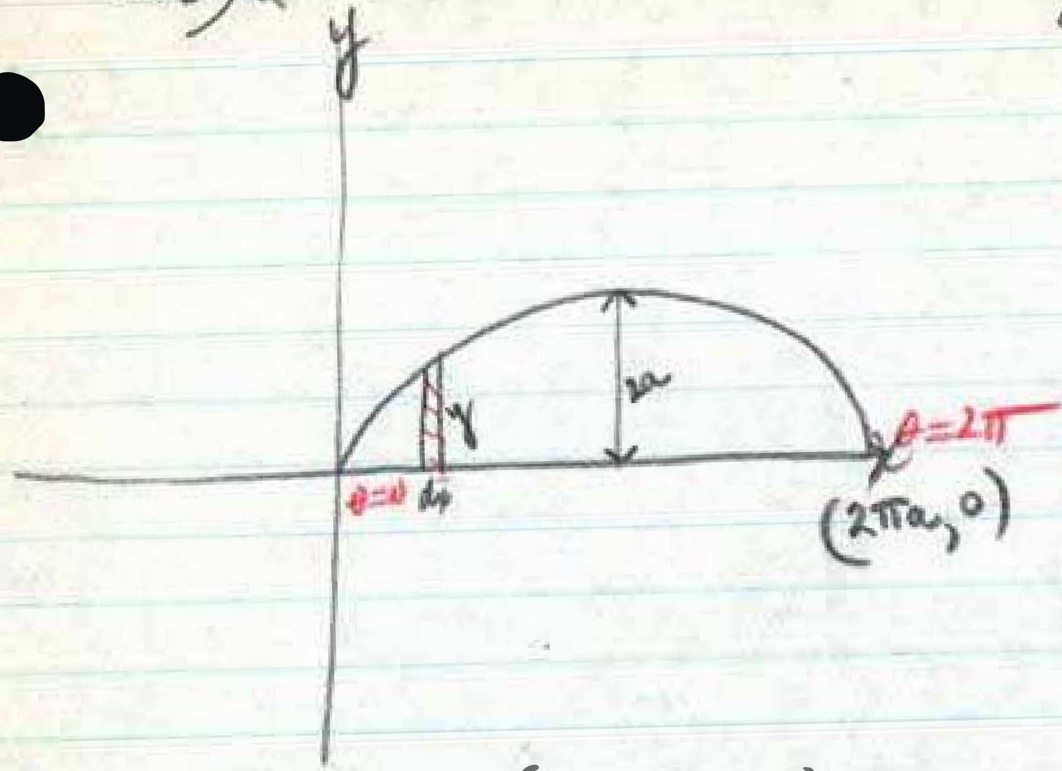
or $18x^4 - 12x^2 + 26 - 108x^4 - 36x^2 = 0$
$$-90x^4 - 50x^2 + 26 = 0$$
$$-45x^4 - 25x^2 + 13 = 0$$

$a = -45$, $b = -25$, $c = 13$

$$x^2 = \frac{25 \pm \sqrt{625 + \cancel{1840}^{2340}}}{-90} = \frac{25 \pm 50.96}{-90}$$

180
13
54
18
2340

386) 2b




$x = a(\theta - \sin\theta)$

$dx = (a - a\cos\theta)\,d\theta$

$dx = a(1-\cos\theta)\,d\theta$

$y = a(1-\cos\theta)$

Element of area $= y\,dx$

Total Area $= a^2\int_0^{2\pi}[\cancel{a}(1-\cos\theta)]^2\,d\theta$

$= \int_0^{2\pi}(a - a\cos\theta)^2\,d\theta$

$= \int_0^{2\pi}(a^2 - 2a^2\cos\theta + a^2\cos^2\theta)\,d\theta$

$= a^2\theta - 2a^2\sin\theta + \frac{a^2\theta}{2} + \frac{a^2}{4}\sin 2\theta\Big]_0^{2\pi}$

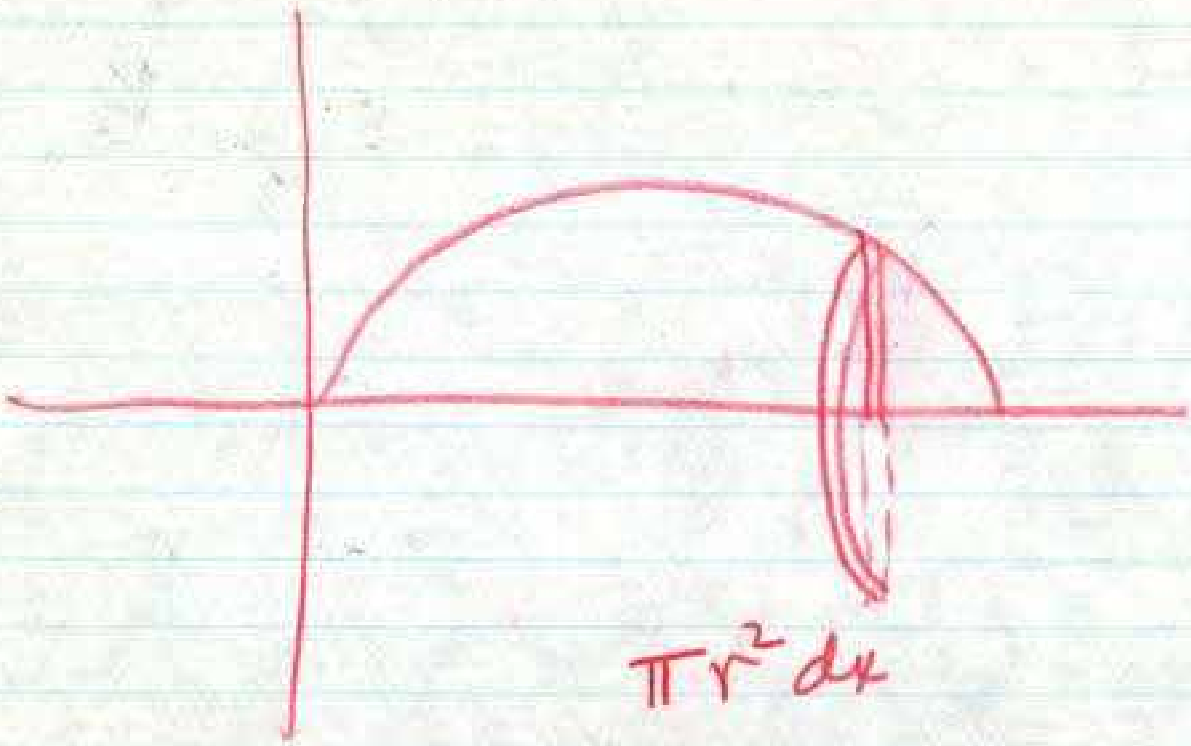
$= \left(a^2 2\pi + 2a^2\sin 2\pi + \frac{2\pi a^2}{2} + \frac{a^2}{4}\sin 4\pi\right)$

$- \left[0 + 2a^2\sin 0 + 0 + \frac{1}{4}\sin 0\right]$

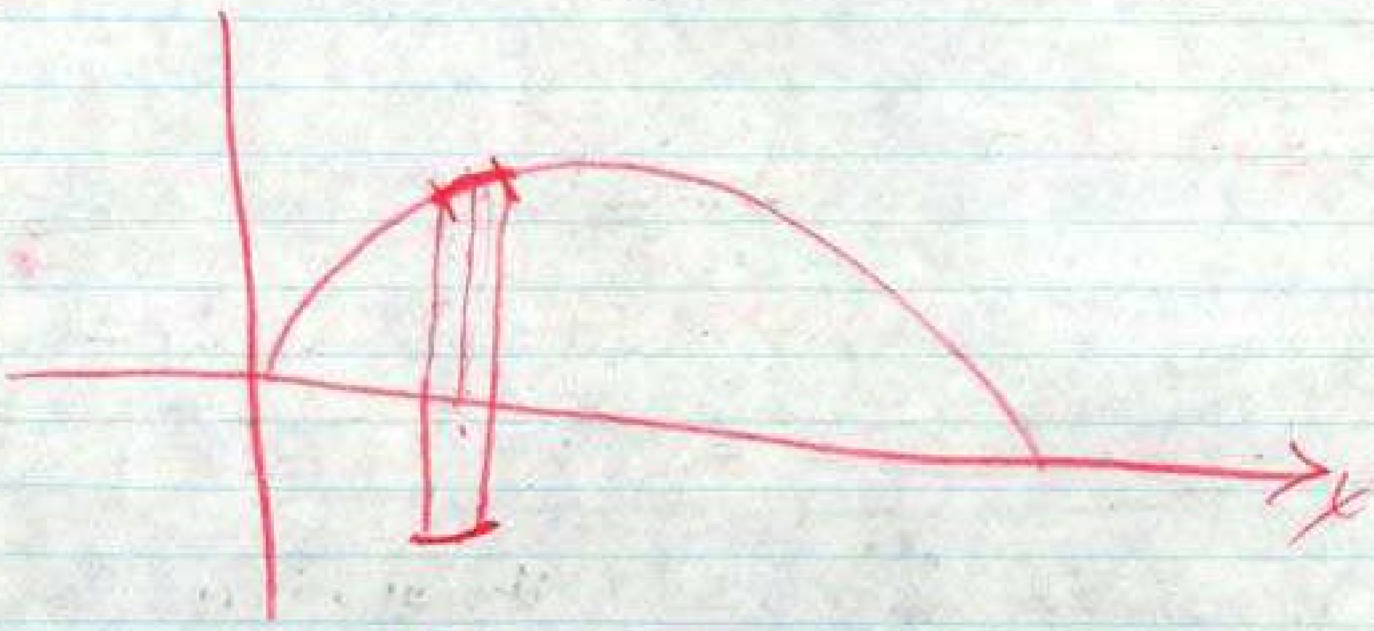
$= \left[\frac{3}{2}2\pi a^2 + \left(2a^2 + \frac{a^2}{4}\right)\sin 2\pi\right] - 0$

$= 3\pi a^2 + \left(2a^2 + \frac{a^2}{4}\right)\sin 2\pi$

$= 3\pi a^2$



$\pi r^2 \, dx$

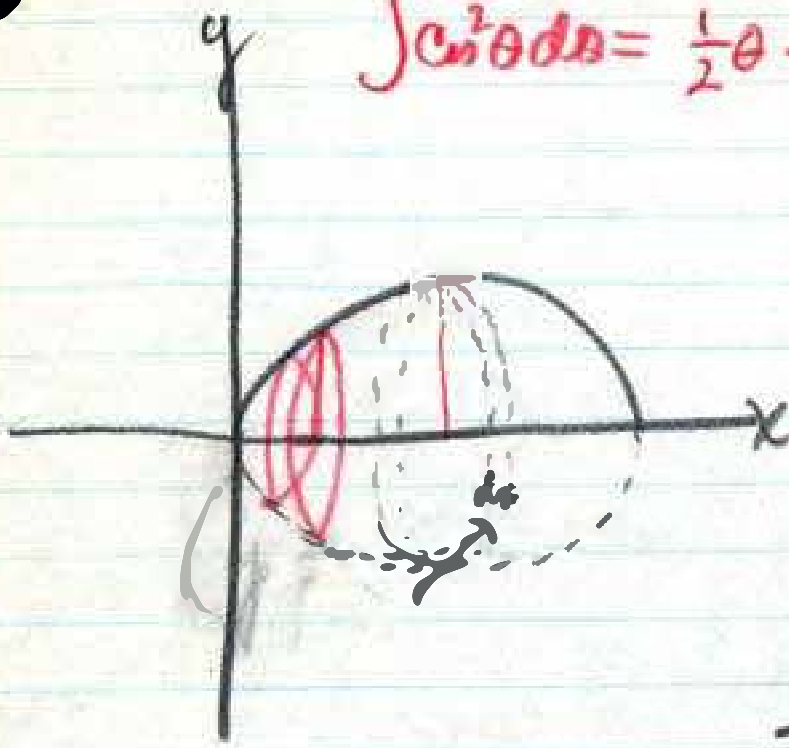


$ds \cdot 2\pi y$

$ds = \sqrt{(dx)^2 + (dy)^2}$

$$dS = 2\pi \cdot a^2(1-\cos\theta)\sqrt{a^2(1-\cos\theta)^2 + a^2 \sin^2\theta \, d\theta^2}\; d\theta$$

386) 1c




El. of Volume = $\cancel{2}\pi r^2\,dx$

$\int \cos^2\theta\,d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$

$r = y = a(1-\cos\theta)$

$dx = (a(1-\cos\theta))\,d\theta$

Total Vol. = $\cancel{2}\pi \int_0^{2\pi} (a - a\cos\theta)^3\,d\theta$

$dy = a\sin\theta\,d\theta$

$= \cancel{2}\pi \int_0^{2\pi} (a^3 - 3a^3\cos\theta + 3a^3\cos^2\theta - a^3\cos^3\theta)\,d\theta$

$= 2\pi\left(a^3\theta + 3a^3\sin\theta + \frac{3a^3}{2}\theta + \frac{3a^3}{\cancel{2}\,4}\sin 2\theta - \frac{a^3}{3}\sin\theta(\cos^2\theta + 2)\right)\Big|_0^{2\pi}$

$\frac{d}{d\theta}\sin\theta = \cos\theta$

$\int \cos\theta\,d\theta = \sin\theta$

$= 2\pi(a^3 2\pi + 0 + 3a^3\pi + 0 - 0) - (0)$

$= 4\pi^2 a^3 + 6\pi^2 a^3 = \frac{10}{\cancel{2}}\pi^2 a^3 =$

$\boxed{\text{Total Surface} = 2\pi r(r+a) = 2\pi r^2 + 2\pi a r}$

Element of Surface = $(2\pi r^2 + 2\pi a r)\,dx$

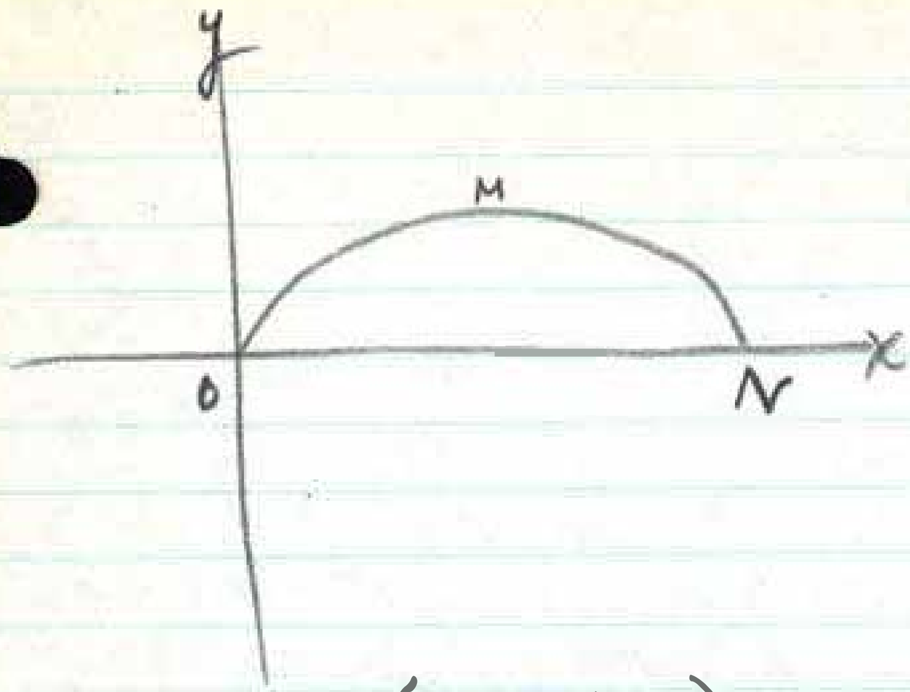
Total Surface = $2\pi \int_0^{2\pi} (r^2 + ar)\,dx$

$= 2\pi \int_0^{2\pi} \left[(a - a\cos\theta)^2 + a(a - a\cos\theta)\right](a - a\cos\theta)\,d\theta$

$= 2\pi (a - a\cos\theta)^3 + a(a - a\cos\theta)^2 \Big]_0^{2\pi}$

from 1b + 1c

$=$




$$s = \int_0^{2\pi} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{2\pi} \sqrt{1 + \left(\frac{a \sin\theta}{a(1-\cos\theta)}\right)^2} (a - a\cos\theta)\, d\theta$$

$$= \int_0^{2\pi} \sqrt{1 + \frac{\sin^2\theta}{(1-2\cos\theta+\cos^2\theta)}}\; (a[1-\cos\theta])\, d\theta$$

$x = a(\theta - \sin\theta)$

$dx = (a - a\cos\theta)\, d\theta$

$y = a(1-\cos\theta)$

$dy = a \sin\theta\, d\theta$

$$= \int_0^{2\pi} \sqrt{[(1-2\cos\theta+\cos^2\theta)+\sin^2\theta]}\; d\theta$$

$\cancel{\cos 2\theta}$

$$= a\int_0^{2\pi} \sqrt{(1-2\cos\theta+1\ )}\quad d\theta$$

$\cancel{\cos\theta}$

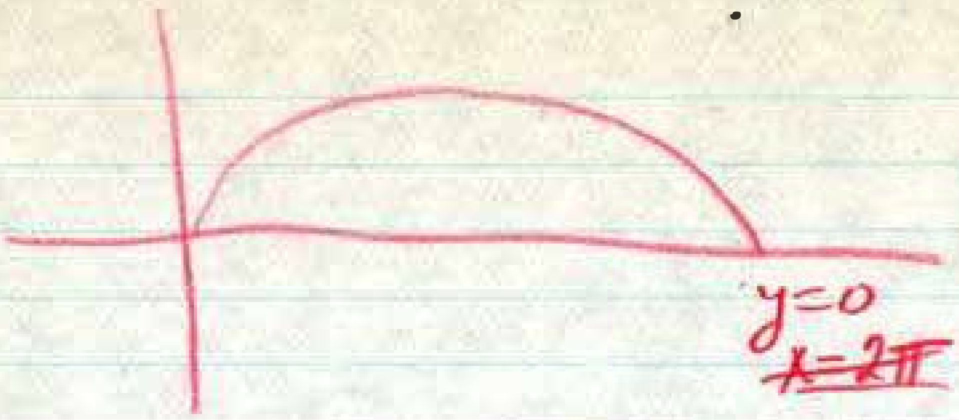
$$= a\int_0^{2\pi} \sqrt{(2-2\cos\theta)}\; d\theta$$

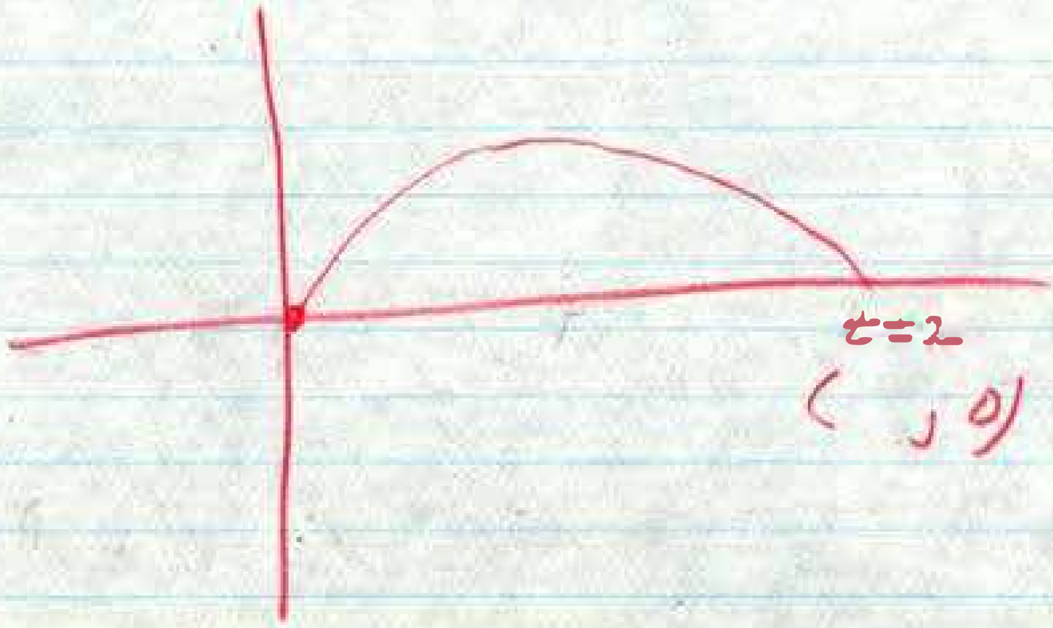
$$= a \int_0^{2\pi} \sqrt{2\,\frac{(1-\cos\theta)}{2} \cdot 2}\; d\theta$$

$$= 2a \int \sqrt{\frac{1-\cos\theta}{2}}\; d\theta$$

$2a\left(-2\cos\frac{\theta}{2}\right)\Big]$

$$= 2a \int \left(\sin\frac{\theta}{2}\right) d\theta$$




$\cos \pi t = 1$

$\pi t = 0, \quad \pi t = 2\pi$

---

$$1 + 2 + 3 + 4 + \cdots$$

$$1,\ 3,\ 6,\ 10,\ \cdots$$

(Converges) $1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10{,}000} + \cdots$

$a = 1$
$r = 1/10$

$$\underbrace{1,\ 1.1,\ 1.11,\ 1.111,\ 1.1111,}\ \cdots$$

$S_1$
$S_2$
$S_3$
$\vdots$
$S_n$

$$a + ar + ar^2 + ar^3 + \cdots$$

$r < 1$

$$\frac{a}{1-r} = \frac{1}{1 - 1/10} = \frac{1}{9/10} = \frac{10}{9} = 1\frac{1}{9}$$

$$\boxed{\lim_{n\to\infty} S_n}$$

Read M&B Infinite Series

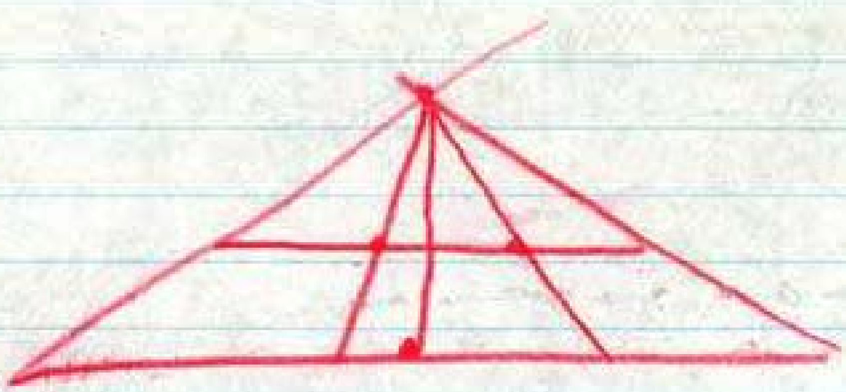
429 1a, b

2a, b

430 4a, b, d, h

5 a, c, e

6 a, b, d, f, g



1 2 3 4 5 6 7 — — —

2 4 6 8 10 12 14 — — —

$$\underbrace{1 + \tfrac{1}{3} + \tfrac{1}{5} + \tfrac{1}{7} + \cdots \qquad + \tfrac{1}{\ } + \cdots}$$

$u_1 + u_2 + u_3 + \cdots + u_n + \cdots$

$S_1 = u_1$

$S_2 = u_1 + u_2$

$\vdots$

$S_n = u_1 + u_2 + \cdots + u_n$

$\left\{ \lim_{n\to\infty} S_n = A \right\}$ Series converges to $A$

$\lim_{n\to\infty} S_n$ does not exist, series diverges

$1 - 1 + 1 - 1 + 1 - 1 + \cdots$

$1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10{,}000} + \frac{1}{100{,}000} + \cdots$ $\quad a + ar + ar^2 + ar^3 + \cdots$ Geom. Series

$1,\ 1.1,\ 1.11,\ 1.111, \cdots$

Geom. Series conv. if $|r| < 1$

$\left.\begin{matrix} a = 1 \\ r = 1/10 \end{matrix}\right\}$ conv. to $\frac{1}{9/10} = \frac{10}{9}$ $\quad$ to $\frac{a}{1-r}$

---

$u_1 + u_2 + u_3 + u_4 + \cdots + u_n + \cdots$

<u>Nec. cond. for conv.</u> $\quad u_n \to 0 \;\}\; \underline{u_n \not\to 0}$

---

~~$200 + 200 + 200 + \cdots$~~

$\frac{1}{1000000000} + \frac{1}{10000} + \frac{1}{1000\,000} + \frac{1}{}$

M + B 429) 1a

$$\frac{2}{2}\,(\text{or } 1),\ \frac{4}{3},\ \frac{6}{4},\ \frac{8}{5} + \cdots + \frac{2n}{n+1} + \cdots$$

1b

$$\frac{1}{1},\ \frac{4}{4},\ \frac{7}{9},\ \frac{10}{16} + \cdots + \frac{3n-2}{n^2} + \cdots$$

---

429) 2a

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \cdots + \frac{2n-1}{2n} + \cdots$$

$S_1 = 1/2$
$S_2 = 5/4$

2b)

$$\sqrt{2} + \frac{\sqrt{3}}{4} + \frac{2}{9} + \frac{\sqrt{5}}{16} + \cdots + \frac{\sqrt{n+1}}{n^2} + \cdots$$

---

430) 4a

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \cdots + \frac{1}{\sqrt{n+1}}$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots \text{ div.}$$

X $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$, ~~However, as $n$ gets larger, $S_n \to \infty$~~

~~Therefore, series diverges.~~

Is this ~~variant of harmonic~~ series?

Ser. div. by compar

---

4b

$$\frac{1}{2} - \frac{4}{5} + \frac{9}{10} - \frac{16}{17} + \cdots + \frac{n^2}{n^2+1}$$

$$\lim_{n\to\infty}\left(\frac{n^2}{n^2+1}\right) = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n^2}}\right) = 1 \neq 0$$ , $\therefore$ series diverges

4d) $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\cdots\cdots+\frac{1}{2n}$

X $\lim_{n\to\infty}\left(\frac{1}{2n}\right)=\lim_{(n\to\infty)}\frac{\frac{1}{n}}{2}=0$ , $\therefore$ ~~series is convergent~~ ✓

4h $\frac{1\cdot 2}{3\cdot 4}+\frac{2\cdot 3}{4\cdot 5}+\frac{3\cdot 4}{5\cdot 6}+\frac{4\cdot 5}{6\cdot 7}+\cdots\cdots+\frac{n(n+1)}{(n+2)(n+3)}+$ —

$$\lim_{n\to\infty}\left(\frac{n(n+1)}{(n+2)(n+3)}\right)=\lim_{n\to\infty}\left(\frac{1\left(1+\frac{1}{n}\right)}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)}\right)=\frac{1}{1}=1$$

Ratio Test $\rho=\left(\frac{12}{30}\div\frac{6}{20}\right)=\frac{12}{30}\times\frac{20}{6}=\cancel{\frac{4}{3}}>1$ $\therefore$ series is divergent ✓

4 3o) 5a $\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots\cdots+\frac{1}{(n+1)^2}$

$\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+$ —

~~geometric series, ... , + $\rho=\frac{1}{(n+1)}<1$~~ $\lim_{n\to\infty}$ ~~$(r^n=0)$~~

$\therefore$ series is convergent

Comparison ~~c known convergent geometric series shows individual members are less c~~ known series.

5e) $\frac{1}{2\sqrt{2}+1}+\frac{1}{3\sqrt{3}+1}+\frac{1}{4\sqrt{4}+1}+\frac{1}{5\sqrt{5}+1}+\cdots\cdots+\frac{1}{(n+1)\sqrt{n+1}+1}$

$\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+$ — . conv.

~~As $n\to\infty$, $\lim S_n=0$, ∴ series is convergent~~ conv. (by compar.)

5e) $\frac{1}{1\cdot 4}+\frac{1}{2\cdot 5}+\frac{1}{3\cdot 6}+\frac{1}{4\cdot 7}+\cdots\cdots+\frac{1}{n(n+3)}$

$\lim_{n\to\infty}\left(\frac{1}{n(n+3)}\right)=0$ , $\therefore$ ~~series is convergent~~ series conv.

Compare with $\frac{1}{1.1}+\frac{1}{2.2}+\frac{1}{3.3}+$ —

[redacted] mcB

Page 430) 6a

$$\frac{3}{5} + \frac{5}{8} + \frac{7}{11} + \frac{9}{14} + \cdots + \frac{2n+1}{3n+2} + \frac{2n+3}{3n+5} + \cdots$$

$$\lim_{n\to\infty}\left(\frac{2n+3}{3n+5}\cdot\frac{3n+2}{2n+1}\right) = \lim_{n\to\infty}\left(\frac{(2+3/n)\,(3+2/n)}{(3+5/n)\,(2+1/n)}\right) = 1$$

Test ratio $\rho = \frac{5}{8} \div \frac{3}{5} = \frac{5}{8} \times \frac{5}{3} = \frac{25}{24} > 1 \therefore$ divergent

$$\lim_{n\to\infty}\left(\frac{2n+1}{3n+2}\right) = \frac{2+\frac{1}{n}}{3+\frac{2}{n}} = 2/3 \neq 0 \quad \therefore \text{ divergent}$$

---

6b) $\frac{2^{-1}}{1^3} + \frac{2^{-2}}{2^3} + \frac{2^{-3}}{3^3} + \frac{2^{-4}}{4^3} + \cdots\cdots + \frac{2^{(-n)}}{n^3}$

$$\lim_{n\to\infty}\left(\frac{2^{-n}}{n^3}\right) = \lim_{n\to\infty}\left(\frac{1}{2^n n^3}\right) = 0 \quad \therefore \text{ series is convergent}$$

---

6d) $\frac{2}{9} + \frac{2}{12} + \frac{2}{15} + \frac{2}{18} + \cdots\cdots + \frac{2}{3n+6}$

$= 2\left(\frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} + \cdots\right)$

$$\lim_{n\to\infty}\left(\frac{2}{3n+6}\right) = \lim_{n\to\infty}\left(\frac{\frac{2}{n}}{3+\frac{6}{n}}\right) = 0 \quad \therefore \text{ series } \cancel{\text{converges}} \text{ div.}$$

$2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + \cdots \rightarrow$ conv.

$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots$ (conv.)

6g) $\frac{\sqrt[3]{2}}{1}+\frac{\sqrt[3]{3}}{2}+\frac{\sqrt[3]{4}}{3}+\frac{\sqrt[3]{5}}{4}+\dots+\frac{\sqrt[3]{n+1}}{n}$

When $n=1000$, $S=\frac{\sqrt[3]{1000+1}}{1000}$

Thus as $n$ gets larger, $S$ gets smaller

& $\lim_{n\to\infty}\left(\frac{\sqrt[3]{n+1}}{n}\right)=0$,

$\therefore$ series is ~~convergent~~ div.

$\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots+\frac{1}{n}\right)$ div.

$$\lim_{n\to\infty}\left(\frac{\sqrt[3]{n+1}}{n}\right)=\lim_{n\to\infty}\left(\frac{\sqrt[3]{n+1}}{\sqrt[3]{n^3}}\right)=\lim_{n\to\infty}\sqrt[3]{\frac{n+1}{n^3}}$$

$$=\lim\sqrt[3]{\frac{1+1/n}{n^2}}$$

$$=0$$

4/6) 1

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \cdots + \frac{n}{(n+1)^n}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{n}{(n+1)^n}\right) = \lim_{n\to\infty}\left(\frac{1}{\left(1+\frac{1}{n}\right)^n}\right) = 1$$

$\therefore$ series is convergent. $\therefore$ convergent

~~Test Ratio fails because~~ $\rho = 1$ $\boxed{\left(\frac{3}{4} \div \frac{1}{2} = 1\right)}$ — $\frac{u_{n+1}}{u_n} = \rho$ $\left(\frac{3}{8} \div \frac{1}{2} = \frac{3}{8} \times \frac{2}{1} = \frac{6}{8} < 1\right)$

4/6) 2

$$\frac{2}{3} + 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)^3 + 4\left(\frac{2}{3}\right)^4 + \cdots + nr^n$$

This is geometric series, where $r = \frac{2}{3}$ $(|r| < 1)$, & $r^n$ decreases as $n$ increases,

$\lim_{n\to\infty} r^n = 0$, & $\lim_{n\to\infty} S_n = 0$, & series is convergent

4/6) 3

$$1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \cdots + \frac{1}{\lfloor n}$$

(Test Ratio – $\frac{1}{6} \div \frac{1}{2} = \frac{2}{6} < 1$)

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{\lfloor n} = 0$, $\therefore$ series is convergent

Comparison Test

Also series is term by term less than or equal to the convergent geometric series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

4/6) 4

$$\frac{\lfloor 2}{10} + \frac{\lfloor 3}{10^2} + \frac{\lfloor 4}{10^3} + \cdots + \frac{\lfloor n+1}{10^n}$$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{\lfloor n+1}{10^n} = \frac{\lfloor 1 + \frac{1}{n}}{10^1} = \frac{1}{10}$ $< 1$ Note $\neq 0$ (only not divergent)

$\therefore$ series is convergent

or Test-Ratio – $\rho = \frac{u_{n+1}}{u_n} = \frac{\frac{6}{100}}{\frac{2}{10}} = \frac{6}{100} \times \frac{10}{2} = \frac{6}{20} < 1$

$\therefore$ Convergent.

417) 7 $\quad \frac{1}{3} + \frac{1}{3}\cdot\frac{2}{5} + \frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7} + \cdots\cdots \frac{\lfloor n}{3\cdot 5\cdot 7\cdots\cdots(2n+1)}$

For every $\lfloor n$, $S_n > 0$, and $S_1 > S_2 > S_3 > S_4$, etc.

$\therefore$ series is convergent.

* Ask about M r B page 431 ↑

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419) 1 $\quad \sqrt{630} = \sqrt{625+5} = 25\sqrt{1+\frac{1}{5}}$

$$z = 0.2 \quad , n = \frac{1}{2}$$

Using binomial series, $\quad (1+z)^n = 1 + \frac{n}{1}z + \frac{n(n-1)}{1\cdot 2}z^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}z^3 + \ldots$

$$\sqrt{630} = 25\left[1 + \frac{1}{2}(.2) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{1\cdot 2}(.2)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-1\frac{1}{2})}{1\cdot 2\cdot 3}(0.2)^3 + \ldots\right]$$

$$= 25\left[1 + 0.1 - .005 + \frac{\frac{3}{8}\ (.008)}{6} + \ldots\right]$$

$$= 25\left[1 + 0.1 - .005 + .0005\right]$$

$$= 25 + 2.5 - .125 + .0125$$

$$= 26.3875$$

27.5
.125
26.375
.0125
26.3875

$$a^k : a^l = a^{k-l}$$

$$u_1 + u_2 + \cdots + u_n + \cdots \qquad \frac{2^n}{2^{n+1}} = \frac{1}{2}$$

Nec. cond. for conv. is $\lim_{n\to\infty} u_n = 0$

---

$p=1$ $\quad 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ div.

$$1 + \frac{1}{2} + \underbrace{\left(\frac{1}{3} + \frac{1}{4}\right)}_{>1/2} + \underbrace{\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right)}_{>1/2} + \underbrace{\left(\frac{1}{9} + \frac{1}{10} + \cdots + \frac{1}{16}\right)}_{>1/2}$$

$$+ \underbrace{\left(\frac{1}{17} + \cdots + \frac{1}{32}\right)}_{>1/2}$$

---

$$u_1 + u_2 + u_3 + \cdots + u_n + \cdots$$

$$v_1 + v_2 + v_3 + \cdots + v_n + \cdots \quad \text{(conv.)} \; \text{(div.)}$$

$$\left\{1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots + \frac{1}{n^p} + \cdots\right\}$$

$p<1$ $\quad 1 + \frac{1}{2^{1/3}} + \frac{1}{3^{1/3}} + \frac{1}{4^{1/3}} + \cdots$ $\therefore$ ser. div.

$p>1$ $\quad 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$ ser. conv.

$r = 1/2$

$$1 + \underbrace{\left(\frac{1}{2^2} + \frac{1}{3^2}\right)}_{<1/2} + \underbrace{\left(\frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2}\right)}_{<1/4} + \underbrace{\left(\frac{1}{8^2} + \cdots + \frac{1}{15^2}\right)}_{<1/8} + \cdots$$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$\cancel{u_1 + u_2 + u_3}$$

$$\frac{1}{1.4} + \frac{1}{2.5} + \frac{1}{3.6} + \frac{1}{4.7} + \cdots + \frac{1}{n(n+3)} + \text{conv.}$$

$$\begin{cases} \frac{1}{1.1} + \frac{1}{2.2} + \frac{1}{3.3} + \frac{1}{4.4} + \cdots + \frac{1}{n.n} + \cdots \\ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \quad (\text{conv.}) \end{cases}$$

---

Compar. Test {

Ratio test $\longrightarrow u_1 + u_2 + \cdots + u_n + u_{n+1} + \cdots$

$$\lim_{n \to \infty} \left( \frac{u_{n+1}}{u_n} \right) = r$$

$$\frac{1}{\sqrt[4]{2} - 1} + \frac{1}{\sqrt[4]{3} - 1} + \frac{1}{\sqrt[4]{4} - 1} + \cdots \quad \text{ser. div.}$$

$$\frac{1}{\sqrt[4]{2}} + \frac{1}{\sqrt[4]{3}} + \frac{1}{\sqrt[4]{4}} + \cdots \quad \text{ser. div.}$$

{ Cahen's Test
Raabe's Test

$$\frac{2}{9}+\frac{2}{12}+\frac{2}{15}+\frac{2}{18}+\cdots+\frac{2}{3n+6}+\cdots \qquad \text{(div.)}$$

$$\frac{2}{3}+\frac{2}{6}+\frac{2}{9}+\frac{2}{12}+\cdots+\frac{2}{3n}+\cdots$$

$$\frac{2}{3}\Big(\underbrace{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}+\cdots}_{\text{div.}}\Big)$$

$$\lim_{n\to\infty}\left(\frac{\cancel{2}}{3n+6}\,\frac{3n}{\cancel{2}}\right)=\cancel{\lim_{n\to\infty}\Big(\frac{2}{\;}}$$

$$=\lim_{n\to\infty}\left(\frac{3}{3+6/n}\right)=1$$

---

4/13

p. 435) 1a, b, c, d, e, f, h, j, k, m, n, o

436) 2a, 2b, 2c

438) 1, 2, 3, 4, 5, 7

439) 9, 11

$u_1 - u_2 + u_3 - u_4 + \text{———}$ $\left(\lim_{n\to\infty} u_n = 0\right)$

$|u_2| < |u_1|$

$|u_3| < |u_2|$

$|u_4| < |u_3|$

$1 + x + x^2 + x^3 + \text{———} + x^n + x^{n+1} + \text{———}$

$$\lim_{n\to\infty}\left(\frac{x^{n+1}}{x^n}\right) = \lim(x) = x$$

$|x| < 1$ ser. conv.

$|x| > 1$ " div.

$|x| = 1$, $x = 1$ (div.) or $x = -1$ (div.)

$1 - 1 + 1 - 1 + 1 - 1$

$a + ar + ar^2 + ar^3 + \text{———}$ $\frac{a}{1-r}$

large power $r$ $|r| < 1$

$\left(\frac{1}{2}\right)$

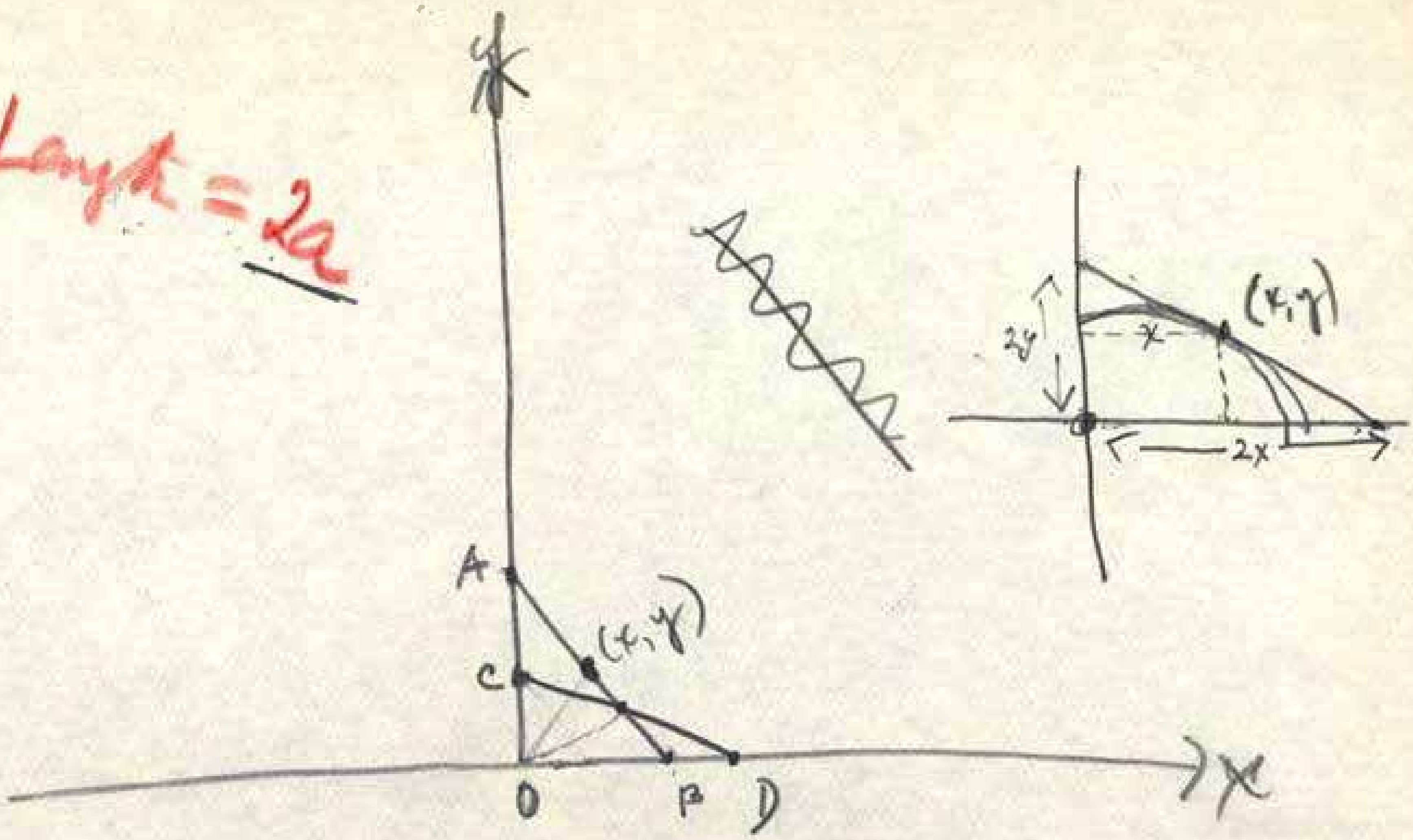
$2 \cdot 3 \cdot 4 \cdot \text{———} n$

$2 \cdot 2 \cdot 2 \cdot \text{———}$

$$\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)\left(\frac{3}{7}\right)\left(\frac{4}{9}\right) \cdots \frac{n}{(2n+1)}$$

$(2 + 1/n)$

Length = $\underline{2a}$




$AB = CD = 2a$

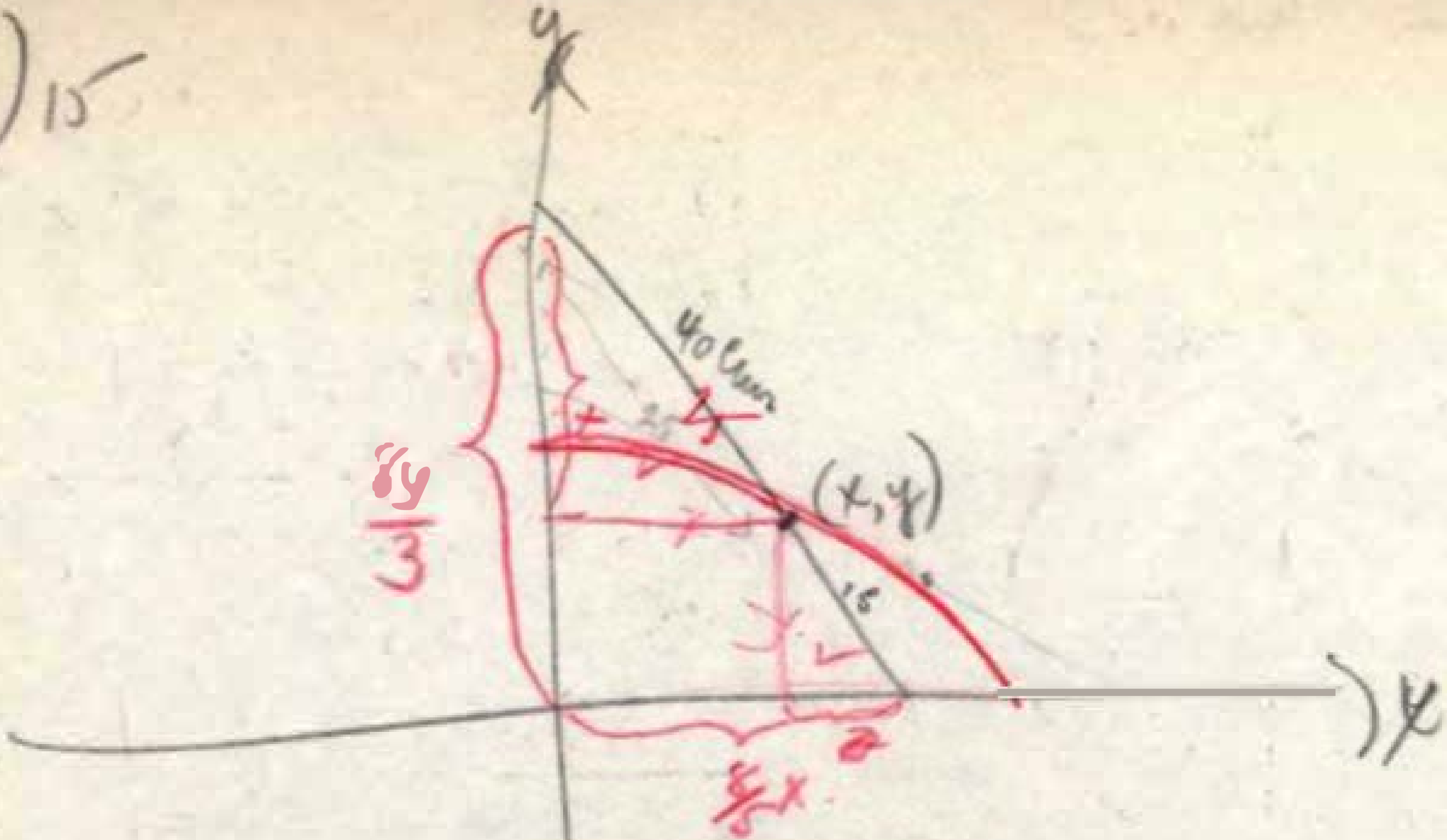
$\cancel{x^2 + y^2 = 2a}$

$(2x)^2 + (2y)^2 = (2a)^2$

$4x^2 + 4y^2 = 4a^2$

$x^2 + y^2 = a^2$

306) 15




$\frac{z}{15} = \frac{x}{25}$

$z = \frac{15x}{25} = \frac{3x}{5}$

$\frac{t}{y} = \frac{25}{15}$

$t = \frac{25y}{15} = \frac{5y}{3}$

$40^2 = \frac{64y^2}{9} + \frac{64}{25}x^2$

$\cancel{40} \cdot \cancel{40} = \cancel{8}\frac{y^2}{9} \cdot \cancel{8} + \cancel{\frac{8}{25}}x^2 \cdot \cancel{8}$

$25 = y^2/9 + x^2/25$

$1 = \frac{y^2}{225} + \frac{x^2}{625}$

416) 1 $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\dots+\frac{n}{2^n}+\frac{n+1}{2^{n+1}}+\dots$

~~This is geometric series, $r=\frac{1}{2}<1$, $\therefore$ convergent~~

ask about $\lim_{n\to\infty} u_n = \cancel{\lim_{n\to\infty}\left(\frac{n}{2^n}\right) = \frac{1}{\infty}} =$ $\lim_{n\to\infty}\left(\frac{\cancel{\frac{n+1}{2^{n+1}}}^{\,1+1/n}}{2}\cdot\frac{\cancel{2^n}}{\cancel{n}}\right) = \frac{1}{2}$

~~ask~~

---

416) 2 $1\left(\frac{2}{3}\right)+2\left(\frac{2}{3}\right)^2+3\left(\frac{2}{3}\right)^3+4\left(\frac{2}{3}\right)^4+\dots + n(2/3)^n + (n+1)\left(\frac{2}{3}\right)^{n+1}+\dots$

~~This is geometric series, $r=\frac{2}{3}<1$, $\therefore$ convergent~~

$\lim_{n\to\infty}\left[\frac{\cancel{(n+1)(2/3)^{n+1}}}{\cancel{n(2/3)^n}}\right] = (1+1/n)\cdot 2/3 = 2/3$ $\therefore$ abs. conv.

---

416) 3 $1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\dots+\frac{1}{\lfloor n}+\frac{1}{\lfloor n+1}+\dots$

~~2! 3! 4!~~

Comparison with known convergent geometric series,

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$, $\frac{1}{2^{n+1}}$ shows series

to be tested to be smaller term for term, $\therefore$ series is convergent

$\lim_{n\to\infty}\left(\frac{1}{\cancel{(n+1)!}}\cdot\frac{n!}{1}\right)$

---

416) 4 $\frac{\lfloor 2}{10}+\frac{\lfloor 3}{10^2}+\frac{\lfloor 4}{10^3}+\dots\frac{\lfloor n+1}{10^n}+\frac{\lfloor n+2}{10^{n+1}}+\dots$ $\frac{n+1}{n+2}=0$

~~This is geometric series, $r=\frac{1}{10}<1$, $\therefore$ convergent~~ $\lim\left[\frac{\cancel{(n+2)!}}{\cancel{10^{n+1}}}\cdot\frac{\cancel{10^n}}{\cancel{(n+1)!}}\right]$ $10$

abs. div.

---

417) 7 $\frac{1}{3}+\frac{1}{3}\cdot\frac{2}{5}+\frac{1\cdot 2\cdot 3}{3\cdot 5\cdot 7}+\dots\frac{\lfloor n}{3\cdot 5\cdot 7\dots(2n+1)}+\frac{\lfloor n+1}{3\cdot 5\cdot 7\dots(2n+1)(2n+3)}$

$\lim_{n\to\infty} u_n = 0$ (obvious, although how derived from ↑ ? ask

$u_n = \frac{\lfloor n}{3\cdot 5\cdot 7\cdots(2n+1)}$ Then $\frac{u_{n+1}}{u_n} =$ $\lim_{n\to\infty} =$

$u_{n+1} = \frac{\lfloor n+1}{3\cdot 5\cdot 7\dots(2n+3)}$ ask Schaum 155

$$- - - - + \frac{L^n}{3.5.7 - - - (2n+1)} + \frac{L^{n+1}}{3.5.7 - - (2n+1)(2n+3)} +$$

$$\rho = \lim_{n\to\infty}\left(\frac{\cancel{(n+1)}}{\cancel{(2n+1)}(2n+3)} \cdot \frac{\cancel{2n+1}}{\cancel{Ln}}\right)$$

$$= \frac{1}{2} < 1 \quad \therefore \text{converges}$$

$$\left(1+\frac{2}{n}\right)^n$$

$$\lim_{n\to\infty}\left\{\frac{\cancel{(n+2)^{n+1}}}{\cancel{[(n+1)^2]^{n+1}}} \cdot \frac{\cancel{(n^2)^{n}}}{\cancel{(n+1)^{n}}}\right\}$$

$$\cancel{(n+2)^n} \cdot (n+2)$$

$$\cancel{(n+1)^{2n+2}}$$

$$\cancel{(n+1)^{2n}} \cdot (n+1)^2$$

let $\frac{2}{n} = \frac{1}{t}$

$$\left(1+\frac{1}{t}\right)^{2t}$$

$$e = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t$$

$$= \lim\left[\frac{\left(\frac{1+2/n}{1+1/n}\right)^n \cdot \frac{1+2/n}{\cancel{(n+1)}}}{\cancel{\left(1+1/n\right)^{2n}} \quad \frac{\cancel{(n+1)}}{1+1/n}(n+1)}\right]$$

$$\left(\frac{n+2}{n+1}\right)^n$$

$$\left(\frac{1+2/n}{1+1/n}\right)$$

$$\left(\frac{n+1}{n}\right)^{2n}$$

$$\left[\left(1+\frac{1}{n}\right)^n\right]^2$$

$$\lim\left[\frac{\cancel{e^2}\, e}{\cancel{e^2}\, e(n+1)}\right] = 0$$

M + B

435) 1a $\quad 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \cdots \pm \frac{1}{\sqrt{n}} \mp \frac{1}{\sqrt{n+1}} \pm \text{———}$

Alternating series in which each term is numerically less than the preceding one + the limit of the $n^{th}$ term is 0 as $n \to \infty$

(Theorem 6)

$$\lim_{n\to\infty} u_n = 0$$

$\therefore$ Converges

---

1b) $\quad \frac{1^3}{1!} + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \cdots + \frac{n^3}{n!} + \frac{(n+1)^3}{(n+1)!} + \cdots \quad \left(1+\frac{1}{n}\right)^3 \quad \cancel{\left(\frac{n+1}{n}\right)^3}$

$$\lim_{n\to\infty}\left(\frac{n^3}{n!}\right) = 0 \qquad \rho = \lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left[\frac{(n+1)^3}{(n+1)!}\cdot\frac{n!}{n^3}\right] = 0$$

$n+1$

(quot. ÷ by smaller #s)

$$\frac{a^3}{b^3} = \left(\frac{a}{b}\right)^3 \qquad \rho = < 1 \quad \therefore \text{Converges}$$

$$\frac{(n+1)^3}{n^3} = \left(\frac{n+1}{n}\right)^3$$

---

c) $\quad \frac{3}{4} + \frac{3\cdot 6}{4\cdot 6} + \frac{3\cdot 6\cdot 9}{4\cdot 6\cdot 8} + \frac{3\cdot 6\cdot 9\cdot 12}{4\cdot 6\cdot 8\cdot 10} + \cdots + \left(\frac{(3)(6)(9)(12)\,\cancel{(3n)}}{(4)(6)(8)(10)\,\cancel{(2n+2)}}\right) + \cdots \quad \frac{3}{2+2/n}$

$$\lim_{n\to\infty} u_n \neq 0, \text{ therefore \textit{series} diverges}$$

---

d) $\quad \left(\frac{2}{1}\right)^1 + \left(\frac{3}{4}\right)^2 + \left(\frac{4}{9}\right)^3 + \left(\frac{5}{16}\right)^4 + \cdots \left(\frac{n+1}{n^2}\right)^n + \left[\frac{(n+2)}{(n+1)^2}\right]^{n+1} + \cdots$

$$\lim_{n\to\infty} u_n = 0$$

$$\rho = \lim_{n\to\infty}\left[\frac{n+2}{(n+1)^2}\right]^{n+1} \div \left(\frac{n+1}{n^2}\right)^n$$

$= < 1$, $\therefore$ convergent series

435) e

$$\left(\frac{3}{1} - \frac{3^2}{2}\right) + \left(\frac{3^3}{3} - \frac{3^4}{4}\right) + \cdots + \left(\frac{3^n}{n} - \frac{3^{(n+1)}}{n+1}\right) + \cdots$$

Does not conform to Theorem 6
Grouping by 2 leads $S_n$ to greater minus value with no approach to limit $\Big\}$ ∴ diverges

---

f) $$\frac{1!}{3} - \frac{2!}{3^2} + \frac{3!}{3^3} - \frac{4!}{3^4} + \cdots + \frac{n!}{3^n} - \frac{(n+1)!}{3^{(n+1)}} + \cdots$$

Grouping $[u_1 - u_2] + [u_3 - u_4] \ldots$ shows that $S_n$ does not approach a limit, because $u_1 > u_2$ + $u_3 > u_4$, etc. Also this is oscillating series which does not conform to Theorem 6

∴ diverges

---

h) $$\frac{1}{1\cdot 3^1} + \frac{1}{2\cdot 3^2} + \frac{1}{3\cdot 3^3} + \frac{1}{4\cdot 3^4} + \cdots + \frac{1}{n(3^n)} + \cdots$$

$$\lim_{n\to\infty} u_n = 0$$

~~Geometric series, $r = \frac{1}{3}$~~ $< 1$

Comparison with known convergent geometric series, as

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n}$ shows given series to be smaller (term for term), ∴ convergent

M + B

435) 1 j

$$\frac{2}{\pi^2}+\frac{3}{\pi^3}+\frac{4}{\pi^4}+\frac{5}{\pi^5}+\cdots\frac{n+1}{\pi^{(n+1)}}+\frac{n+2}{\pi^{n+2}}+\cdots$$

~~Geometric series, $r=\frac{1}{\pi}<1$, $\therefore$ convergent~~

$$\lim_{n\to\infty}\left[\frac{\cancel{n+2}\ 1+2/n}{\cancel{\pi^{n+2}}\ \pi}\div\frac{\cancel{n+1}\ 1+1/n}{\cancel{\pi^{n+1}}}\right]=\frac{1}{\pi}\quad \text{ser. conv.}\quad \lim\left[\frac{}{}\right]$$

435) 1 k $\quad \frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\frac{1}{7\cdot9}+\cdots+\frac{1}{(2n-1)(2n+1)}+\frac{1}{(2n+1)(2n+3)}+\cdots$

$$\lim_{n\to\infty}\mu_n = 0$$

$$\mu_n=\frac{1}{(2n-1)(n+1)}$$

$$\mu_{n+1}=\frac{1}{(2n-1)(2n+3)}$$

~~$\frac{1}{1\cdot3}+\frac{1}{3\cdot3}+\frac{1}{}$~~

$$\rho=\lim_{n\to\infty}\left(\frac{1}{(2n+1)(2n+3)}\div\frac{1}{(2n-1)(2n+1)}\right)=1$$

Test fails

Comparison with Harmonic Series

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots$$

shows given series to be term for term smaller than harmonic series, $\therefore$ it is convergent

(conv.) $\quad \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots+\frac{1}{n^2}+\cdots$

$$\frac{1}{1^2}+\cancel{\frac{1}{2^2}}+\frac{1}{3^2}+\cancel{\frac{1}{4^2}}+\frac{1}{5^2}+\cdots$$

435) Prob. $\frac{1}{1^2+2} + \frac{2}{2^2+2} + \frac{3}{3^2+2} + \dots \frac{n}{n^2+2} + \dots$

$\lim_{n \to \infty} u_n = 0$ $\qquad u_n = \frac{n}{n^2+2}$

$u_{n+1} = \frac{n+1}{(n+1)^2+2}$

$\rho = \lim_{n\to\infty}\left(\frac{n+1}{(n+1)^2+2} \div \frac{n}{n^2+2}\right) = \lim_{n\to\infty}\left(\frac{n+1}{n^2+2n+4} \cdot \frac{n^2+2}{n}\right)$

$\rho = \frac{1+\frac{1}{n}}{n+2+\frac{4}{n}} \cdot \frac{n+\frac{2}{n}}{1} = \frac{\frac{1}{n}+\frac{1}{n^2}}{1+\frac{2}{n}+\frac{4}{n^2}} \cdot \frac{1+\frac{2}{n^2}}{\frac{1}{n}} = 1$ (Test fails)

Comparison Test with

~~$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$, etc., shows term for term, given~~
~~(Harmonic series - divergent) series to be smaller,~~

~~$\therefore$ converges~~

Also p test, $p = 2 > 1$, $\therefore$ convergent, Also series is term by term less than convergent p series
$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$

$\frac{1}{1^2} + \cancel{\frac{1}{2^2}} + \cancel{\frac{1}{3^2}} + \dots + \frac{1}{n^2}$

$$\frac{1}{1+2} + \frac{1}{2+1} + \frac{1}{3+2/3} + \dots + \overset{u_n}{\frac{1}{n+2/n}}$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \underset{V_n}{\frac{1}{n}} + \dots \longrightarrow \text{div.}$$

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