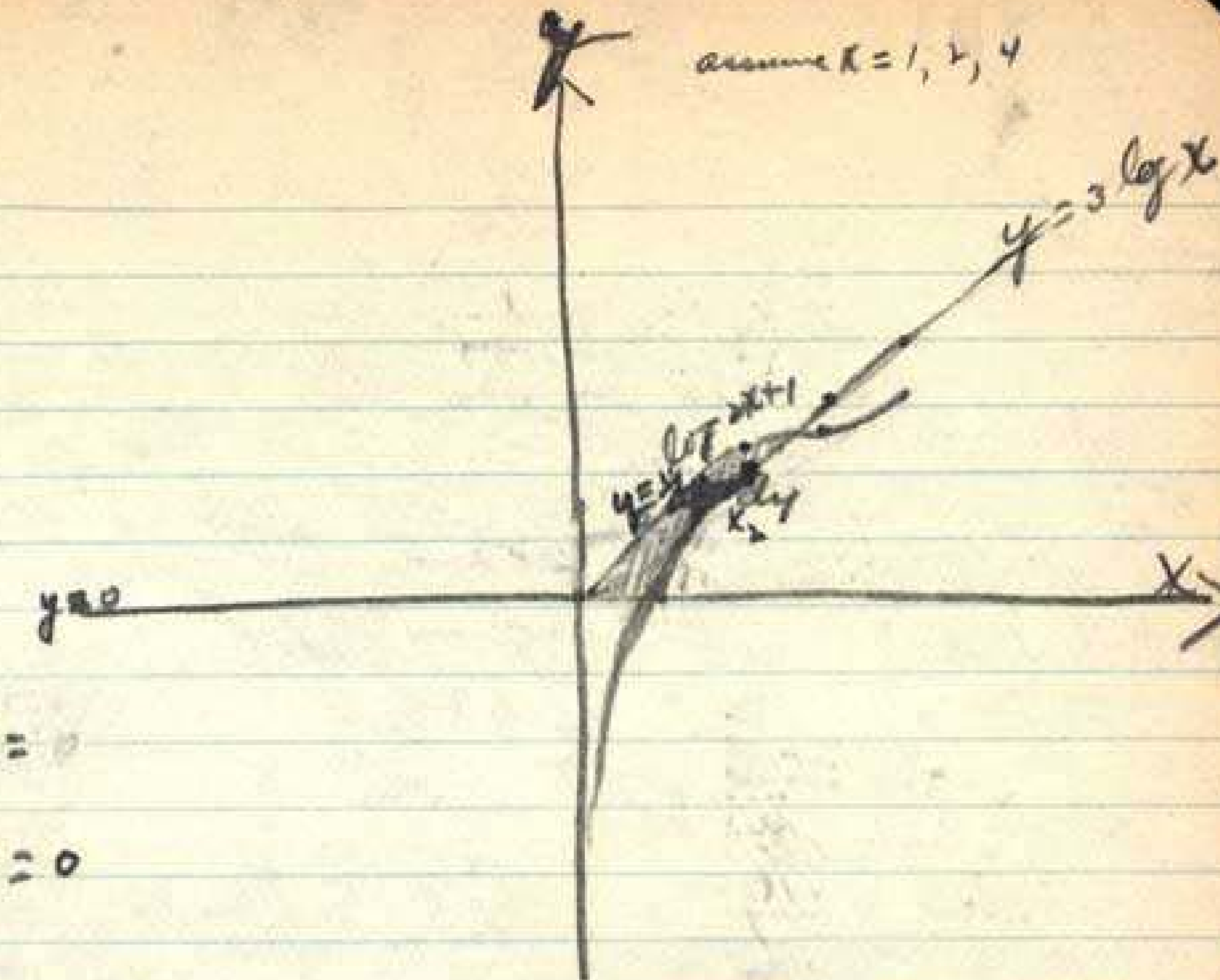
297) 2h

I $\quad y = \log 2x + 1 \quad (x =$

II $\quad y = 3 \log x \quad (x = e^{3y})$

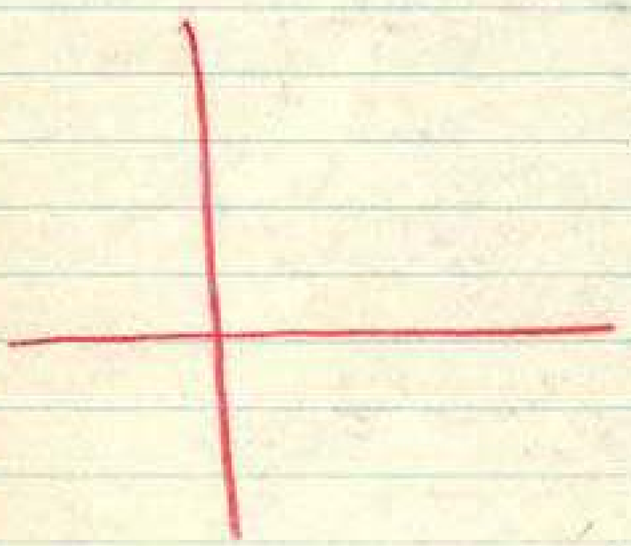
$\quad y = 0$

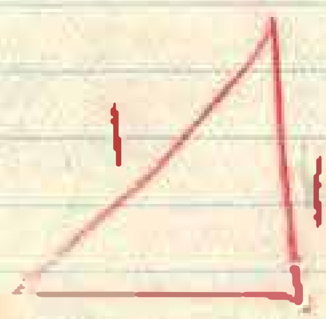


I If $x=0,\ y=1$	II If $x=0,\ y=$
$x=1,\ y=1.69$	$x=1,\ y=0$
$x=2,\ y=2.38$	$x=2,\ y=2.07$
$x=3,\ y=2.79$	$x=3,\ y=3.27$

Element of area $= (x_2 - x_1)\, dy$

Total area $= \int (x_2 - x_1)\, dx$





arcsin $\frac{1}{3} = A$

$\sin A = 1/3$

arcsin $0 = \cancel{A}\ 0$

~~sin A = 0~~

arcsin $\frac{\sqrt{3}}{2} = 60°$

arcsin $\frac{\sqrt{3}}{2} = \pi/3$

see page 76

303)1a

$$y^2 = 4x$$

$$2y = 4\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y}{2}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{y^2}{4}$$

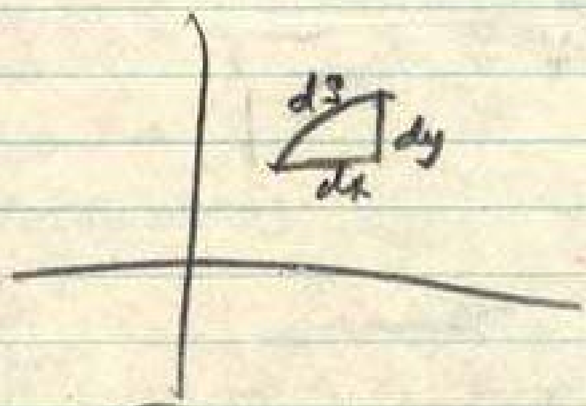
$$\int\sqrt{1+(dx/dy)^2}\,dy$$

$$ds = \sqrt{1+\frac{y^2}{4}}\,dy$$

$$ds = \sqrt{(dx)^2+(dy)^2} = \sqrt{1+(dy/dx)^2}\,dx$$

$$= \sqrt{(dx/dy)^2+1}\,dy$$

$$= \sqrt{\frac{4+y^2}{4}}\,dy$$



$$= \frac{1}{2}\sqrt{4+y^2}\,dy$$

$$S = \int_0^{-2a} \frac{1}{2}\sqrt{4+y^2}\,dy \quad (a=2,\ u=y)$$

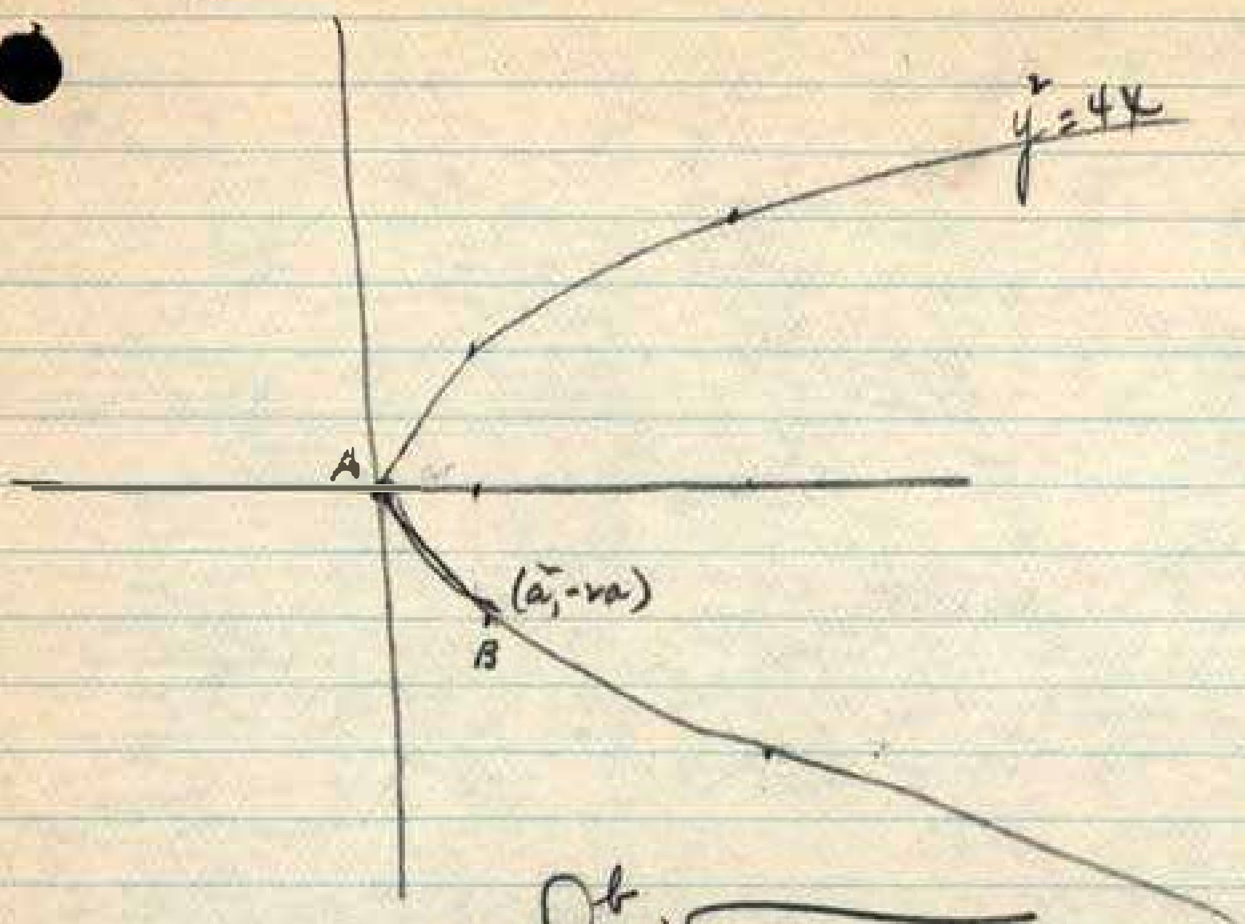
$$= \frac{1}{2}\left[\frac{y}{2}\sqrt{y^2+4} + 2\ln\left(y+\sqrt{y^2+4}\right)\right]_0^{-2a}$$

$$= -a\sqrt{4a^2+4} \pm 2\ln\left(-2a+\sqrt{4a^2+4}\right)$$

$$= \frac{1}{2}\left[\sqrt{4a^4+4a^2} \mp 2\ln\left(-2a+\sqrt{4a^2+4}\right)\right]$$

$$= \sqrt{a^4+a^2} + \ln\left(-a+\sqrt{a^2+1}\right)$$

303) 1a



To find the length of AB

$$y^2 = 4x$$

$$2y\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{4}{y^2} = \frac{4}{4x} = \frac{1}{x}$$

$$ds = \sqrt{1+\frac{4}{y^2}}\,dx$$

$$= \sqrt{\frac{y^2+4}{y^2}}\,dx$$

$$= \frac{1}{y}\sqrt{y^2+4}\,dx$$

$$= \frac{1}{2\sqrt{x}}\sqrt{4x+4}\,dx$$

$$= \frac{1}{\sqrt{x}}\sqrt{(x+1)}\,dx$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$S = \int_0^{a^2} \sqrt{1+\frac{1}{x}}\,dx$$

$$S = \int_0^{a^2} \sqrt{\frac{(x+1)x}{x^2}}\,dx$$

$$S = \int_0^{a^2} \frac{\sqrt{x^2+x}}{x}\,dx$$

For. 170
Pg. 228

$a=0$
$b=1$
$c=1$

$$\cancel{= \sqrt{x^2+x} + \frac{1}{2}\int dx}$$

$$= \sqrt{x^2+x} + \frac{1}{2}\log\left(\sqrt{x^2+x} + x + \frac{1}{2}\right)$$

303) 12

$$x^2 = 6y - y^2$$

$$2x = 6\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$= \frac{dy}{dx}(6 - 2y)$$

$$\frac{dy}{dx} = \frac{2x}{6-2y} = \frac{x}{3-y}$$

$$x^2 = 6y - y^2$$

$$2x\frac{dx}{dy} = 6 - 2y$$

303) 1 c

$$x^2 = 6y - y^2$$

$$x = \pm\sqrt{6y - y^2}$$

$$\frac{dx}{dy} = \frac{6-2y}{2(6y-y^2)^{\frac{1}{2}}} = \frac{3-y}{\sqrt{6y-y^2}}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{9-6y+y^2}{6y-y^2}$$

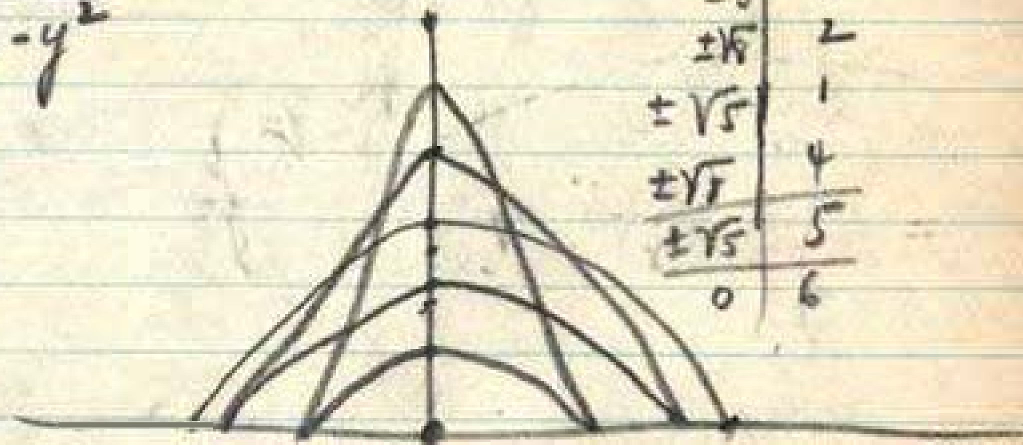
$$ds = \sqrt{1 + \frac{9-6y+y^2}{6y-y^2}}\ dy$$

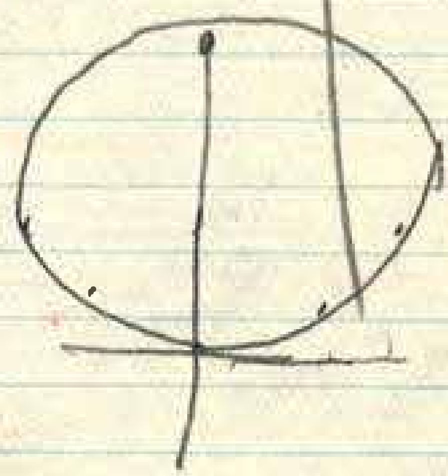
$$S = \int \qquad dy$$

Let $u = 6y - y^2$

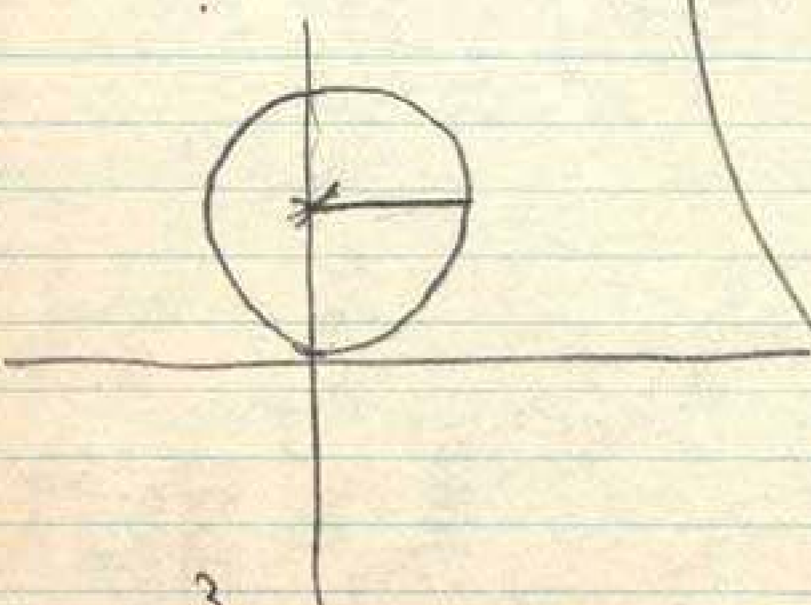
$$\frac{du}{dy} = (6-2y)$$

x	y
0	0
± 3	3
$\pm\sqrt{8}$	2
$\pm\sqrt{5}$	1
$\pm\sqrt{8}$	4
$\pm\sqrt{5}$	5
0	6





	y
0	0
	1
	2
	3



$$x^2 + y^2 - 6y + 9 = 0 + 9$$

$$x^2 + (y-3)^2 = 9$$

$(0, 3)$

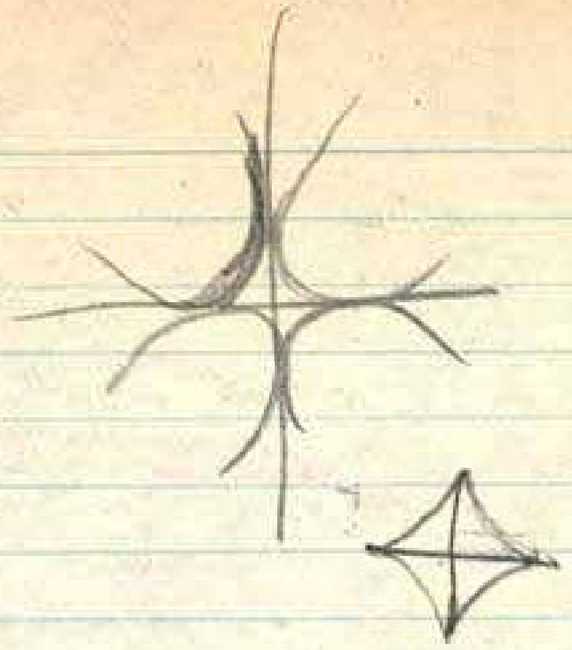
$$2x = 6\frac{dy}{dx} - 2y\frac{dy}{dx} = (6-2y)\,dy/dx$$

$$\frac{dy}{dx} = \frac{2x}{6-2y} = \frac{x}{3-y}$$

$$S = \int_0^3 \sqrt{1 + \frac{x^2}{9-6y+y^2}}\ dx$$

$$= \int_0^3 \sqrt{\frac{9\cancel{-6y}\cancel{+y^2}\cancel{+6y}\cancel{-y^2}}{9-6y+y^2}}\ dx = \int_0^3 \sqrt{\frac{9}{9-x^2}}\ dx$$

303) 1q $\quad x^{2/3}+y^{2/3}=a^{2/3}$



Differentiating, $\frac{2}{3}x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx}=0$

$$\frac{2}{3\sqrt[3]{x}}+\frac{2}{3\sqrt[3]{y}}\frac{dy}{dx}=0$$

$$S=4\int_0^a a^{1/3}\cdot x^{-1/3}\,dx$$

$$\frac{dy}{dx}=-\frac{2}{3\sqrt[3]{x}}\cdot\frac{3\sqrt[3]{y}}{2}=-\sqrt[3]{\frac{y}{x}}$$

$$\left(\frac{dy}{dx}\right)^2=\left(\frac{y}{x}\right)^{\frac{2}{3}}$$

$$ds=\sqrt{1+\left(\frac{y}{x}\right)^{\frac{2}{3}}}\,dx=\sqrt{1+\left(\frac{a^{\frac{2}{3}}-x^{\frac{2}{3}}}{x}\right)^{\frac{2}{3}}}\,dx$$

$$\sqrt{1+\frac{a^{2/3}-x^{2/3}}{x^{2/3}}}\,dx$$

$$=\sqrt{\frac{a^{2/3}}{x^{2/3}}}\,dx=\frac{a^{1/3}}{x^{1/3}}\,dx$$

$$S=\int_0^a\sqrt{1+\left(\frac{a^{\frac{2}{3}}-x^{\frac{2}{3}}}{x}\right)^{\frac{2}{3}}}\,dx$$

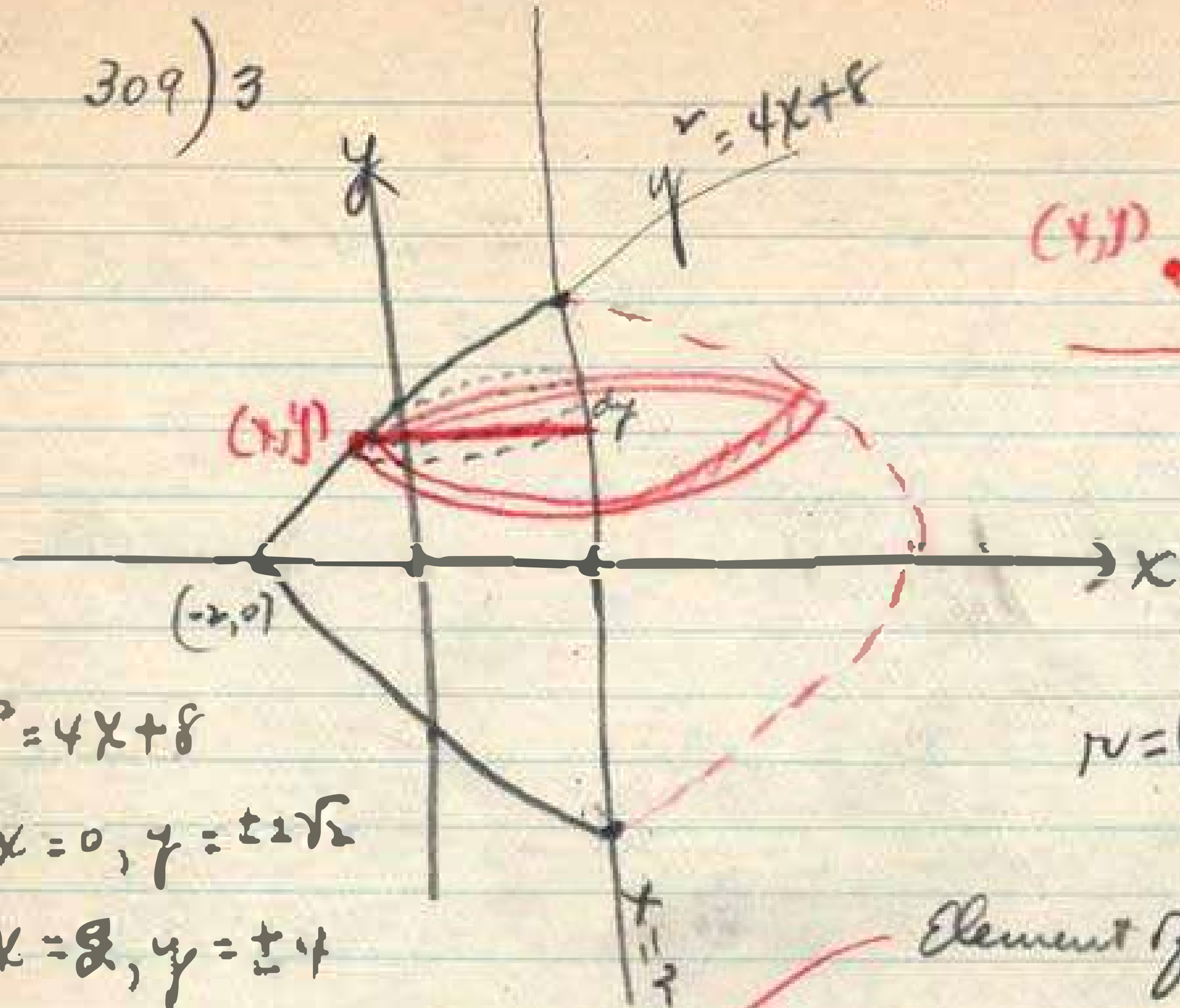
Let $u=\left(\frac{a^{\frac{2}{3}}-x^{\frac{2}{3}}}{x}\right)^{\frac{1}{3}}$

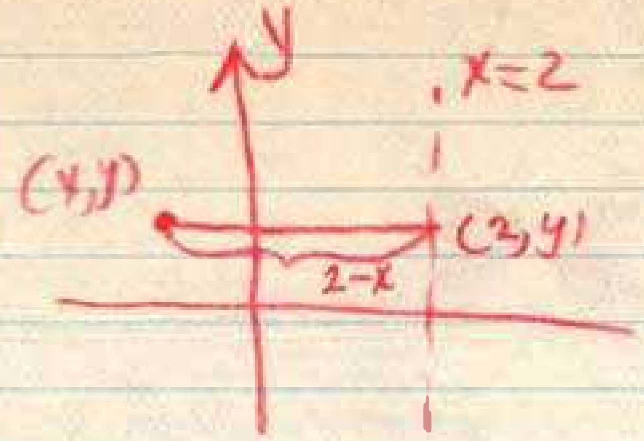
then $\int\sqrt{a^2+u^2}=\frac{u}{2}\sqrt{u^2+a^2}+\frac{a^2}{2}\ln\left(u+\sqrt{u^2+a^2}\right)+C$

$$=\frac{\left(\frac{a^{\frac{2}{3}}-a^{2/3}}{a}\right)^{\frac{1}{3}}\sqrt{u^2+a^2}}{2}+\frac{1}{2}\ln\left(0+\sqrt{\left(\frac{a^{2/3}-a^{2/3}}{x}\right)^{2/3}+1}\right)$$

$$=0$$

309) 3





$y^2 = 4x+8$

If $x=0,\ y=\pm 2\sqrt{2}$

$x=8,\ y=\pm 4$

If $y=0,\ x=-2$

$x = \dfrac{y^2-8}{4}$

$r = \cancel{(4-x)}\ 2-x$

Element of Volume $= \pi r^2 dy$

Total Volume $= \cancel{\int_0^4 \pi(4-x)^2 dy}$

$= \cancel{\pi\int_0^4 \left(4-\frac{(y^2-8)}{4}\right)^2 dy}$

$= \cancel{\pi\int_0^4 \left(\frac{16-y^2+8}{4}\right)^2 dy}$

$\pi(4-4x+x^2)dy$

$V = 2\pi\int_0^4 (4-4x+x^2)\,dy$

$= 2\pi\int_0^4 \left[4-(y^2-8)+\frac{y^4-16y^2+64}{16}\right]dy$

$2\pi\int_0^4 \left(4-y^2+8+\frac{y^4}{16} \cdots y^2+4\right)$

$= 2\pi\int_0^4 \left(16-2y^2+\frac{1}{16}y \cdots\right.$

$= \frac{\pi}{16}\int_0^4 (24-y^2)^2 dy$

$= \frac{\pi}{16}\int_0^4 (576-48y^2+y^4)\,dy$

$= \frac{\pi}{16}\left(576y-\frac{48y^3}{3}+\frac{y^5}{5}\right)\Big]_0^4$

$= \frac{\pi}{4}\left(576-\frac{256}{3}+\frac{256}{5}\right)$

$= \frac{\pi}{4}\left(\frac{8128}{15}\right)$

$= \frac{2032}{15}\pi$

$z = (1 + u)^{1/u}$

$\lim_{u \to 0} z$

u	z
$1/10$	$(1.1)^{10} = 2.594$
$1/100$	$(1.01)^{100} = 2.704$
$1/1000$	$(1.001)^{1000}$

$\downarrow$

$e = 2.71828$

$\log 1.1 = 0.04139$

10 " $= 0.4139$

$.00432$

$.43200$

$\log a^m = m.\log a$

$1^\circ = (\pi/180)^{rad}$

$$\left\{ \begin{array}{l} \frac{d}{dx}(\log_a u) = \frac{1}{u} \cdot \frac{du}{dx} \cdot \log_a e \\ \boxed{\frac{d}{dx}(\log_e u) = \frac{1}{u} \cdot \frac{du}{dx}} \end{array} \right.$$

$\log \sqrt{a} = \log a^{1/2}$

$= \frac{1}{2} \log a$

$\log \sqrt{2e}$

$$\left\{ \frac{d}{dx}(\sin u) = \cos u \, \frac{du}{dx} \right.$$

$$\frac{d}{dx}(\sin u^\circ) = \frac{d}{dx}\left[\sin\left(\frac{\pi}{180}u\right)\right] = \cos\left(\frac{\pi}{180}u\right)^\circ \cdot \pi/180 \, \frac{du}{dx}$$

$$= \cos u \cdot \frac{du}{dx} \cdot \frac{\pi}{180}$$

$\log_e 0 = b\ ,\ e^b = 0$

$\log_e 0$ (blank)

$\log_e 0 = -\infty$

$\log_e a = b,\ e^b = a$

$\log_e 1$

$\log_{10} 1/10 = -1$

$\log_{10} 1/100 = -2$

$\log_{10} \cancel{1/100000\ldots} = -20$

$y = \log_e 2x+1$

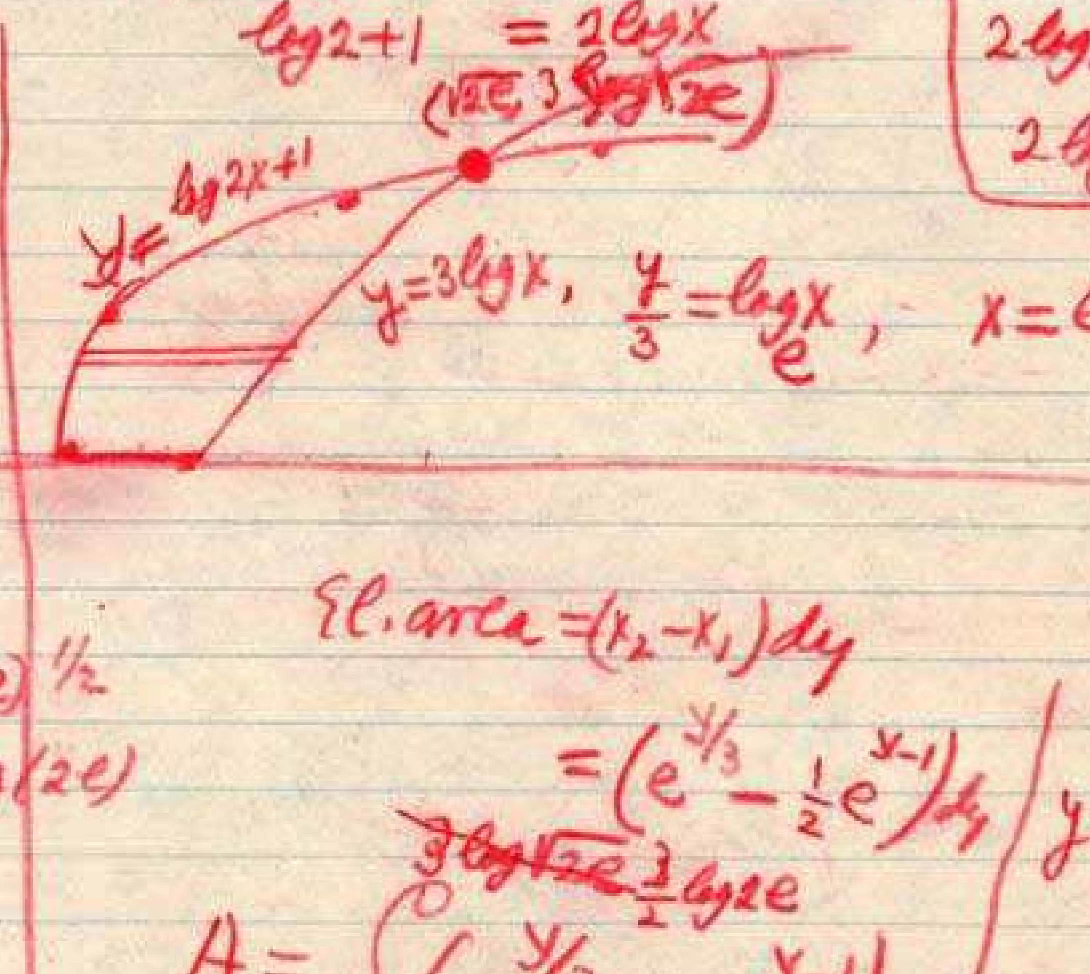
$\log 2x+1 = 3\log x$

$\log 2 + \log x + 1 = 3\log x$

$\log 2 + 1 = 2\log x$

$2\log x = \log 2 + \log e$

$2\log x = \log 2e$



$\log_e 2x = y-1$

$e^{y-1} = 2x$

$y = 3\log x,\ \frac{y}{3} = \log_e x,\ x = e^{y/3}$

$\log x^2 = \log 2e$

$x^2 = 2\cdot e$

$x = \sqrt{2e}$

$\log\sqrt{2e} = \log(2e)^{1/2}$

$= \frac{1}{2}\log(2e)$

El. area $= (x_2 - x_1)\,dy$

$= \left(e^{y/3} - \frac{1}{2}e^{y-1}\right)dy$

297/4

$y = 3\log_e x$

$$A = \int_0^{\cancel{3\log\sqrt{2e}}\ \frac{3}{2}\log 2e} \left(e^{y/3} - \frac{1}{2}e^{y-1}\right)dy$$

x	y
1	0
e	3
e^2	6

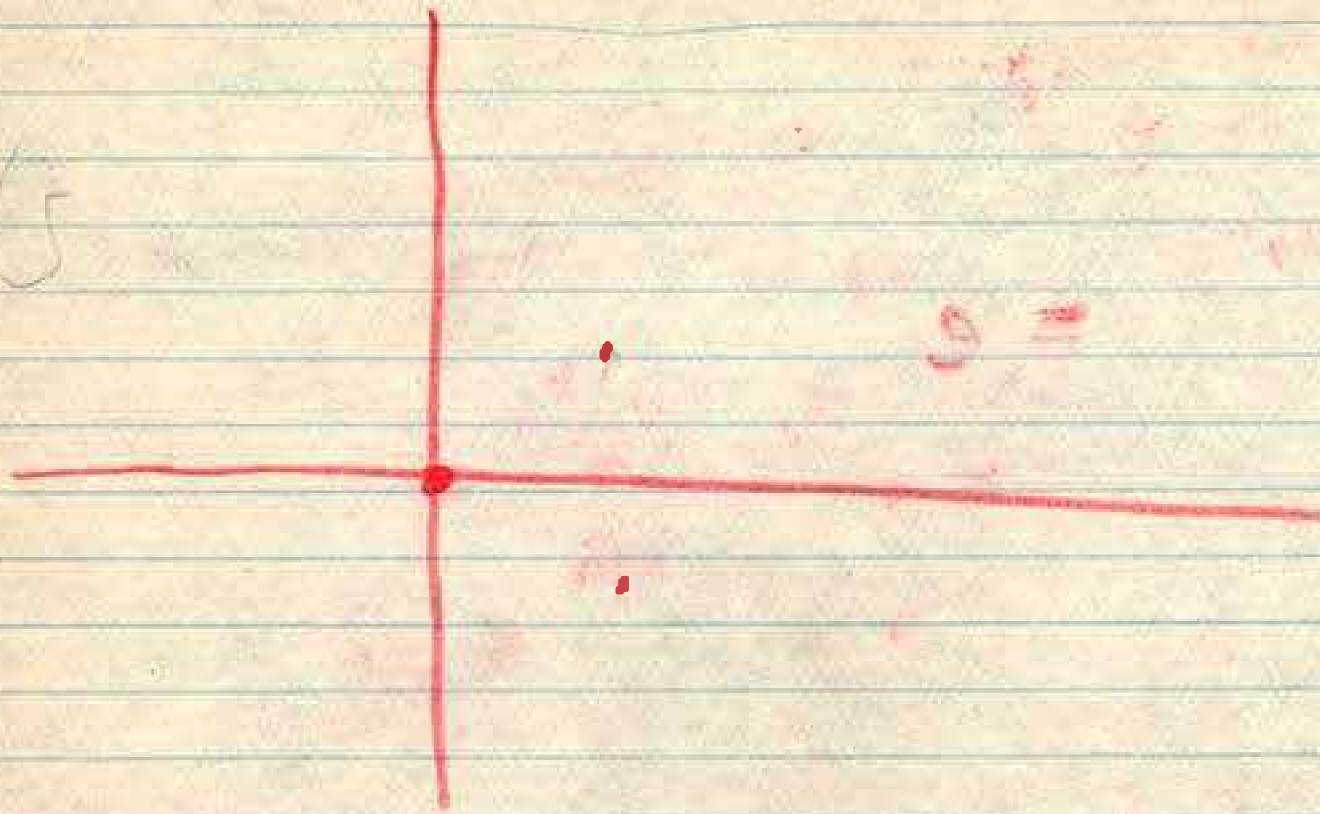
x	y
$1/2$	1
$e/2$	2
$e^2/2$	3
$\frac{1}{2e}$	

$= 3e^{y/3} - \frac{1}{2}e^{y-1}\Big]_0^{\frac{3}{2}\log 2e}$

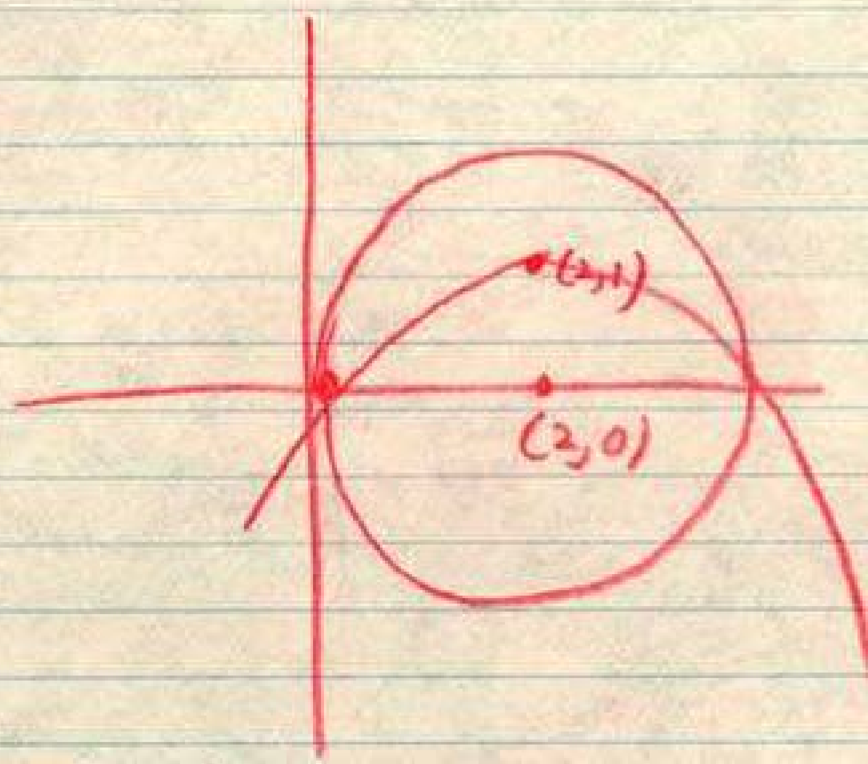
$\cancel{e^{y/3} = \frac{1}{2}e^{y-1}}$

$$\boxed{= \left(3e^{\frac{1}{2}\log 2e} - \frac{1}{2}e^{\frac{3}{2}\log 2e - 1}\right) - \left(3 - \frac{1}{2}e^{-1}\right)}$$

M.B. 297/1i



$y = \pm\sqrt{4x - x^2}$



$$y^2 + x^2 - 4x \overset{+4}{} = 0 \; {}^{+4}$$

$$y^2 + (x-2)^2 = 4$$

$$y = \frac{4x - x^2}{4}$$

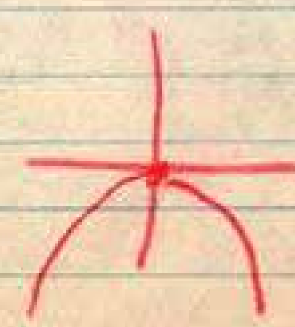
$$x^2 - 4x \overset{+4}{} + 4y = 0 \; {}^{+4}$$

$$(x-2)^2 + 4y = 4$$

$$(x-2)^2 = -4y + 4$$

$$(x-2)^2 = -4(y-1)$$

$x^2 = -4y$



$$e^{\log_e a}$$

$$e^{\frac{1}{2}\log 2e} = e^{\log\sqrt{2e}} = \sqrt{2e}$$

$$e^{\frac{3}{2}\log 2e - 1} = e^{\log (2e)^{3/2} - \log e}$$

$$= e^{\log \frac{2e\sqrt{2e}}{e}}$$

$$= e^{\log 2\sqrt{2e}} = 2\sqrt{2}\,e$$

$$\left.\begin{array}{l} 1)\ \log_e e^{2/3} \\ 2)\ e^{\log \sqrt[3]{e}} \\ 3)\ e^{\log 2e - 2} * \\ 4)\ \log_e (e^2+1)^2 \end{array}\right\}$$

$= 2/3$ ✓

$= \cancel{e^{2/3}}\ e^{1/3}$

$= e^{.69-2} = e^{-.31}$ ✓

$= \log_e (e^4 + 2e^2 + 1)$

or $= 2 \log_e (e^2+1)$

$= 2 \log_e ([2.71828]^2 + 1)$.

$= 2 \log (7.39 + 1)$

$= 2 \log 8.39 = \cancel{1.25}$

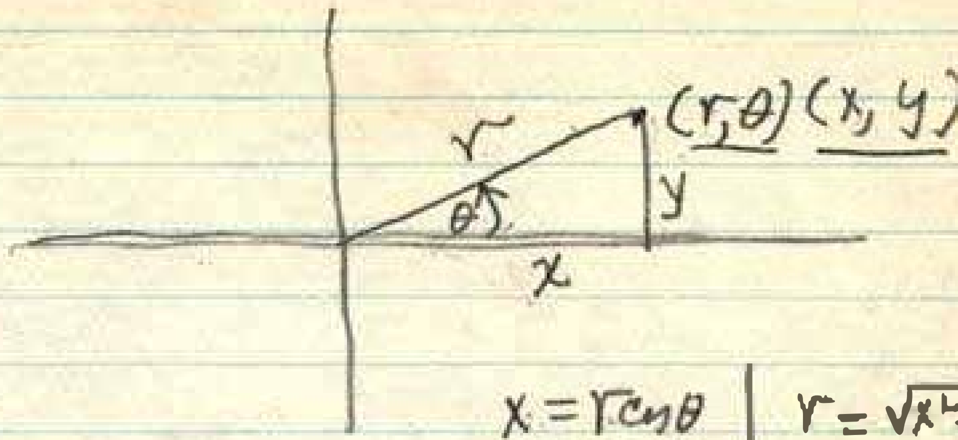
$e^{a+b} = e^a \cdot e^b$

$\log_e \sqrt[3]{e} = \log_e e^{1/3} = 1/3$

$\log_e 2e = \log_e 2 + \log_e e$

$= \log_e 2 + 1$

$* \quad e^{\log_e 2 + 1 - 2} = e^{\log_e 2 - 1} = e^{\log_e 2} \cdot e^{-1} = 2 \cdot e^{-1} = 2/e$

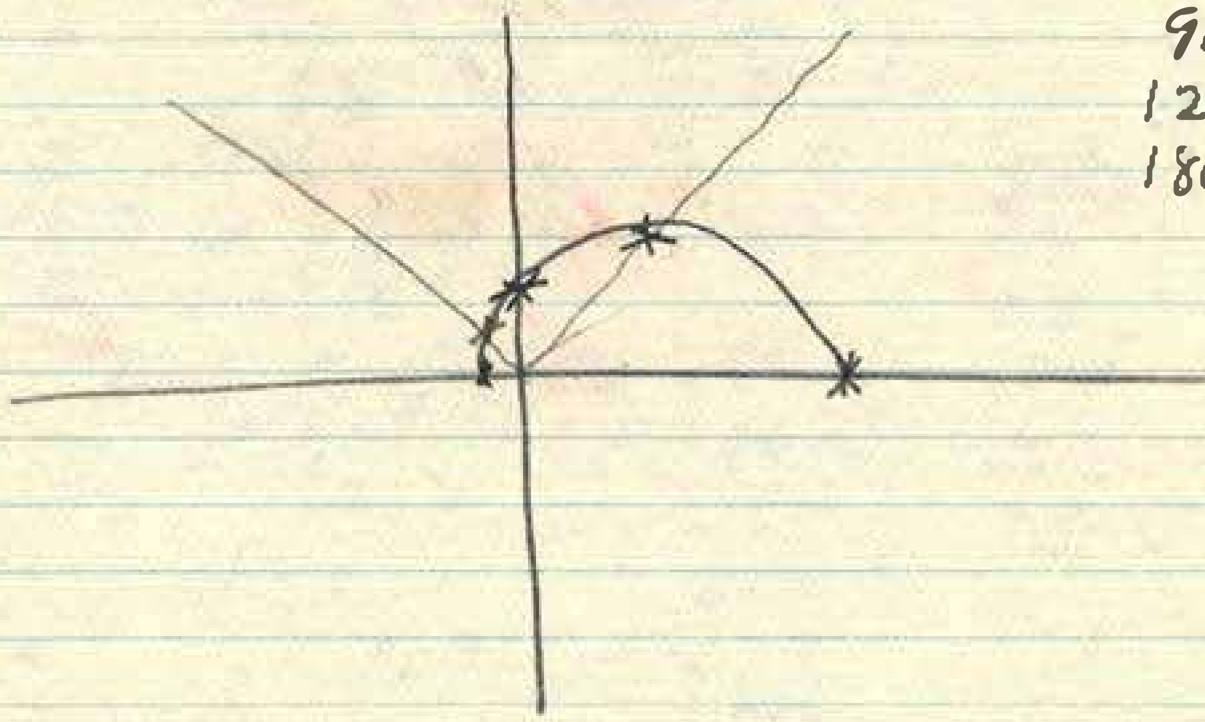


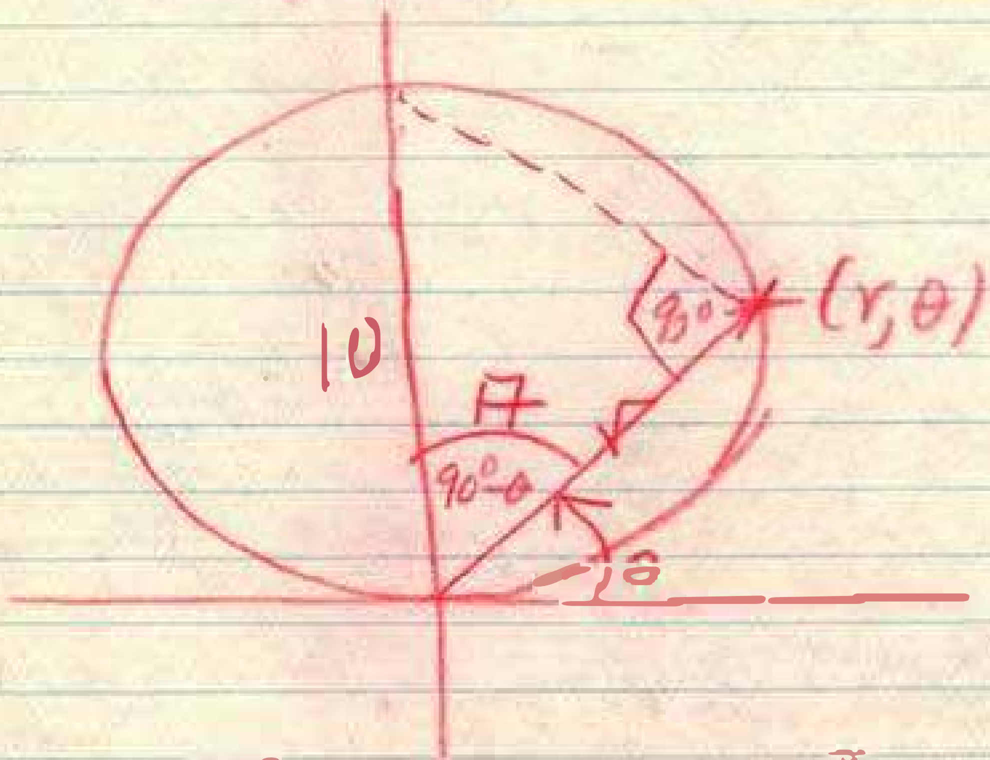
$x = r\cos\theta$ | $r = \sqrt{x^2+y^2}$

$y = r\sin\theta$ | $\theta = \arctan y/x$

$r = 2+\cos\theta$

θ	r
0°	3
60°	$2\frac{1}{2}$
90°	2
120°	$1\frac{1}{2}$
180°	1





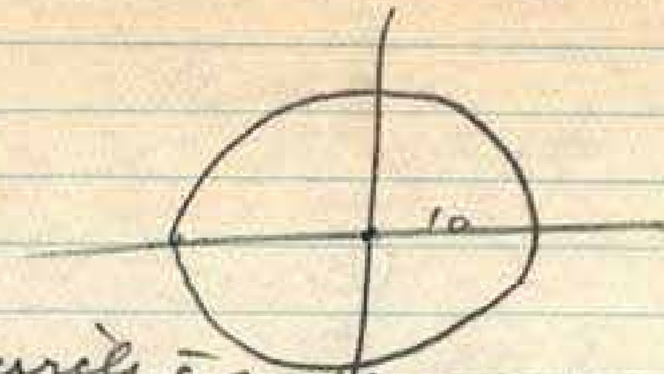
$$r = 10\sin\theta = 10\cos(90^\circ-\theta)$$

$$\frac{r}{10} = \cos(90^\circ-\theta)$$

$$\frac{r}{10} = \cos A$$

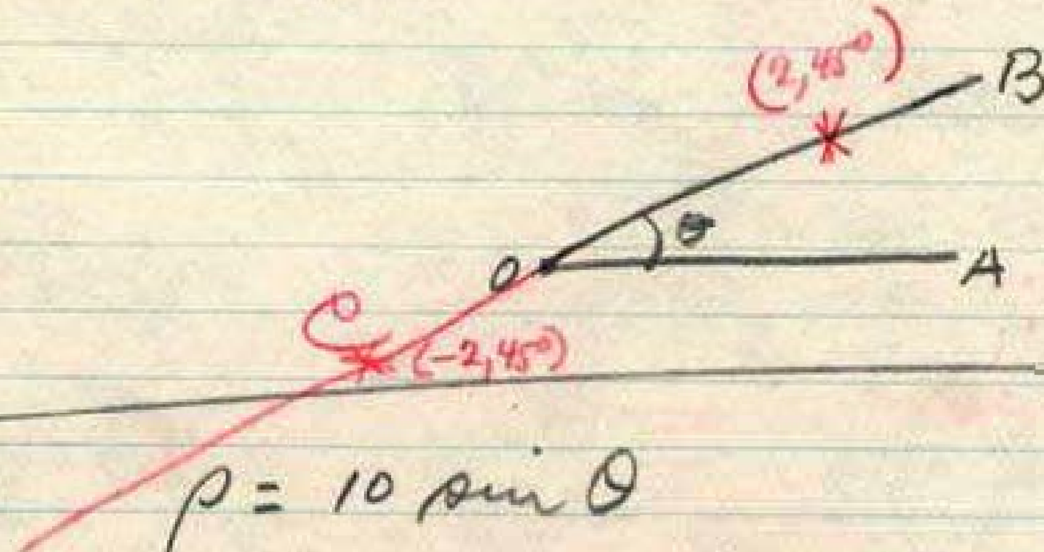
354) 1

$\rho = 10$



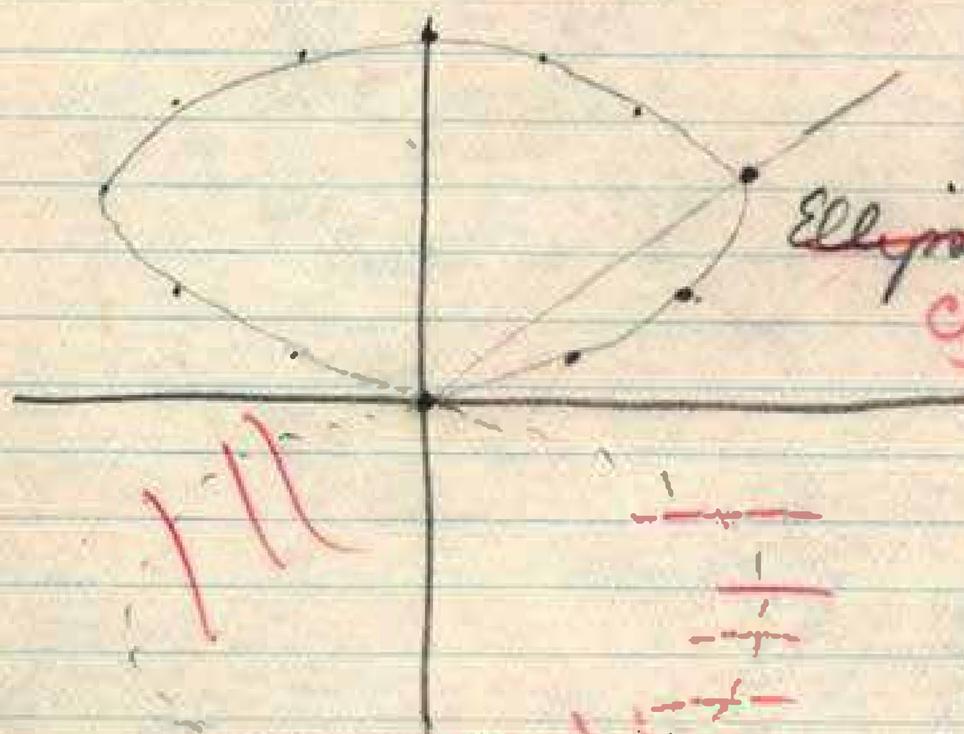
Locus equal circle $\bar{c}$ radius $= 10$ ✓

354) 2 $\theta = 45^\circ$



Locus is $\mathbb{C}$ OB to ∞ (radius vector)

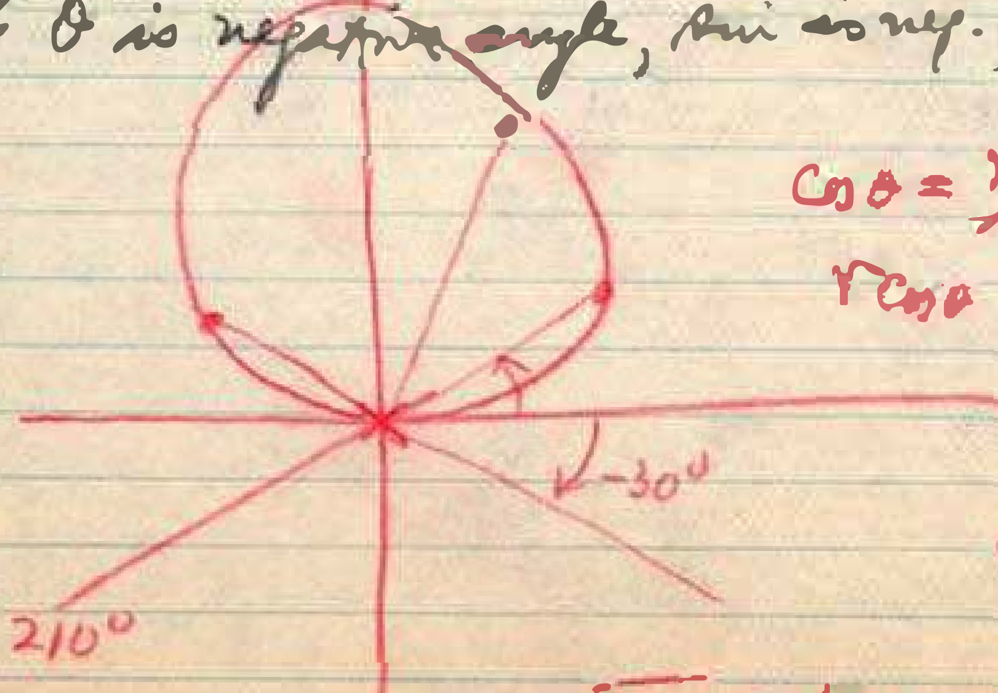
354) 3 $\rho = 10 \sin\theta$



~~Ellipse~~ circle

θ	sin	ρ
0°	.000	0.0
15°	.258	2.58
30°	.5	5.
45°	.707	7.07
60°	$\frac{\sqrt{3}}{2}$ ~~.866~~	~~8.66~~ $5\sqrt{3}$
75°	.966	9.66
90°	1.0	10.00
105°	.966	9.66
120°	.866	8.66
135°	.707	7.07
150°	.5	5.0
165°	.258	2.58
180°	0	0

If θ is negative angle, sin is neg., & we get ellipse on neg. side of y axis



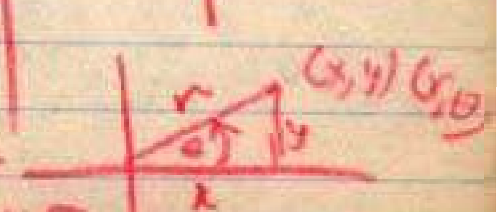
$\cos\theta = \frac{x}{r}$

$r\cos\theta = y$

θ	$\sin\theta$	ρ
−30°	−1/2	−5
210°	−1/2	−5

$r = \sqrt{x^2+y^2}$

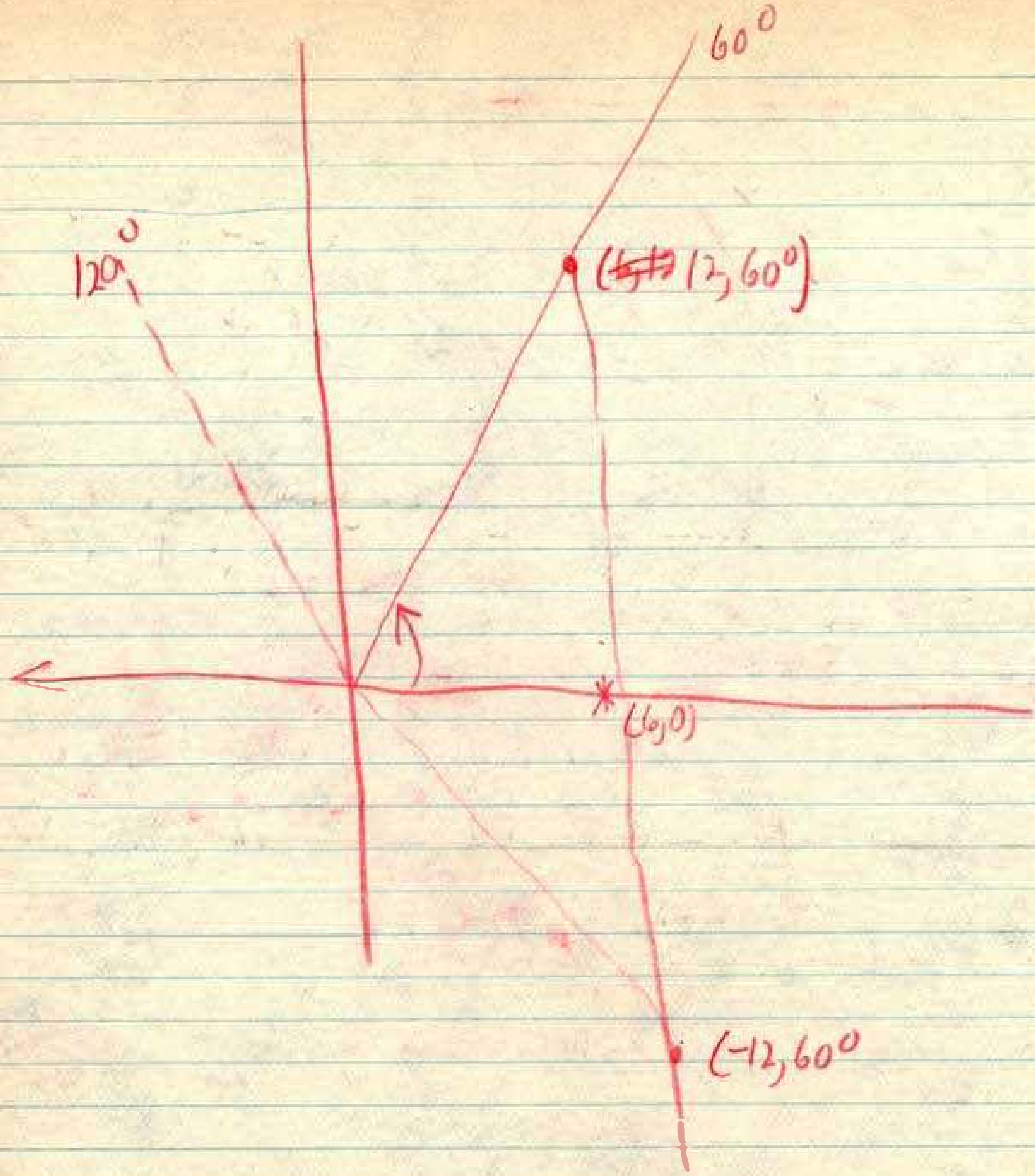
$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$



$\sqrt{x^2+y^2} = \frac{10y}{\sqrt{x^2+y^2}}$,

$x^2+y^2-10y = 0$ $+25$ $+25$

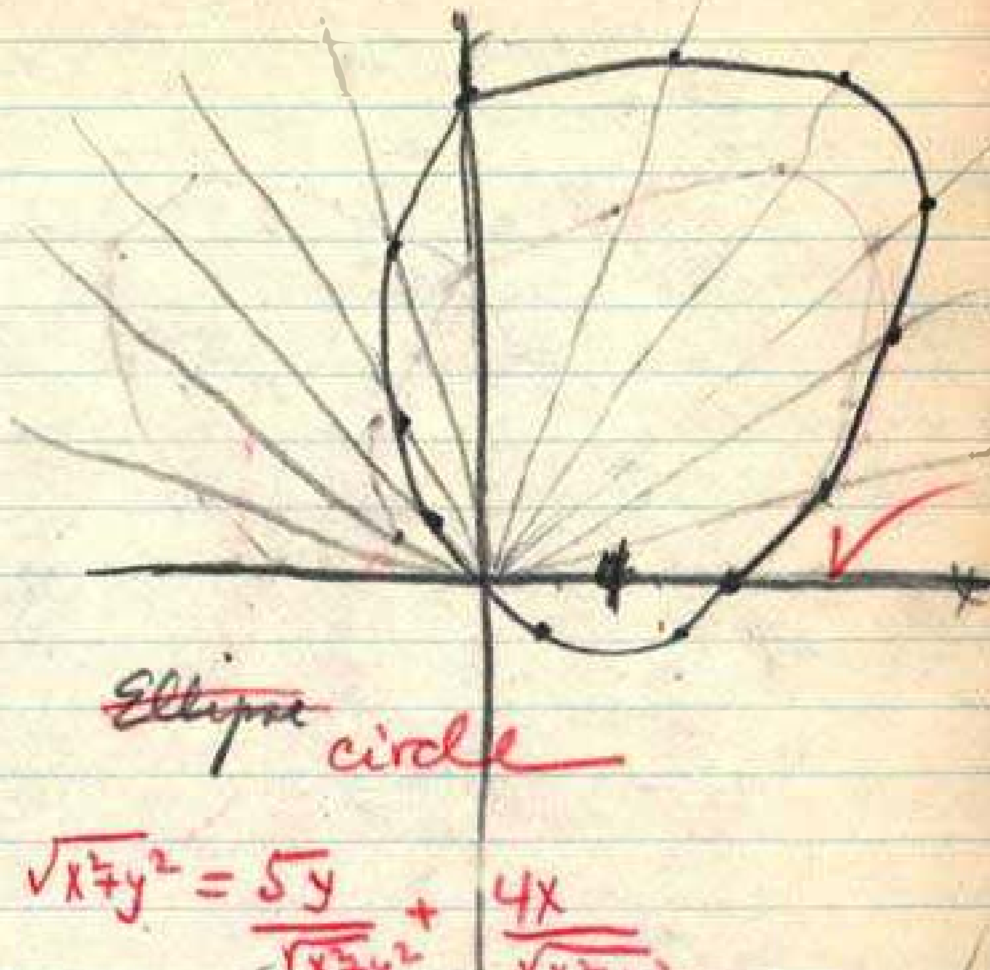
$x^2+(y-5)^2 = 25$



60°
120°
(~~6~~ 12, 60°)
(6,0)
(-12, 60°

354) 5 $\rho = 5\sin\theta + 4\cos\theta$

θ	$5\sin\theta$	$4\cos\theta$	ρ
0°	0	4	4
15°	1.29	3.932	5.222
30°	2.5	3.464	5.964
45°	3.535	2.828	6.363
60°	4.330	2.	6.330
75°	4.830	1.018	5.848
90°	5.	0	5.000
105°	4.830	-1.018	3.812
120°	4.330	-2.	2.330
135°	3.535	-2.828	0.707
150°	2.5	-3.464	-0.964
165°	1.29	-3.932	-2.642
180°	0	-4.	-4.000



~~Ellipse~~ circle

$$\sqrt{x^2+y^2} = \frac{5y}{\sqrt{x^2+y^2}} + \frac{4x}{\sqrt{x^2+y^2}}$$

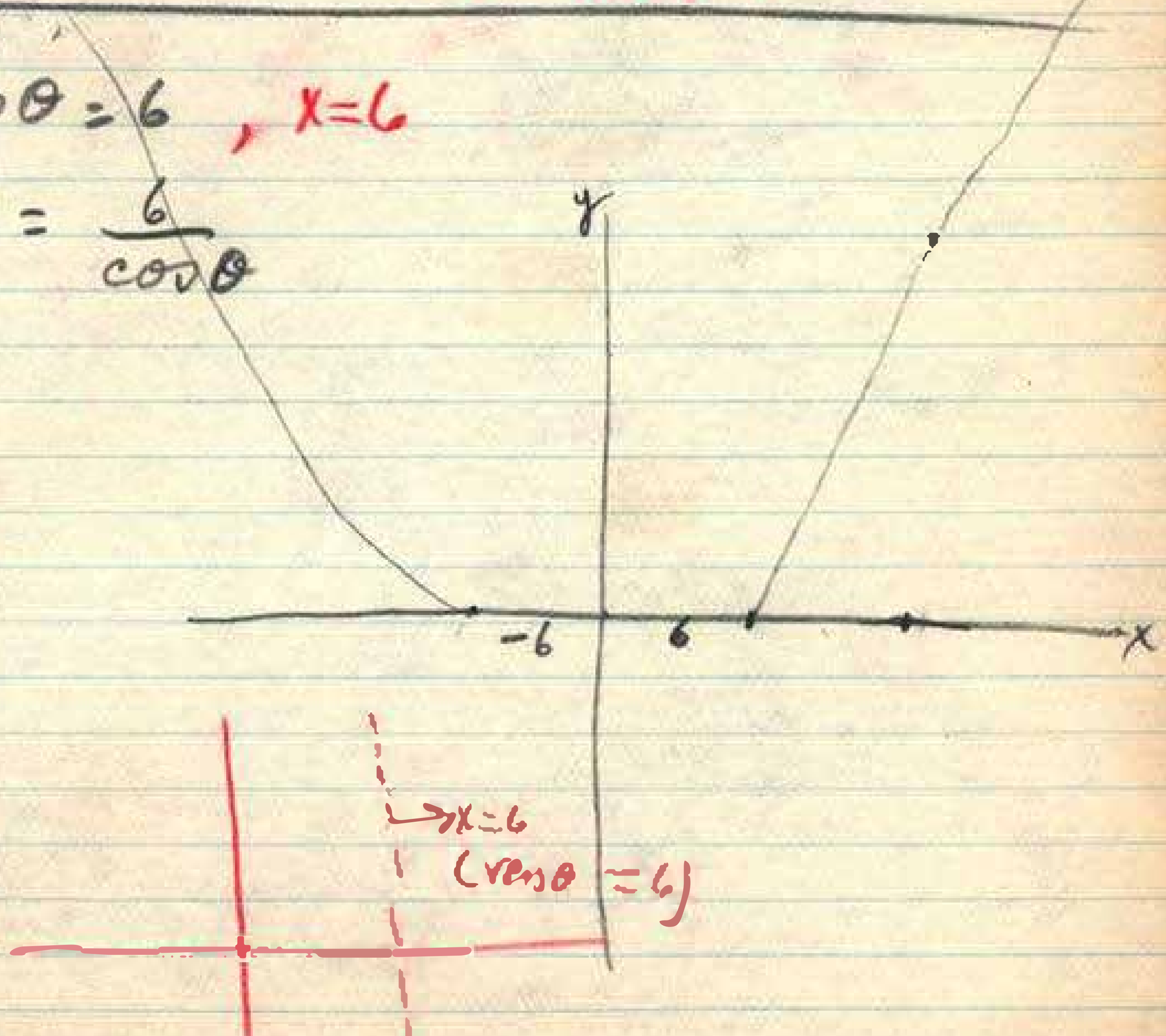
$$x^2+y^2-4x-5y=0$$

354) 6 $\rho\cos\theta = 6$, $x=6$

$$\rho = \frac{6}{\cos\theta}$$

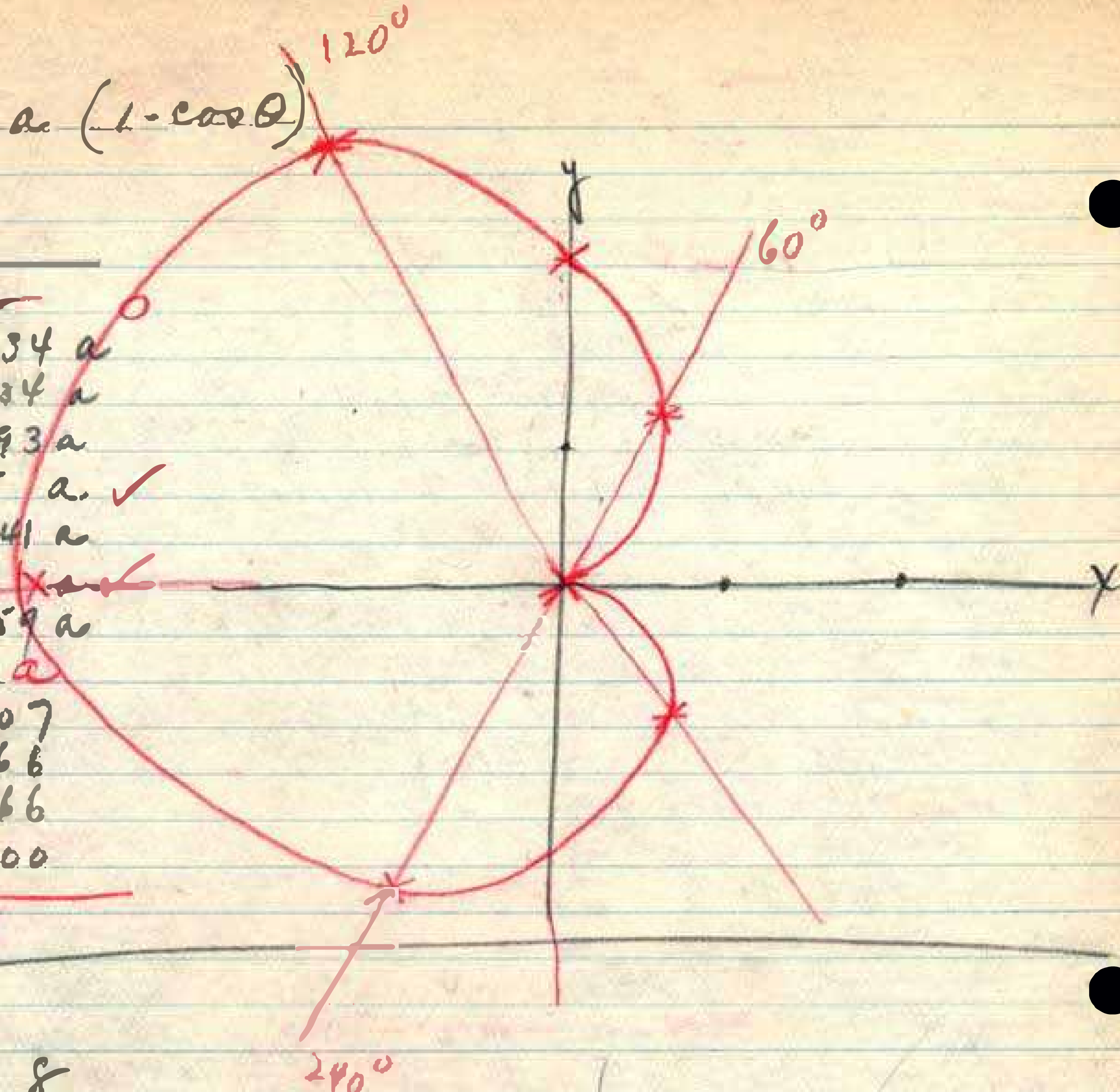
θ	$\cos\theta$	ρ
0°	1	6
60°	.5	12
90°	0	∞
120°	-.5	-12
180°	-1	-6

see Text 353



354)8 $\rho = a(1-\cos\theta)$

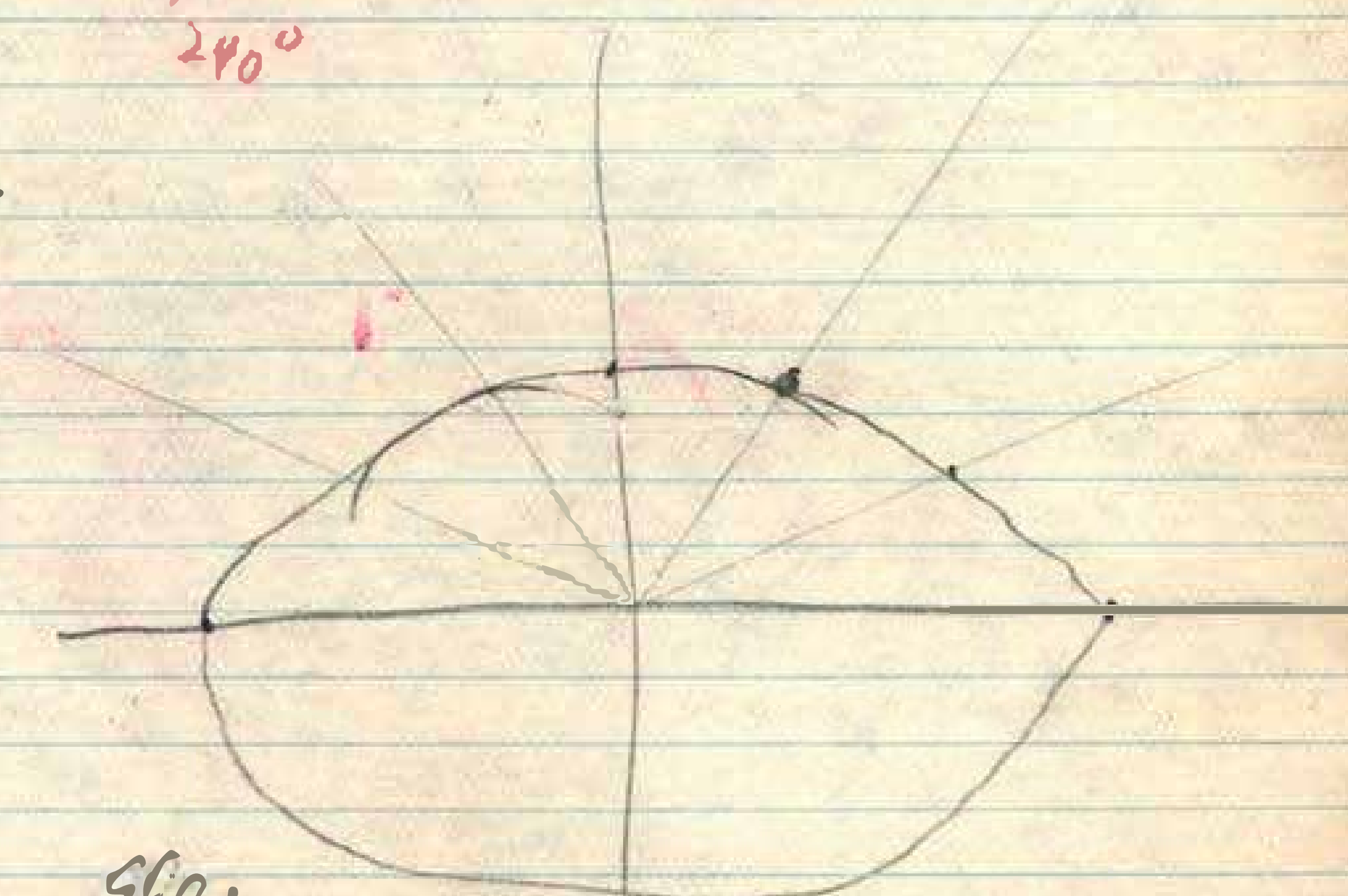
θ	ρ
0°	~~1 a~~
15°	0.034 a
30°	0.134 a
45°	0.293 a
60°	.5 a ✓
75°	.741 a
90°	1. ~~a~~
105°	1.259 a
120°	1.5 a
135°	1.707
150°	1.866
165°	1.966
180°	2.000
240°	1.5a
270°	a
300°	.5a
360°	0



354)16

$$\rho = \frac{8}{1+\sin\theta}$$

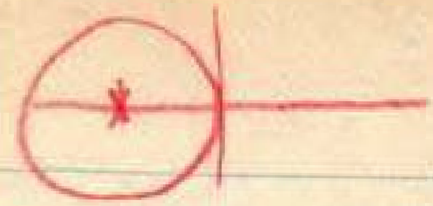
θ	ρ
0°	8
30°	5.3
60°	4.2
90°	4.
120°	4.2
150°	5.3
180°	8



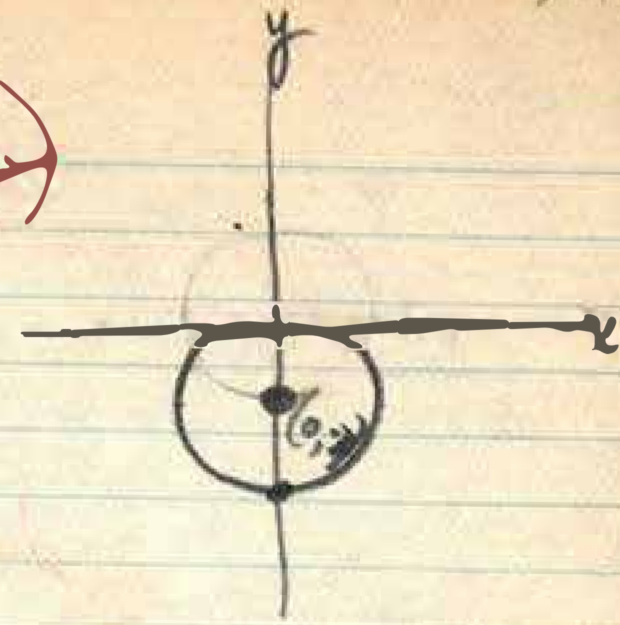
Ellipse

If θ is neg., lower half of ellipse is supplied.

357) 1



(circle of radius = 2 with center at ...)



$$x^2+y^2+4x=0$$

$$(\rho^2\cos^2\theta)+(\rho^2\sin^2\theta)+4\rho\cos\theta=0$$

$$\rho^2(\cos^2\theta+\sin^2\theta)+4\rho\cos\theta=0$$

$$\rho^2+4\rho\cos\theta=0$$

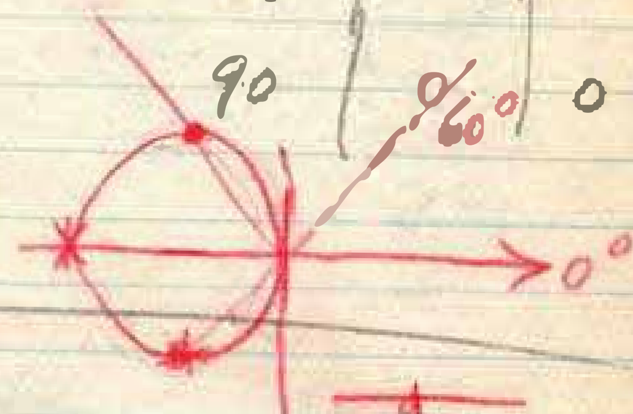
$$\rho^2=-4\rho\cos\theta$$

$$\boxed{\rho=-4\cos\theta}$$

$$x^2+4x^{+4}+y^2=0^{+4}$$

$$(x+2)^2+y^2=4$$

θ	$\cos\theta$	ρ
0°	1	-4
60°	0.5	-2
90	0/60°	0



120°	2
180°	4
240	2

357) 5

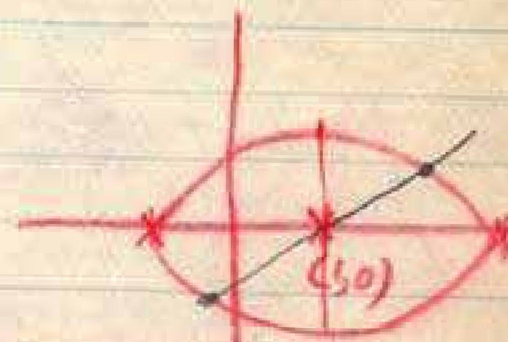
$$3x^2+4y^2-6x-9=0$$

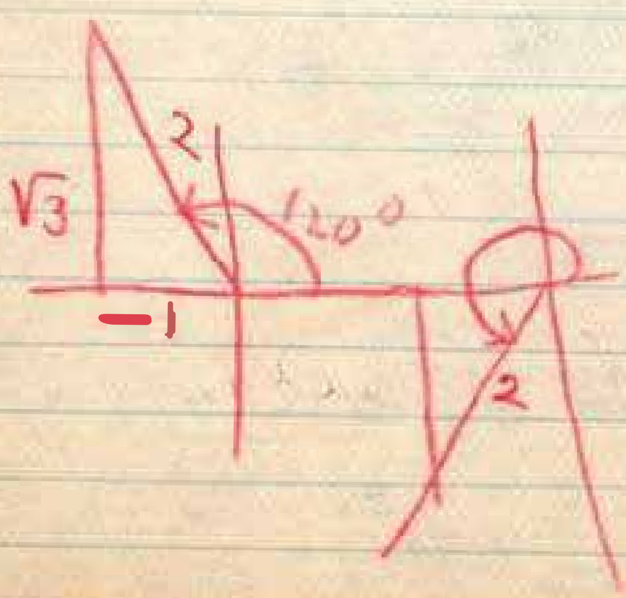
$$3(\rho^2\cos^2\theta)+4(\rho^2\sin^2\theta)-6\rho\cos\theta-9=0$$

$$\rho^2(3\cos^2\theta+4\sin^2\theta)=6\rho\cos\theta+9$$

$$\frac{\cancel{\rho}}{6\cancel{\cos\theta}}=\frac{9}{(3\cos^2\theta+4\sin^2\theta)}$$

$$\cancel{\rho=\frac{54\cos\theta}{3\cos^2\theta+4\sin^2\theta}}$$

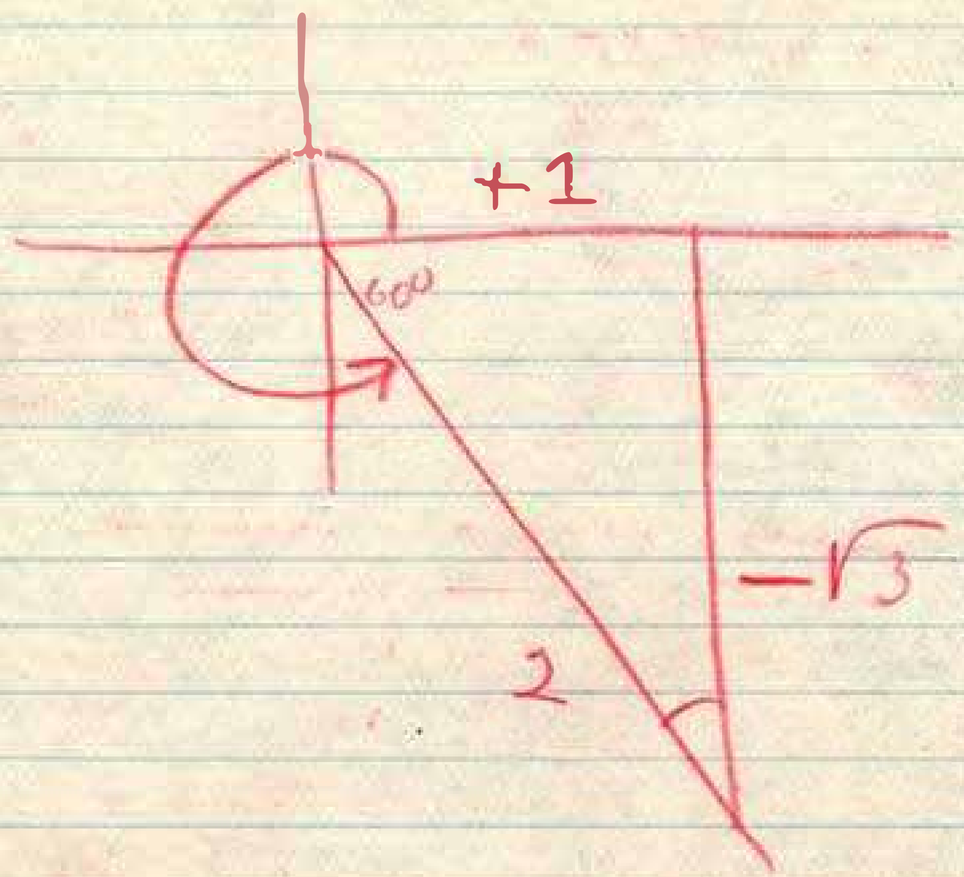




$$3(x^2-2x^{+1})+4y^2=9^{+3}$$

$$3(x-1)^2+4y^2=12$$

$$\frac{(x-1)^2}{4}+\frac{y^2}{3}=1$$



$\sqrt{3} = 1.732$

$\sqrt{3}/2 = .86$

$1 + (-.86) = .14$

$$\rho^2 \underbrace{(3\cos^2\theta + 4\sin^2\theta)} - (6\cos\theta)\rho - 9 = 0$$

$$\rho = \frac{\cancel{6\cos\theta \pm 6\sqrt{36\cos^2\theta + 36(3\cos^2\theta + 4\sin^2\theta)}}}{2(3\cos^2\theta + 4\sin^2\theta)}$$

$$\cancel{\rho = 6\cos\theta \pm \sqrt{144\cos^2\theta +}}$$

$$\rho = \frac{6\cos\theta \pm 6\sqrt{4\cos^2\theta + 4\sin^2\theta}}{2(3\cos^2\theta + 4\sin^2\theta)} = \frac{6\cos\theta \pm 12}{2(\quad)}$$

$$\rho = \frac{3\cos\theta \pm 6}{3\cos^2\theta + 4\sin^2\theta}$$

$$r = \frac{8}{1+\sin\theta}$$

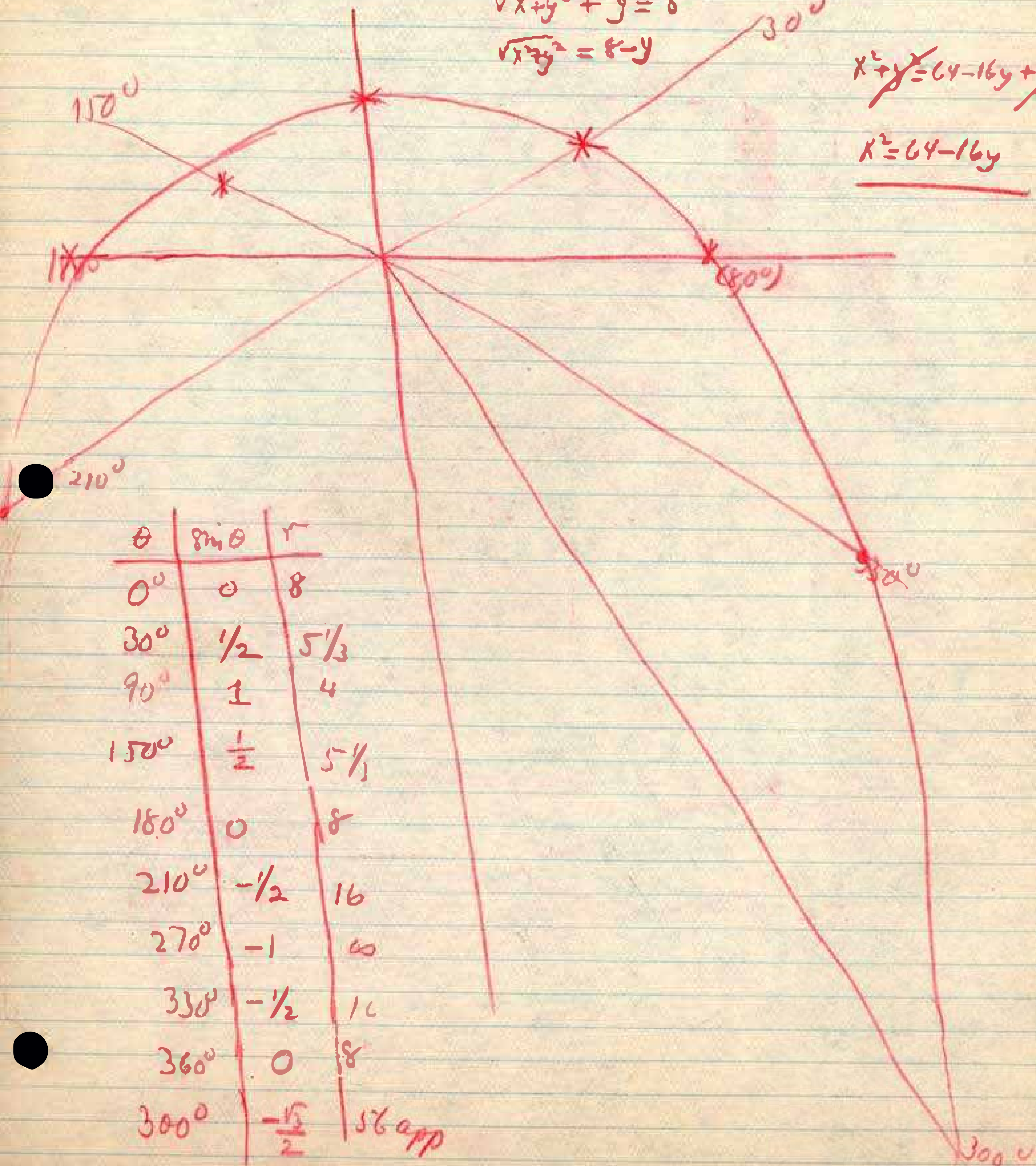
$$r + r\sin\theta = 8$$

$$\sqrt{x^2+y^2} + y = 8$$

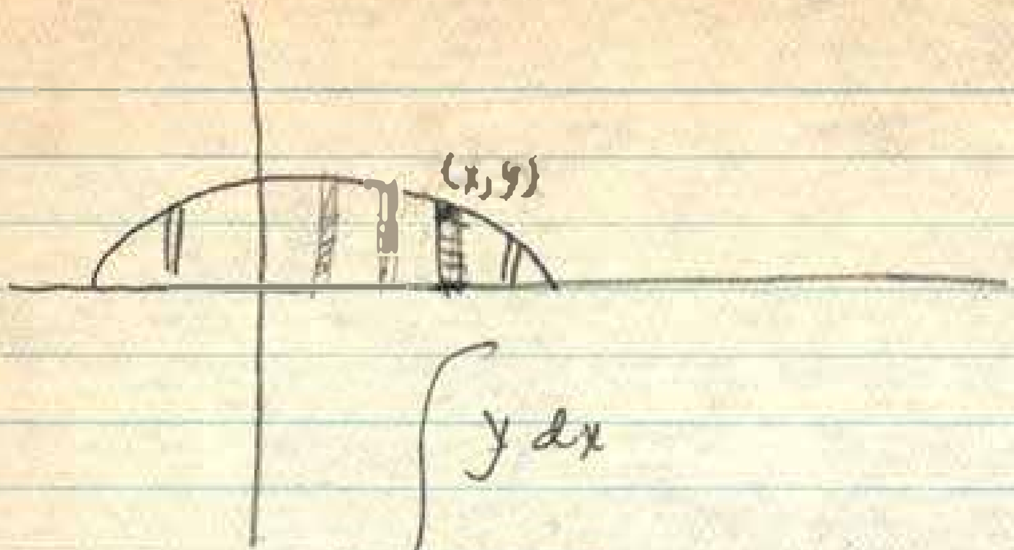
$$\sqrt{x^2+y^2} = 8 - y$$

$$x^2 + \cancel{y^2} = 64 - 16y + \cancel{y^2}$$

$$\underline{x^2 = 64 - 16y}$$



θ	$\sin\theta$	r
0°	0	8
30°	1/2	5 1/3
90°	1	4
150°	$\frac{1}{2}$	5 1/3
180°	0	8
210°	−1/2	16
270°	−1	∞
330°	−1/2	16
360°	0	8
300°	$-\frac{\sqrt{3}}{2}$	56 app



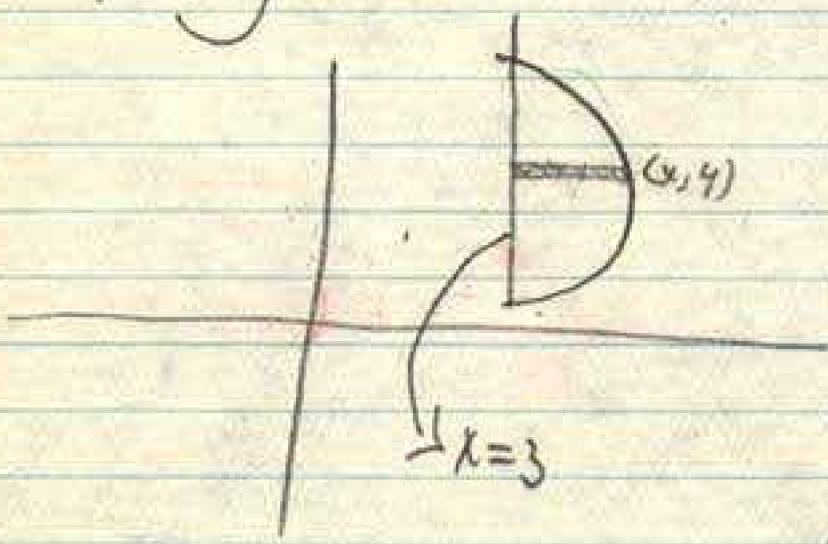
$$\int y\,dx$$

$$\int \cos^2\theta\,d\theta$$

$$= \int \frac{1+\cos 2\theta}{2}\,d\theta$$

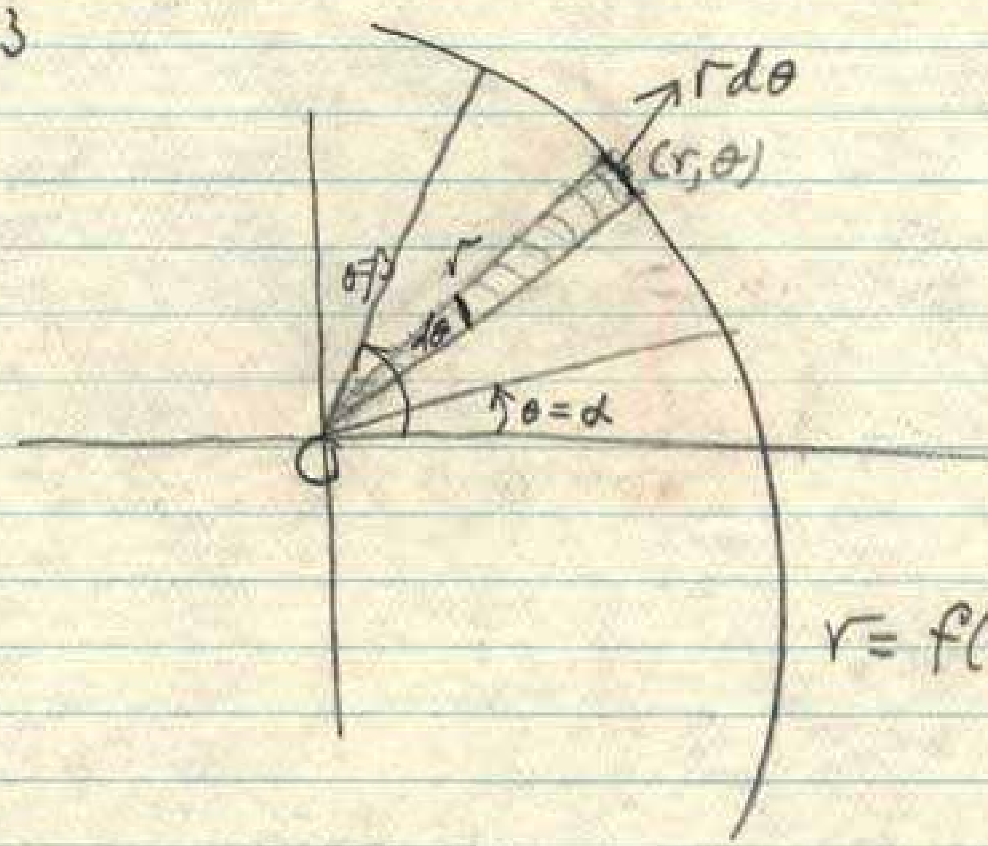
$$= \frac{1}{2}\int (1+\cos 2\theta)\,d\theta$$

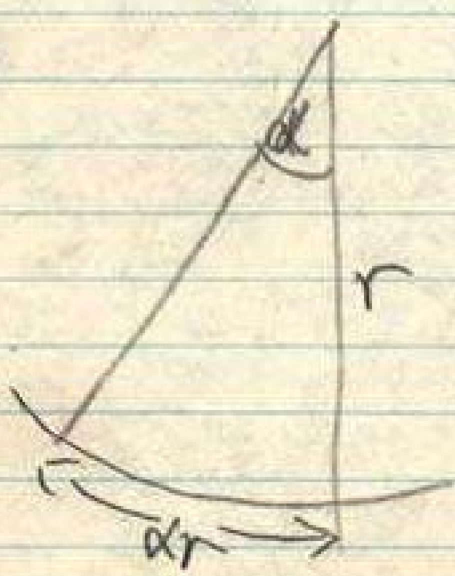
$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)$$

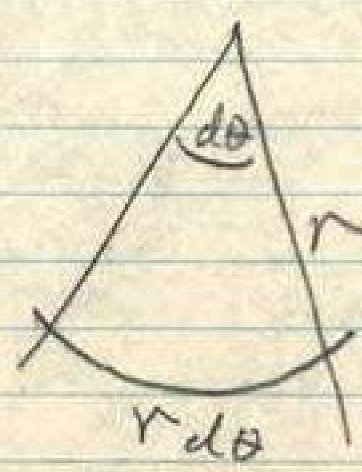


$$\int (x-3)\,dy$$

d

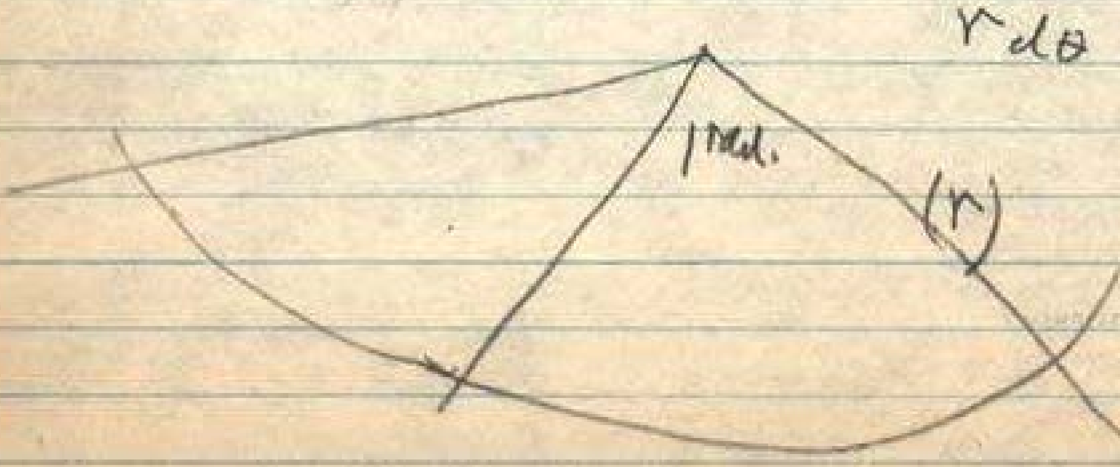


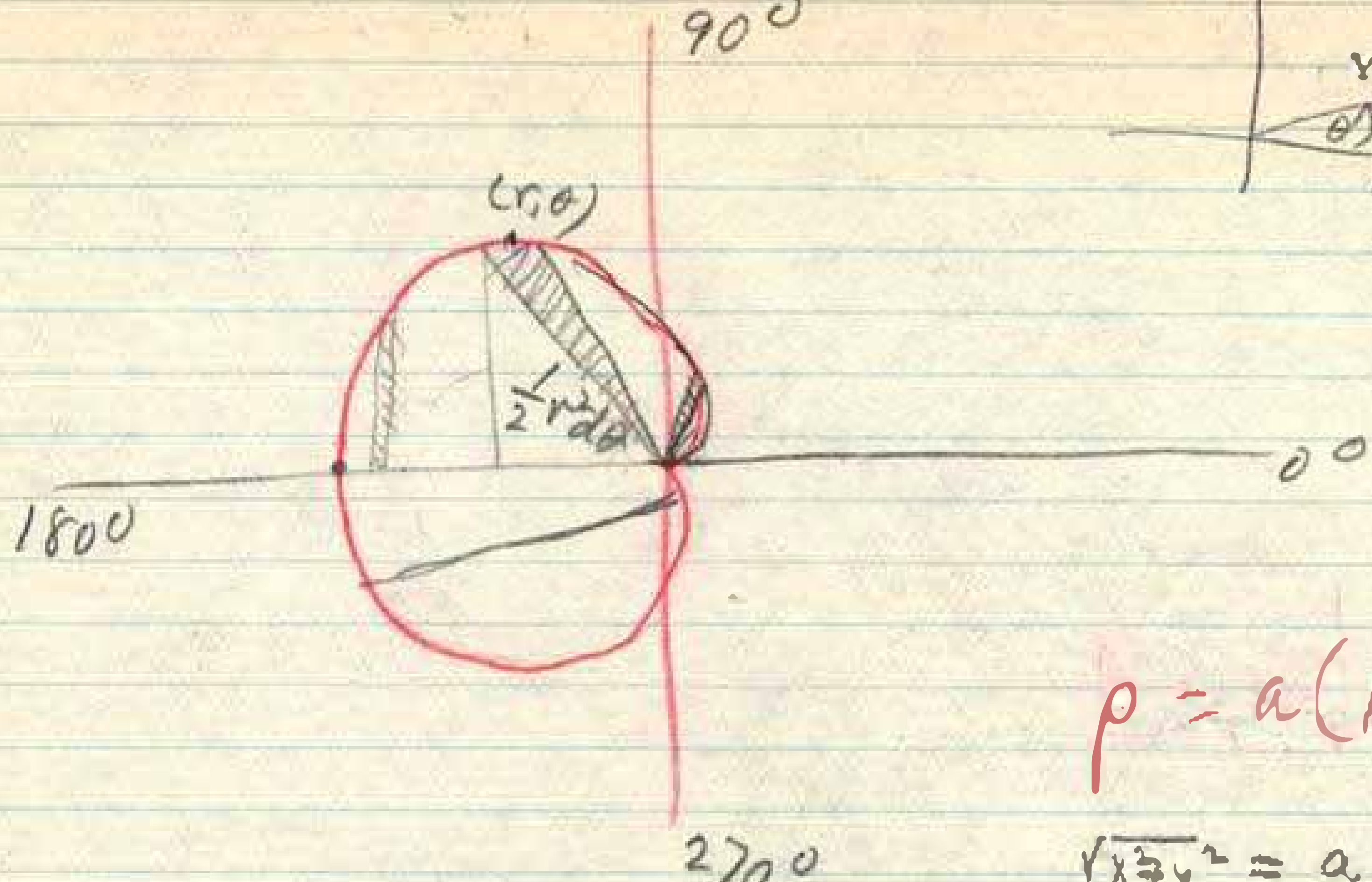


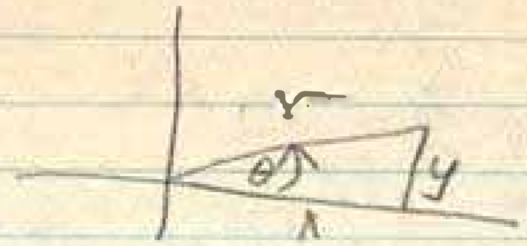


El. area $= \frac{1}{2} r^2\,d\theta$

$$\text{Area} = \int_{\theta=\alpha}^{\theta=\beta} \frac{1}{2} r^2\,d\theta$$







$\rho = a(1-\cos\theta)$

$\sqrt{x^2+y^2} = a - \frac{ax}{\sqrt{x^2+y^2}}$

$x^2+y^2 = a\sqrt{x^2+y^2} - ax$

$x^2+y^2+ax = a\sqrt{x^2+y^2}$

$\boxed{(x^2+y^2+ax)^2 = a^2(x^2+y^2)}$

El. area $= \frac{1}{2} r^2 \, d\theta$

$$\text{Area} = \int_0^{\pi} r^2 \, d\theta = \int_0^{\pi} \left[a(1-\cos\theta)\right]^2 d\theta$$

$$= a^2 \int_0^{\pi} (1 - 2\cos\theta + \cos^2\theta) \, d\theta$$

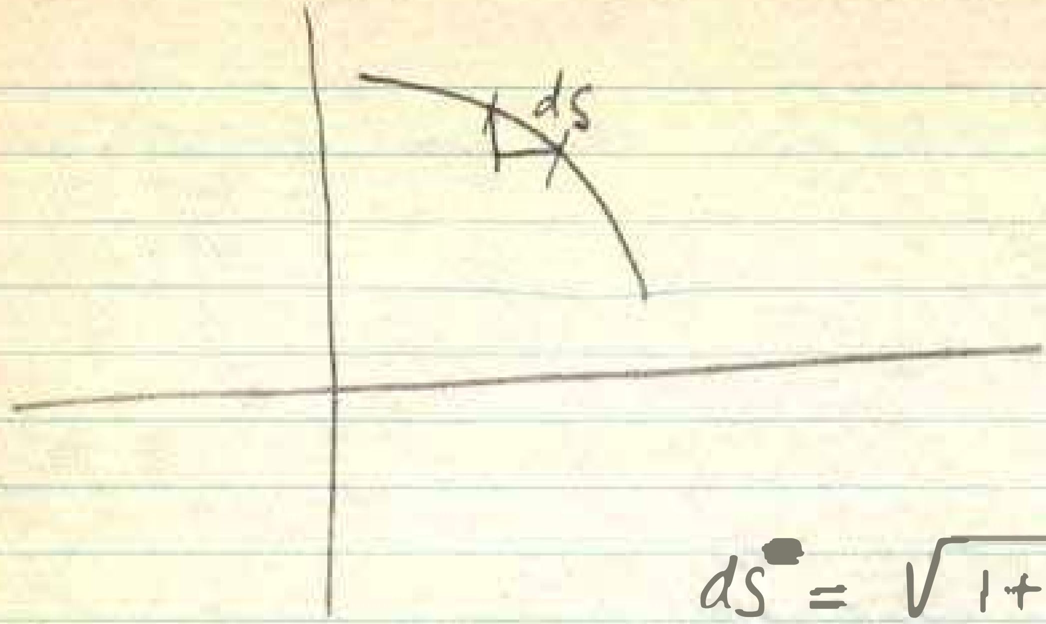
$$= a^2 \left[\tfrac{3}{2}\theta - 2\sin\theta + \cancel{\tfrac{1}{2}\theta} + \tfrac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$a^2\left(\frac{3\pi}{2}\right) - a^2(0) = \cancel{\frac{a^2\theta}{2}}$$

$\frac{3\pi a^2}{2}$

$180^\circ = \pi^{\text{rad}}$

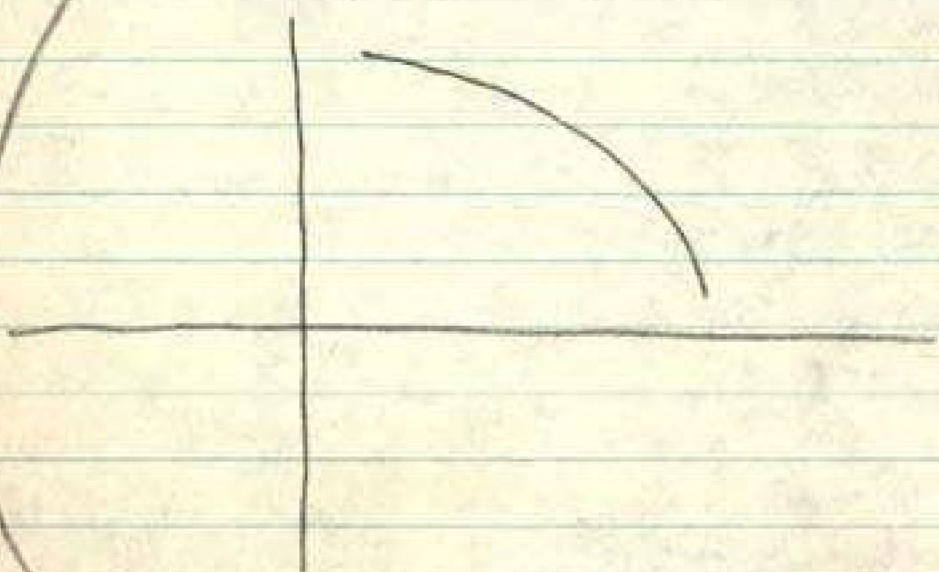
$45^\circ = \pi/4 \text{ rad.}$



$$ds = \sqrt{1 + (dy/dx)^2}\; dx$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{\cancel{dx^2}\left(1 + \left(\frac{dy}{dx}\right)^2\right)}\; dx$$



$$ds = \sqrt{(dr)^2 + r^2 (d\theta)^2}$$

$$= \sqrt{\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right)(d\theta)^2}$$

$$= \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$$

$x = r \cdot \cos\theta$, $dx = -r \sin\theta\, d\theta + \cos\theta \cdot dr$

$y = r \sin\theta$ $\quad$ $dy = r \cos\theta\, d\theta + \sin\theta \cdot dr$

$$dx^2 = r^2 \sin^2\theta\, d\theta^2 - 2r \sin\theta \cos\theta\, d\theta\, dr + \cos^2\theta\, dr^2$$

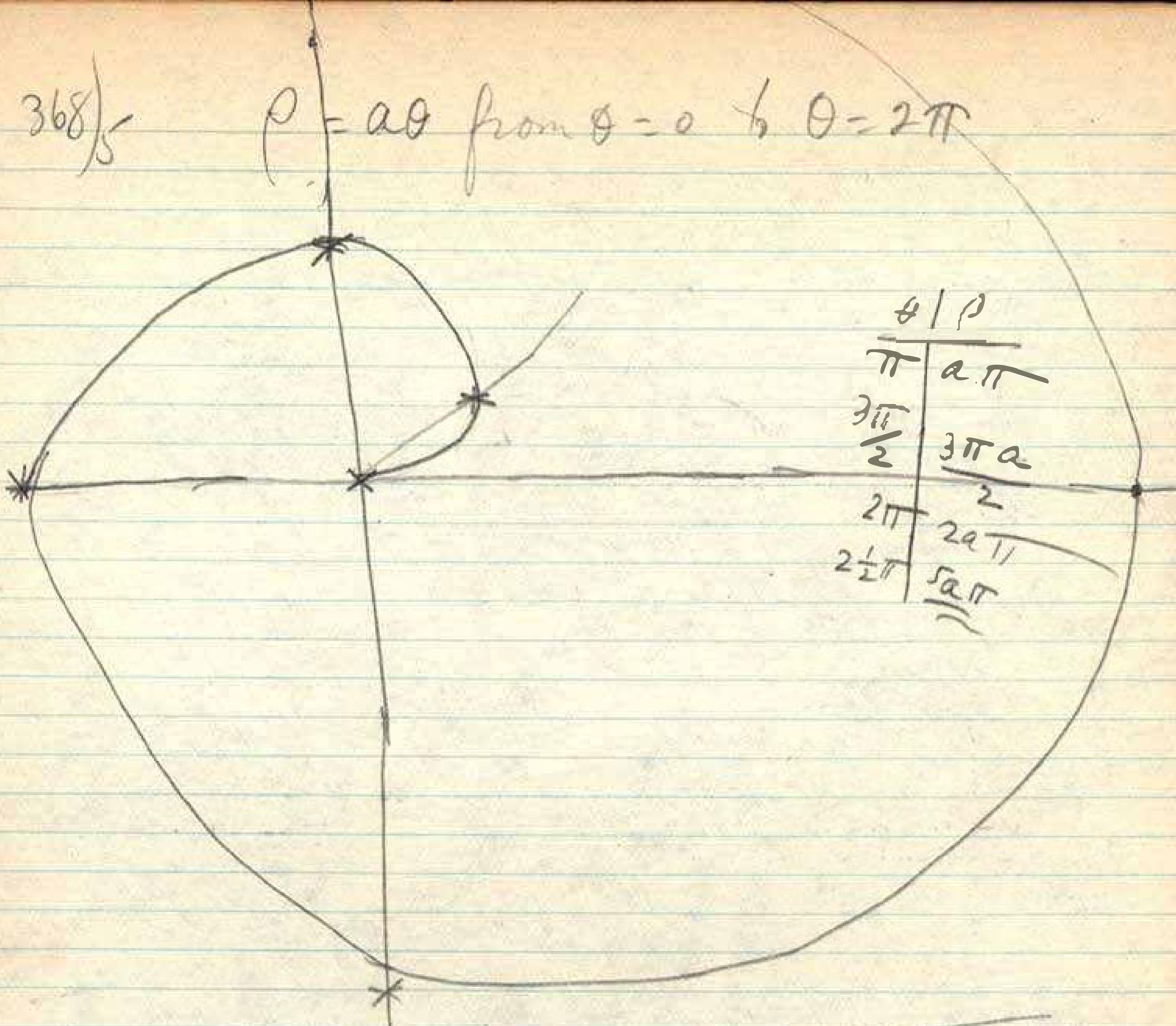
$$dy^2 = r^2 \cos^2\theta\, d\theta^2 + 2r \cos\theta \sin\theta\, d\theta\, dr + \sin^2\theta\, dr^2$$

$$dx^2 + dy^2 = r^2 d\theta^2 + dr^2$$

$$s = a\left[\pi\sqrt{4\pi^2+1} + \frac{1}{2}\ln\left(2\pi + \sqrt{4\pi^2+1}\right)\right] - a\left[\frac{1}{2}\ln\left(0 + \sqrt{1}\right)\right]$$

$- \quad 0$

368/5 $\rho = a\theta$ from $\theta = 0$ to $\theta = 2\pi$



θ	ρ
π	$a\pi$
$\frac{3\pi}{2}$	$\frac{3\pi a}{2}$
2π	$2a\pi$
$2\frac{1}{2}\pi$	$5a\pi$

$$ds = \sqrt{a^2\theta^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$$

$$= \sqrt{a^2\theta^2 + a^2}\ d\theta$$

$$= a\sqrt{\theta^2 + 1}\ d\theta$$

$$s = a\int_0^{2\pi} \sqrt{\theta^2+1}\ d\theta$$

$$= \left[\frac{\theta}{2}\sqrt{\theta^2+1} + \frac{1}{2}\ln\left(\theta + \sqrt{\theta^2+1}\right)\right]_0^{\pi}$$

$$\begin{cases} x = t-1 \\ y = 4-t^2 \end{cases}$$

$dx/dt = 1$

$dy/dt = -2t$

t	x	y
0	-1	4
1		
2		
3		
-1		
-2		
-3		

$dy/dx = -2t$

$t=3$

$dy/dx = -6$

$t = x+1$

$y = 4-(x^2+2x+1)$

$y = 3-x^2-2x$

$dy/dx = -2x-2$

$$\frac{dy}{dx} = -2(x+1)$$
$$= -2x-2$$

357) 10

x	y
0	± 4
2	± 6
-2	0

$$r(1-\cos\theta) = 4$$

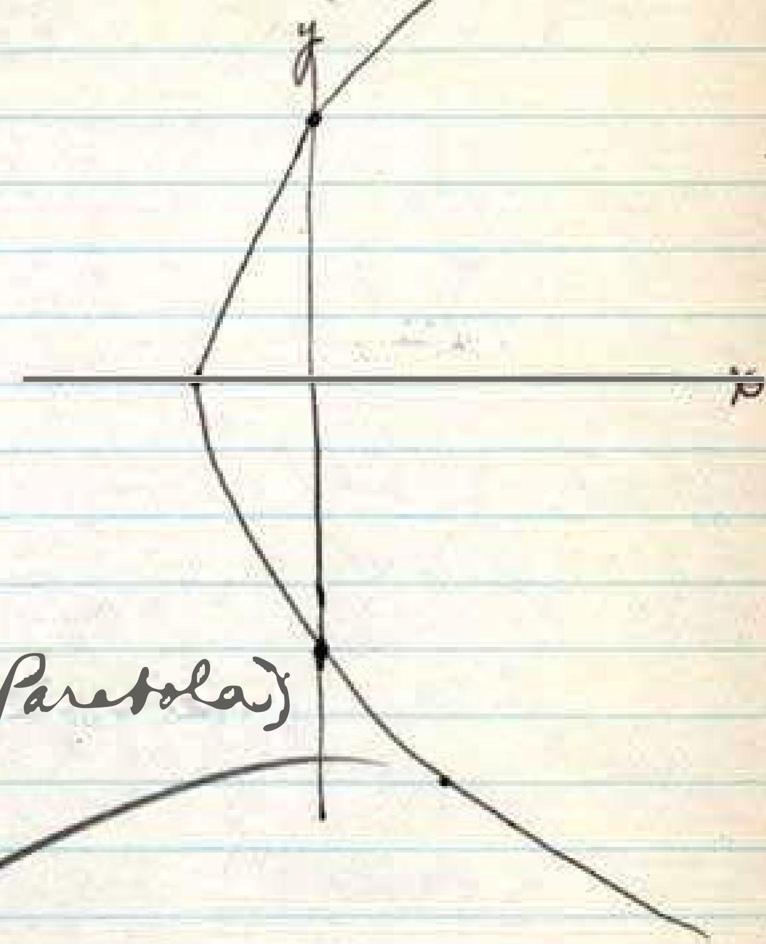
$$\sqrt{x^2+y^2}\left(1 - \frac{x}{\sqrt{x^2+y^2}}\right) = 4$$

$$\sqrt{x^2+y^2} - x = 4$$

$$\sqrt{x^2+y^2} = 4 + x$$

$$x^2+y^2 = 16 + 8x + x^2$$

$$y^2 = 8x + 16$$ (Eq. of Parabola)



357) 11

$$r(2-\cos\theta) = 3$$

$$\sqrt{x^2+y^2}\left(2 - \frac{x}{\sqrt{x^2+y^2}}\right) = 3$$

$$2\sqrt{x^2+y^2} - x = 3$$

$$2\sqrt{x^2+y^2} = 3 + x$$

$$4x^2+4y^2 = 9 + 6x + x^2$$

$$3x^2 - 6x + 4y^2 = 9$$

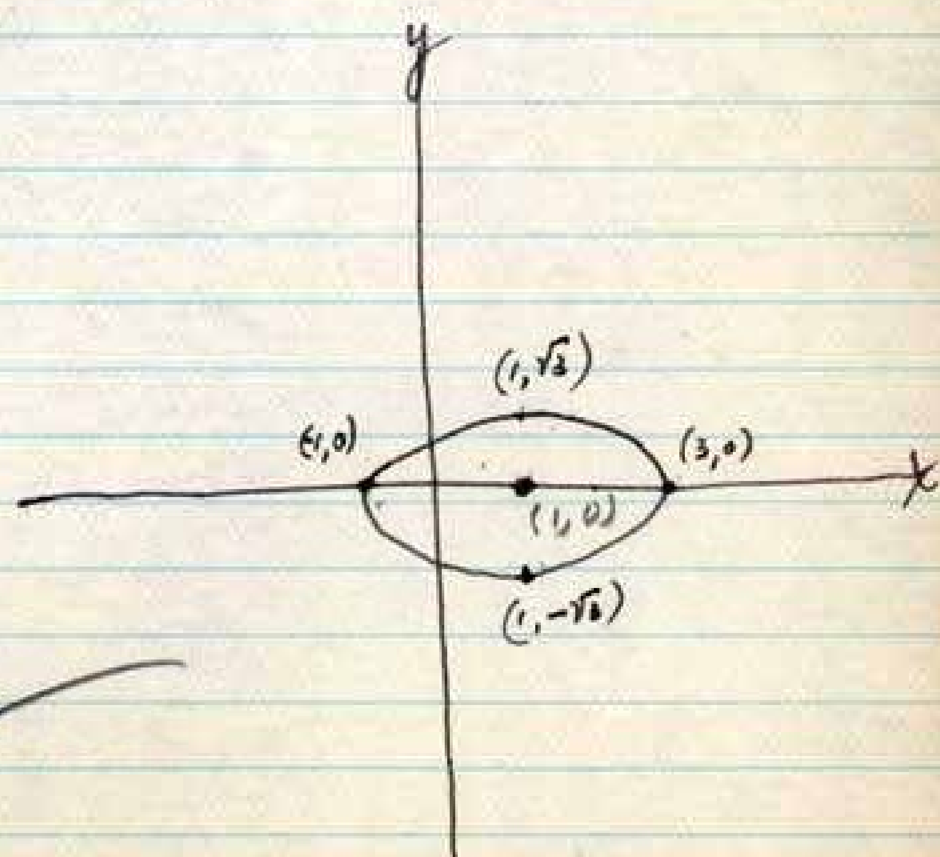
$$3(x^2-2x+1) + 4y^2 = 12$$

$$\frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$h = 1$, $k = 0$

$a = 2$, $b = \sqrt{3}$



$r\cos\theta = x$

358) 9 I $4r\cos\theta = 3$

II $2r = 3$

$4x = 3$

$$\begin{cases} 4r = \dfrac{3}{\cos\theta} \\ 4r = 6 \end{cases}$$

$$\frac{3}{\cos\theta} = 6$$

$$6\cos\theta = 3$$

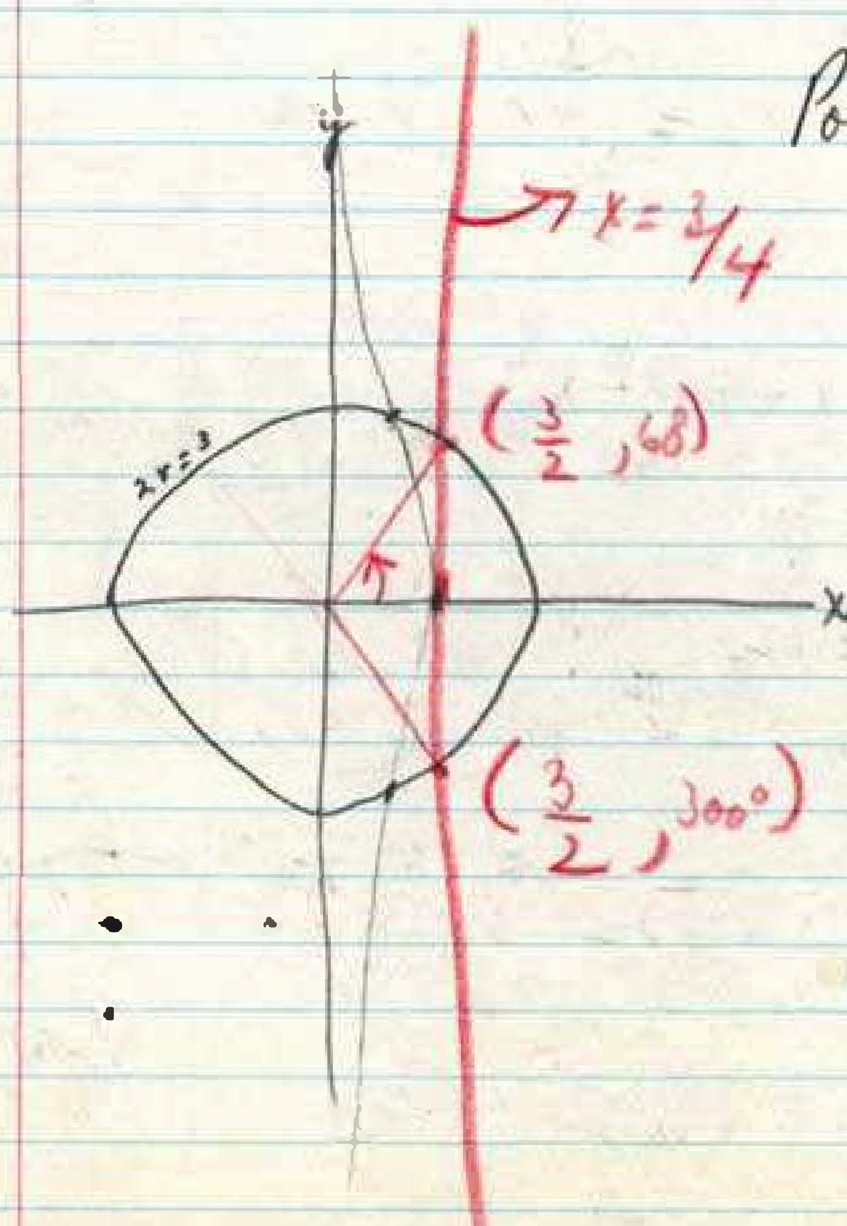
$$\cos\theta = \frac{1}{2}$$

$$\therefore \theta = 60°, 300°$$

I $r = \dfrac{3}{4\cos\theta}$ ⟶

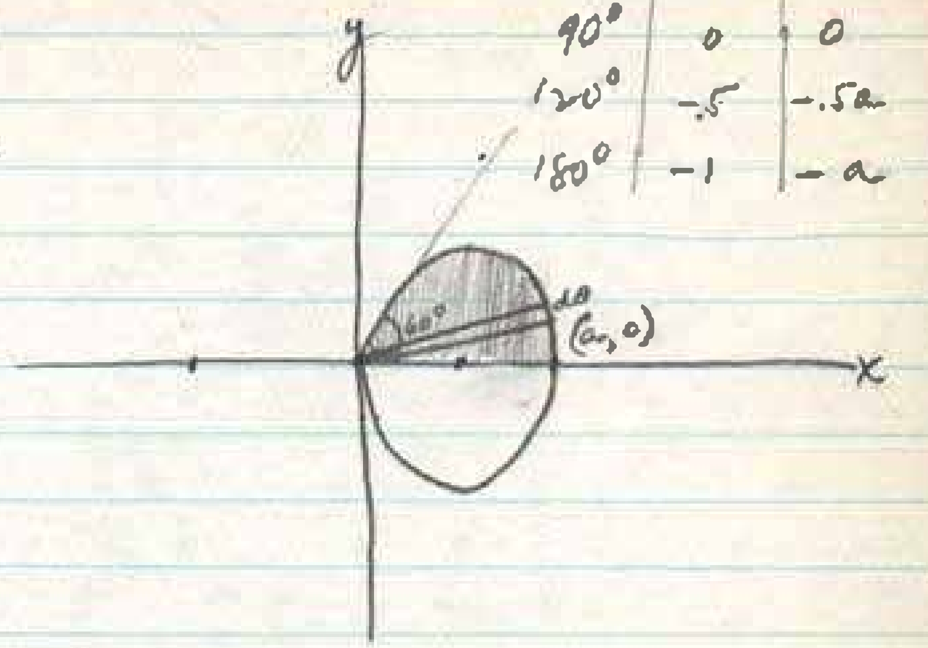
θ	$\cos\theta$	r
0°	1	3/4
60°	.5	3/2
90°	0	∞
120°	−0.5	−3/2
180°	−1	−3/4

II Circle c radius 3/2



Points of intersection when r of parabola equals +3/2 + ~~[illegible]~~

θ	$\cos\theta$	r
$0°$	1	a
$60°$	$.5$	$.5a$
$90°$	0	0
$120°$	$-.5$	$-.5a$
$180°$	-1	$-a$



366) $r = a\cos\theta$

$$\boxed{\begin{aligned}\sqrt{x^2+y^2} &= a\frac{x}{\sqrt{x^2+y^2}}\\ x^2+y^2 &= ax\\ x^2-ax+y^2 &= 0\\ \left(x-\frac{a}{2}\right)^2+y^2 &= \frac{a^2}{4}\end{aligned}}$$

Center at $\left(\frac{a}{2}, 0\right)$ & Radius $\frac{a}{2}$

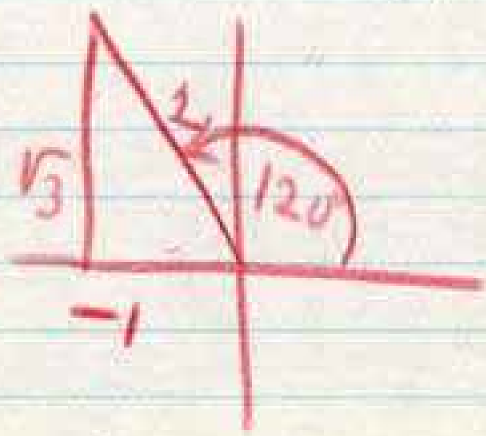
El. of Area $= \frac{1}{2} r^2 \, d\theta$

Total Area $= \int \frac{1}{2} r^2 \, d\theta$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{2}(a\cos\theta)^2 \, d\theta$$

$$= \frac{a^2}{2}\int_0^{\frac{\pi}{3}} \cos^2\theta \, d\theta$$

$$= \frac{a^2}{2}\int_0^{\frac{\pi}{3}} \frac{1+\cos 2\theta}{2} \, d\theta$$



$$= \frac{a^2}{\cancel{4}\ 2}\int_0^{\frac{\pi}{3}} \frac{(1+\cos 2\theta)}{2} \, d\theta$$

$$= \frac{a^2}{\cancel{4}\ 2}\left(\frac{\theta}{2} + \frac{1}{\cancel{2}\ 4}\sin 2\theta\right)\Bigg]_0^{\frac{\pi}{3}}$$

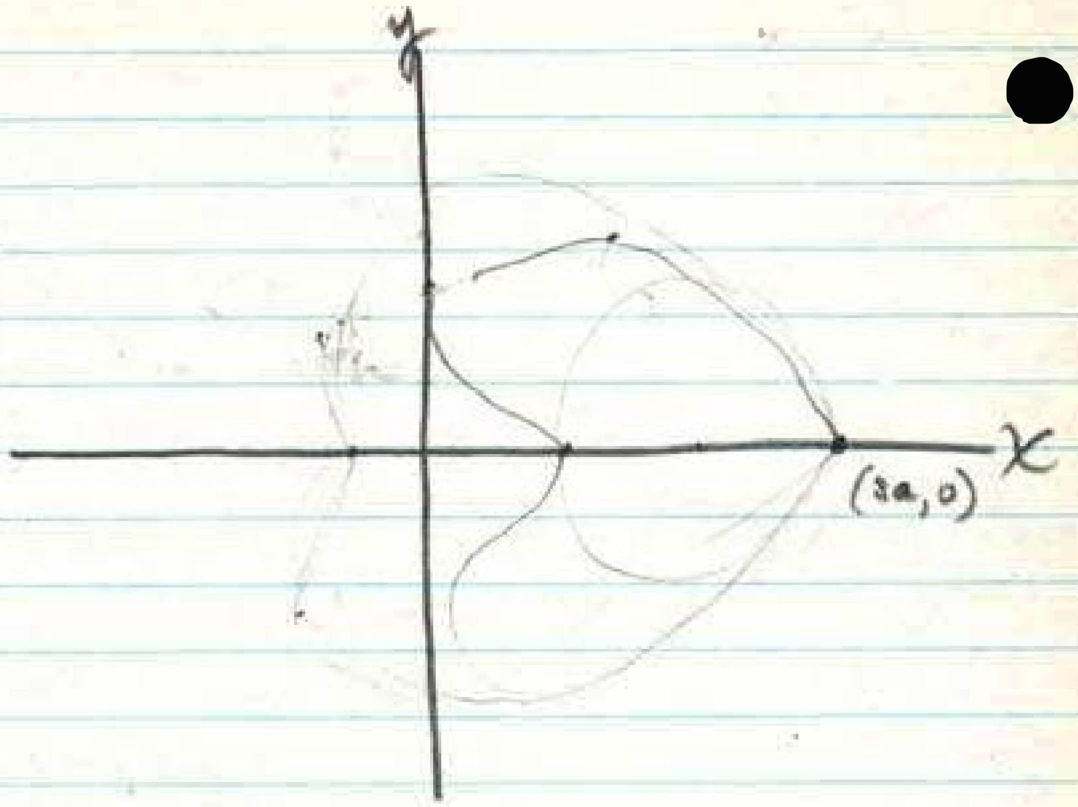
$$= \frac{a^2}{4}\left(\frac{\pi}{3} + \cancel{\frac{1}{2}\cdot\frac{\pi}{3}}\ + \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right) - \frac{a^2}{4}(0+0)$$

$$= \frac{a^2\pi}{12} + \frac{a^2\sqrt{3}}{\cancel{24}\ 16} = \cancel{\frac{2a^2\pi + a^2\pi}{24}}$$

$$= \cancel{\frac{a^2\pi}{8}}$$

366) 7 $r = a(2+\cos\theta)$

θ	$\cos\theta$	r
$0°$	1	$3a$
$60°$	.5	$2.5a$
$90°$	0	$2a$
$120°$	$-.5$	$1.5a$
$180°$	-1	$1a$
$240°$	$-.5$	$1.5a$
$270°$	0	$2a$
$300°$	0.5	$2.5a$
$360°$	1	$3a$

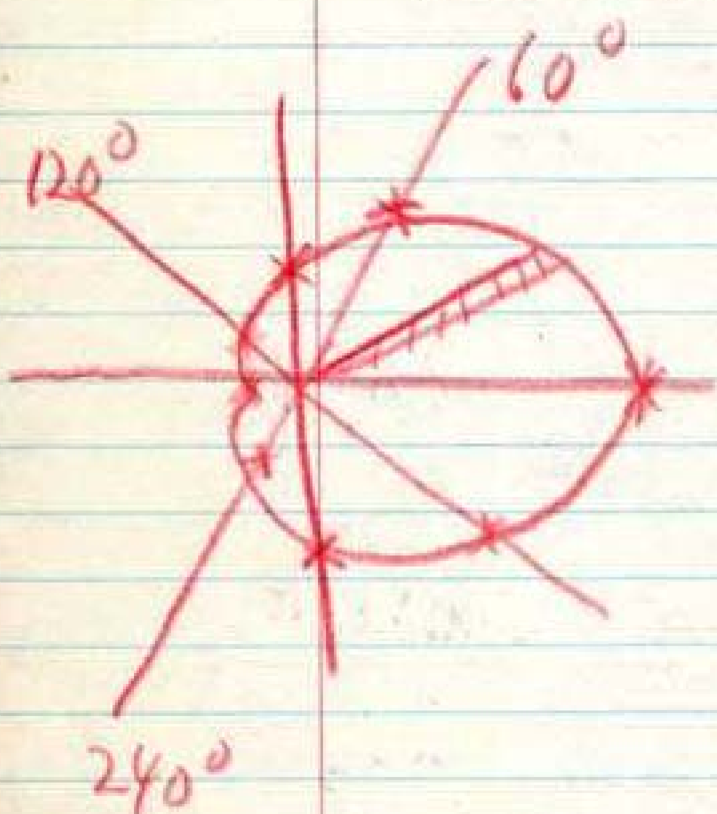


$$\text{El. of Area} = \frac{1}{2}r^2\,d\theta = \frac{1}{2}\left[a(2+\cos\theta)\right]^2 d\theta$$

$$= \frac{(2a + a\cos\theta)^2}{2}\,d\theta$$

$$= \frac{(4a^2 + 4a^2\cos\theta + a^2\cos^2\theta)}{2}\,d\theta$$

$$\text{Total Area} = \int_0^{2\pi} \frac{(4a^2 + 4a^2\cos\theta + a^2\cos^2\theta)}{2}\,d\theta$$



$$= \left. \frac{4a^2\theta + 4a^2\sin\theta + \frac{a^2}{2}\theta + \frac{a^2}{4}\sin 2\theta}{2} \right]_0^{2\pi}$$

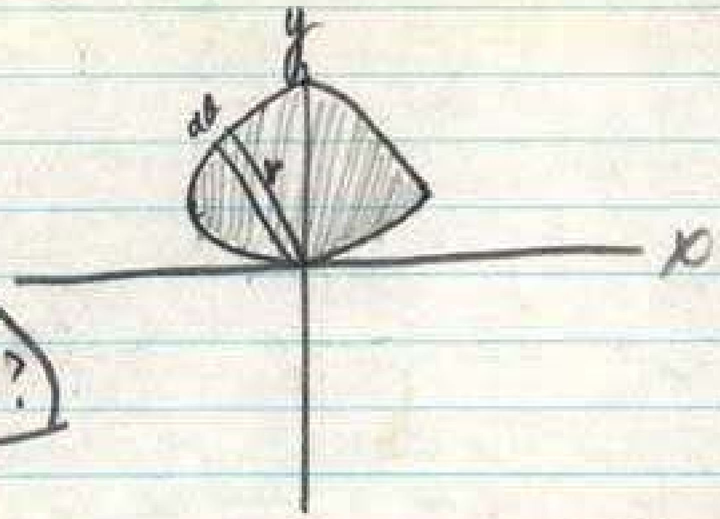
$$= \left[\frac{8a^2\pi + 0 + a^2\pi + 0}{2}\right] - \left[0 + 0 + 0 + 0\right]$$

$$= \frac{8a^2\pi + a^2\pi}{2}$$

$$= \frac{9a^2\pi}{2}$$

r must be plotted against θ & not 3θ

367) 9 $r = a \sin 3\theta$



∗ Why 3 leaved rose?

θ	3θ	$\sin 3\theta$	r
0°	0°	0	0
10°	30°	½	a/2
30°	90°	1	a
50°	150°	½	a/2
60°	180°	0	0
70°	210°	−½	−½ a
90°	270°	−1	−a
110°	330°	−½	−½ a
120°	360°	0	0
130°	390°	½	½ a
150°	450°	1	a
170°	570°	½	½ a
180°	540°	0	0

$$\text{El. of Area} = \tfrac{1}{2} r^2 \, d\theta$$

$$= \frac{(a \sin 3\theta)^2}{2} d\theta$$

$$= \frac{a^2 \sin^2 3\theta}{2} d\theta$$

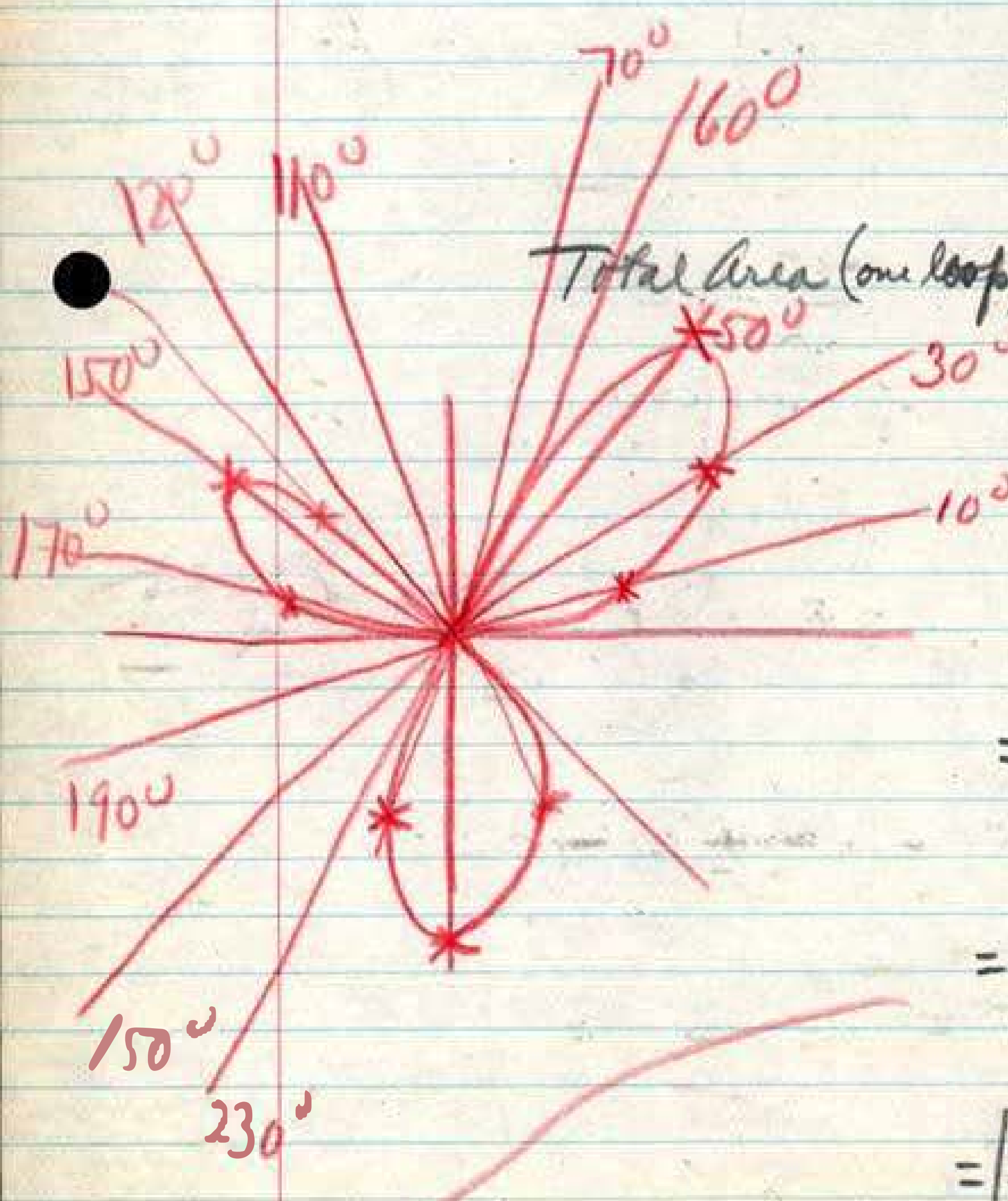
$$\text{Total Area (one loop)} = \int_0^{\pi/3} \frac{a^2 \sin^2 3\theta}{2} d\theta$$

$$= \frac{a^2}{2} \int_0^{\pi/3} \sin^2 3\theta \, d\theta$$

$$= \frac{a^2}{2} \int_0^{\pi} \left(\frac{1}{2} - \frac{1}{2}\cos 6\theta\right) d\theta$$

$$= \frac{a^2}{2}\left(\frac{\theta}{2} - \frac{1}{12}\sin 6\theta\right)\Bigg]_0^{\pi/3}$$

$$\cancel{= \left[\frac{a^2\pi}{4} - 0\right] - 0 = \frac{a^2\pi}{4}}$$



$$= \frac{a^2}{2}\left[\frac{\pi}{6} - 0\right] - \frac{a^2}{2}\left[0 - 0\right]$$

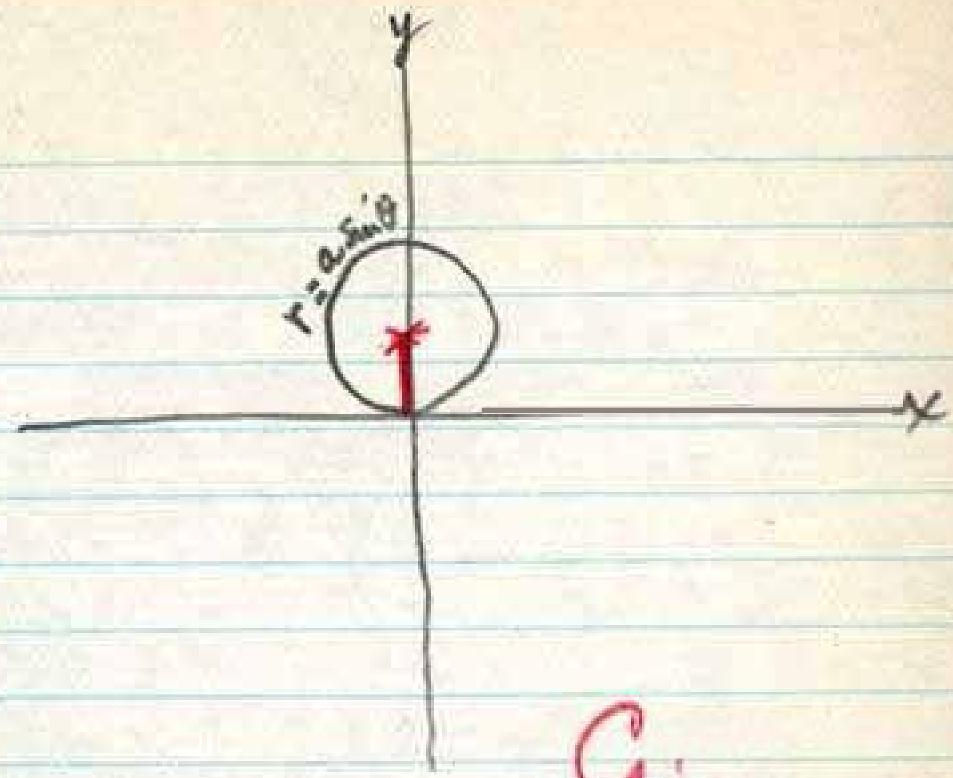
$$= \frac{\pi a^2}{12}$$

MrB

30 a) 3a $\quad r = a \sin\theta$

θ	r
0	0
30°	$a/2$
90°	a
150°	$a/2$
180°	0

$\frac{dr}{d\theta} = a.\cos\theta$



$\int \sin\theta \, d\theta = -\cos\theta$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

$$s = \int_0^{\pi} \sqrt{(a\sin\theta)^2 + (a[\cos\theta])^2} \, d\theta$$

$$= \int_0^{\pi} \sqrt{a^2\sin^2\theta + a^2\cos^2\theta} \, d\theta$$

$$= \int_0^{\pi} \sqrt{a^2(\sin^2\theta + \cos^2\theta)} \, d\theta$$

$$= a\int_0^{\pi} \sqrt{1} \, d\theta = a\theta \Big]_0^{\pi}$$

$$= a\pi$$ ✓

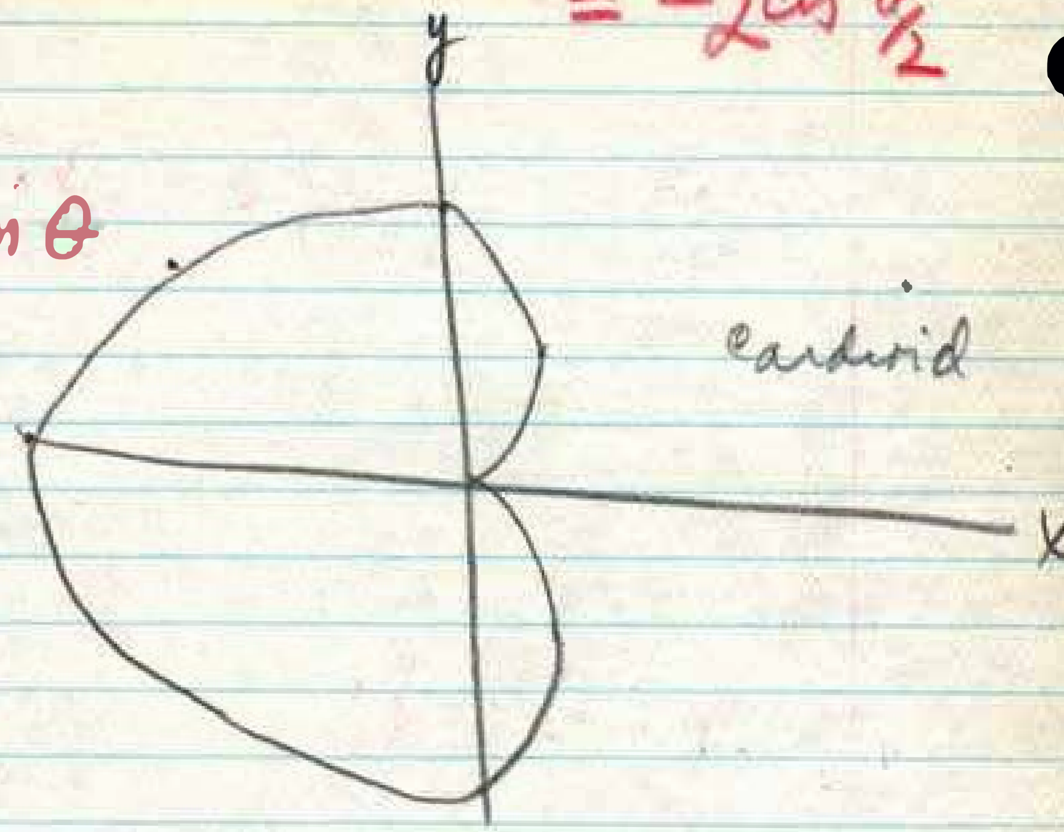
MrB

303) 3b $\quad r = a(1-\cos\theta)$

$\int \sin\frac{\theta}{2}\,d\theta = -2\cos\frac{\theta}{2}$

θ	$\cos\theta$	r
$0°$	1	0
$60°$	.5	$a/2$
$90°$	0	a
$120°$	$-.5$	$\frac{3}{2}a$
$180°$	-1	$2a$
$240°$	$-.5$	$\frac{3}{2}a$
$270°$	0	a
$300°$	.5	$a/2$
$360°$	1	0

$\frac{dr}{d\theta} = a\sin\theta$



$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

$$S = \int_0^{2\pi} \sqrt{(a-a\cos\theta)^2 + (\cancel{a} + a\sin\theta)^2}\,d\theta$$

$$= \int_0^{2\pi} \sqrt{(a^2 - 2a^2\cos\theta + a^2\cos^2\theta + \cancel{a^2} + \cancel{2a^2\sin\theta} + a^2\sin^2\theta)}\,d\theta$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

$2\sin^2\frac{\theta}{2} = 1-\cos\theta$

$\sqrt{2}\sin\frac{\theta}{2} = \sqrt{1-\cos\theta}$

$$= \int_0^{2\pi} \sqrt{\cancel{(-2a^2\cos\theta + a^2\cos^2\theta + 2a^2\sin\theta + a^2\sin^2\theta)}}\,d\theta$$

$2a^2 - 2a^2\cos\theta$

$$\cancel{= a\int_0^{2\pi} \sqrt{(-2\cos\theta + \cos^2\theta + 2\sin\theta + \sin^2\theta)}\,d\theta}$$

$$\cancel{= a\int_0^{\pi} \sqrt{[2(\sin\theta - \cos\theta) + 1]}\,d\theta}$$

$$= a\sqrt{2}\int_0^{2\pi} \sqrt{1-\cos\theta}\,d\theta = a\sqrt{2}\cdot\sqrt{2}\int_0^{2\pi} \sin\frac{\theta}{2}\,d\theta$$

$$= 2a\left[-2\cos\frac{\theta}{2}\right]_0^{2\pi} = 2a\left[2-(-2)\right] = 8a$$

M r B

303) 3 d

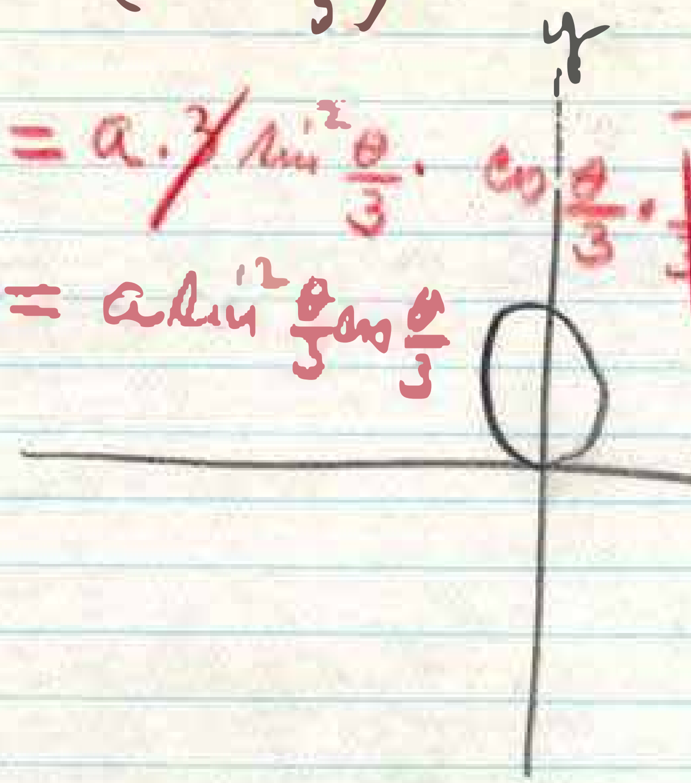
$\sin 60^\circ = \sqrt{3}/2$

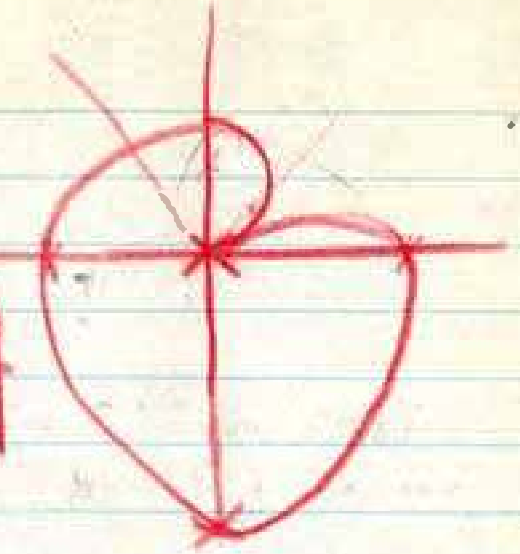
$r = a\left(\sin^{\cancel{3}}\frac{\theta}{3}\right)^3$

$\frac{dr}{d\theta} = a\cdot\cancel{3}\sin^2\frac{\theta}{3}\cdot\cos\frac{\theta}{3}\cdot\frac{1}{\cancel{3}}$

$= a\sin^2\frac{\theta}{3}\cos\frac{\theta}{3}$

θ	$\frac{\theta}{3}$	$\sin^3\frac{\theta}{3}$	r
0°	0°	0	0
45°	15°	$\frac{1}{64}$	$\frac{1}{64}a$
90°	30°	$\frac{1}{2}$	$\frac{1}{8}a$
180°	60°		$\rightarrow \frac{3\sqrt{3}}{8}a$
270°	90°	1.0	a
360°	120°		$\rightarrow \frac{3\sqrt{3}}{8}a$
450°	150°	.5	$\frac{1}{8}a$
540°	180°	0	0





$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$

$= \sqrt{a^2\sin^6\frac{\theta}{3} + \left(3a\sin^2\frac{\theta}{3}\right)^2}\,d\theta$

$= \sqrt{\left(a^2\sin^6\frac{\theta}{3} + 9a^2\sin^4\frac{\theta}{3}\right)}\,d\theta$

$S = \int_0^{\pi}\sqrt{a^2\left(\sin^6\frac{\theta}{3} + 9\sin^4\frac{\theta}{3}\right)}\,d\theta$

$= a\int_0^{\pi}\sqrt{\left(\sin^6\frac{\theta}{3} + 9\sin^4\frac{\theta}{3}\right)}\,d\theta$

$ds = \sqrt{a^2\sin^6\frac{\theta}{3} + a^2\sin^4\frac{\theta}{3}\cos^2\frac{\theta}{3}}\,d\theta$

$= \sqrt{a^2\sin^4\frac{\theta}{3}}\,d\theta = a\sin^2\frac{\theta}{3}\,d\theta$

$S = \frac{a}{\cancel{2}}\int_0^{3\pi}\frac{1-\cos\frac{2\theta}{3}}{\cancel{2}}\,d\theta = \frac{a}{2}\left[\theta - \frac{3}{2}\sin\frac{2\theta}{3}\right]_0^{\frac{3\pi}{2}} = \frac{a}{2}\left[\frac{3\pi}{2} - 0\right] = \frac{3\pi a}{4}$

3.46
3.46
2076
1384
1038
12.1716

$4\sqrt{x}$
$x\sqrt{x}$
$4x$

1.73
1
3
1
2.00
1.89
$11\ 00$

173
6
9.38

$4x$
$2x$

$\frac{4}{4} = 1$
$\frac{4}{2} = 2$
$\frac{4}{1} = 4$

$64 - 128$
$64 \quad \frac{4}{.1} = 40$
$\frac{4}{.01} = 400$

$\sqrt{x} = x^{\frac{1}{2}}$

$x^{\frac{1}{2}}$
x

$4x^{1/4}$
x

$\frac{4}{0} = x$
$x \cdot 0 = 4$

1.73
3
5.19

$16\overline{)112}$ 7

$$y^2 = (16t^2 - 8t^4 + t^6)$$

$$y = (4t - t^3)$$

t^2
t^2
t^2

$48 - 72 + 27$
27
75

$$\boxed{\boxed{\sqrt{x \cdot y}} = 4x - x^2}$$

$xy^2 = 16x^2 - 8x^3 + x^4$
$y^2 = 16x - 8x^2 + x^3$

373) 1b

$$\begin{cases} x = 2t^2 - 2 \\ y = t - 3 \end{cases}$$

$t = -1$

t	x	y
0	-2	-3
1	0	-2
2	6	-1
3	16	0
4	30	1
-1	0	-4
-2	6	-5
-3	16	-6
-4	30	-7

$x = 2t^2 - 2$

$2t^2 = x + 2$

$t^2 = \frac{x+2}{2}$

$y = t - 3$

$t = y + 3$

$t^2 = y^2 + 6y + 9$

$\frac{x+2}{2} = y^2 + 6y + 9$

$x + 2 = 2y^2 + 12y + 18$

Equation $\boxed{2y^2 + 12y - x + 16 = 0}$

$2(y^2 + 6y^{+9}) - x + 16 = 0^{+18}$

$2(y+3)^2 = x + 2$

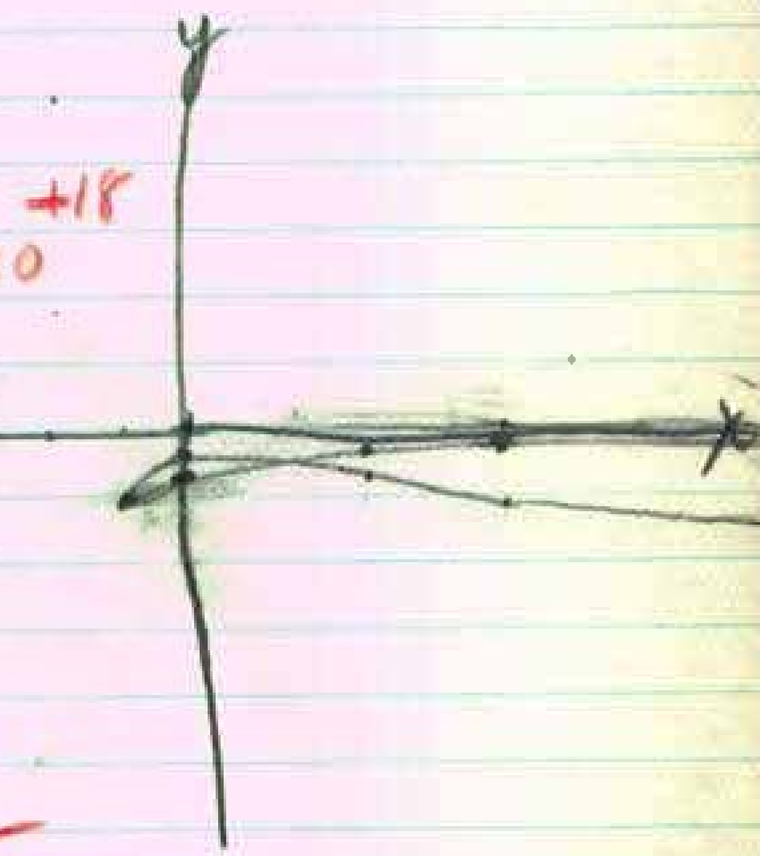
$(y+3)^2 = \frac{1}{2}(x+2)$

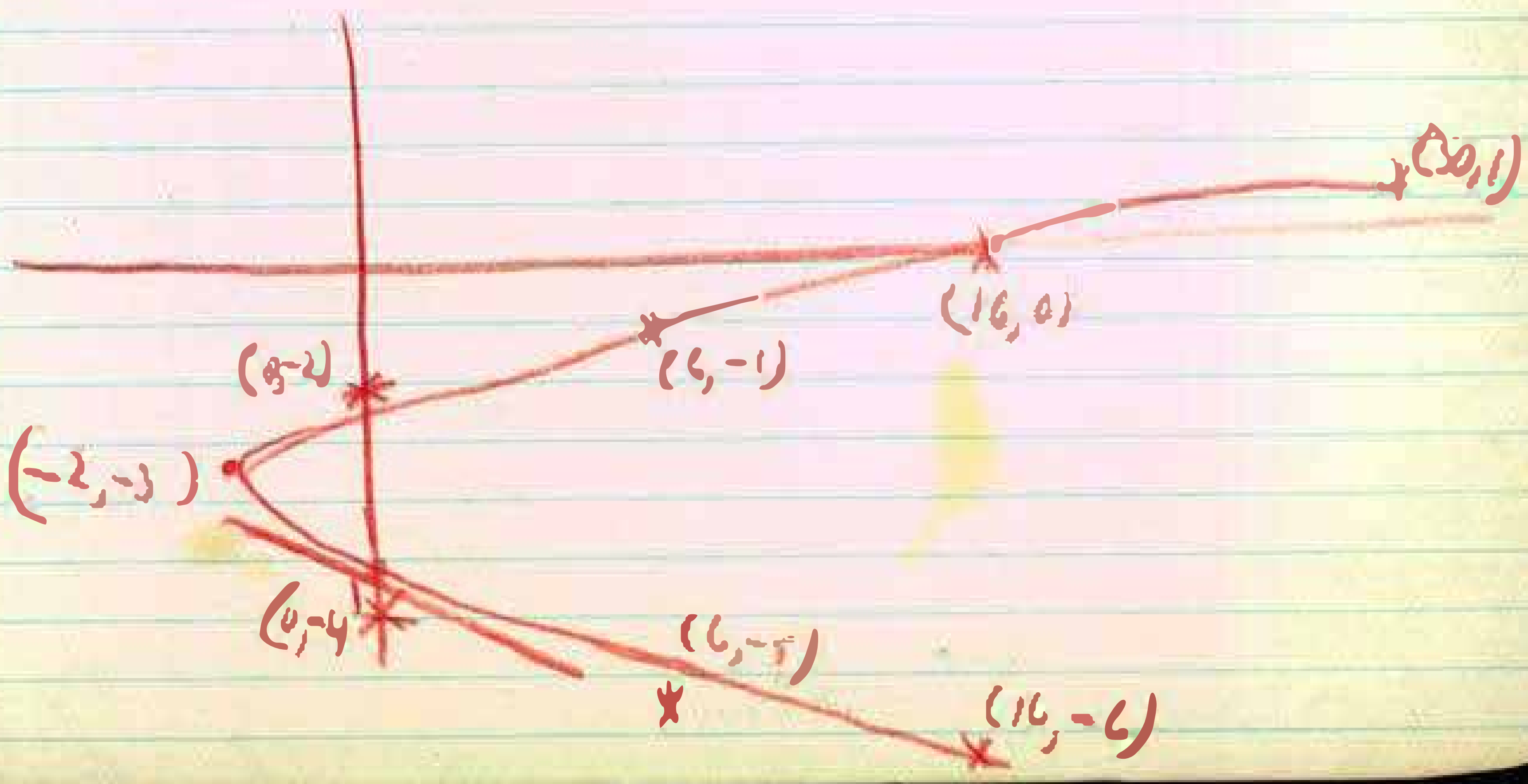
$\frac{dx}{dt} = 4t$

$\frac{dy}{dt} = 1$

$\frac{dy}{dx} = \frac{1}{4t}$ ✓

When $t = -1$, slope $= \frac{1}{-4} = -\frac{1}{4}$ ✓





373) 1c

$$\begin{cases} x = 3\cos\theta \\ y = \sin\theta \end{cases}$$

$$\frac{x}{3} = \cos\theta$$

$\theta = 60°$

$$\begin{cases} \frac{x^2}{9} = \cos^2\theta \\ y^2 = \sin^2\theta \end{cases}$$

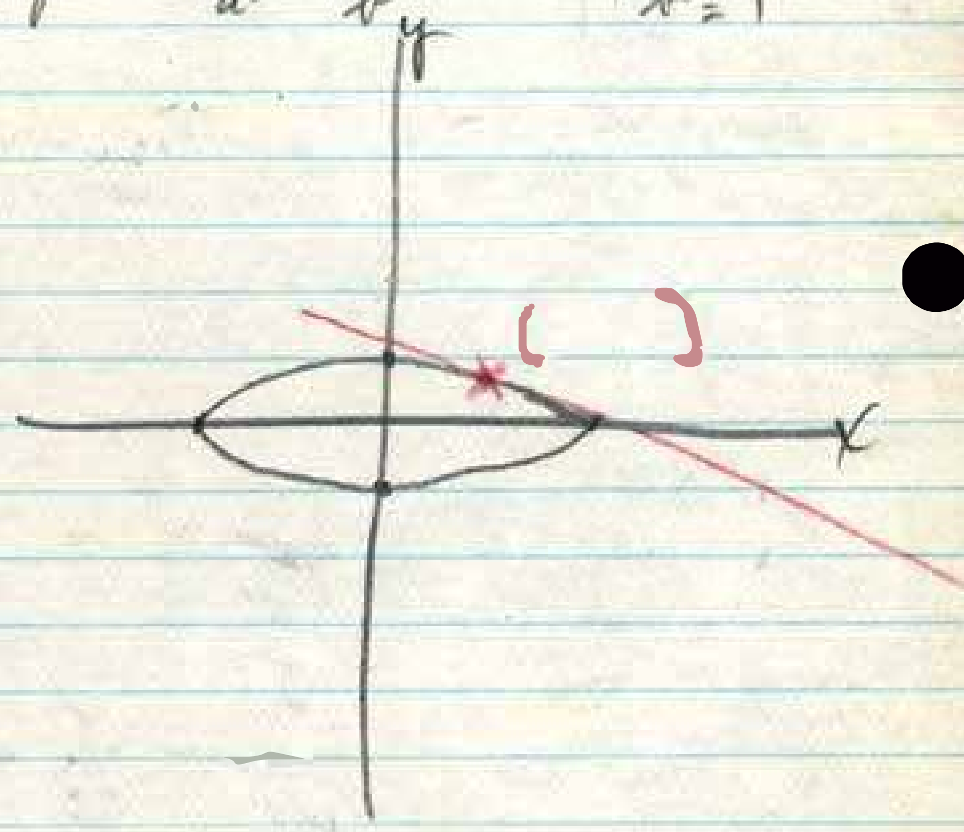
$$\frac{x^2}{9} + y^2 = \cos^2\theta + \sin^2\theta = 1$$

x	y
0	±1
±3	0

θ	x	y

Equation $x^2 + 9y^2 = 9$

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a = 3$, $b = 1$



When $\theta = 60°$, $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

$\cos\theta = \frac{1}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$$\frac{dx}{d\theta} = -3\sin\theta$$

$$\frac{dy}{d\theta} = \cos\theta$$

$$\frac{dy}{dx} = -\frac{\cos\theta}{3\sin\theta}$$

$$= -\frac{1}{3}\cot\theta$$

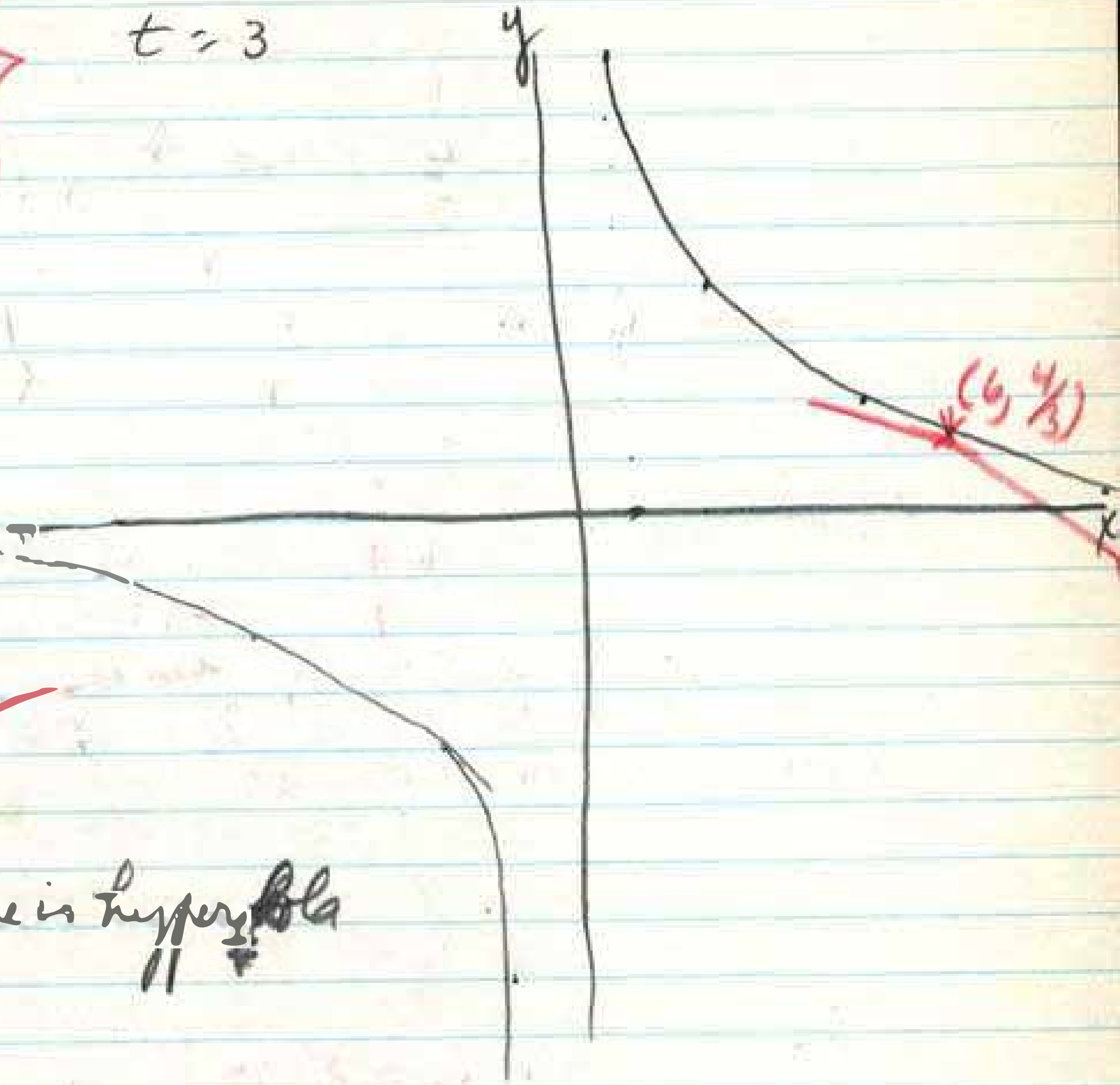
at $60° = \frac{dy}{dx} = -\frac{\frac{1}{2}}{\frac{3\sqrt{3}}{2}}$

$$= -\frac{1}{2} \cdot \frac{2}{3\sqrt{3}}$$

$$= -\frac{1}{3\sqrt{3}} = -\frac{\sqrt{3}}{9}$$

373) 1e

$$\left.\begin{aligned} x &= 2t \\ y &= \frac{4}{t} \end{aligned}\right\} \quad t = 3$$



$$t = \frac{x}{2}$$

$$t = \frac{4}{y}$$

$$\frac{x}{2} - \frac{4}{y} = 0$$

Equation $xy - 8 = 0$

x	y
1	8
2	4
4	2
8	1
−1	−8
−2	−4
−4	−2
−8	−1

Figure is hyperbola

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \frac{d}{dt} 4t^{-1} = -4t^{-2} = \frac{-4}{t^2}$$

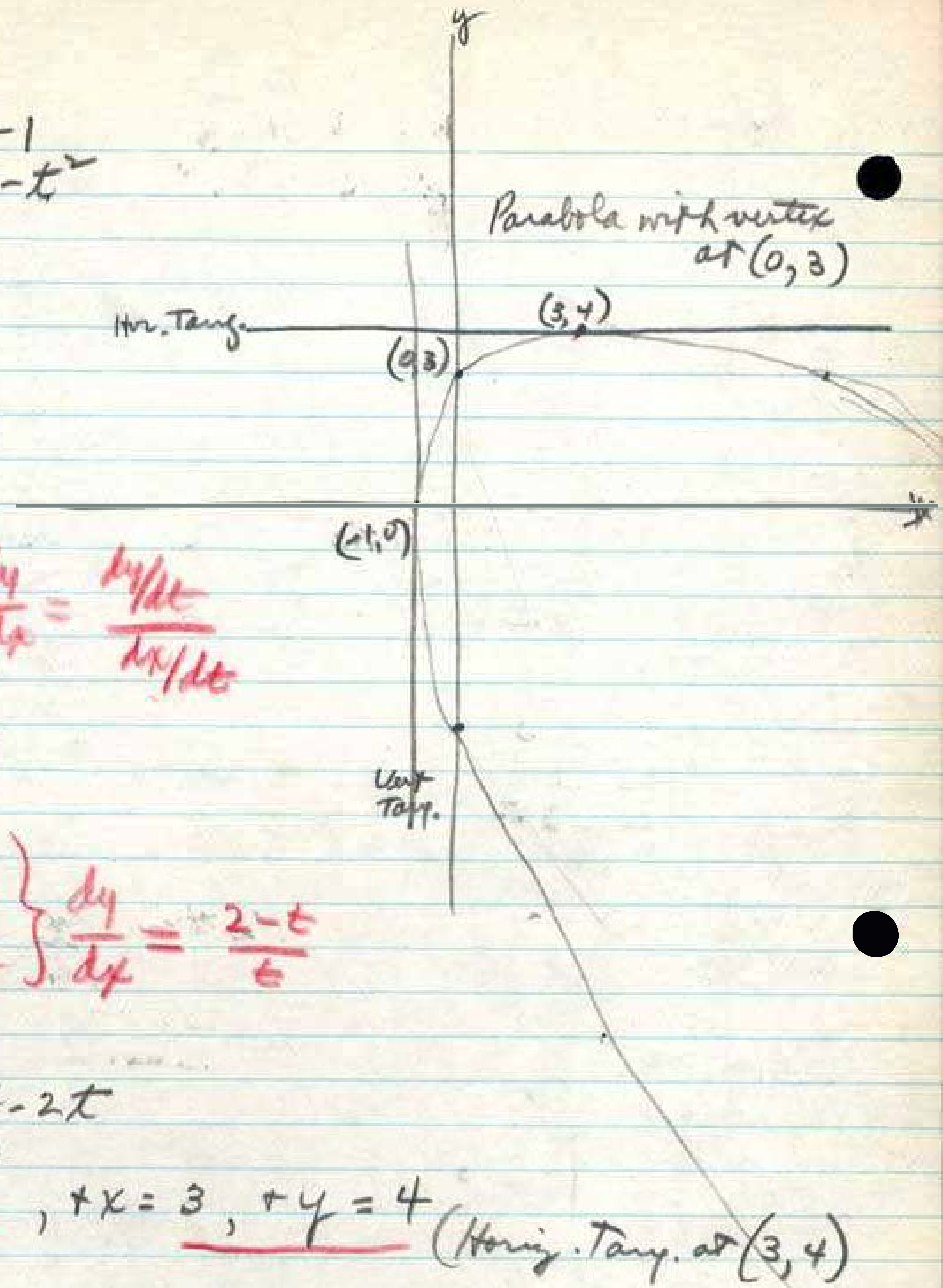
$$\frac{dy}{dx} = \frac{-4}{2t^2} = -\frac{2}{t^2}$$

when $t = 3$, slope $= -\frac{2}{9}$

373) 4a

$x = t^2 - 1$
$y = 4t - t^2$

t	x	y
0	-1	0
1	0	3
2	3	4
3	8	3
4	15	0
-1	0	-5
-2	3	-12
-3	8	-21



$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dx}{dt} = 2t \checkmark$$

$$\frac{dy}{dt} = 4 - 2t \checkmark \quad \Big\} \frac{dy}{dx} = \frac{2-t}{t}$$

If $\frac{dy}{\cancel{dx}} = 0 \Rightarrow 0 = 4 - 2t$

$-2t = -4$

$t = 2$, $+ x = 3$, $+ y = 4$ (Horiz. Tang. at (3, 4) $\checkmark$

If ~~$\frac{dx}{dt} = 0$~~ $\rightarrow t = 0$, $x = -1$, $y = 0$ (Vert. Tang. at $(-1, 0)$

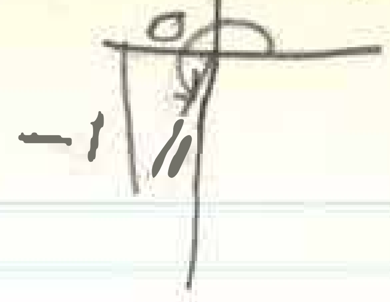
If $\frac{dy}{dx} = \infty$, $t = 0$

$t^2 = x + 1$, $t = \pm\sqrt{x+1}$

$y = 4\sqrt{x+1} - x - 1$	$y = -4\sqrt{x+1} - x - 1$
$y + x + 1 = 4\sqrt{x+1}$	$y + x + 1 = -4\sqrt{x+1}$
$y^2 + x^2 + 1 + 2xy + 2x + 2y = 16x + 16$	
$\boxed{x^2 + y^2 + 2xy - 14x + 2y - 15 = 0}$	

373) 4 e

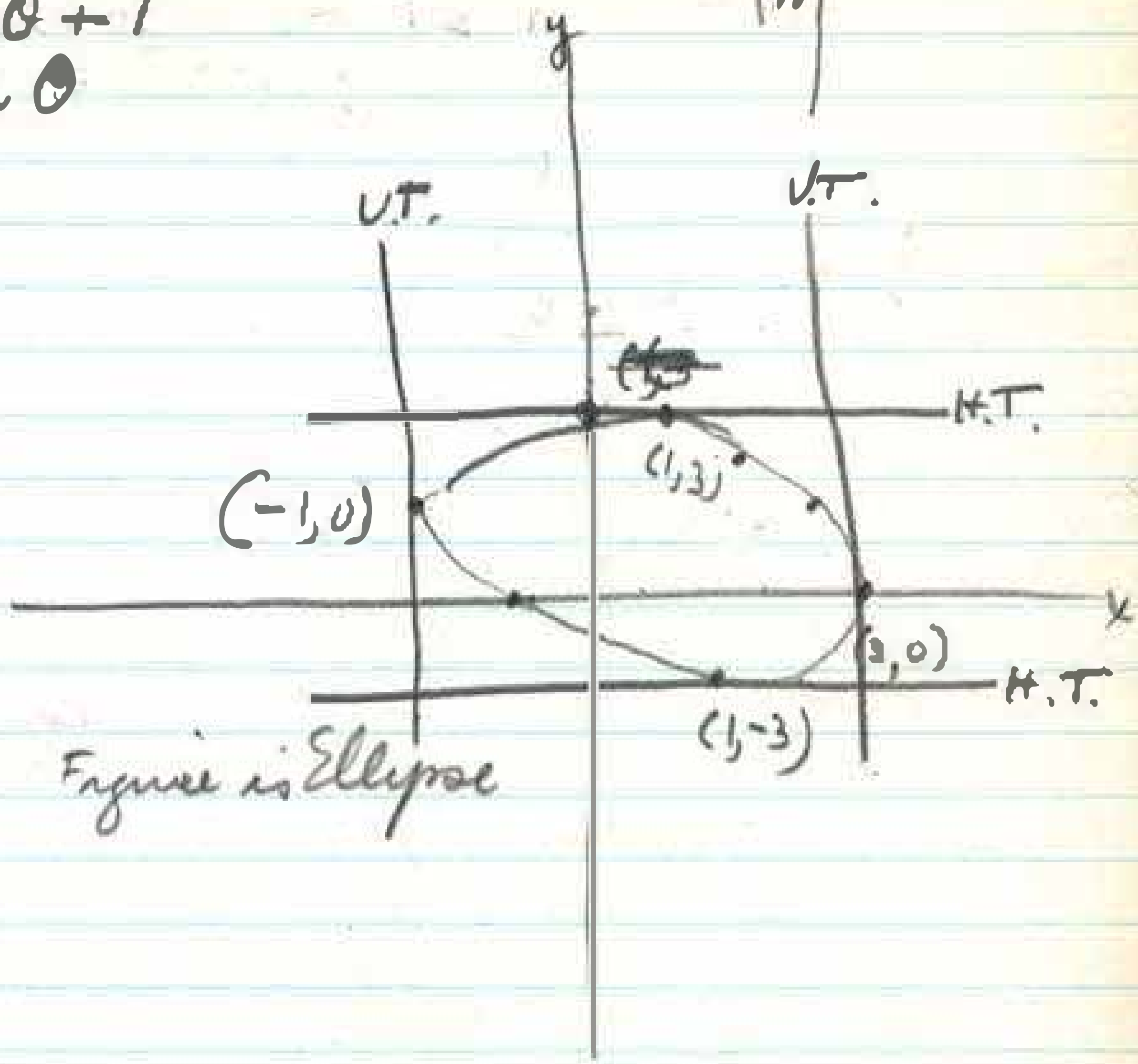
$$x = 2\cos\theta + 1$$
$$y = 3\sin\theta$$



$$\frac{dx}{d\theta} = -2\sin\theta$$

$$\frac{dy}{d\theta} = 3\cos\theta$$

$$dy/dx = -\frac{3}{2}\,\frac{\cos\theta}{\sin\theta}$$



θ	x	y
0°	3	0
30°	2.8	1.5
60°	2	$\frac{3}{2}\sqrt{3}$
90°	1	3
120°	0	2.6
150°	−2.8	1.5
180°	−1	0
270°	1	−3
240°	2	$-\frac{3}{2}\sqrt{3}$
360°		0

Setting $\frac{dx}{d\theta} = 0$, $-2\sin\theta = 0$

Slope infin

$\sin\theta = 0$

Therefore $\theta = 0°, 180°$

If $\frac{dy}{dx} = 0$, $3\cos\theta = 0$

$\cos\theta = 0$

Therefore $\theta = 90°, 270°$

$$\frac{dy}{dx} = \left(\frac{dy/dt}{dx/dt}\right)$$

Discuss examples page 372

Ex. pg. 372

$$\frac{dy}{dx} = \frac{\cos\theta - \cos 2\theta}{-\sin\theta + \sin 2\theta}$$

Hor. tan. $\rightarrow$ $\cos\theta - \cos 2\theta = 0$

$$\cos\theta - (\cos^2\theta - \sin^2\theta) = 0$$

$$\cos\theta - \cos^2\theta + \sin^2\theta = 0$$

$$\cos\theta - \cos^2\theta + 1 - \cos^2\theta = 0$$

$$-2\cos^2\theta + \cos\theta + 1 = 0$$

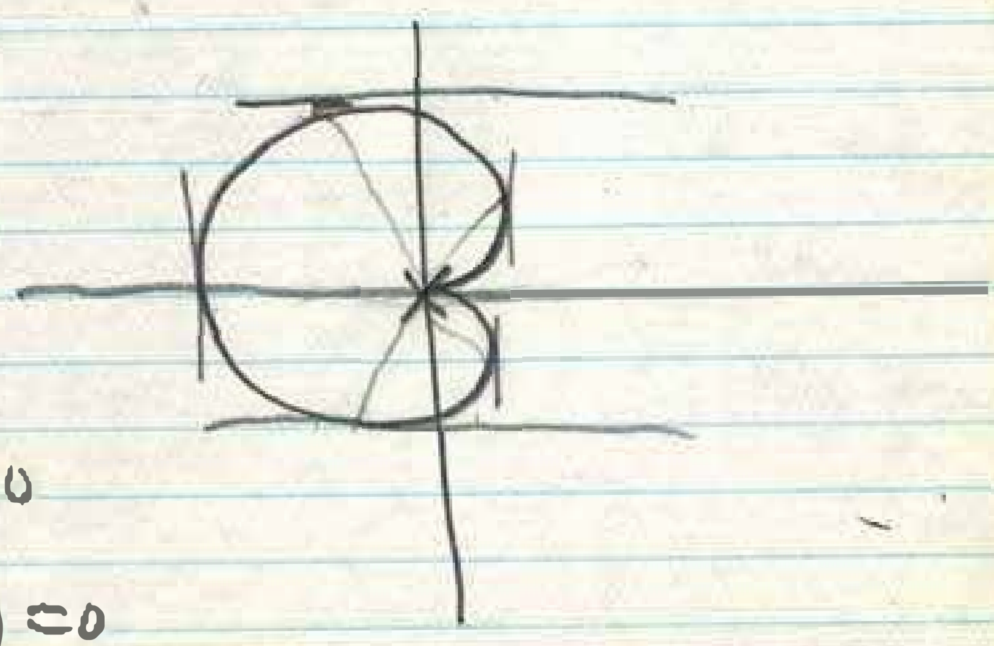
$$2\cos^2\theta - \cos\theta - 1 = 0$$

$$\cos\theta = \frac{1 \pm \sqrt{1+8}}{4}$$

$\cos\theta = 1 \longrightarrow \theta = 0^\circ$

$\cos\theta = -1/2 \longrightarrow \theta = 120^\circ; 240^\circ$

θ	x	y
0°	0	0
120°		



vert. tang. $\rightarrow -\sin\theta + \sin 2\theta = 0$

$$-\sin\theta + 2\sin\theta\cos\theta = 0$$

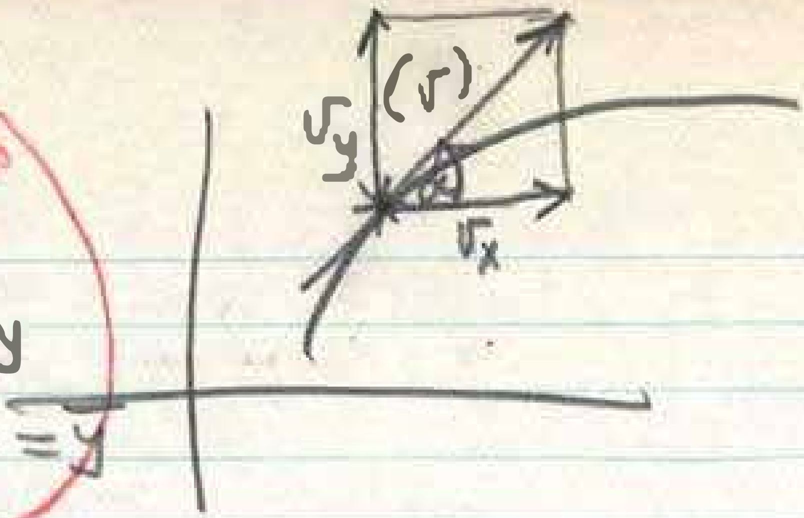
$$\sin\theta(-1 + 2\cos\theta) = 0$$

$\sin\theta = 0^\circ$, $\theta = 0^\circ; 180^\circ$

$\cos\theta = 1/2$ $\theta = 60^\circ, 300^\circ$

Centroid

$$\frac{\int (S(1.\text{area}))\, y}{\text{Area}} = \bar{y}$$



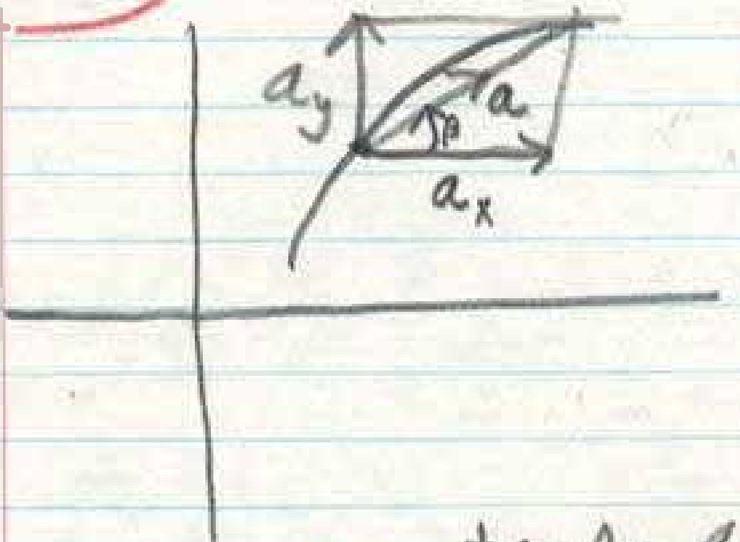
Centroid

$$\frac{\int x\, y\, dx}{\int y\, dx} = \bar{x}$$

$$v_x = \frac{dx}{dt} \qquad \tan\alpha = \frac{v_y}{v_x}$$

$$v_y = dy/dt$$

$$v = \sqrt{v_x^2 + v_y^2}$$

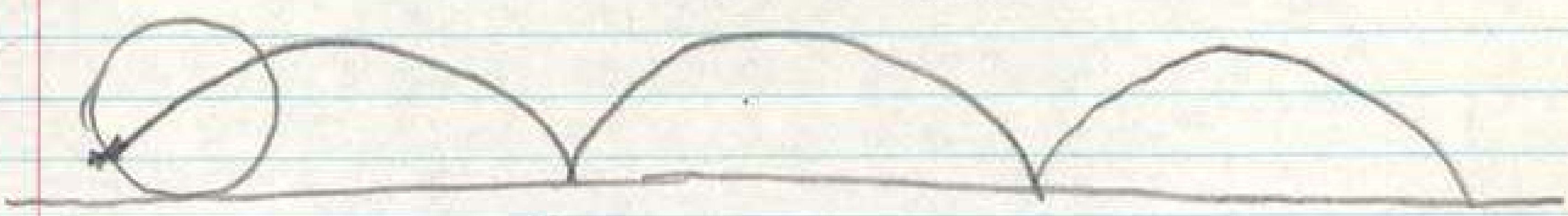


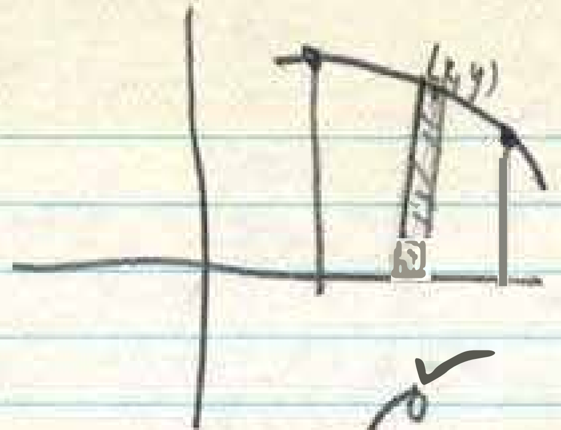
$$\tan\beta = \frac{a_y}{a_x} \qquad \begin{cases} a_x = \dfrac{dv_x}{dt} \\ a_y = \dfrac{dv_y}{dt} \end{cases}$$

$$a = \sqrt{a_x^2 + a_y^2}$$

380/1, 2, 6 b, 7, 8 383/1, 2, 3, 4, 5 Read §227

→ 386/2, 7a, b, c (cycloid)





$$y = t^2 + 1$$

$$x = t^3 - t$$

$$A = \int_{\checkmark}^{\checkmark} y\,dx = \int_{t=\checkmark}^{t=\checkmark} (t^2+1)(3t^2-1)\,dt$$

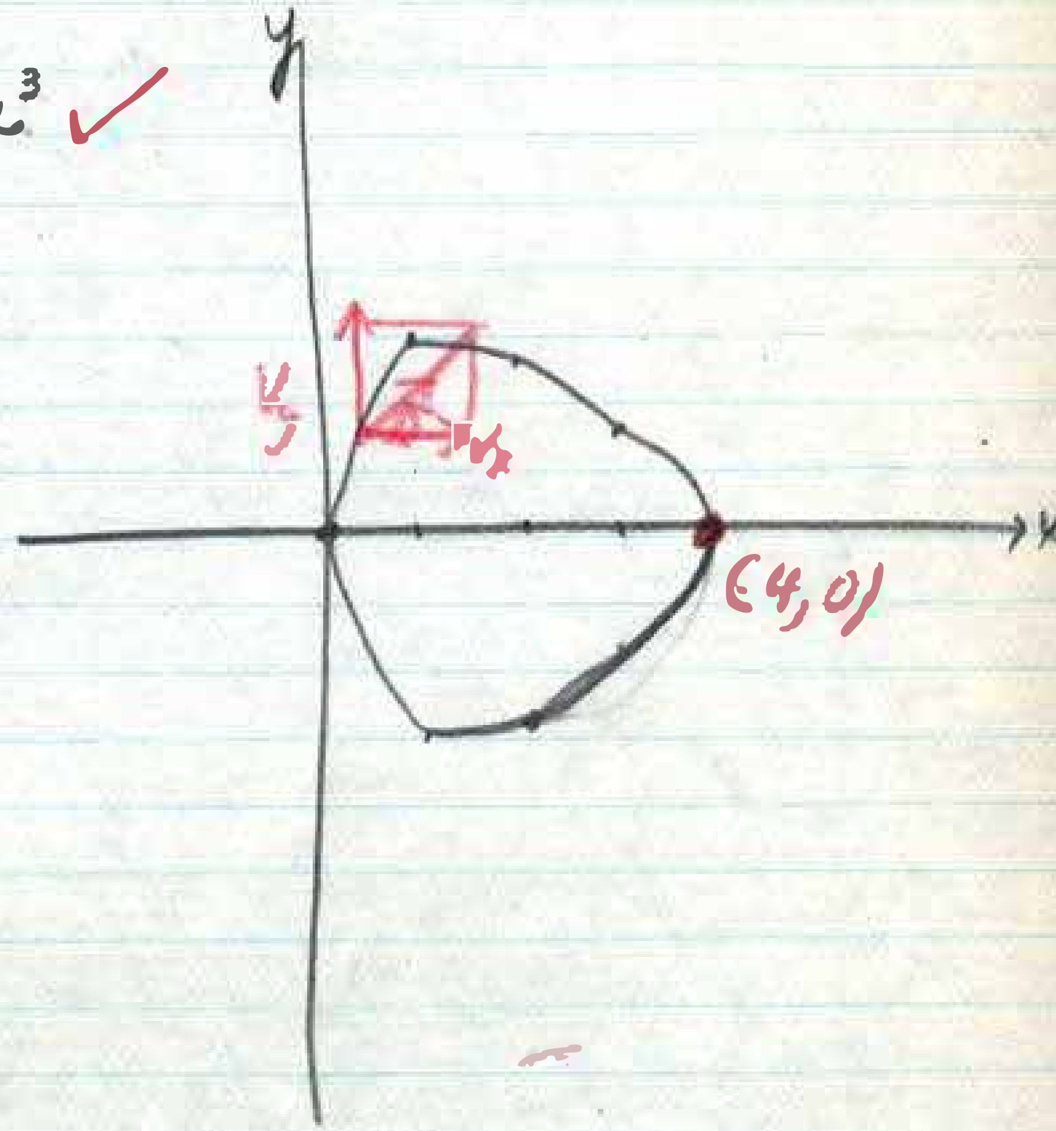
Page 380) 1

a) $\begin{cases} x = t^2 \\ y = 4t - t^3 \end{cases}$ — $t = \pm\sqrt{x}$

$y = 4\sqrt{x} - x\sqrt{x}$

$y^2 = 16x - 8x^2 + x^3$ ✓

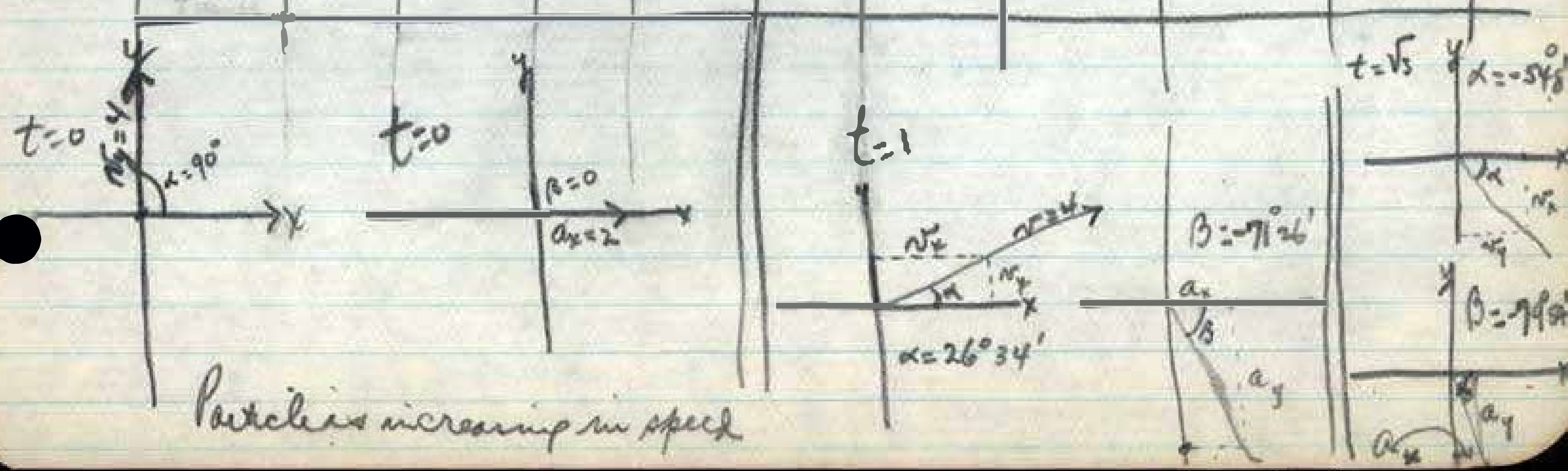
y	x
0	0
± 3	1
$\pm\sqrt{8}$	2
$\pm\sqrt{3}$	3
0	4



b) $v_x = 2t$ | $a_x = 2$

$v_y = 4 - 3t^2$ ✓ | $a_y = -6t$ ✓

t	x	y	v_x	v_y	a_x	a_y	$\tan\alpha$	$\tan\beta$	v	a
When $t = 0$	0	0	0	4	2	0	∞	0	4	2
1	1	3	2	1	2	-6	$\frac{1}{2}$	-3	$\sqrt{5}$	$2\sqrt{10}$
$\sqrt{3}$	3	2.07	3.46	-5	2	$-6\sqrt{3}$	$-1\frac{1}{3}$	$-3\sqrt{3}$	12.17	$4\sqrt{7}$



Particle is increasing in speed

380) 1d Given $v = 4\sqrt{5}$ units per second

But $v = \sqrt{16 - 20t^2 + 9t^4}$

$$4\sqrt{5} = \sqrt{16 - 20t^2 + 9t^4}$$

$$80 = 16 - 20t^2 + 9t^4$$

$$9t^4 - 20t^2 - 64 = 0$$

Substituting $(x = t^2)$ $= 9x^2 - 20x - 64 = 0$

$a = 9$
$b = -20$
$c = -64$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{20 \pm \sqrt{400 + 2304}}{18}$$

$$x = \frac{20 \pm \cancel{41.28}\ 52}{18} = \cancel{\frac{63.28}{18} = 3.4}\ 4$$

$t = 2 \rightarrow x = 4$, $y = 0$ or $\cancel{\frac{-21.28}{18} = -1.19} = \cancel{\ }$

52
52
104
260
2704

36
64
144
216
2304

$+\ y^2 = 16x - 8x^2 + x^3$
or $x(4-x)^2$

$y^2 = 3.4(4. - 3.4)^2 = 3.4(.36) = 1.244$; $y = \pm\sqrt{1.244} = \pm .352$

$y^2 = -1.19(4 + 1.19)^2 = -1.19(5.19)^2 = -$ (quant) $\therefore$ discarded

Therefore, when velocity $= 4\sqrt{5}$ units per sec. / particle is at $(3.4, .352)$ or $(3.4, -.352)$

380) 1e $a = +\sqrt{a_x^2 + a_y^2} = +\sqrt{4 + 36t^2}$

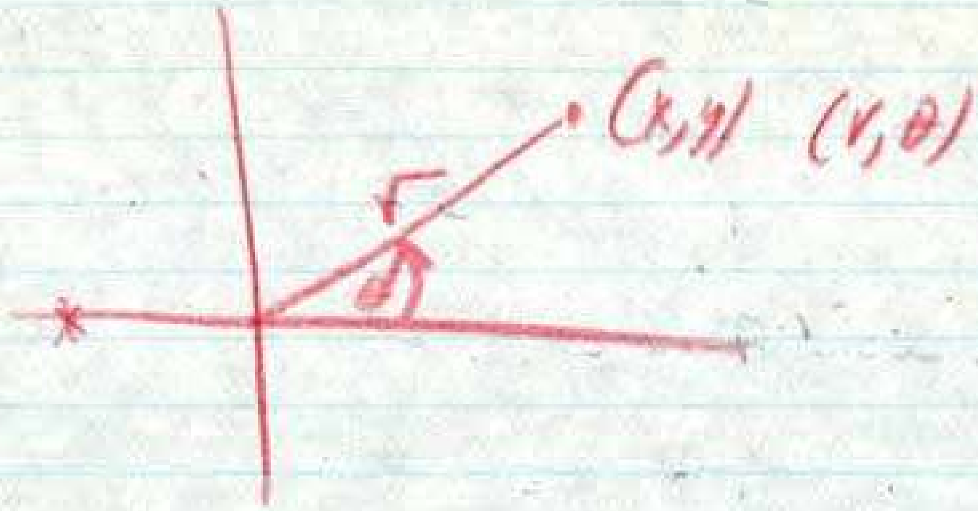
$$\frac{da}{dt} = \frac{72t}{2\sqrt{4 + 36t^2}}$$

~~This is least~~ when $t = 0$

Setting $\frac{da}{dt} = 0$, $72t = 0$

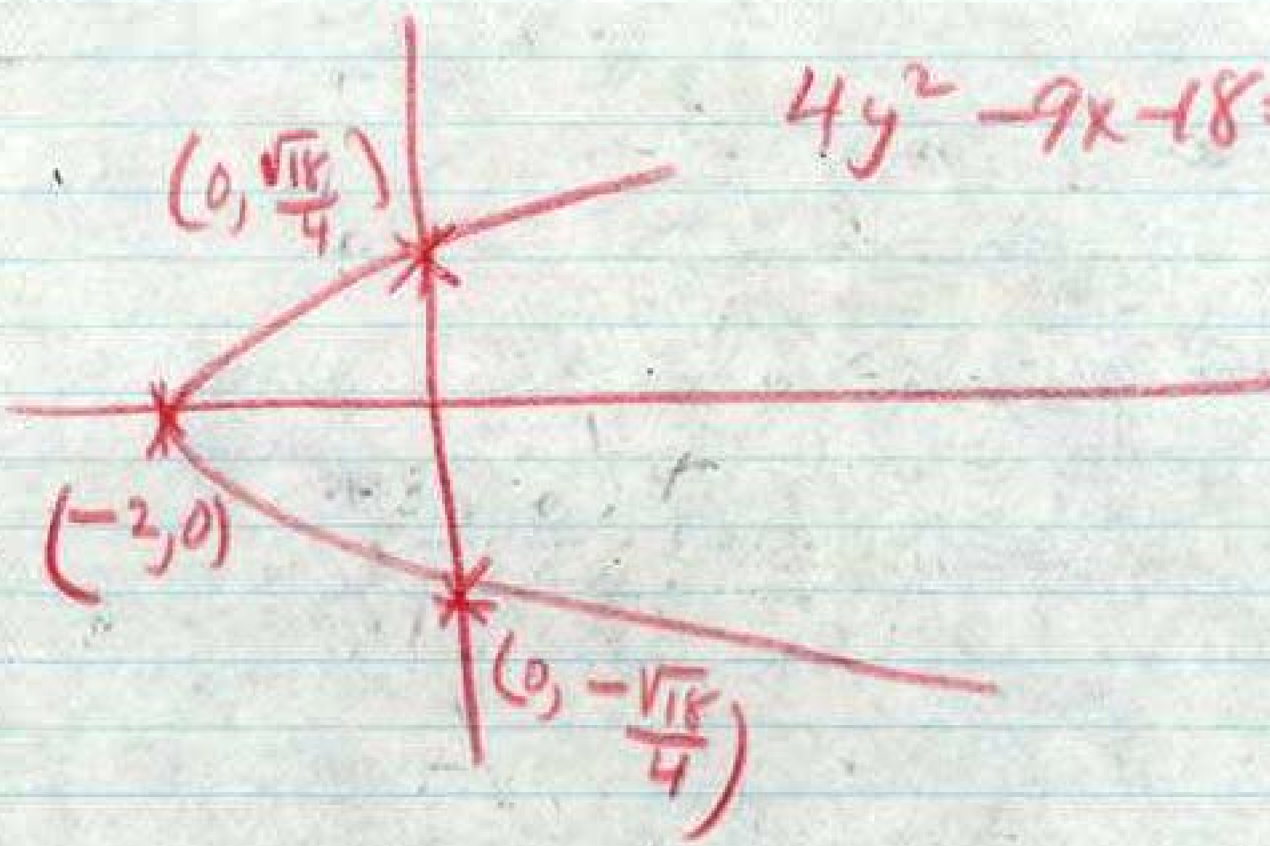
$t = 0$

$\therefore$ Acceleration is least when $t = 0$



$t=2, \quad x = 2\cos 4$

$4y^2 - 9x - 18 = 0$



38)1c

$$v = +\sqrt{v_y^2 + v_x^2} = +\sqrt{(4-3t^2)^2 + (2t)^2}$$

$$= +\sqrt{16 - 24t^2 + 9t^4 + 4t^2}$$

$$= +\sqrt{16 - 20t^2 + 9t^4}$$

$$\frac{dv}{dt} = \frac{-40t + 36t^3}{2\sqrt{16 - 20t^2 + 9t^4}}$$

Setting $\frac{dv}{dt} = 0$, $-40t + 36t^3 = 0$, $t(-10 + 9t^2) = 0$

$$-10 + 9t^2 = 0$$

$t = 0$

$t = \frac{\sqrt{10}}{3}$

$\cancel{t = -\frac{\sqrt{10}}{3}}$

$$9t^2 = 10$$

$$t^2 = \frac{10}{9}$$

$$t = \sqrt{\frac{10}{9}} = \frac{1}{3}\sqrt{10}$$

~~$t<0$~~	dv/dt
$\frac{\sqrt{10}}{3} > t > 0$	$-$
$t > \frac{\sqrt{10}}{3}$	$+$

Speed would be minimum when $t = \frac{1}{3}\sqrt{10}$ sec.

380) 2a $\quad 4y^2 - 9x - 18 = 0$ ✓

$$\cancel{\theta = \frac{x}{\sqrt{x^2+y^2}}}$$

$$4(3\cos t)^2 - 9\left(4\cos^2 t - 2\right) - 18 = 0$$

$$36\cos^2 t - 36\cos^2 t + 18 - 18 = 0$$

or $\quad 4\left(3\cdot\frac{x}{\sqrt{x^2+y^2}}\right)^2 - 9\left[4\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 - 2\right] - 18 = 0$

$$\frac{36\,x}{x^2+y^2} - \frac{36\,x}{x^2+y^2} + 18 - 18 = 0$$

$x = 2\cos 2t$

$y = 3\cos t$

$\cos 2t = 2\cos^2 t - 1$

$x = 4\cos^2 t - 2$

$\cos t = y/3$

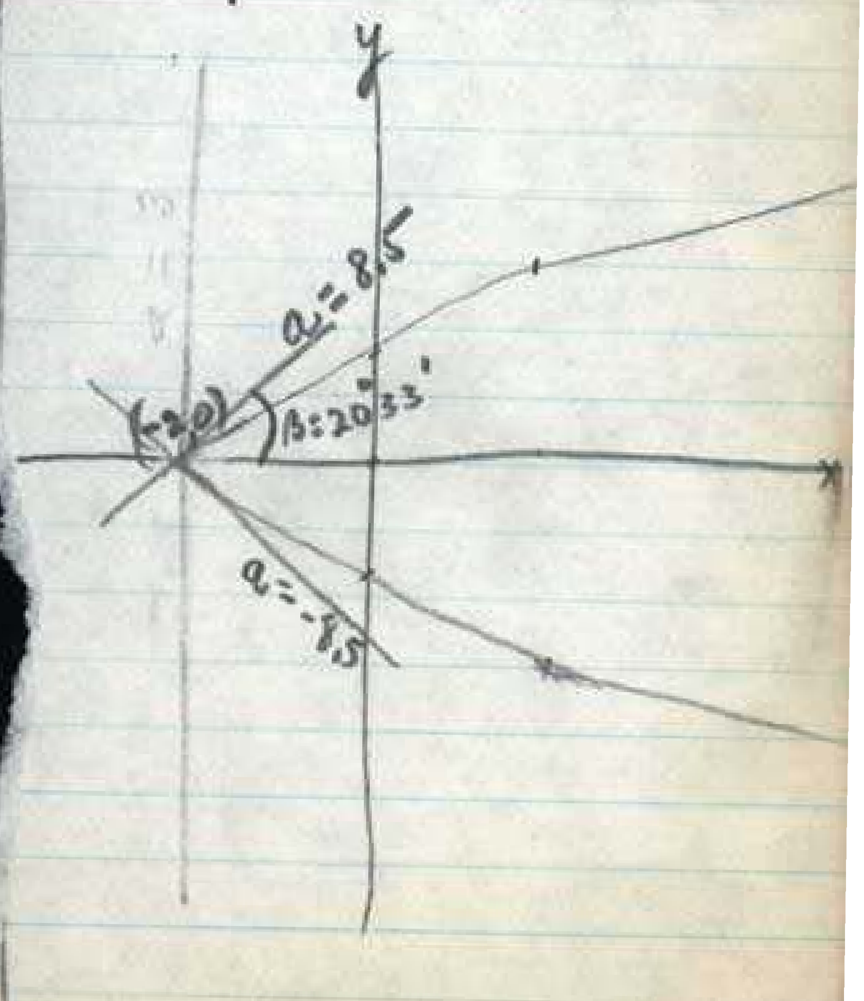
$x = \frac{4y^2}{9} - 2$

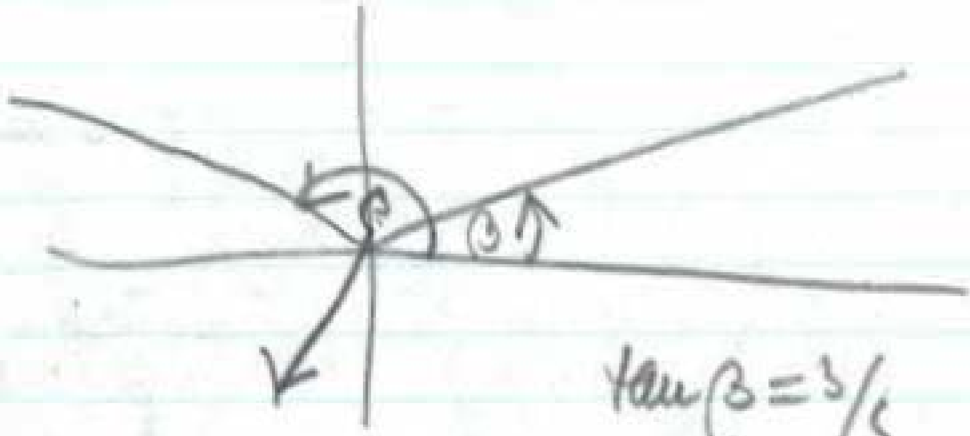
$9x = 4y^2 - 18$

f.e.d.

x	y
-2	0
2	± 3
0	$\pm \frac{3\sqrt{2}}{2}$

$$y^2 = \frac{9x+18}{4}$$





$\tan\beta = 3/2$

$\tan\beta = -3/8$

$$\frac{dr}{dt} = \frac{18\cos t + \ldots}{2\sqrt{9\sin^2 \ldots}}$$

Setting $\frac{dr}{dt} = 0$, 18 ...

18 c...

18 c...

9 c...

3 ...

$a = 3r$

$b = 9$

$c = -16$

cos ...

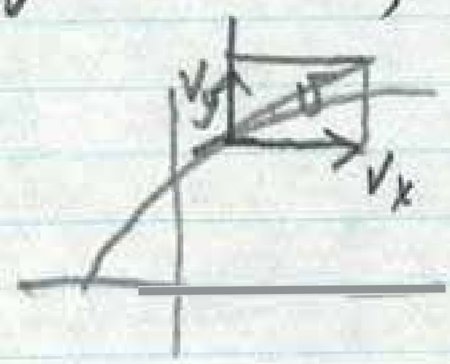
$t = \ldots$

380) 2b

$v_x = \frac{d}{dt}(2\cos 2t) = -4\sin 2t$ ✓

$v_y = \frac{d}{dt}(3\cos t) = -3\sin t$ ✓

If $v = 0$, $\cancel{v = \frac{v_y}{v_x} = \frac{-3\sin t}{4\sin 2t}} = 0$



$v = \sqrt{16\sin^2 2t + 9\sin^2 t}$ * $-3\sin t = 0$

$\sin t = 0$

$t = 0$

Then at $v = 0$, $t = 0$

$a_x = -4\cos 2t(2) = -8\cos 2t = -8$ (when $t = 0$, $\cos 0 = 1$)

$a_y = -3\cos t = -3$ (when $t = 0$, $\cos 0 = 1$)

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{64+9} = \sqrt{73} = 8.5$

$\tan\beta = \frac{a_y}{a_x} = \frac{-3}{-8} = 3/8$ $(20°33')$

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* $16\sin^2 2t + 9\sin^2 t = 0$ $\qquad \sin 2t = 2\sin t \cdot \cos t$

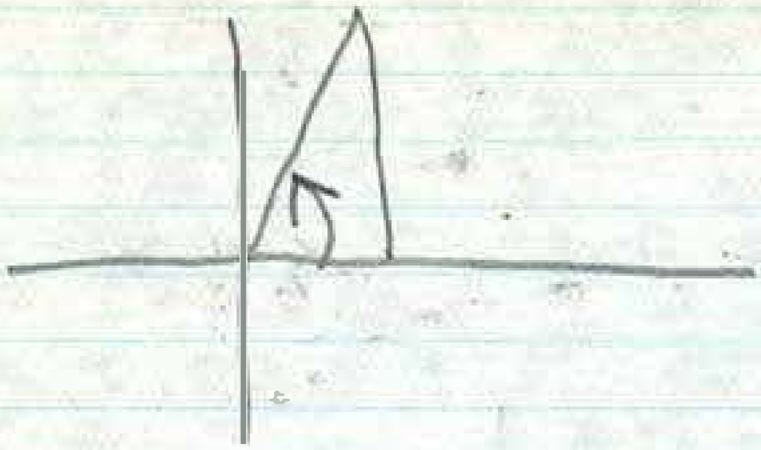
$64\sin^2 t \cdot \cos^2 t + 9\sin^2 t = 0$ $\qquad$ when $t = \pi$,

$a_x = -8$

$\sin^2 t(\underbrace{64\cos^2 t + 9}) = 0$ $\qquad a_y = 3$

$\sin^2 t = 0$, $\sin t = 0$, $t = 0, \pi, 2\pi, 3\pi, ---$ $\qquad \tan\beta = -3/8$

~~tan~~



$$\frac{0}{0} = 15\overline{0}$$

$$\left(\frac{-3\sin t}{-4\sin 2t}\right)_{t=0} = \left(\frac{3\,\cancel{\sin t}}{4\cdot 2\,\cancel{\sin t}\cos t}\right)_{t=0} = 3/8$$