

J O S E P H S C H I L L I N G E R

C O R R E S P O N D E N C E C O U R S E

Subject: Music

Lesson LVII.

VI. Climax and Resistance

The projection of melody is a mechanical trajectory. Its kinetic components are balance, impetus and inertia. Resistance produces impetus, leading either towards the climax, which is a pt (pitch-time) maximum with respect to primary axis, or towards the balance. The impetus is caused by resistance which results from rotation. The geometrical projection of rotation is a circle which extends itself in time projection into a cylindrical or spherical spiral, or ultimately (through time extension) wave motion (plane projection). The kinetic result of rotary motion is centrifugal energy. The discharge of accumulated centrifugal energy is equivalent to a climax. A heavy object attached to a string and put into rotary motion about an axis-point develops considerable energy to move a long distance, when detached from the string.

Overcoming inertia increases mechanical efficiency (gain of kinetic energy). Any body set to move acquires its possible ultimate speed in a certain period of time. The shorter the period from the moment



of the application of the initial force (impetus) till the moment when the body acquires its ultimate speed, the greater is the mechanical efficiency of such motion. Motion is expressible in the wave amplitudes and the projection of kinetic climax is the maximum amplitude. Inert matter does not acquire its maximum amplitude instantaneously when starting from balance, just as the maximum cannot recede to balance (rest) instantaneously. This concerns both velocities (frequencies) and amplitudes.

Mechanical experiences, whether instinctive or intentional, are known to all types of zoological species and are inherited and perfected through the course of evolution. A grown-up animal has a perfect judgment of distances and directions and of the amount of muscular energy necessary to cover them in leaps or flights, without any theoretical knowledge of the law of gravity or mechanics in general. There is no misjudgment in the monkey's flights from tree to tree, or a gazelle leaping over a creek, or an eagle falling on its prey. A certain amount of intentional mechanical efficiency and psycho-physiologic coordination is inherent with every surviving species of the animal world. The relativity of the standards of mechanical efficiency corresponds to the relativity of reflexes, reactions and judgments. The leaping of a human being over a 14



foot rod is the highest achievement in the International Olympics for 1936, and this with the aid of a pole. The mechanical efficiency of an ordinary flea is fifty times greater. A leap of a human being over a rod 50 feet high would seem supernatural, while the respective leap of a flea would be below any low standards of efficiency (the flea leaps about one hundred times its own size).

The standards of mechanical efficiency vary with ages and places, even among human beings. They also vary with different races as well as with different ages. The development of athletic qualities and forms of locomotion imply the raising of the requirements toward the trends of mechanical efficiency.

Geometrical conception of mechanical and bio-mechanical trajectories necessitates the analysis of the corresponding trajectories of nervous impulses and muscular reactions. There are correspondences between the two, and the knowledge of such correspondences leads to scientific production of excitors (in this case, esthetic: music in general, or melody in particular) capable of stimulating the intended reactions. Simple reflexes and reactions project themselves into simple trajectorial patterns; on the other hand excitors having the form of simple trajectories stimulate reactions of the corresponding simplicity. Likewise, this correspondence takes place with the complex patterns.



The intensity interdependence between the excitor and the reaction was formulated in Weber's and Fechner's Psycho-Physiological law. Thus, both the configurations (patterns) and the amplitudes (intensities) have their correspondences between the excitors and the reactions. Judgment based on mechanical experience and mechanical orientation leads higher animals and human beings to certain expectations. In the case of an absolute correspondence between the realization of a mechanical process and the expectation, the resulting reaction is balance (normal satisfaction). A result above expectation stimulates the intensification of activity (positive reaction) and at its extreme, ecstasy. On the other hand, the result of a mechanical process which is below expectation stimulates passivity (negative reaction) and at its extreme, depression. The two opposite poles of reactions, led to their absolute limit, stimulate astonishment, (irrational or zero reaction).

Geometrical projection of the scale of psychological adjectives on a circumference produces the poles of the two rectangular coordinates (the diameters of the circle): 1. normal - absurd; 2. depressing -ecstatic. Producing four new poles on the intermediate arcs of the circumference through addition of another pair of rectangular coordinates





(under  $45^\circ$  to the original pair) we obtain nine poles altogether (including both  $0^\circ$  and  $360^\circ$ ). These nine poles, through the application of the method of evolving concept series, become expressible in adjectives standing for the psychological categories.

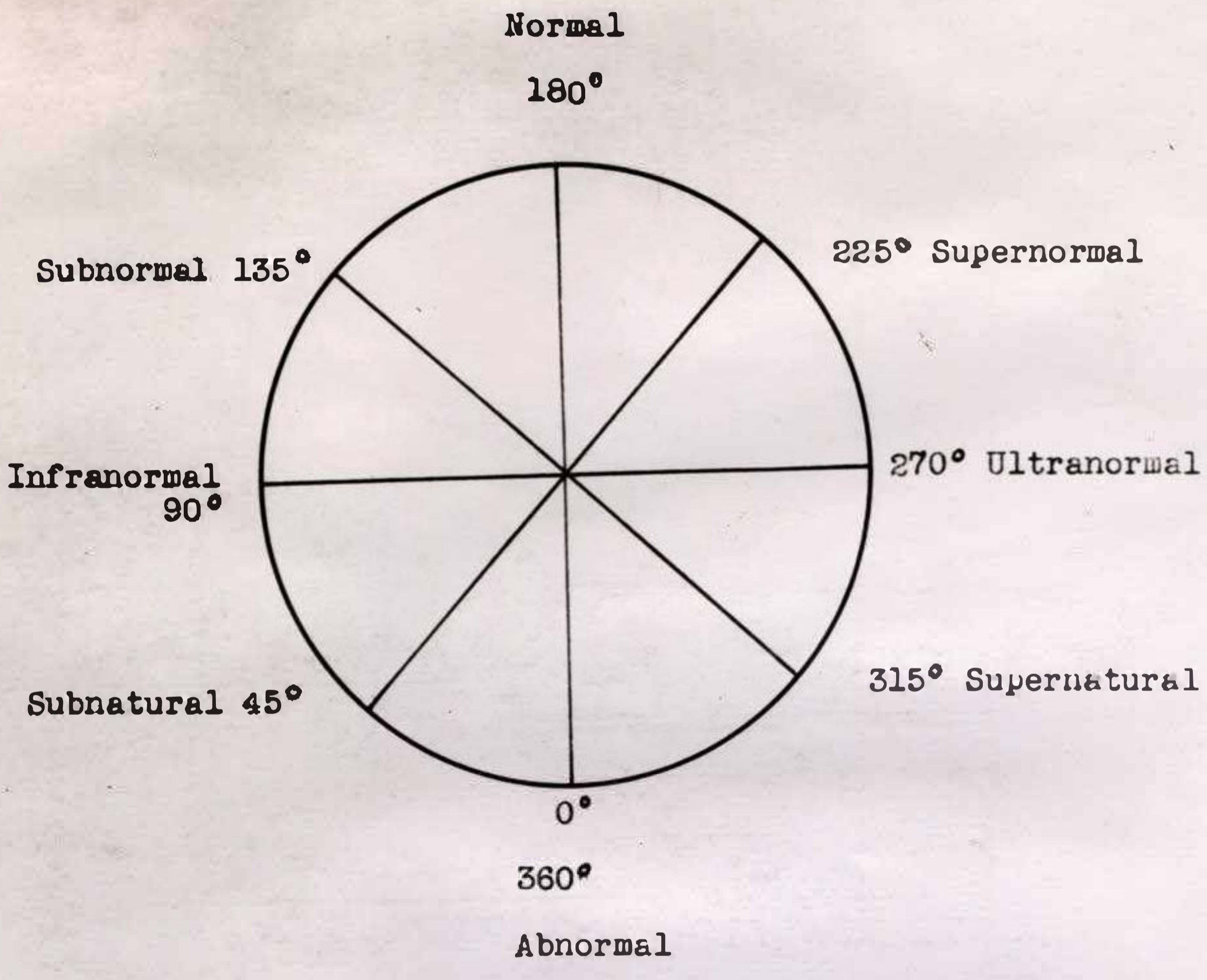
Scale of psychological categories as represented through geometrical projection on a circumference:

The circumference is divided, by the poles of the coordinates, into 8 arcs,  $45^\circ$  each. The geometrical poles correspond to the psychological poles. Arcs represent the transition zones, and the poles -- their absolute expression.

Zero or $360^\circ$ - abnormal	$180^\circ$ - normal
$90^\circ$ - infranormal	$270^\circ$ - ultranormal
$135^\circ$ - subnormal	$315^\circ$ - supernatural
$45^\circ$ - subnatural	$225^\circ$ - supernormal

(please see next page)







The psychological zones within the above limits between the adjacent poles represent:

The zone around the  $0^{\circ}$  or  $360^{\circ}$  stimulates astonishment (zero reaction or delayed reaction).

The zone around  $45^{\circ}$  stimulates either pity or humor.

The zone around  $90^{\circ}$  stimulates depression (pessimism).

The zone around  $135^{\circ}$  stimulates the sense of lyricism (regret, melancholy [pleasant sadness, joyful sadness, controllable, self-imposed sadness]) - close to positive zone - joy of self-destruction, self-sacrifice.

The zone around  $180^{\circ}$  stimulates the sense of quiet contemplation, full psychological balance and satisfaction.

The zone around  $225^{\circ}$  stimulates the sense of heroism and admiration.

The zone around  $270^{\circ}$  stimulates the sense of exaltation, ecstasy and worshipping.

The zone around  $315^{\circ}$  stimulates either the sense of fantastic or the sense of fear (unfavorable surroundings, uncontrollable, unaccountable forces, fear for existence, struggle for survival).

A discus thrower participating in the Olympics and reaching the previous years' record would



stimulate the reaction corresponding to 180 point. The actual reflexes of the spectators would be polite applause. Throwing beyond the expected range would stimulate the reaction corresponding to the zone between  $180^{\circ}$  and  $225^{\circ}$ , culminating into ultimate ecstasy when it reached  $270^{\circ}$  (this would be evidenced by shouting, stamping and whistling, the reactions increasing not only in intensity, but in quantity as well), i.e., the maximum conceivable limit. The clapping reflexes would grow accordingly, from  $180^{\circ}$  to  $270^{\circ}$ . If the disc does not reach the range expected the reaction would be disappointment, increasing toward  $135^{\circ}$  with the sympathetic spectators, while with the range reaching only  $90^{\circ}$  it would lead ultimately toward depression. The spectators will not applaud when the range of disc throwing is near the  $90^{\circ}$  point. It is natural to assume that certain groups of spectators, influenced by their sympathy toward the opponents of the first discus thrower, would produce exactly opposite reactions. These considerations cover the semicircle above the horizon.

The lower zone, on the negative side, i.e., between the  $0^{\circ}$  and  $90^{\circ}$ , stimulates the reaction of laughter and in the case of the discus thrower it would amount to a range of perhaps only a few yards from his position after a long and corresponding preparation to





a throw. When the spectators see a husky, muscular athlete deprived of mechanical efficiency they unquestionably react to it as seeming decidedly humorous.

On the positive side of the lower semicircle between  $315^{\circ}$  and  $360^{\circ}$ , lies the zone of supernatural, where the range of throw of a disc would be beyond any biomechanical possibility. For example, if the range of throw amounted to three miles. In such cases the presence of a trick or a supernatural force would be a necessary ingredient for the logical comprehension of the phenomenon. The usual reaction would be the reaction of smile or laughter transforming into astonishment in the direction of the zero point.

The  $360^{\circ}$  point when reached from the positive side would amount to the absurd caused by an impossible mechanical over-efficiency. Such would be the case when the disc being thrown would never come back, or fall anywhere on the ground, vanishing in the interstellar spaces and thus overcoming the law of gravity.

When zero is reached from the negative side it would mean an impossible mechanical inefficiency. In the case of a disc thrower it would happen when the disc would slip out of the athlete's hands before he actually threw it.



A trajectory expressing a mechanically efficient kinetic process, whether a pendulum or a musical melody, will have the mechanical fundamentals in common. A pendulum cannot start instantaneously at its maximum amplitude; neither can a melody. A pendulum cannot stop instantaneously from its maximum amplitude; neither can a melody. The corresponding effects in both cases will be either supernatural or humorous.

The actual quantitative specifications serving different purposes and expressing different forms of mechanical efficiency vary with times and places. To satisfy any esthetic requirement one has to know the style in which such requirements have to be carried out, beyond which specifications the entire kinetic process, whether efficient or not, will be meaningless. As the standards vary, the coordinates on the circle described above change their absolute positions, i.e., the zero point may move with the entire system, either clockwise or counter-clockwise. If we would assume, with regard to athletic standards,  $180^\circ$  to be a limit of certain mechanical operations when the achievement of the following epoch increases the quantitative value of normal, placing the point of normality to what is  $225^\circ$  on our diagram, the opposite pole of the coordinate will occupy respectively the  $45^\circ$



position. Referring to music in general and melody in particular, we find certain standards become old fashioned and we begin to feel that though they may be charming yet they are entirely inadequate for the purposes of a more mechanically efficient epoch. We feel it in every field concerned with motion, i.e., mechanics.

There is a humorous or a pitiful reaction toward the 1900 horseless carriage and it becomes still more humorous where there is an accumulation of quantities of the symbols of inadequacy, such as the prerequisites of travel required by a horseless carriage (dusters, goggles, safety belts). We have exactly the same picture (i.e., if we are people representing our epoch and not living anachronisms), in melodies composed by a Verdi or a Bellini, where the mechanical efficiency is so low that it ~~makes~~ us smile, if not laugh. The same melodies stimulate entirely different reactions among the octogenarians surviving in our epoch of 400 miles per hour.

In order to achieve an efficient climax it is necessary to accumulate energy that would be effectively discharged into such climax. The means for accumulating energy, as it was described above, are achieved through rotary motion developing centrifugal energy. Trajectories expressing musical pitches of



various frequencies are heard by the listeners in relation to the entire trajectory. It is possible not only to show the range of frequencies (such as a form of direct transition from one frequency to another), but also to show in what way this variation of frequency was achieved. The portion of a melodic trajectory leading toward the climax, without a resistance preceding such a climax, does not produce any dramatic effect. It is the resistance that makes the climax appear dramatic. A portion of melodic trajectory leading from a climax (maximum amplitude) towards balance (minimum amplitude) must be performed in accordance with natural mechanical laws, i.e., it must contain resistance before it reaches the balance (compare with pendulum). The inefficiency or the excess of the forms of resistance (rotary motion) leads to a mechanical abnormality. Abnormal melody stimulates the sense of dissatisfaction or humor. The forms of resistance leading toward climax acquire centrifugal form (increasing amplitude). The forms of resistance leading toward balance acquire centripetal form (decreasing amplitude). The relative period of rotary motion and amplitudes produces various forms and gradations of resistances. For example, the period of rotation may be long, with the amplitude remaining constant; or the period of rotation may be





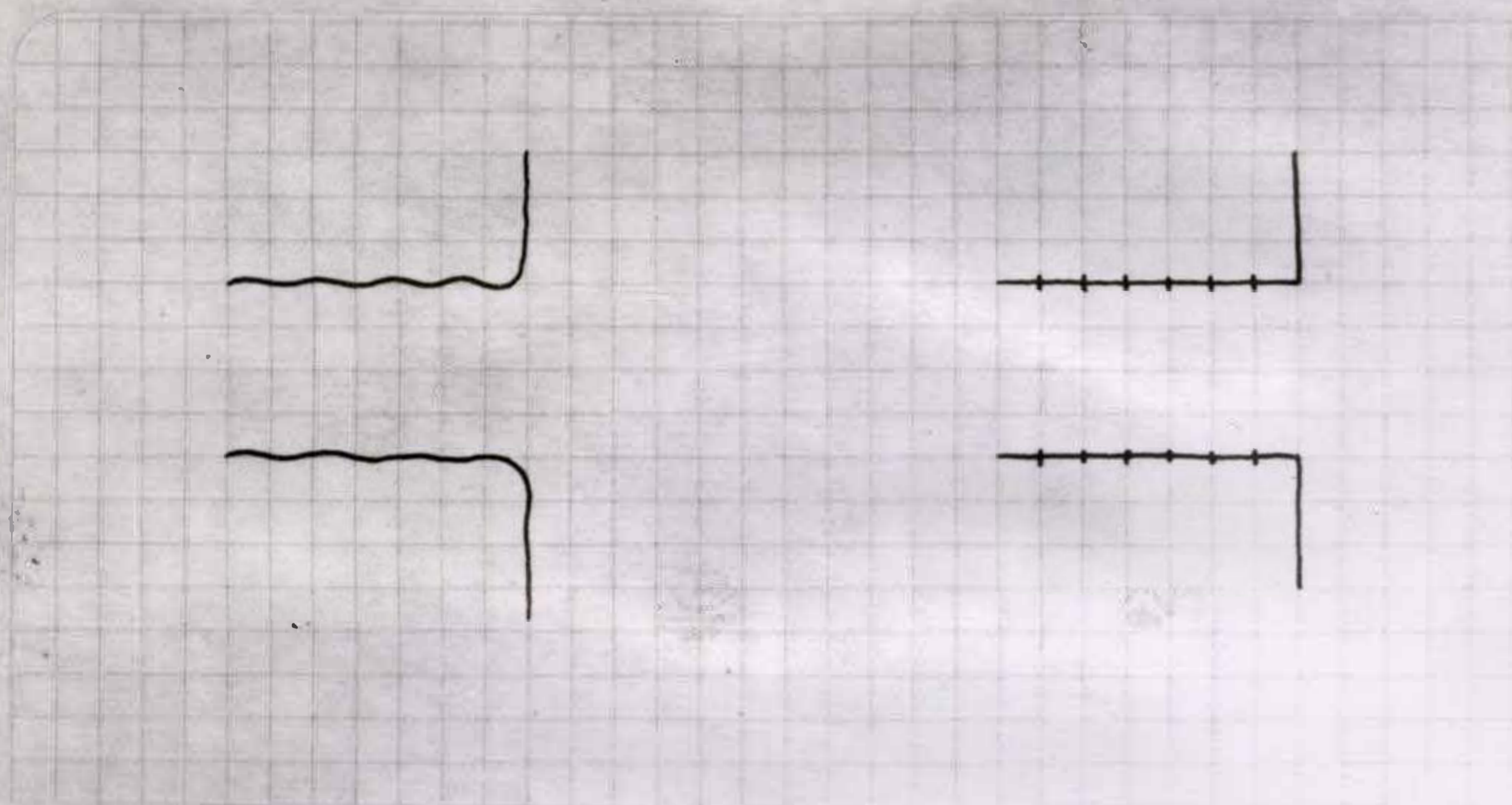
short with rapidly increasing amplitudes. The period of rotation may be short with the correspondingly increasing amplitude. The duration of the rotary period may be in inverse proportion to the amplitude and often the law of squares takes its place.



Lesson LVIII.

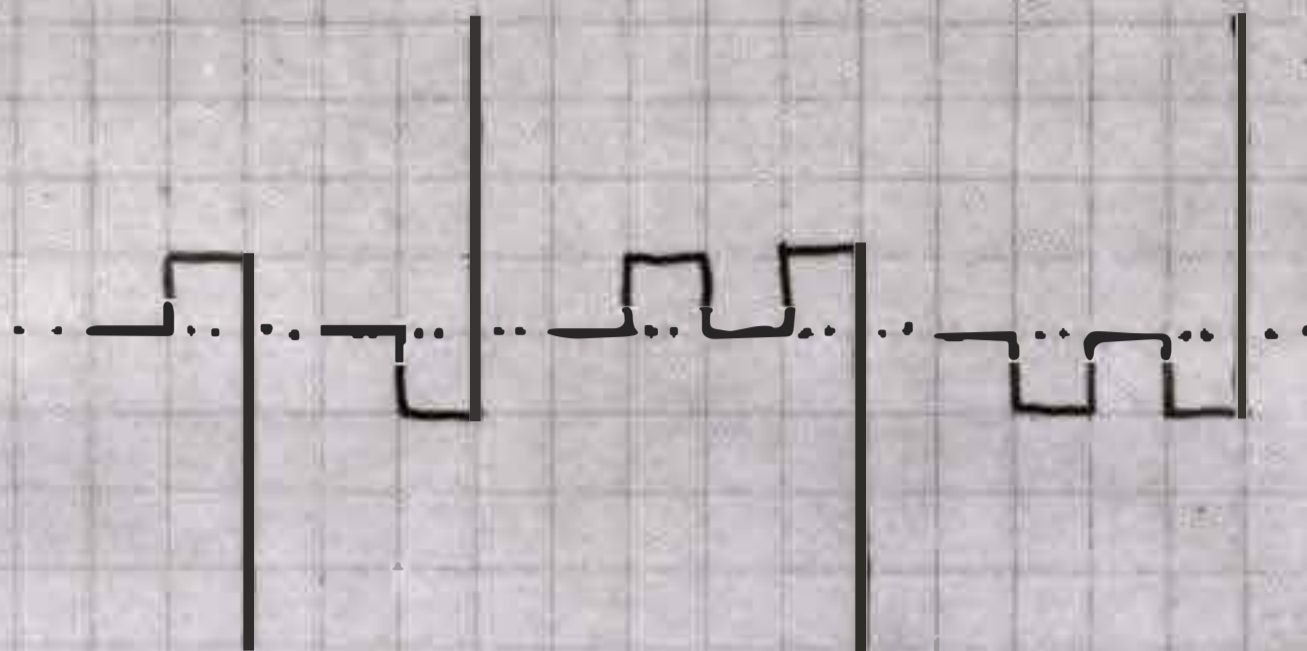
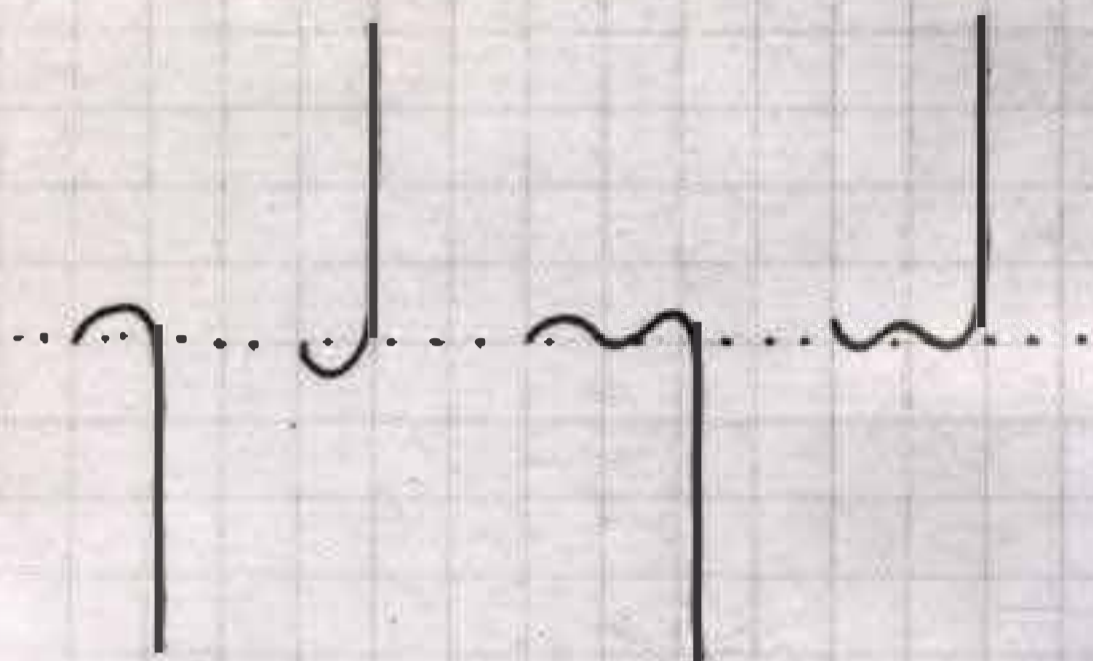
The corresponding forms of resistance as applied to melodic trajectories are:

1. Repetition (correspondences: aiming, rotary motion with infinitesimal amplitudes, affirmation of the axis level as a starting point). Musical form: repeated attacks of the same pitch discontinued by rests or following each other continuously.

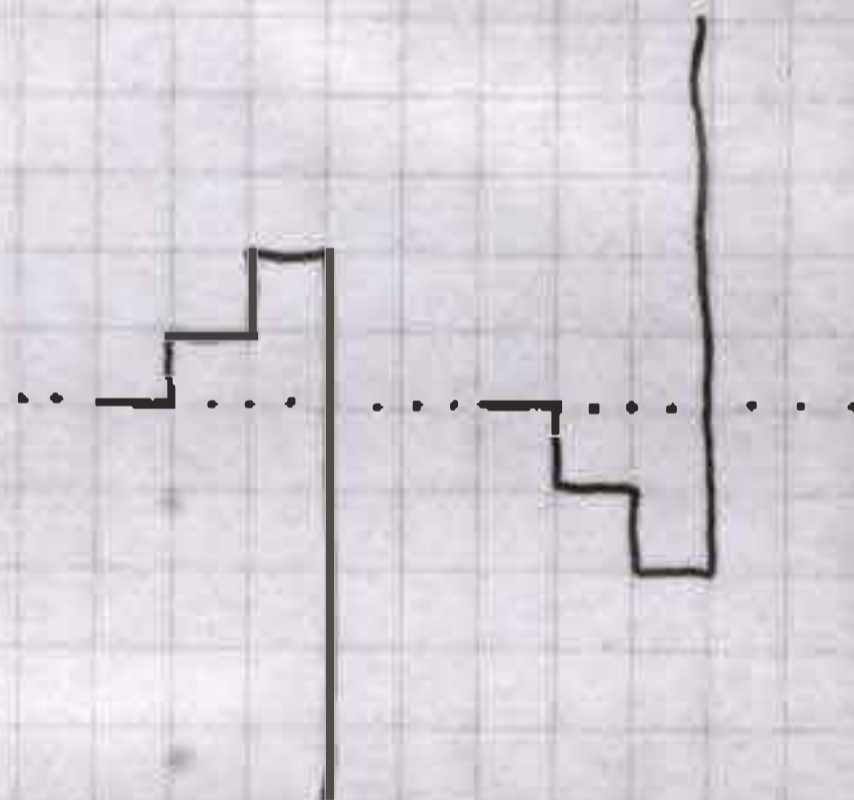
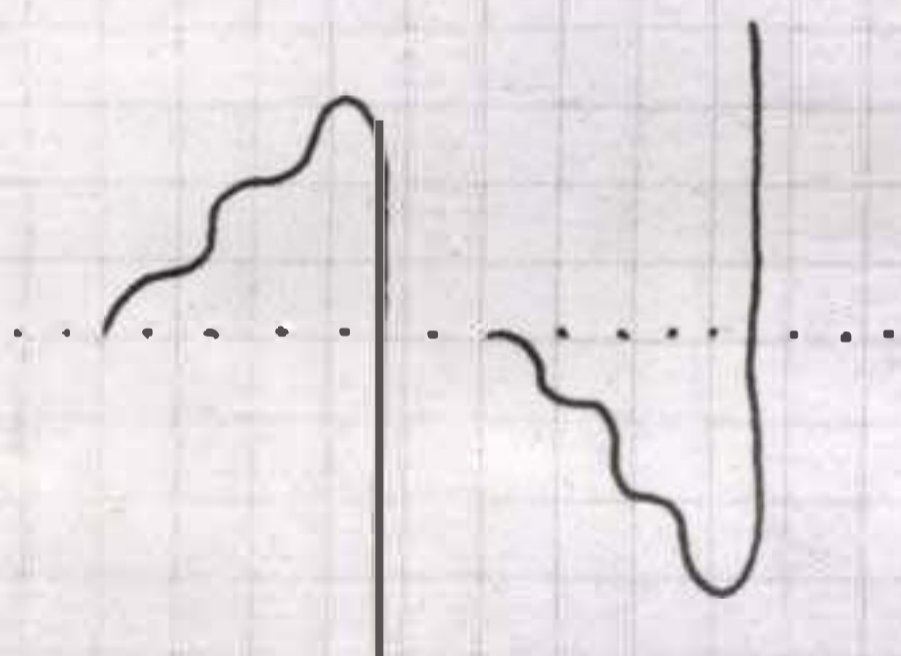
Physical FormMusical Form

2. One phase rotation (correspondences: preliminary contrary motion, initial impulse in archery, artillery, springboard diving, baseball pitching, tennis service, etc.) Musical form: a movement or a group of movements in the direction opposite to the following leap.



Physical FormMusical Form

This form often acquires more than one phase following in one direction which intensifies the resistance.



### 3. Full periodic rotation (one or more periods)

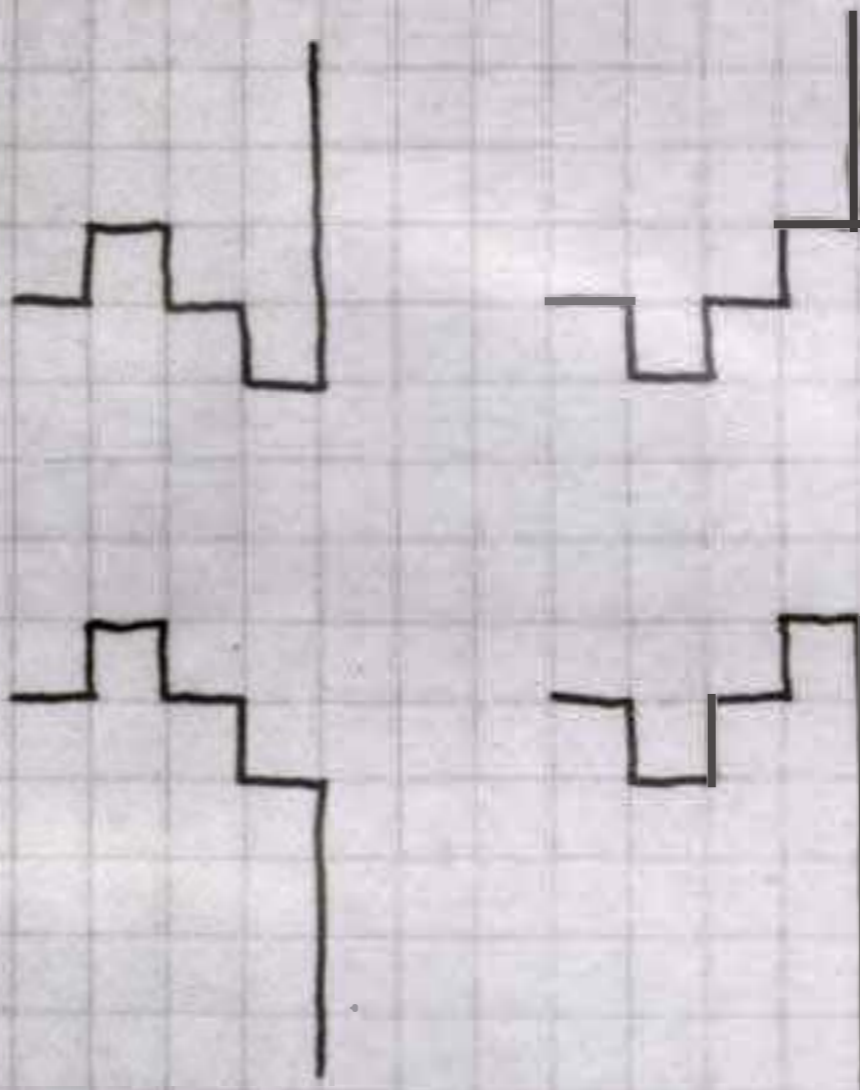
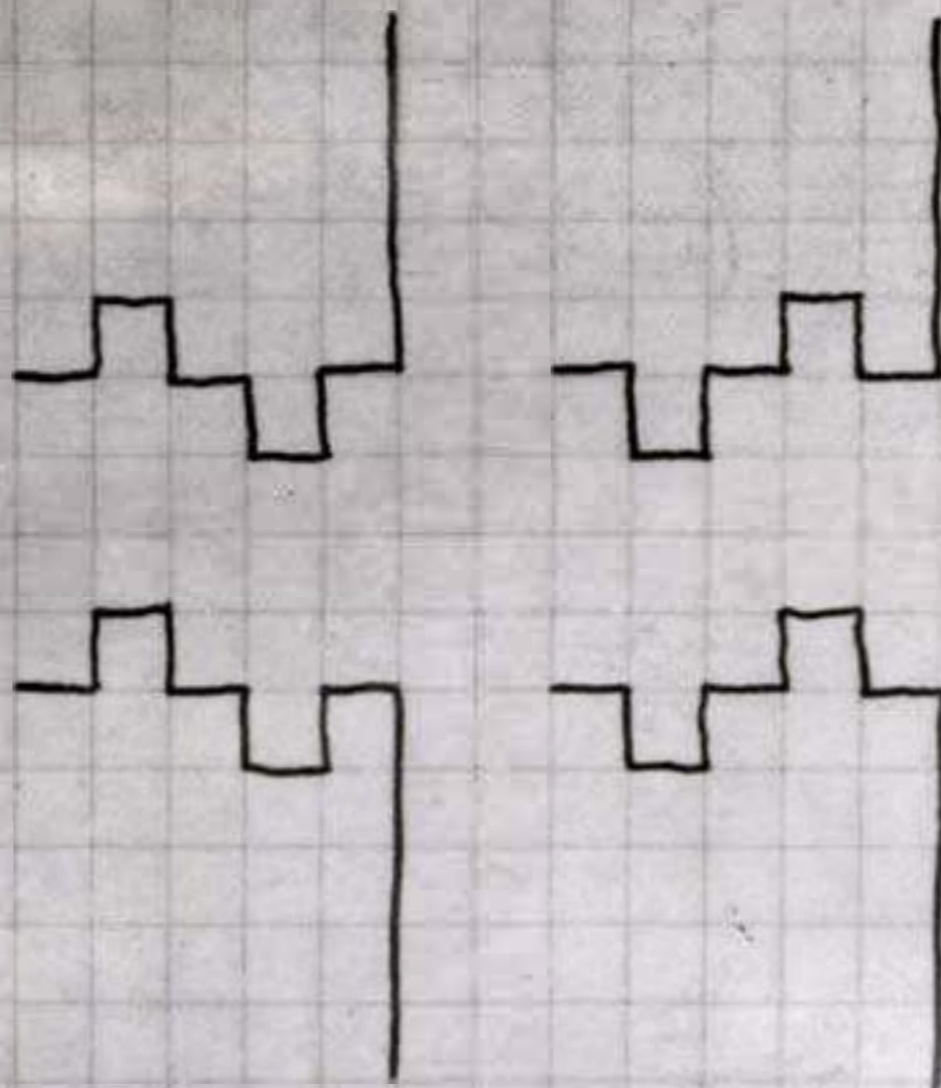
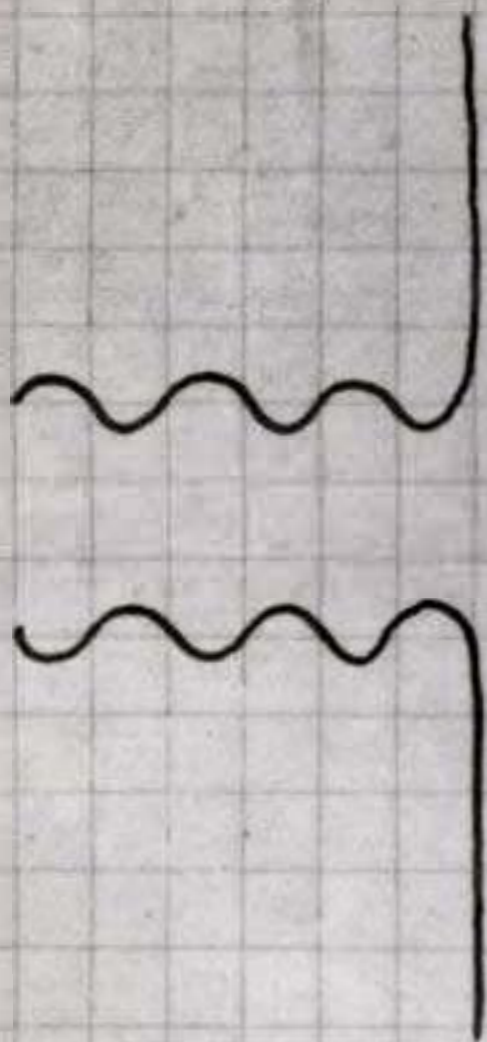
A. Constant amplitude (correspondences: rotation around a stationary point, a top, somersaults - with diving and without - lasso, axis and orbit rotation of the planets, Dervish dances).



**Musical Form: mordente, trill, tied tremolo, groupetto.**

Physical Form

Musical Form



**B. Variable amplitude (correspondences: giroscope, spiral motion, tornado, expansion, contraction). Musical form: expanding and contracting, simple and compound motion.**





Whereas the preceding forms of resistance require only one of the secondary axes, the variable amplitude rotation requires a simultaneous combination of two or three secondary axes. In this case the axis leading towards climax or balance will be considered fundamental and the other axes - complementary.

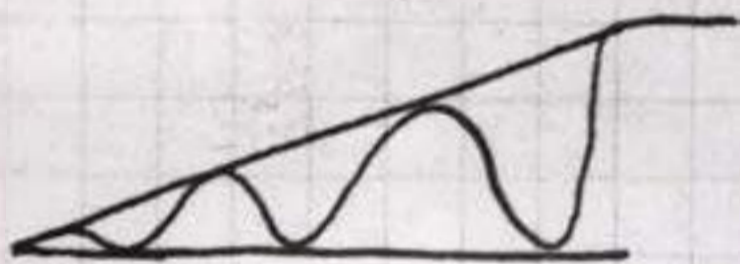
Simultaneous combinations of two axes:

(a) Centrifugal (expanding):

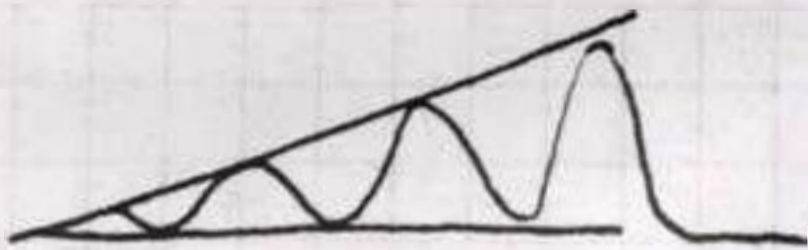
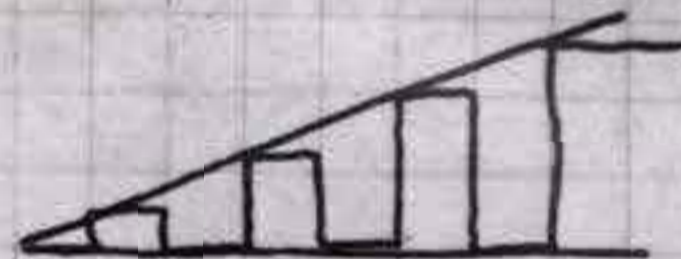
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Physical Form

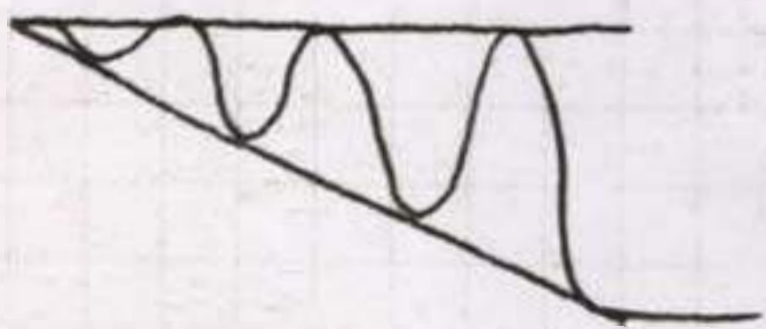
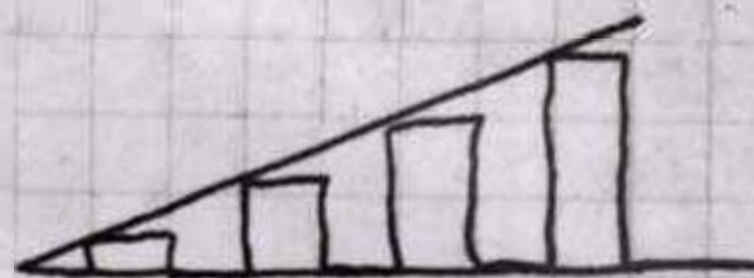
Musical Form



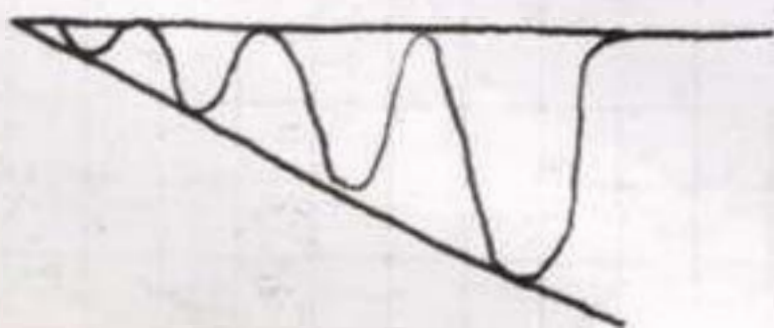
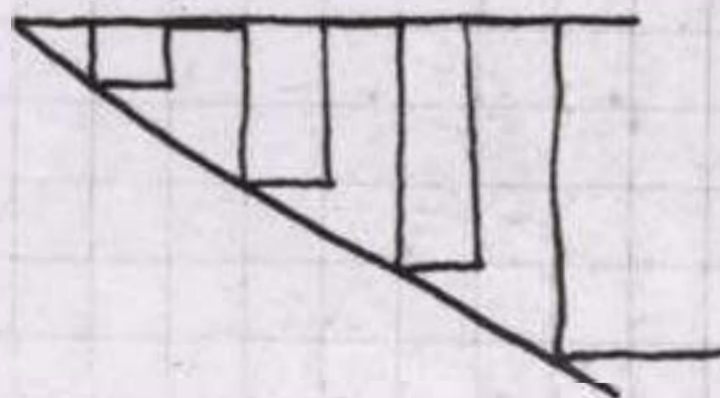
$\frac{a}{o}$



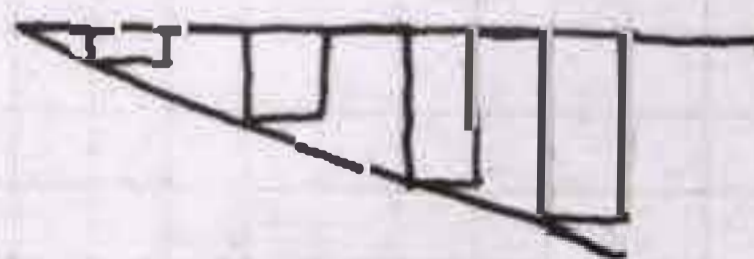
$\frac{o}{a}$



$\frac{o}{a}$



$\frac{o}{a}$



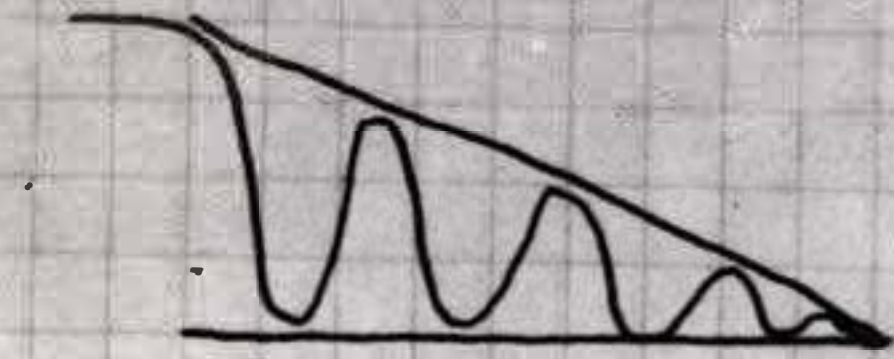


(b) Centripetal (contracting)

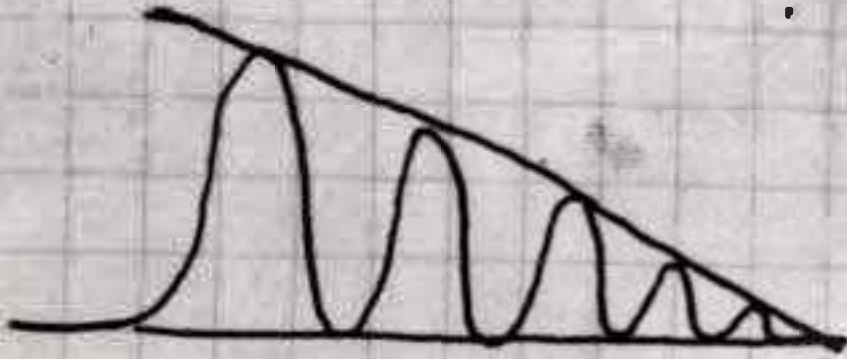
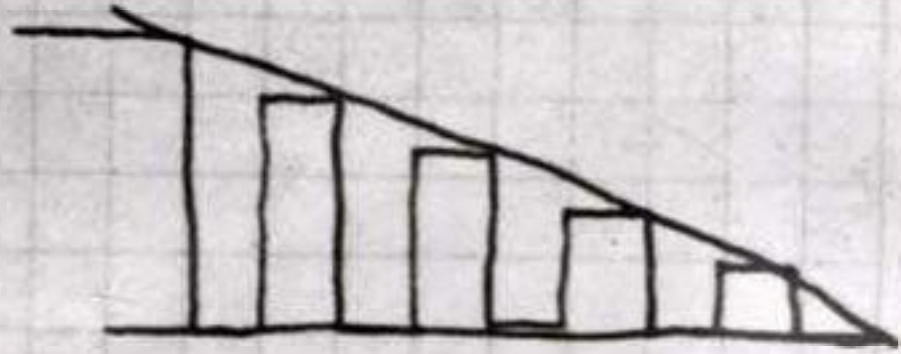
$\frac{b}{a}; \frac{c}{b}; \frac{d}{c}; \frac{e}{d}$

Physical Form

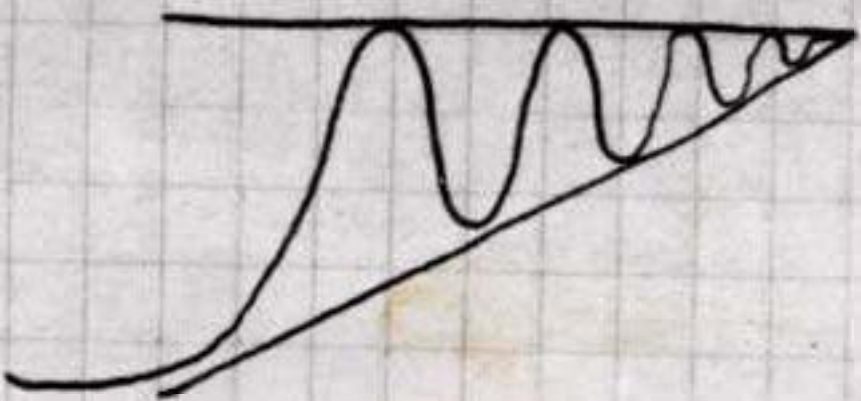
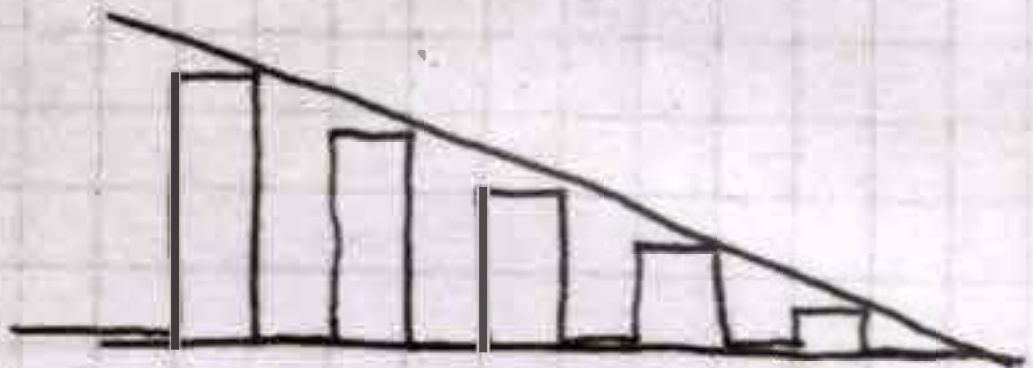
Musical Form



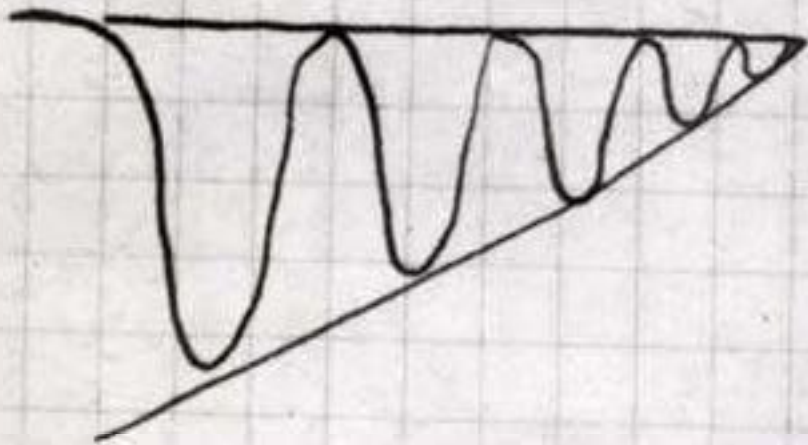
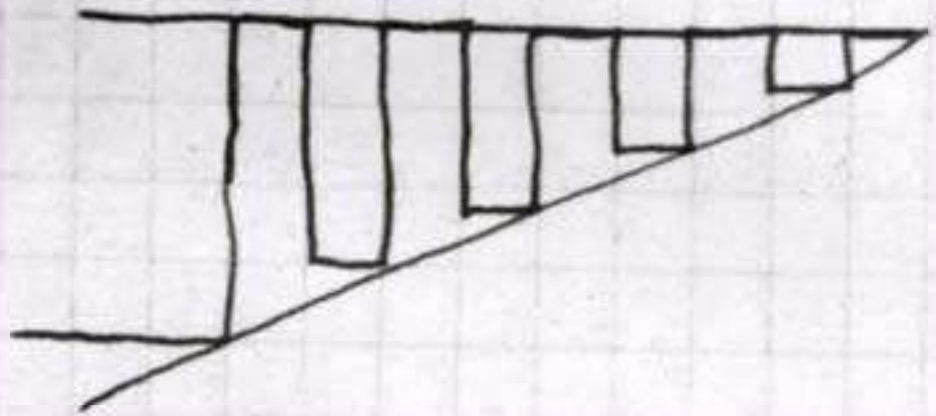
$\frac{a}{b}$



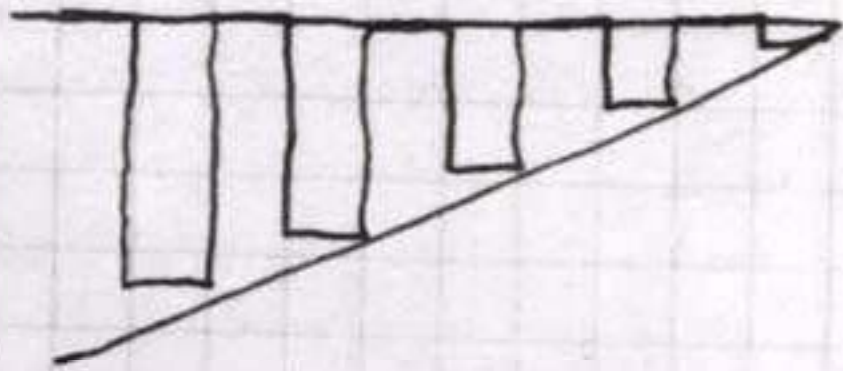
$\frac{a}{b}$



$\frac{a}{b}$



$\frac{a}{b}$



Simultaneous combinations of three axes:

(a) Centrifugal (expanding):

$a + 0 + d; d + 0 \div a$



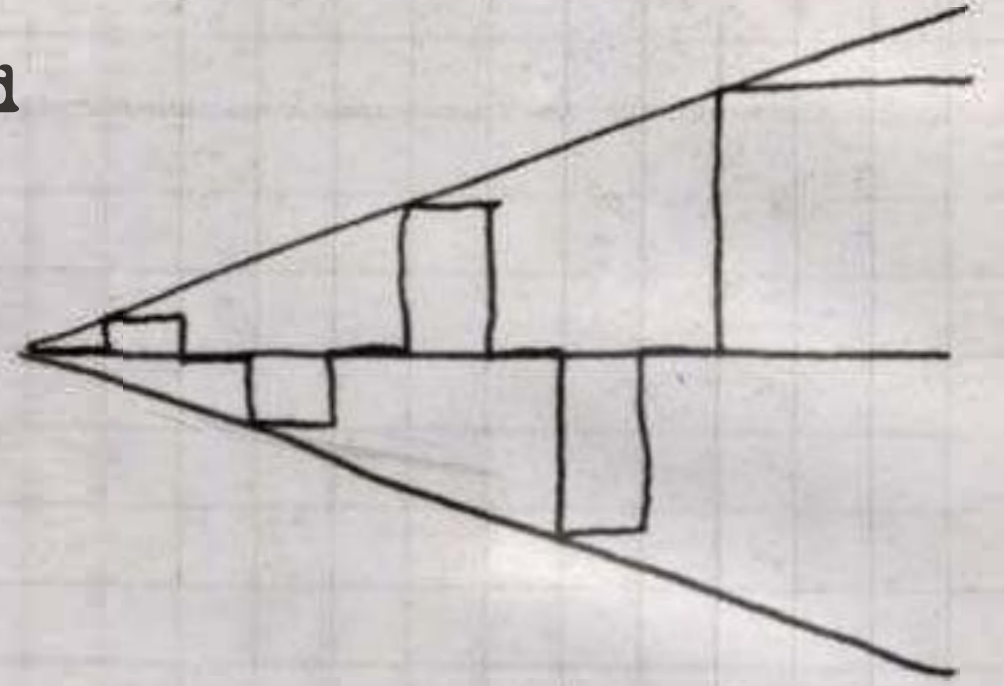
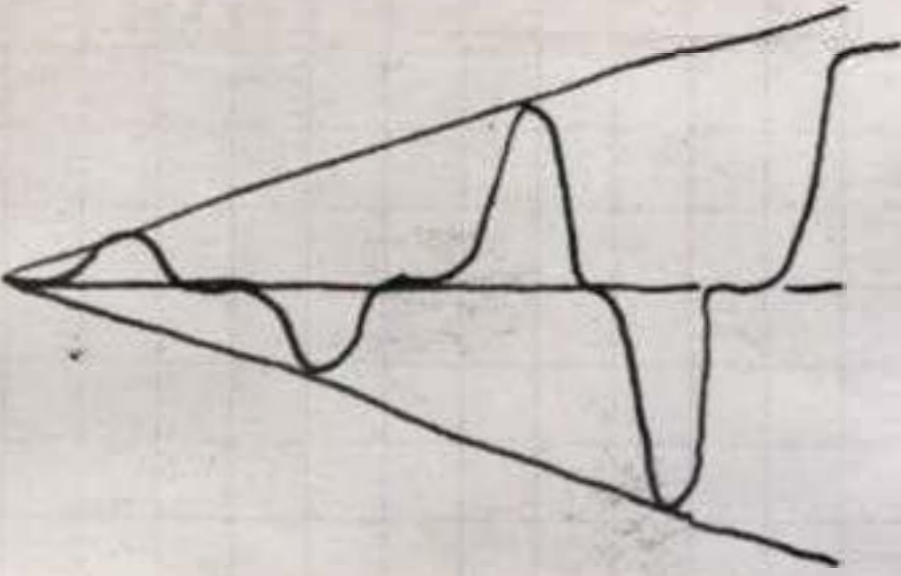
(b) Centripetal (contracting):

$$b \div 0 \div c; \quad c \div 0 \div b$$

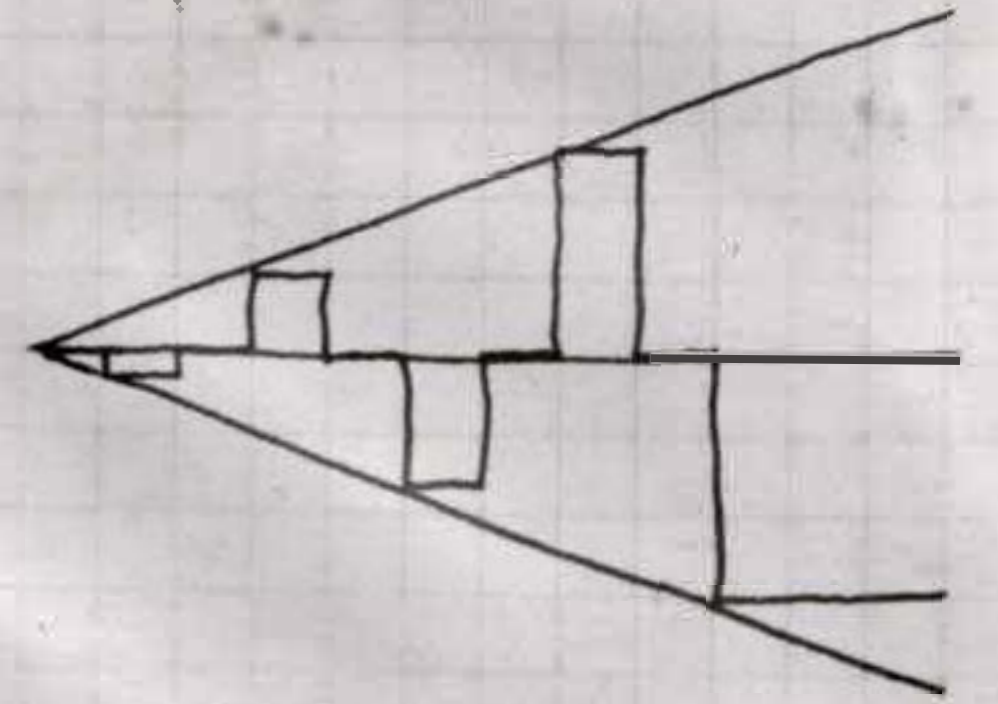
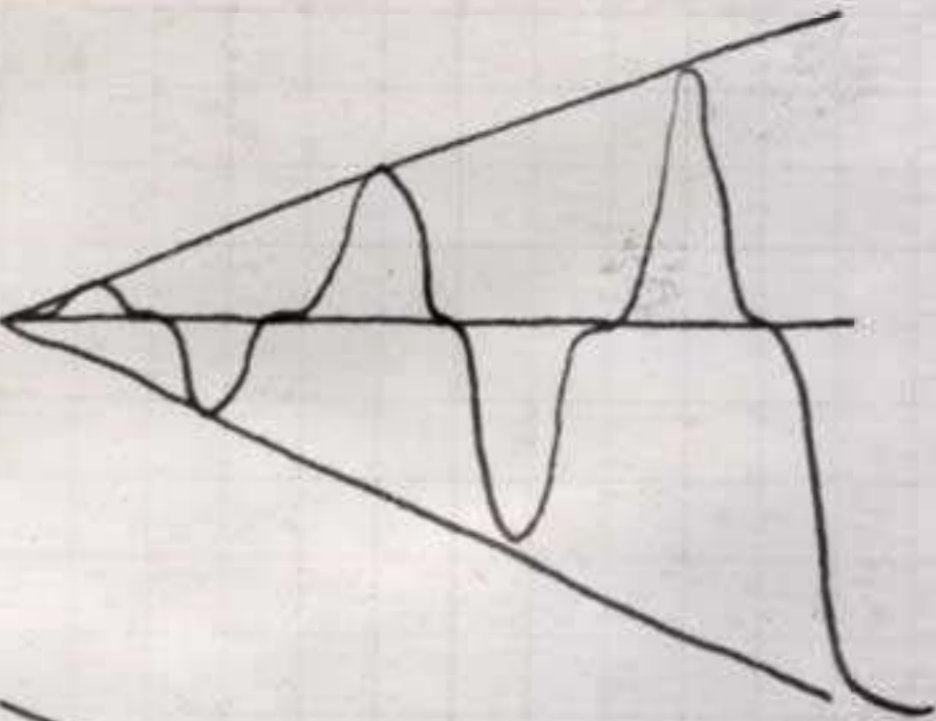
Physical Form

Musical Form

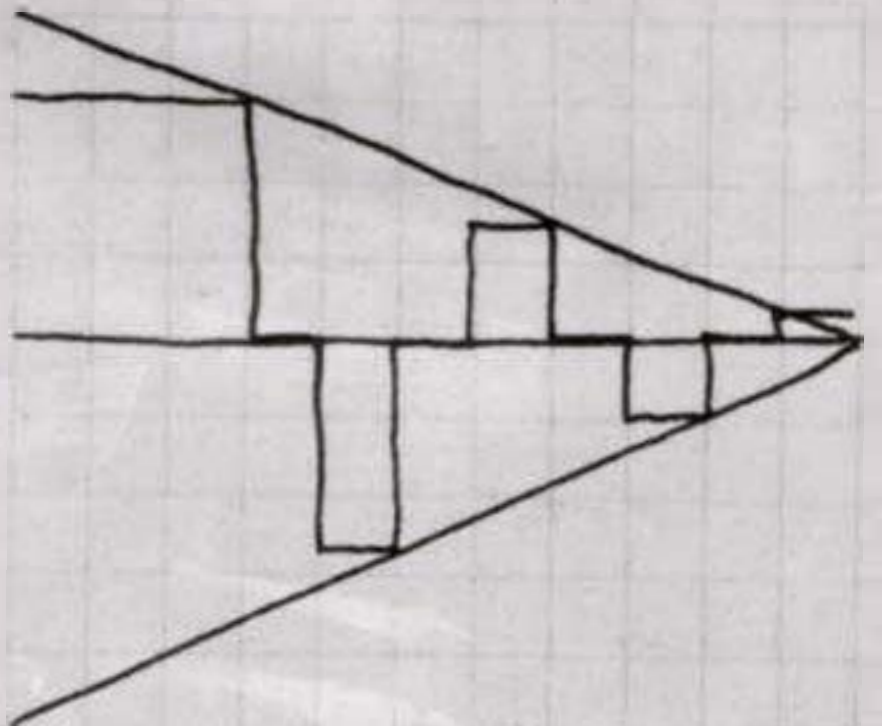
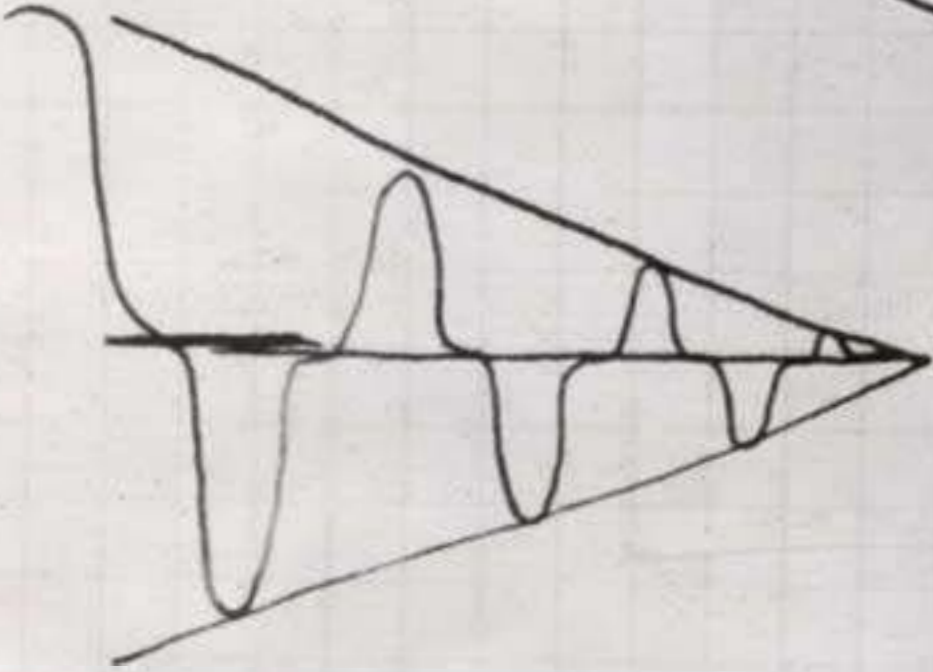
$$a \div 0 \div d$$



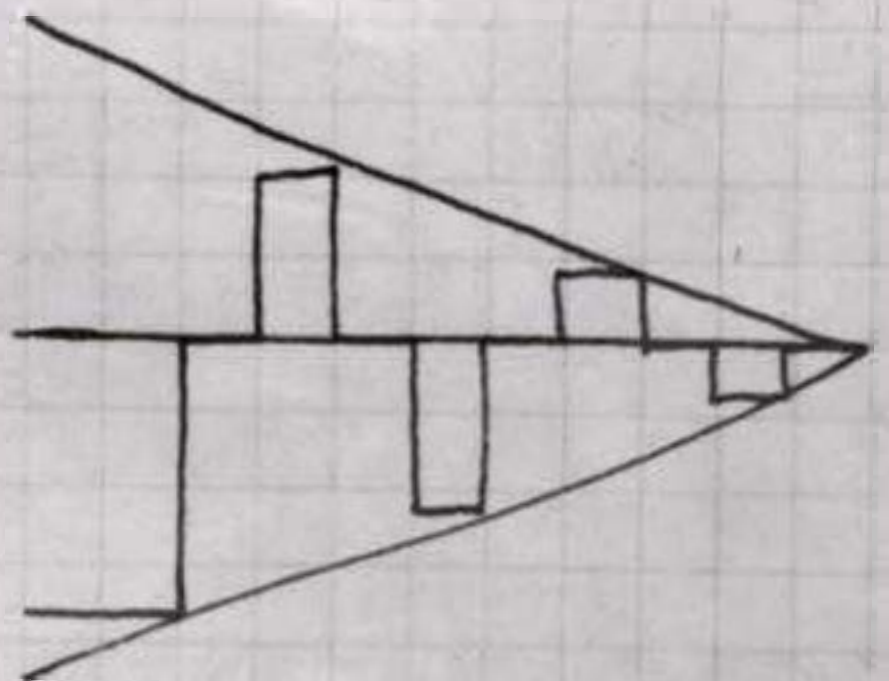
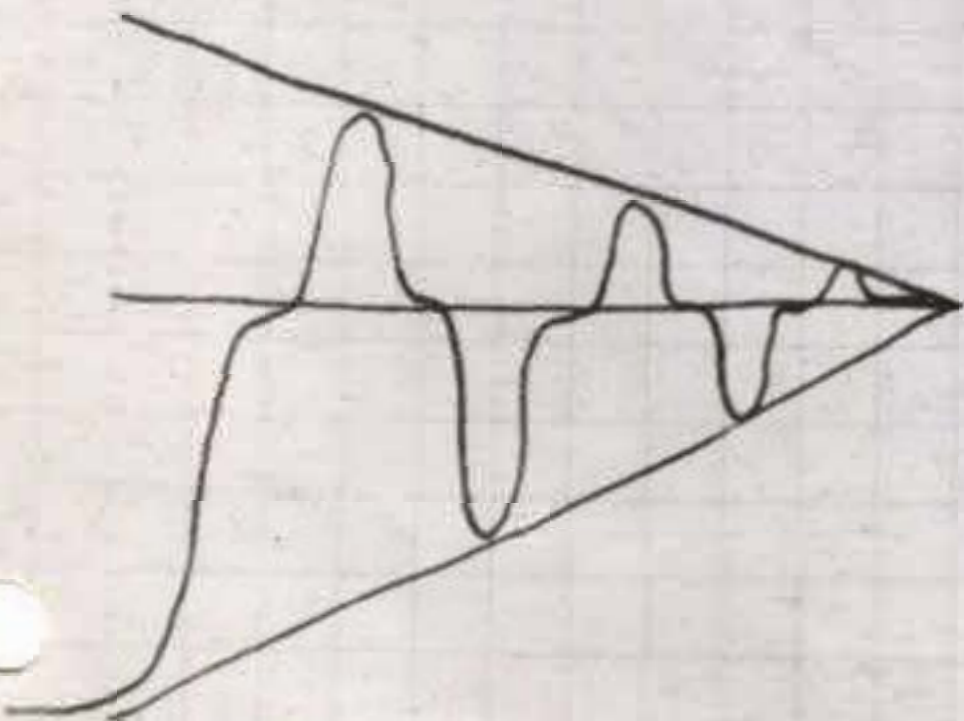
$$d \div 0 \div a$$



$$b \div 0 \div c$$



$$c \div 0 \div b$$





Lesson LIX.

As the interval of a pitch level from the primary axis affects tension (gravity effect where P.A. is a gravitational field) resistance may also result from two parallel secondary axes. The complementary parallel axis may be placed either above or below the fundamental axis. The effect of motion through a pair of parallel axes is that of an extended trajectory (delayed forced inefficiency). In reality it is the usual rotary motion only evolving between the two axis-boundaries.

The correspondences of such motion are: raising and falling, zigzag ascending and descending. Musical form: revolving around alternately progressing points (ascending or descending).

(please see next page)

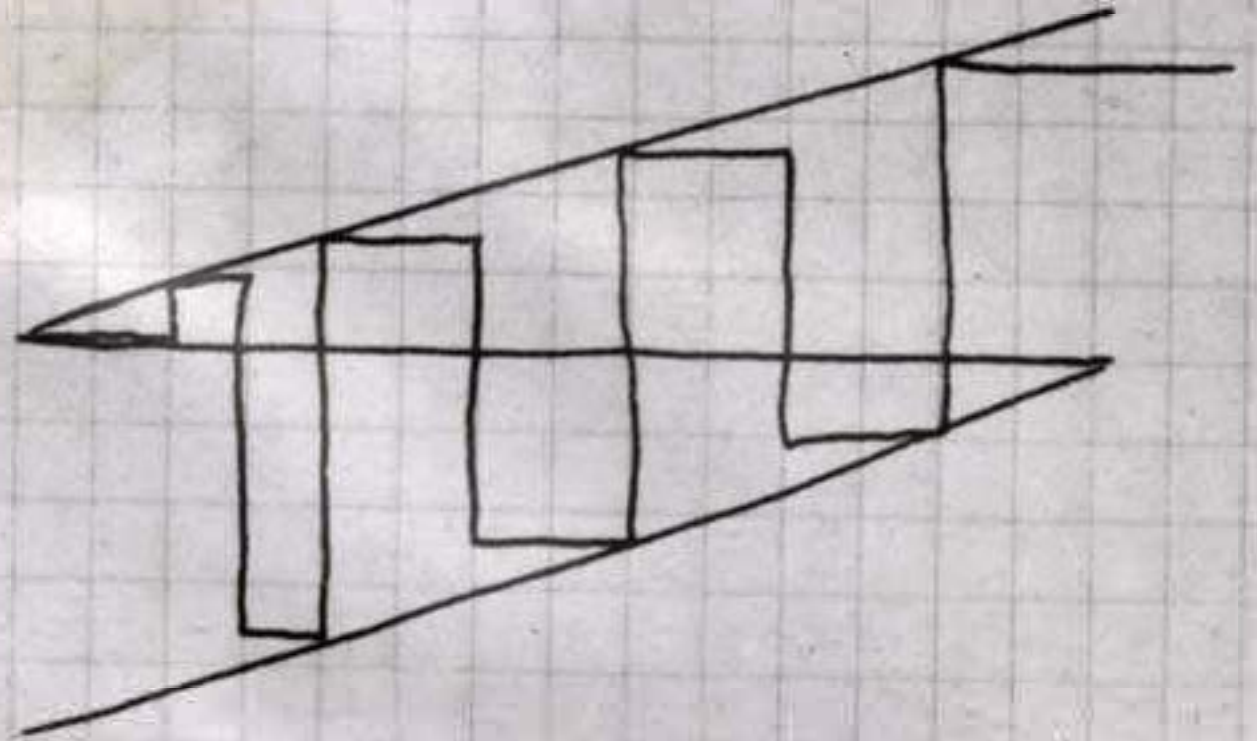
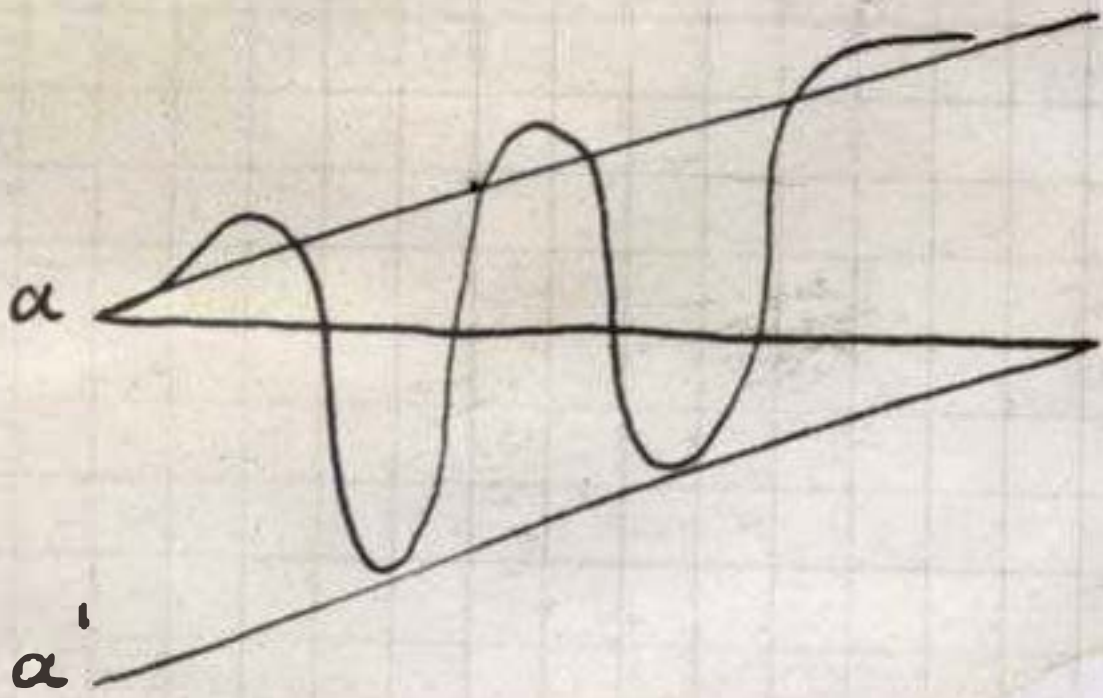
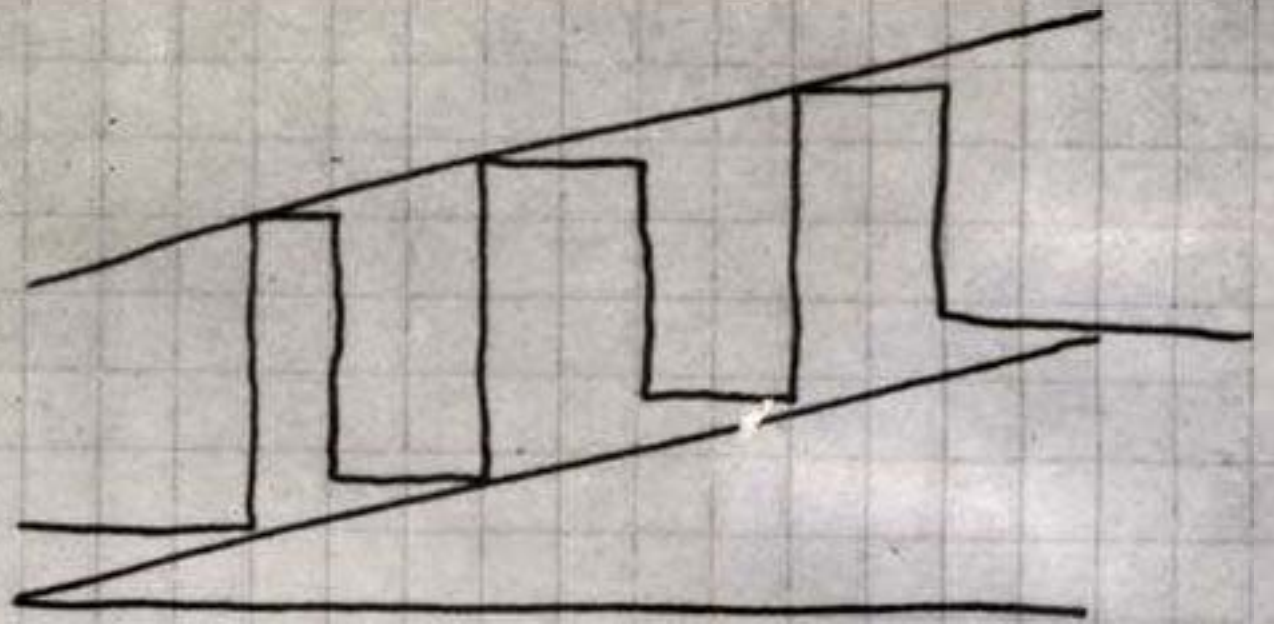
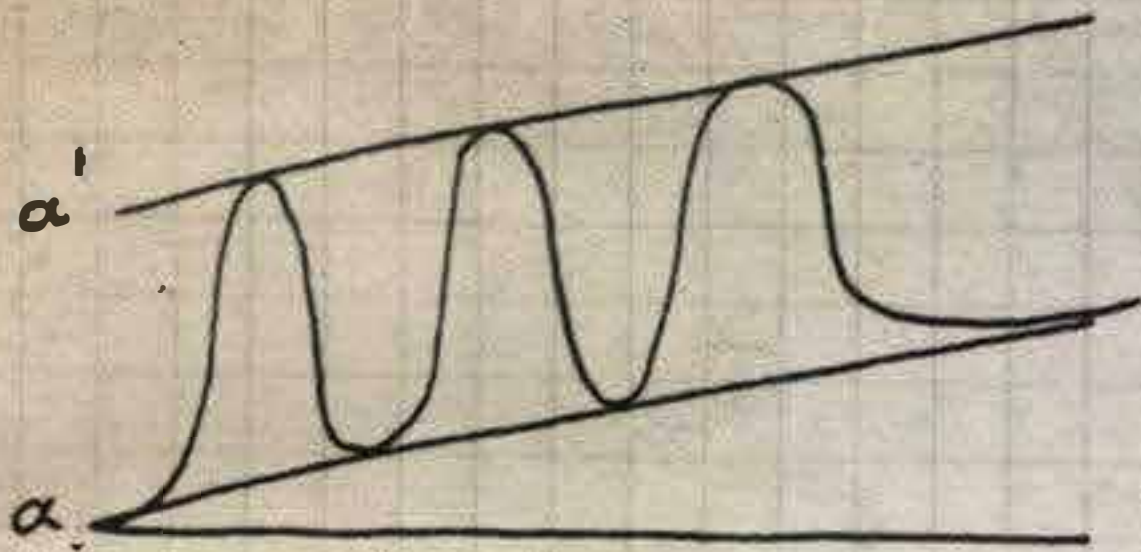




Physical Form

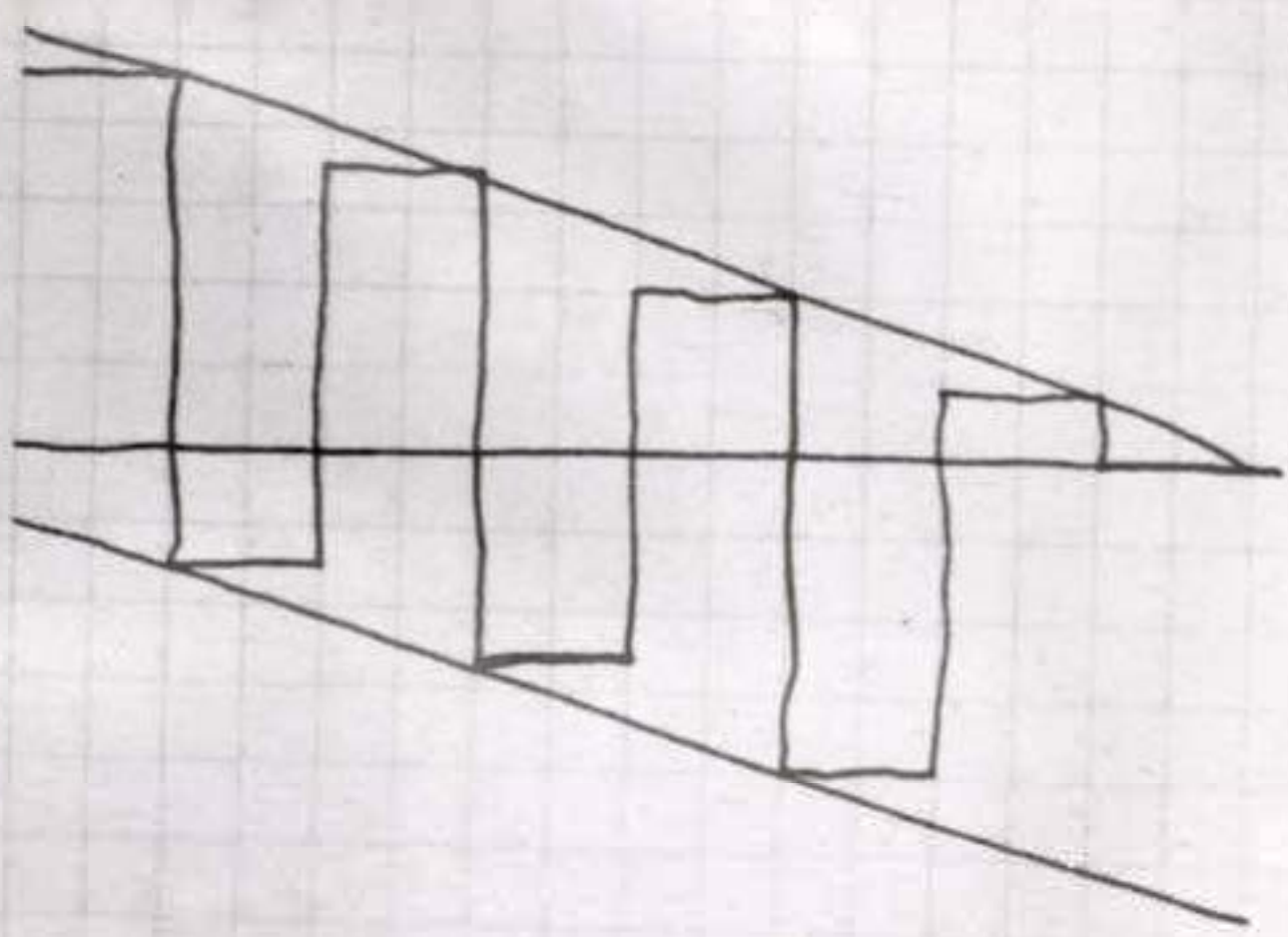
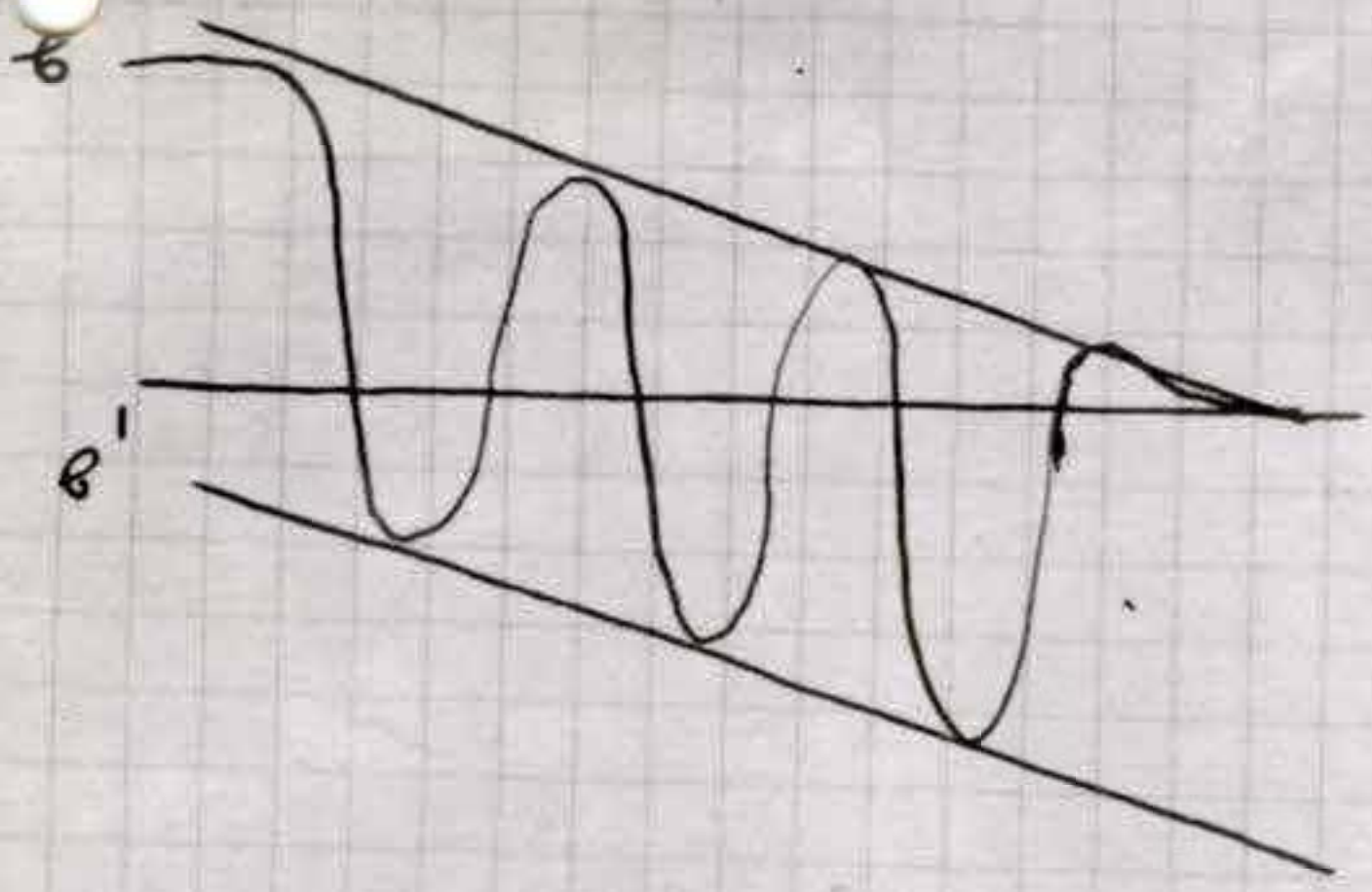
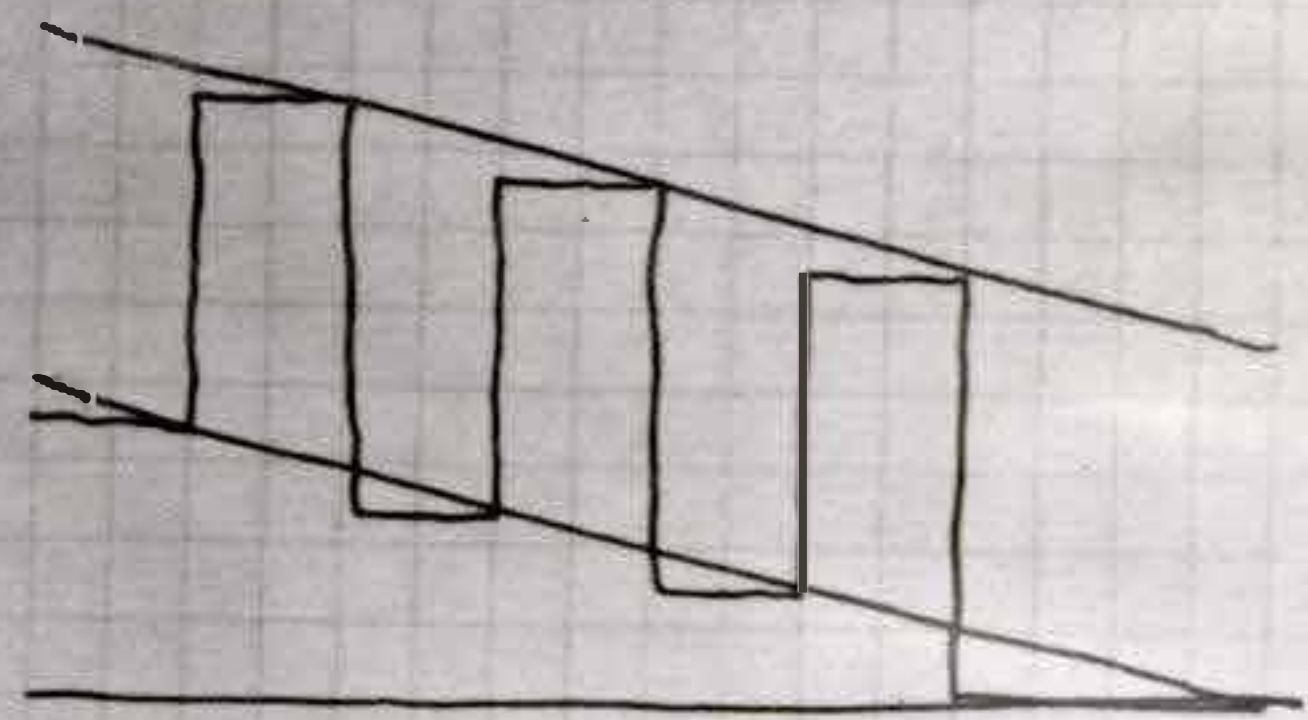
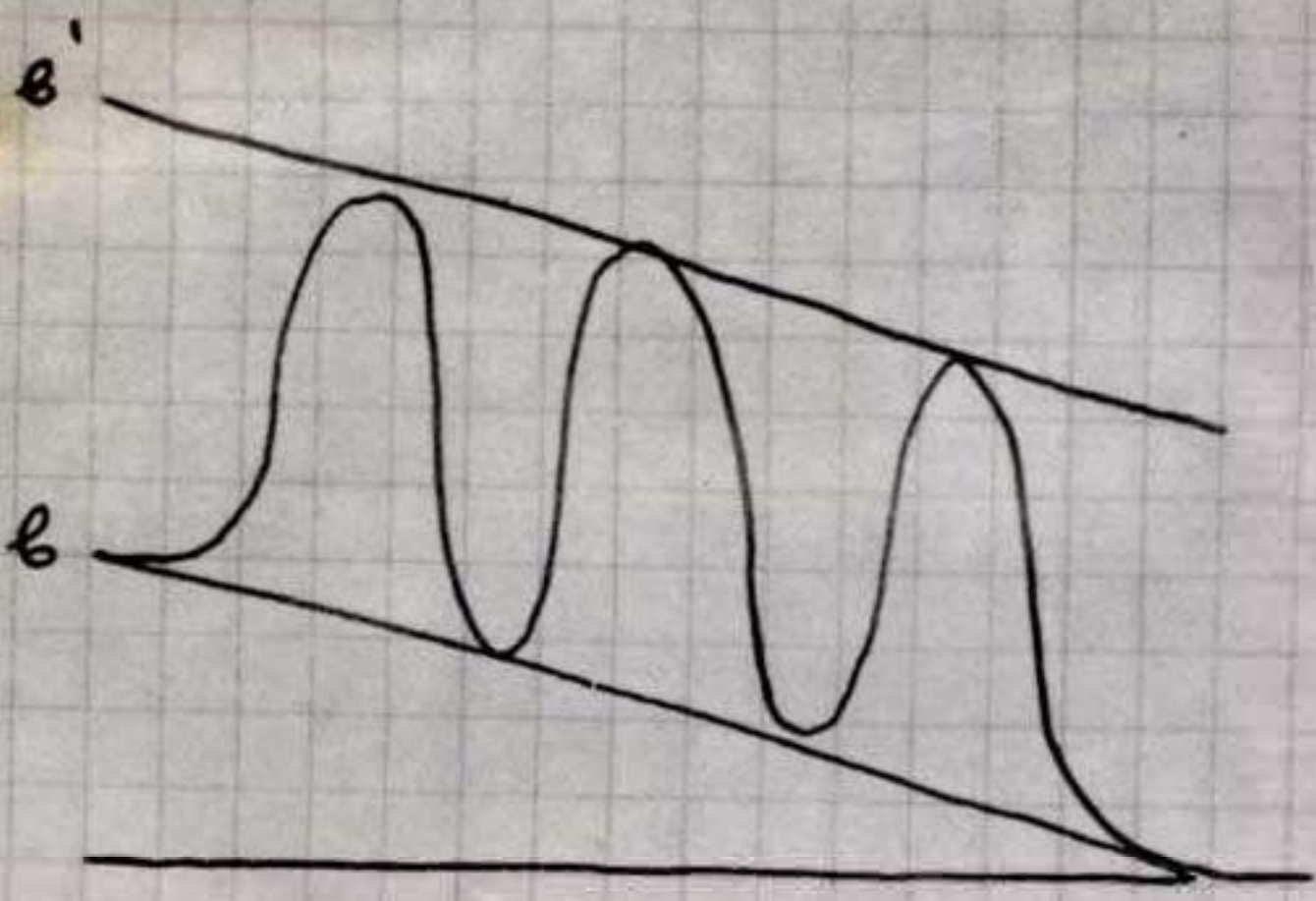
Musical Form

$\frac{a}{a'}$



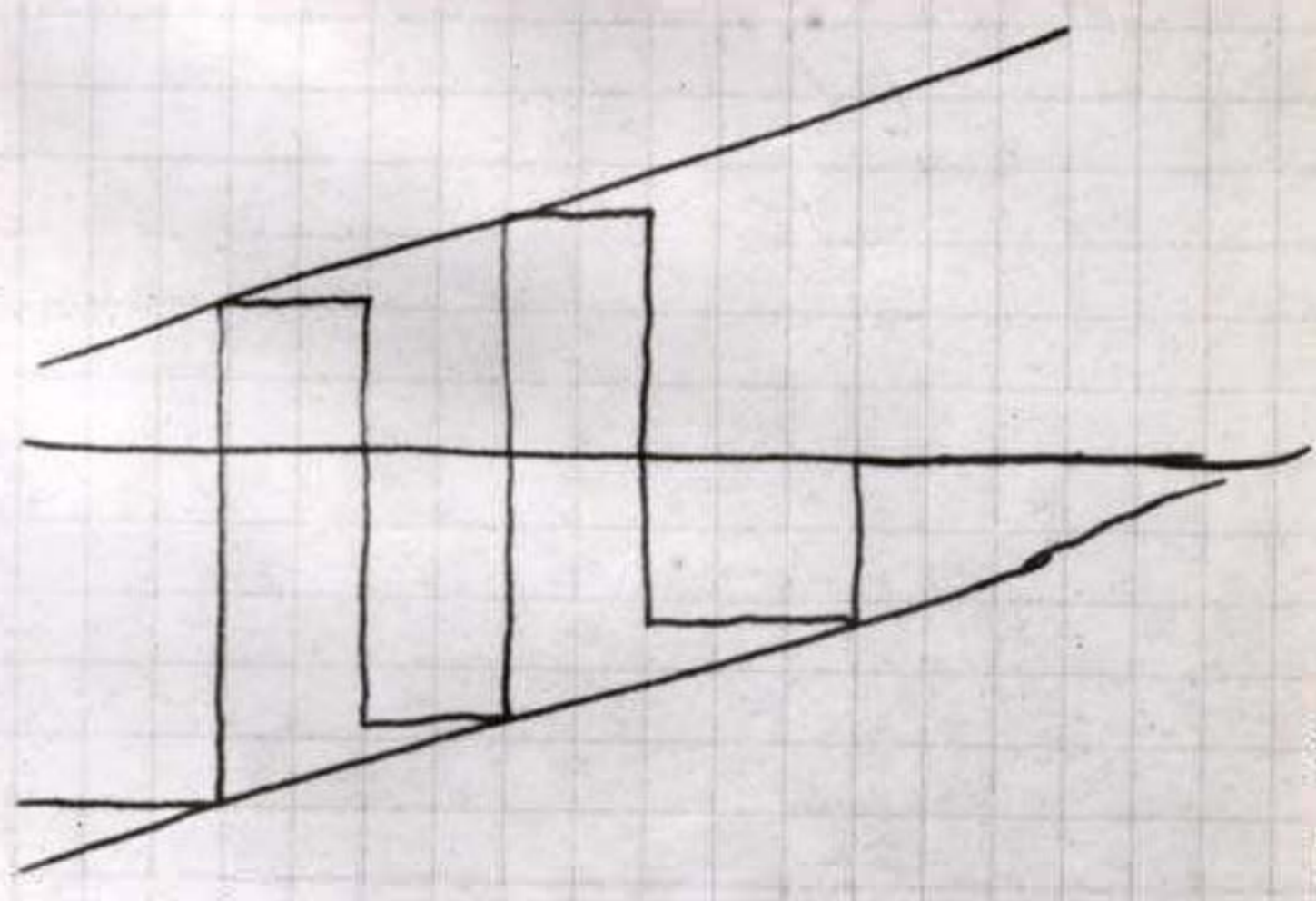
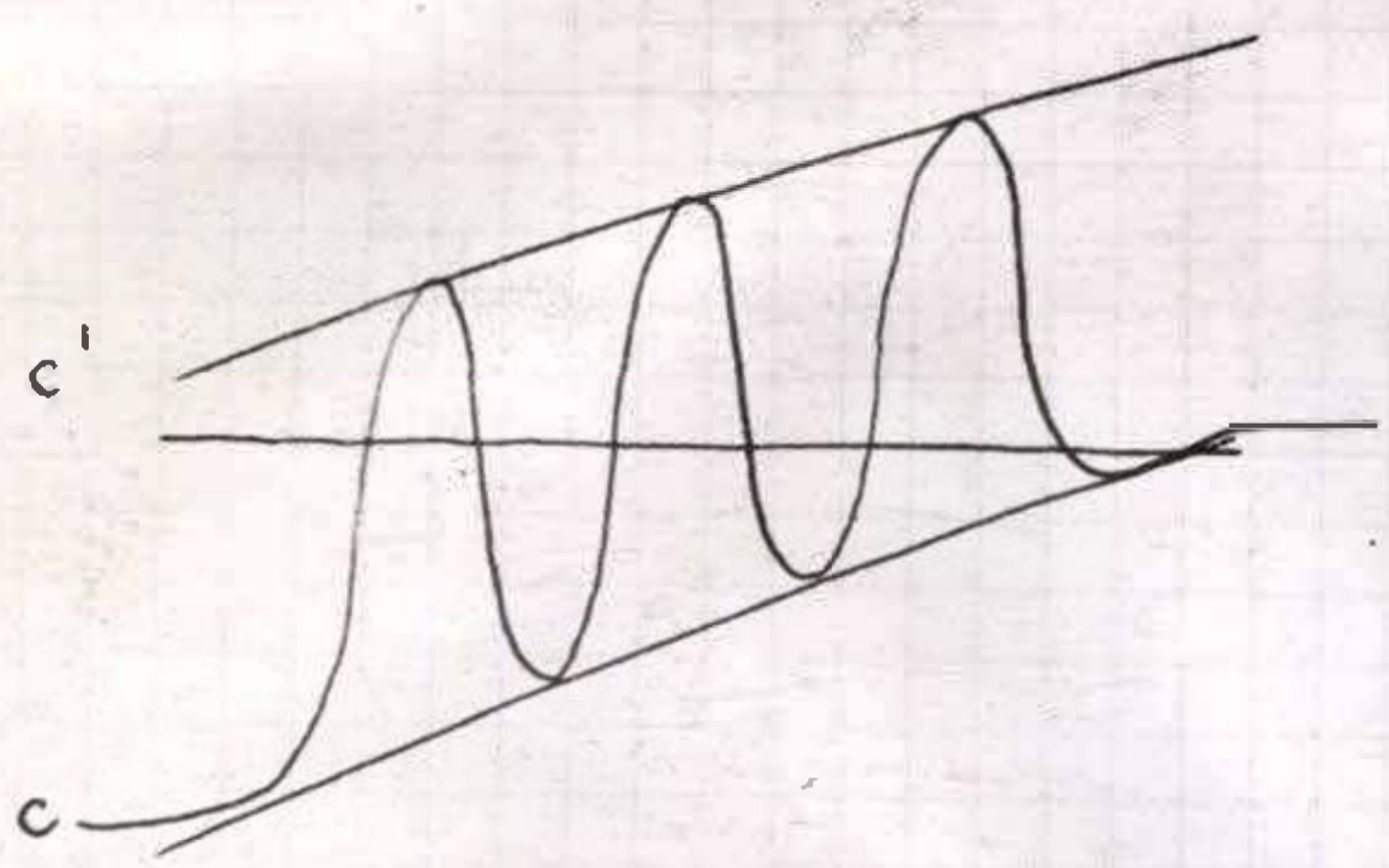
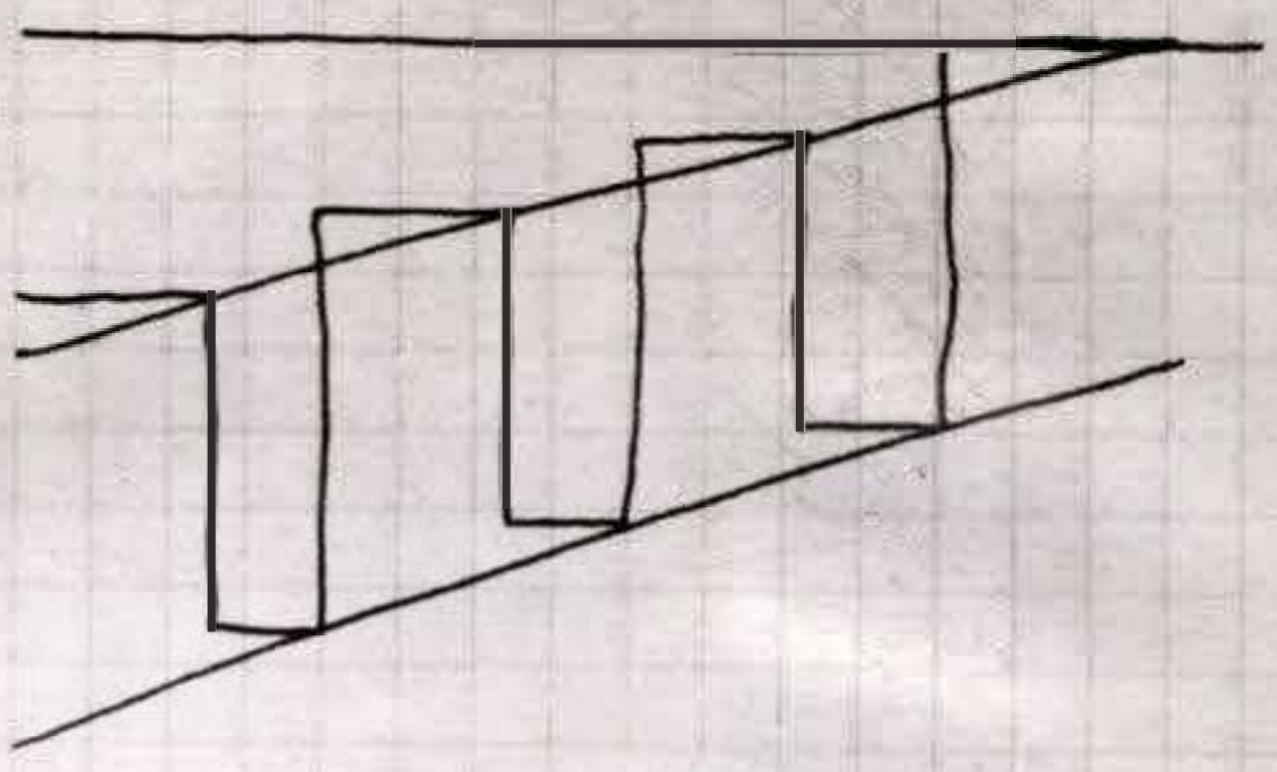
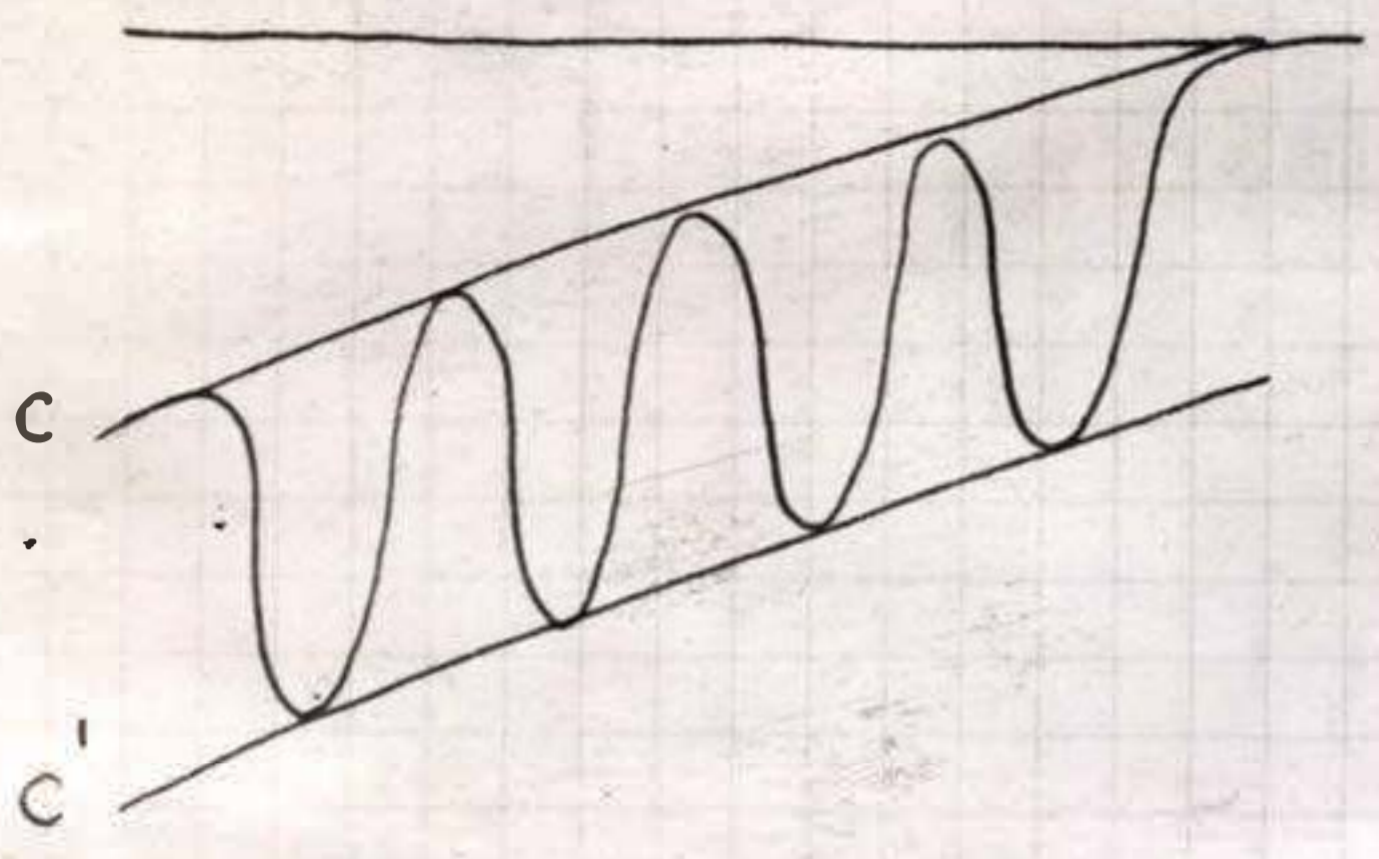


$$\frac{b}{b'}$$



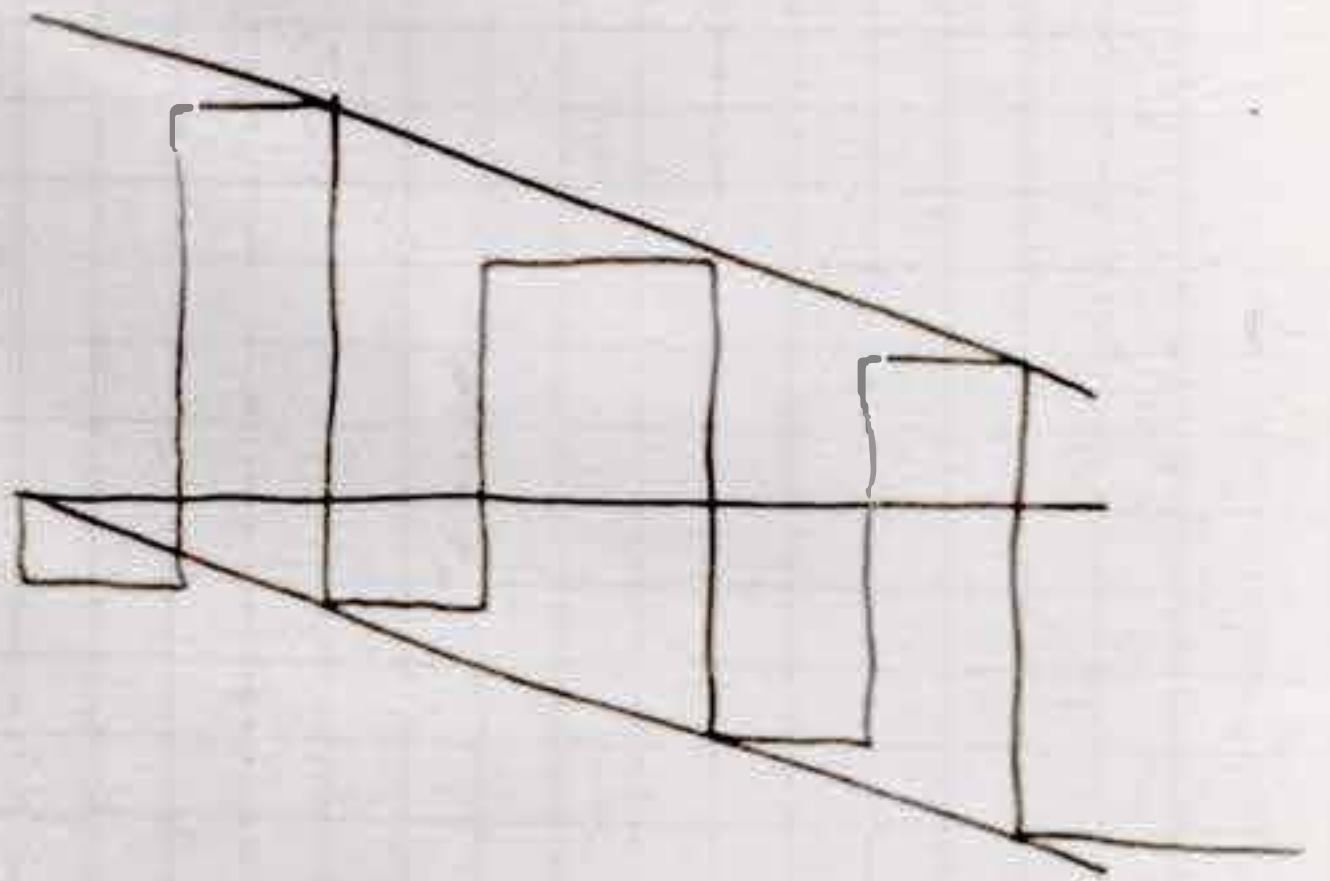
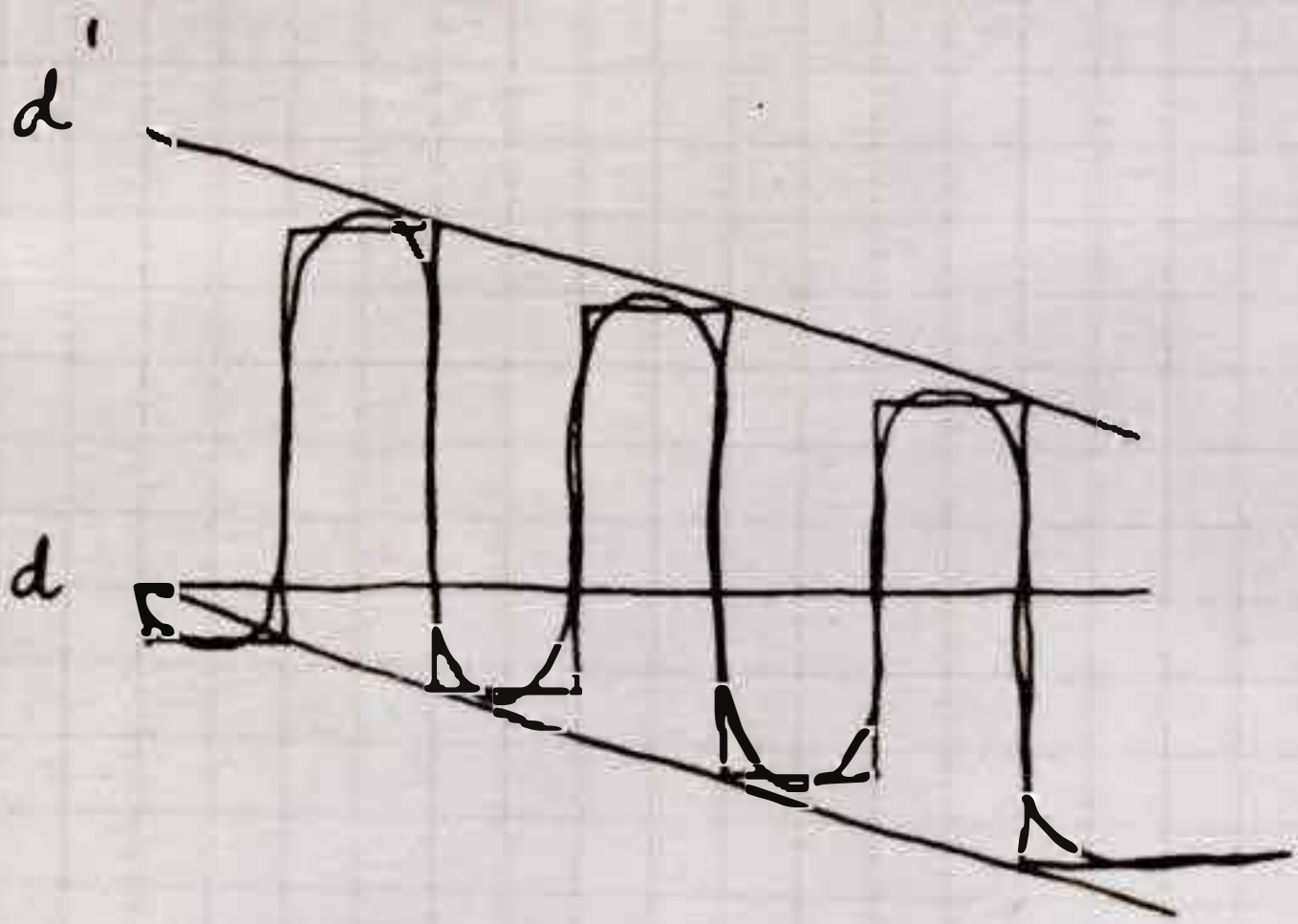
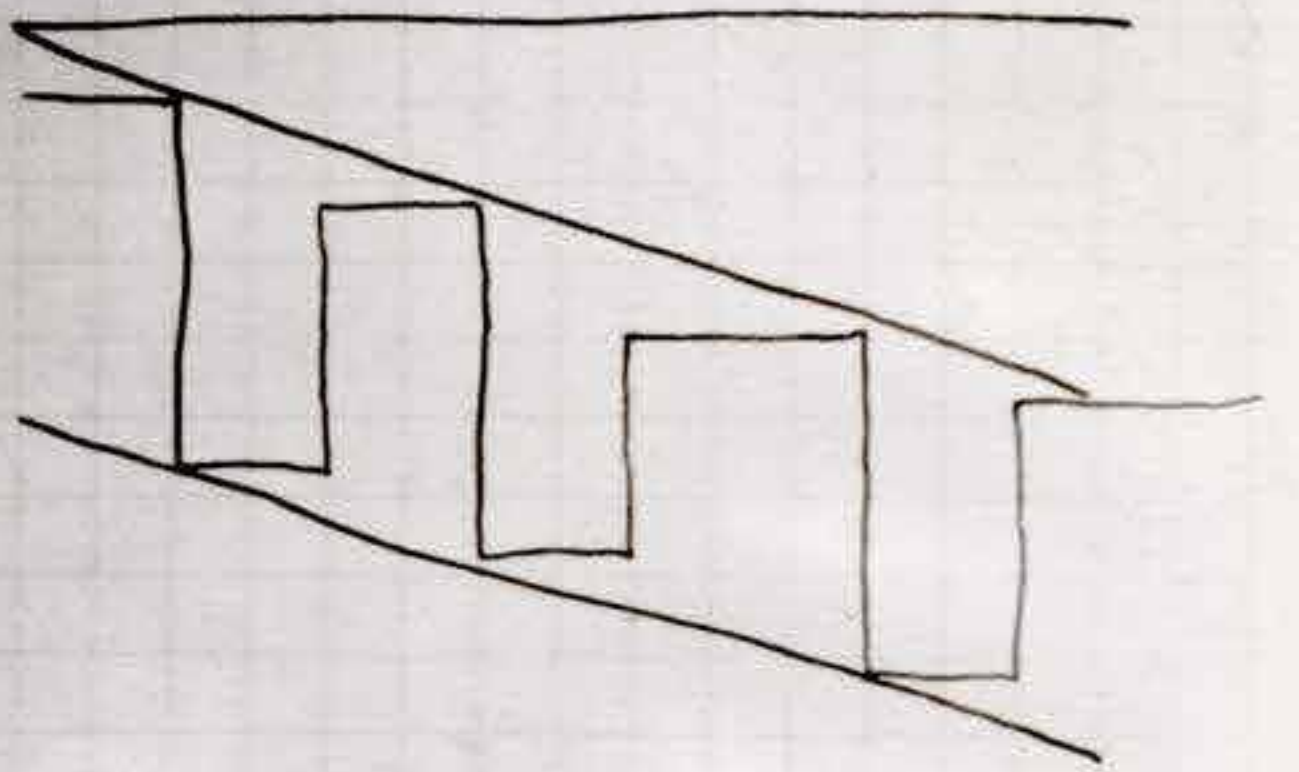
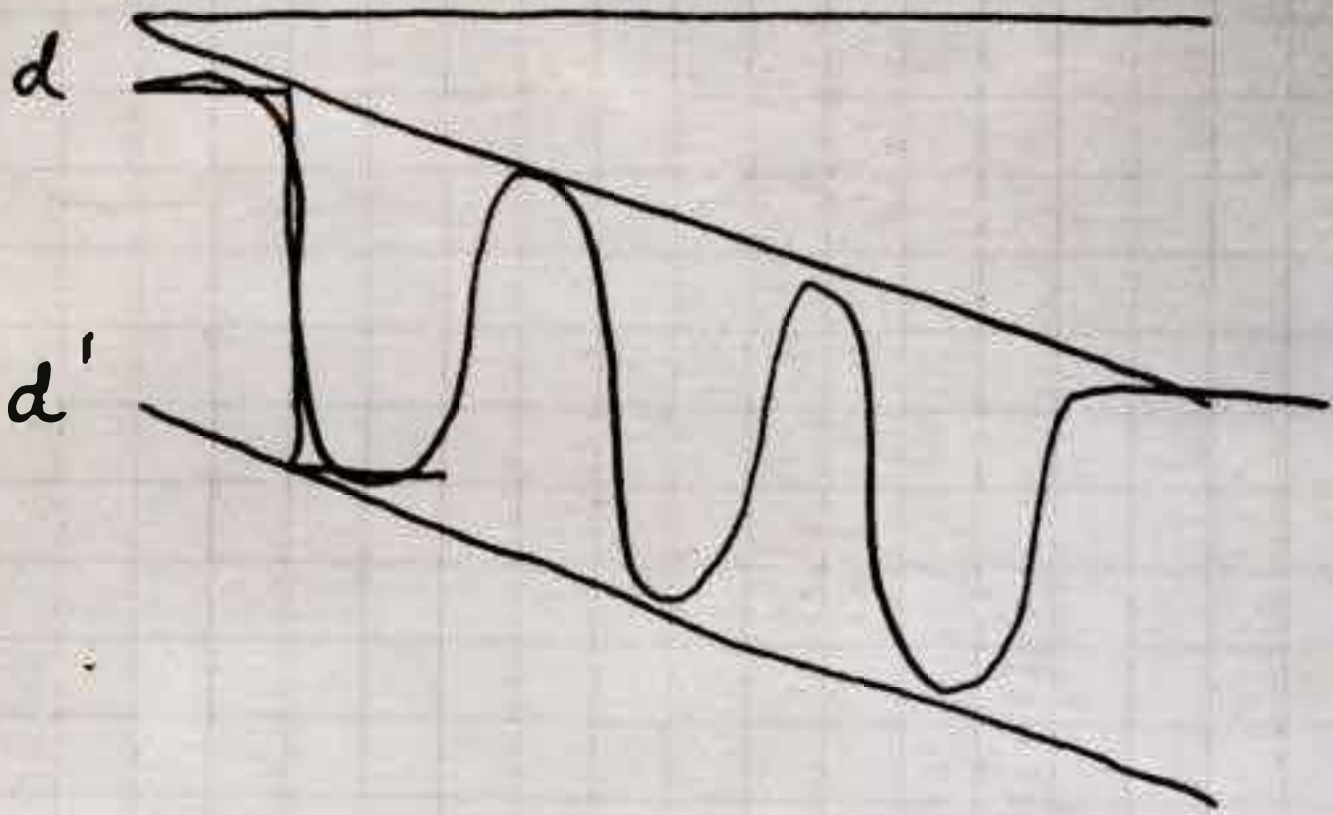


c/c





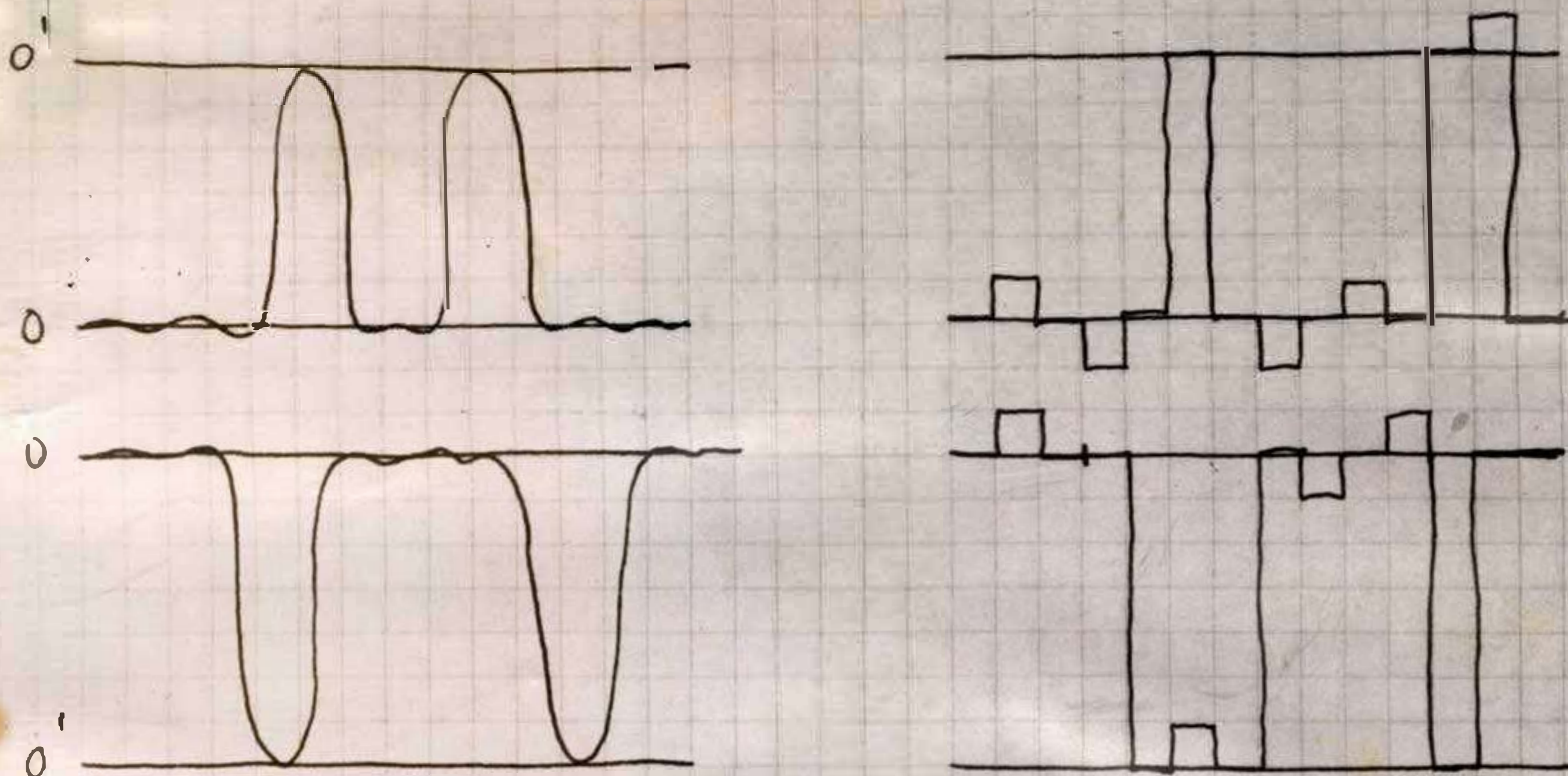
$\frac{d}{d'}$







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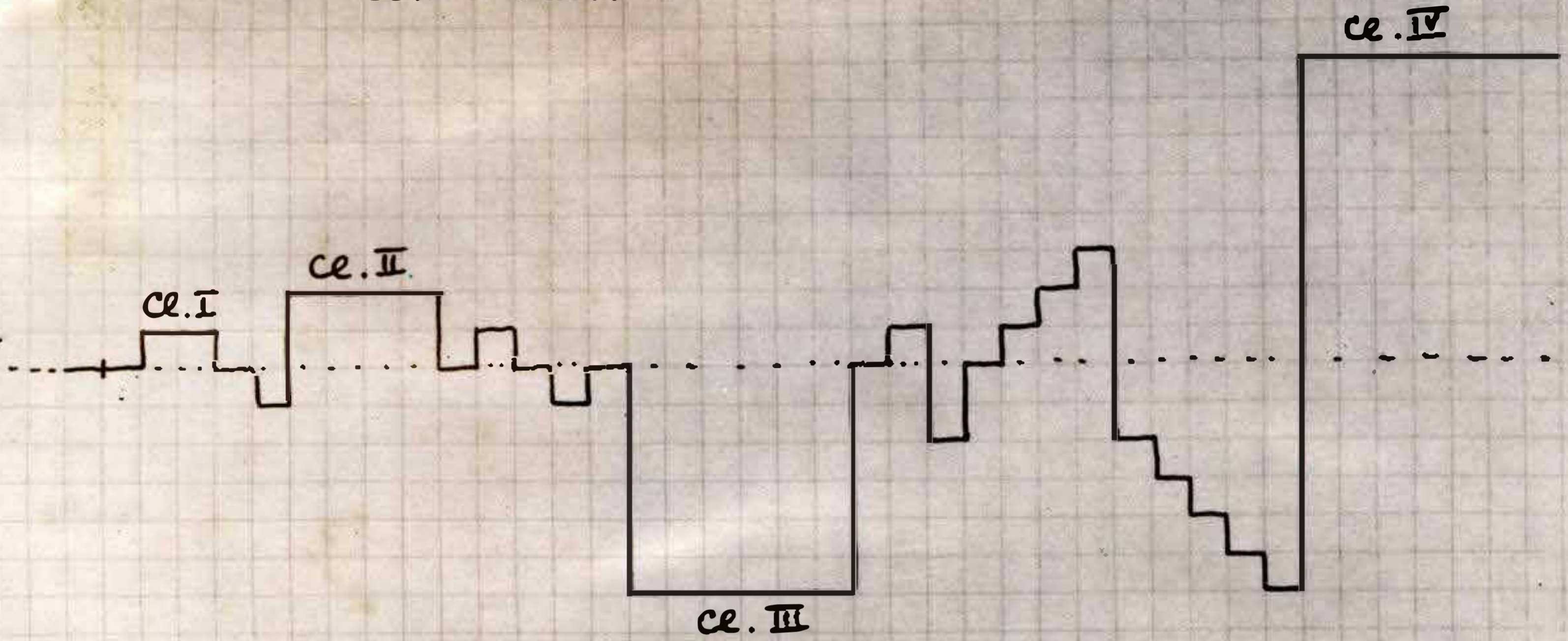


The 1, 2 and 3 forms of resistance produce the respective degrees of resistance. When more than one form is used in successive portions of melodic continuity they must follow one another in the increasing degrees. The opposite arrangement is mechanically inefficient and therefore produces an effect of weakness.

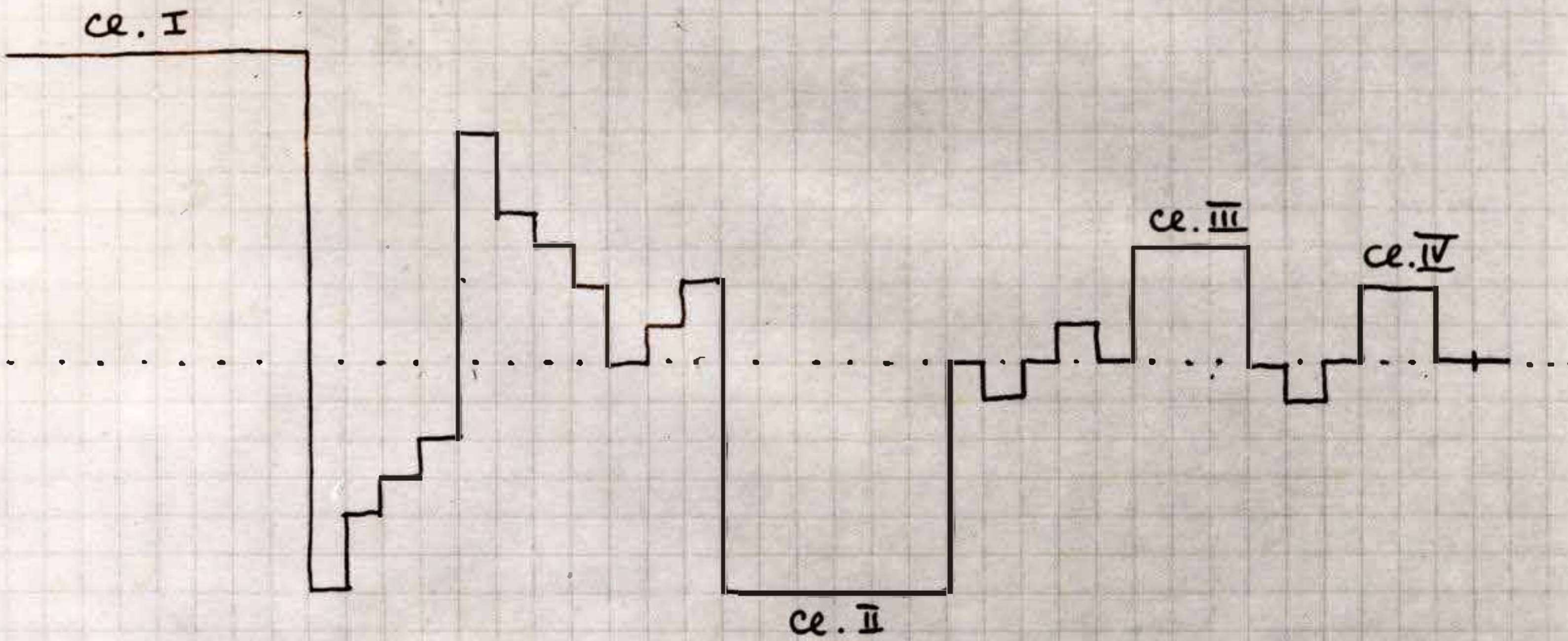
Resistances lead either toward climax or toward balance.



From Balance:



Toward Balance:





Distribution of Climaxes in  
Melodic Continuity.

The distribution of climaxes in melodic continuity must be performed with respect to the total duration of such continuity. The relative intensity of climaxes depends both on time and pitch ratios leading toward the respective climaxes. The natural tendency is the expansion of pitch and the contraction of time. These two components mutually compensate each other.

The climactic gain between the two adjacent climaxes takes place when:

1. The pitch-ratio is increasing and the time-ratio is constant;
2. The time-ratio is decreasing and the pitch-ratio is constant.

The climactic gain reaches its mechanical maximum when both forms are combined (increasing pitch-ratio and decreasing time-ratio).

It is practical to save the last effect for the main climax of the entire melodic continuity and to use it only when the extreme exuberance has to be attained.

As the decreasing time-ratio is characteristic of the continuity with a group of climaxes, rhythmic



material which would appropriately distribute the climaxes must belong to the decreasing series of growth, such as summation or power series. Smaller number values and in inverse correlation serves as material for the distribution of the pitch ratios for a group of successive climaxes.

This description refers to a trajectory moving towards main climax and must be inverted for the opposite direction.





Lesson LX.VII. Superimposition of the Time-Rhythm  
on the Secondary Axes

"Beauty" is the resultant of harmonic relations. In order to obtain a "beautiful" (esthetically efficient) melody it is necessary to establish harmonic relations between its factorial and fractional rhythm. This may be achieved by means of a homogeneous series of factorial-fractional continuity. Rhythm of time durations occurring within the bars must belong to the same series as the rhythm of the secondary axes. Naturally, there are hybrid melodies where fractional and factorial rhythm belong to different series. A homogeneous series is merely an expression of stylistic consistency.

Melodies with structural consistency may be found nearly in every folk lore, as well as in the works of composers who synthesized and crystallized the efforts of their predecessors. Beethoven crystallized the melodic style of the "Viennese School", (which at its time was a revolt against counterpoint and polyphonic writing). Bach, in his melodic themes, (in many cases with an odd number of bars), crystallized the efforts of several centuries. Mediaeval Cantus Firmus was the source of his themes.



Different styles have different evolutionary velocities. "Jazz" having a very high one, (like some of the specimens of the Alpine flora with a very short life-span), has already crystallized its homogeneity. Examples are numerous and may be found more in the "swing" playing than in the printed copies of the songs.

After the series has been selected, the actual composition of the fractional continuity may be accomplished in two ways:

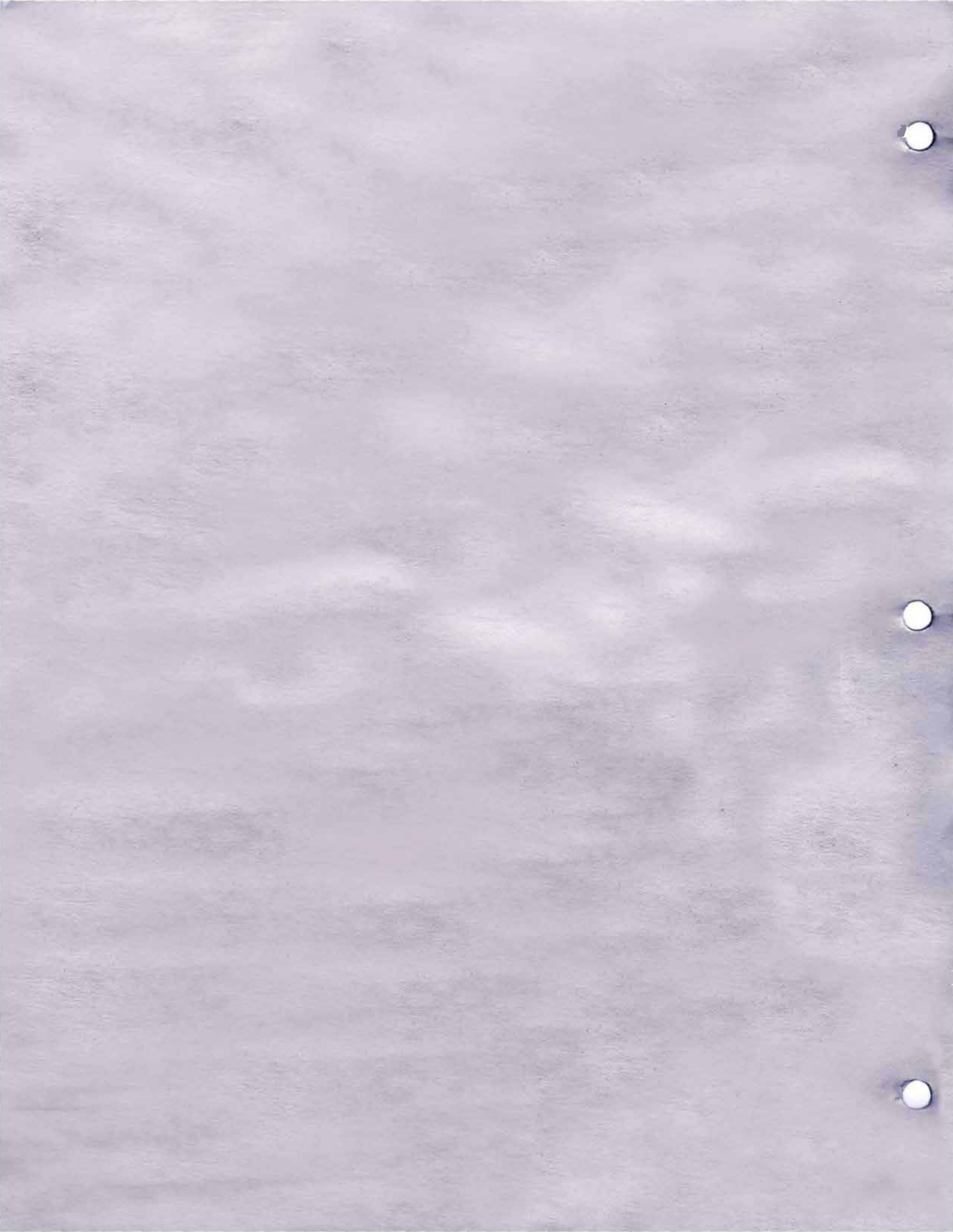
- (1) by using the resultants or the power groups,
- (2) by composing freely from the monomial, binomials, trinomials and quintinomials of a given family, (see "Evolution of Style in Rhythm").

An example of composing fractional continuity in  $\frac{4}{4}$  series:

Suppose we have a trinomial of the Secondary axes,  $a2T + bT + cT$ . In this case  $4T = 16t$ . To satisfy  $16t$  we may use  $r_{\underline{4+3}}$ , or  $(\frac{2+1+1}{4})^2$ , or any of their variations, i.e., the permutations or the resultants.

A free composition according to (2) may give results identical with some of the variations.

The groups of the  $\frac{4}{4}$  series are:



monomial . . . 4

binomials. . . 3 + 1 and 1 + 3

trinomials . . . 2 + 1 + 1, 1 + 2 + 1 and 1 + 1 + 2

the uniform quadrinomial . . . 1 + 1 + 1 + 1

Deciding upon a<sub>2T</sub> being (3+1) + (2+1+1), b<sub>T</sub> being 1+1+2 and c<sub>T</sub> being 1+3, we obtain r<sub>4+3</sub>. By selecting freely various recurrences of the same binomial, like 3+1, we obtain: a<sub>2T</sub> = (3+1) + (3+1), b<sub>T</sub> = 3+1, c<sub>T</sub> = 3+1, or various recurrences of the same trinomial with variations, like: a<sub>2T</sub> = (2+1+1) + (2+1+1), b<sub>T</sub> = 1+2+1, c<sub>T</sub> = 1+1+2, we obtain groups that are not identical with the resultants or the power groups.

When a climax is desired the maximum time value must be placed at the corresponding point of a Secondary axis (in a at the end, in b at the beginning, in c at the beginning and in d at the end). For instance, if a climax is desired on a<sub>2T</sub>, it must be the last term of a rhythmic group of this axis. In the  $\frac{4}{4}$  series it would be:

$$a_{2T} = (2+1+1) + (1+3)$$

$$\text{or } (2+1+1) + (1+1+2)$$

$$\text{or } (2+1+1) + 4 \text{ and the like.}$$

To superimpose a fractional rhythmic group on a factorial group of the Secondary axes, means to

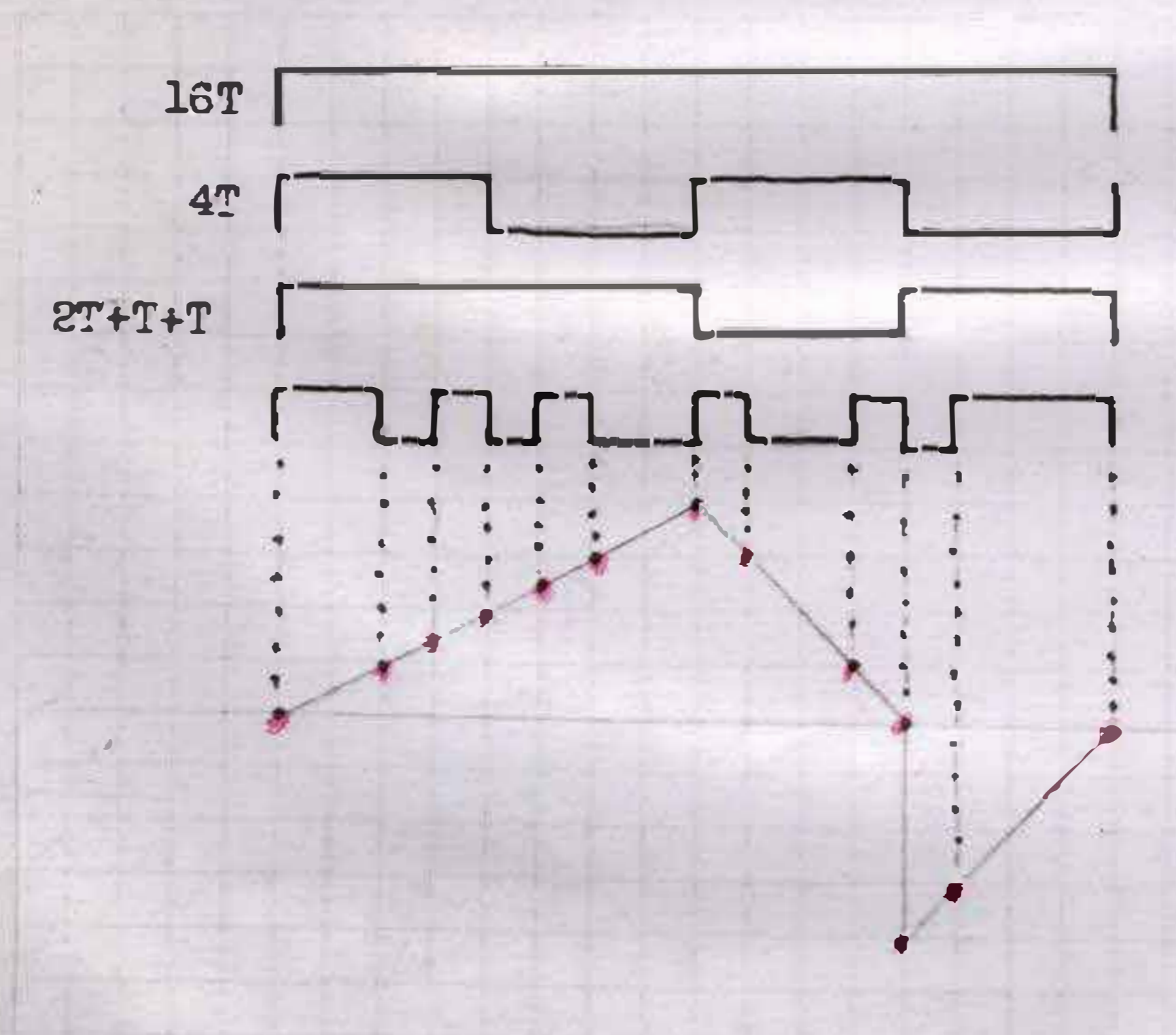


distribute the points of attack on a pitch trajectory  
(the path of a moving point).

Let us assume that a group of secondary axes has been constructed with no reference to any particular logarithmic (tuning) system. Placing the pre-selected fractional group above the axes and dropping perpendiculars from the points of attack, we accomplish the distribution of the points of attack (which become the moments of attack) along the pitch trajectory of a hypothetic tuning system.

Example

$$a2T + bT + ct = (2+1+1) + (1+1+2) + (1+2+1) + (1+3)$$



Thus, the red points are the moments of attack on  
this pitch trajectory.





Here we arrive at the following definition of melody: melody is a resultant trajectory of the axis-group moving through the points of attack. Melody, in the academic sense, i.e., with sudden pitch variations within a tuning system, is a rectangular trajectory. Melody, in the Oriental conception, as well as in any musical actuality, is a curvilinear trajectory, i.e., containing a certain amount of pitch-sliding. We shall deal with composition of a melody in the academic sense as our musical culture leaves the bending of a rectangular trajectory to the instrumental performer.

As the secondary axes form triangles (with respect to primary axis), two forms of rectangular motion through the points of attack are possible:

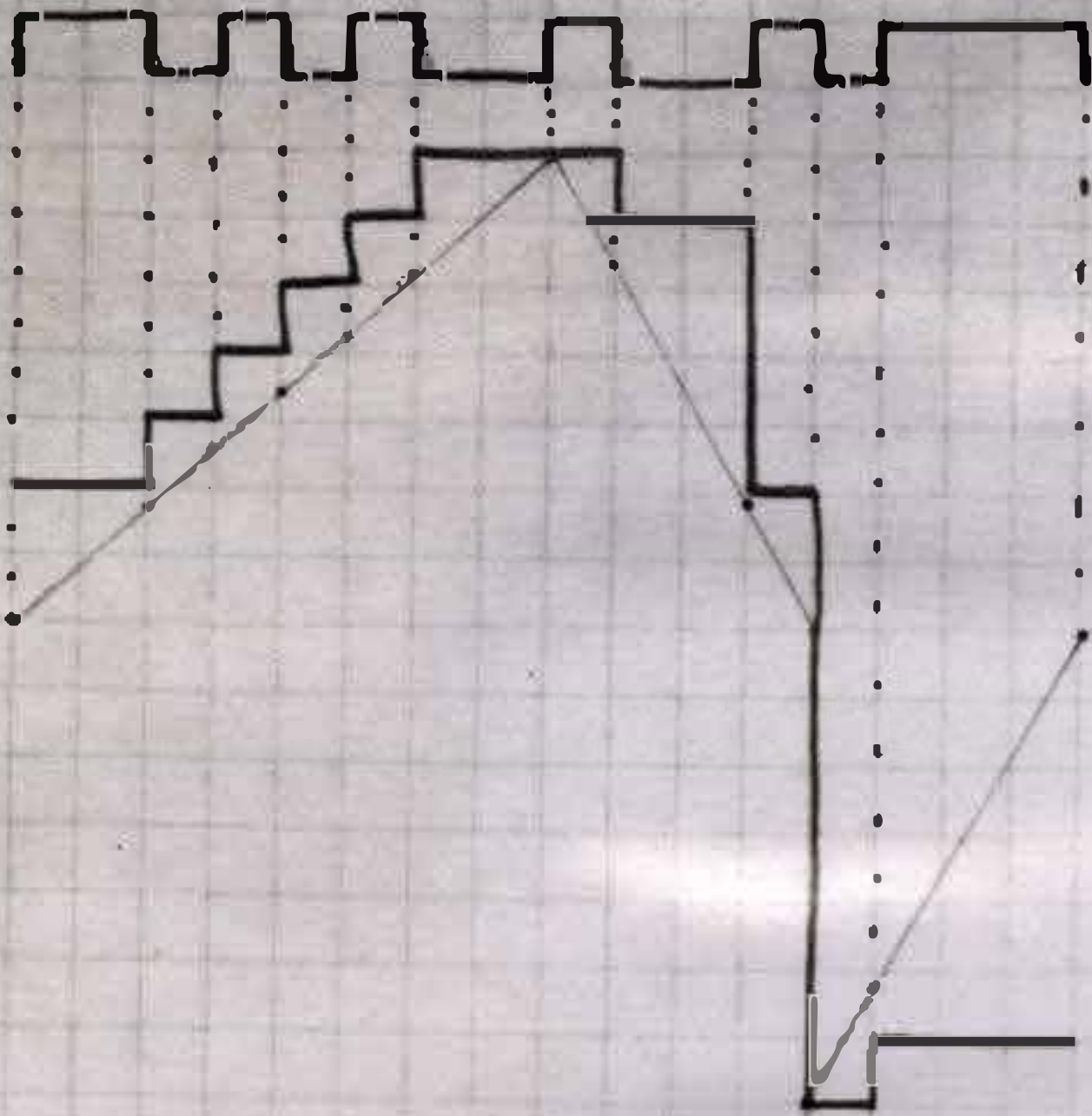
- (1) ascribed (sin phases)
- (2) inscribed (cos phases)

Though in composing melody a free choice of the two may take place, in balancing melody at its end on b or c axes, the ascribed motion produces an incomplete (i.e., unbalanced) cadence, while the inscribed motion produces a complete (i.e., balanced) one. The first one is a device for deviating from balance, i.e., for accumulating tension, a stimulus for the new recapitulation.

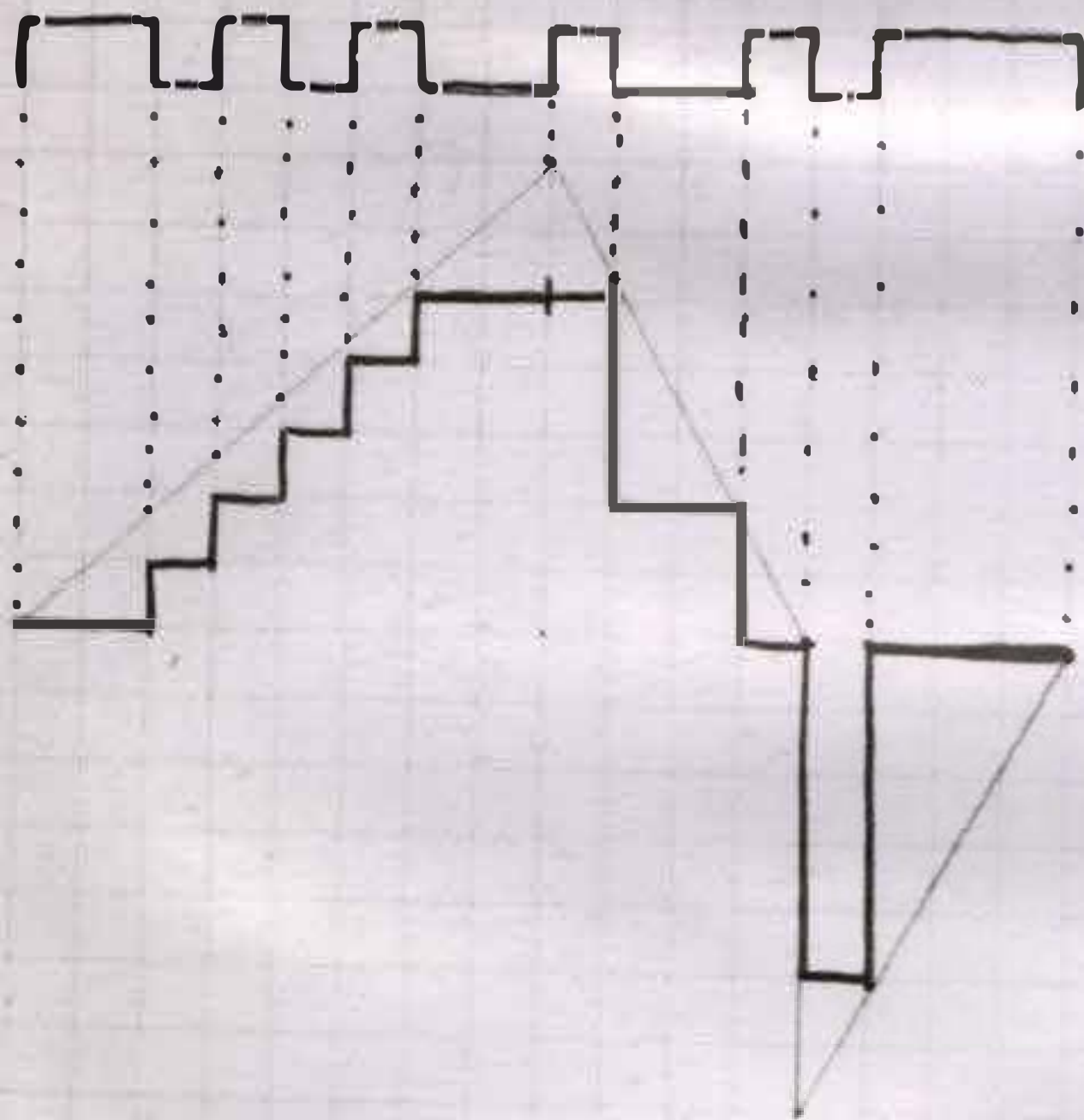
Examples of rectangular trajectories evolved through the axes of the previous example:

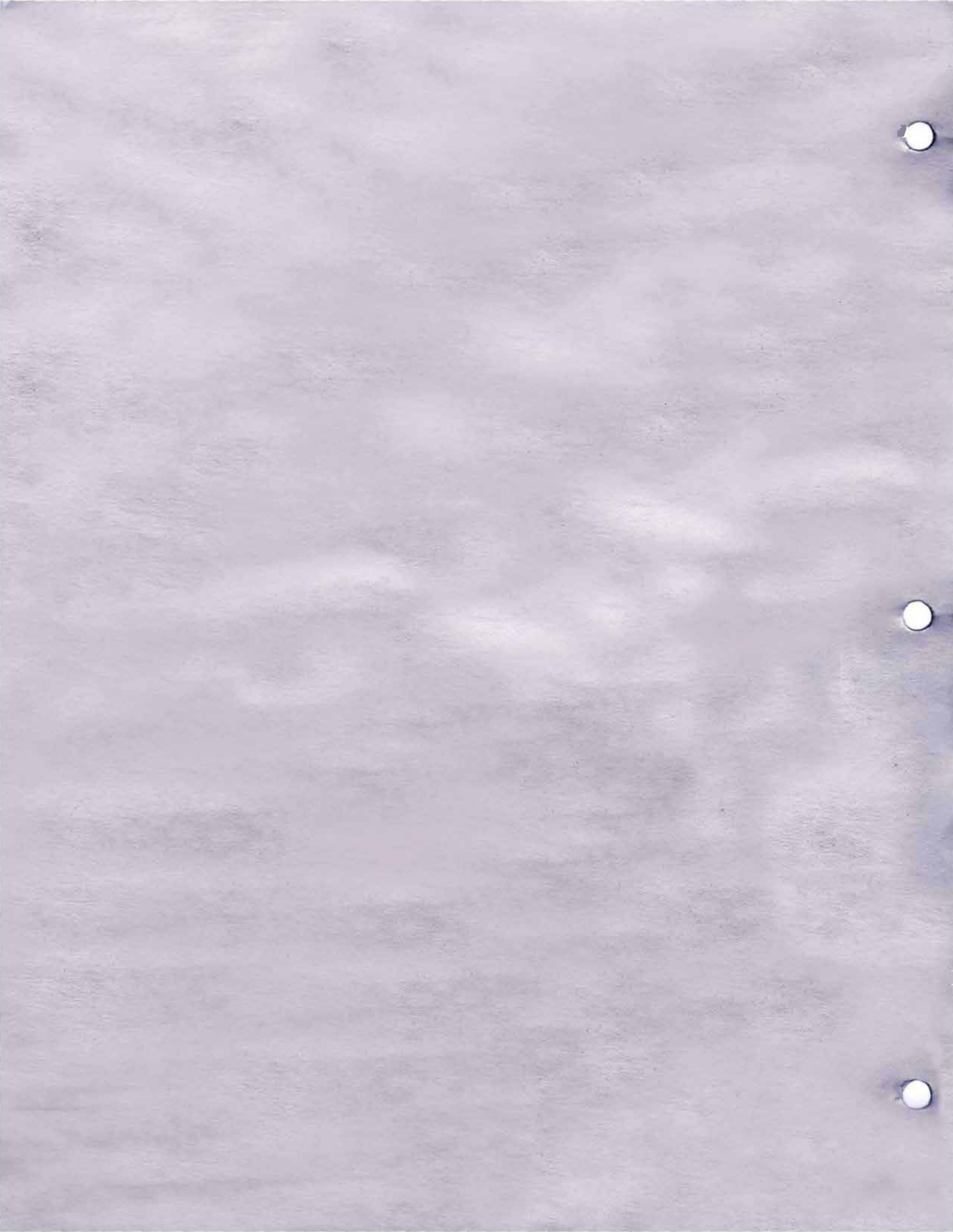


Example I - Ascribed Motion



Example II - Inscribed Motion





These two potential melodies are totally different in their pitch progression. The usual, commonplace composition of pairs varies with respect to the cadence only. Such pairs may be either inscribed or ascribed, but must be identical otherwise; the ending of the first one is ascribed, while the ending of the second - inscribed.



J O S E P H S C H I L L I N G E R

C O R R E S P O N D E N C E C O U R S E

Subject: Music

Lesson LXI.

VIII. Superimposition of Pitch-Rhythm

(Pitch-Scale) on the Secondary Axes

Uniform time-intervals (durations) being geometrically projected produce space-intervals, (extensions). Such uniform time scales are primary selective systems when  $T = r_{a+1}$ . When  $b \neq 1$  they become secondary selective systems, (rhythm-scales).

Uniform pitch-intervals of our tuning system produce logarithms to the base of  $^12\sqrt{2}$  (semitones). Chromatic scale is the primary selective system of pitch in our intonation. Geometrical projection of such scale is a uniformity along the ordinate. Any other pitch-scale within the same tuning system is a secondary selective system, (i.e., a derivative of the primary selective system).

It is easy to see that a pitch-time trajectory moving in either ascribed or inscribed form of motion through the points of intersection of time (abscissa) and pitch (ordinate) uniformities (primary selective systems), is (structurally) the simplest form of melody, i.e., a chromatic scale in uniform rhythm.





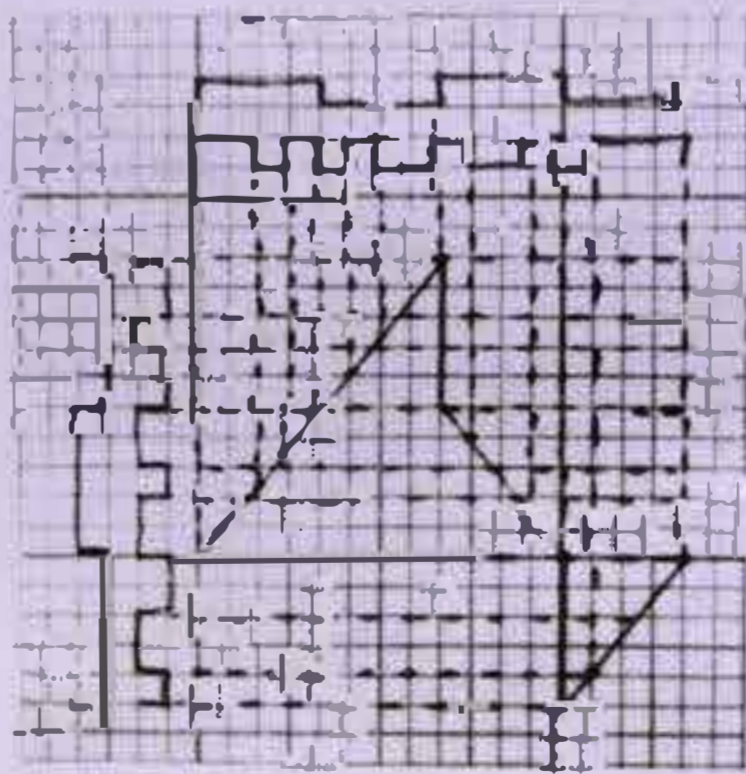
Here we arrive at the following definition of melody: melody is a pitch-time trajectory resulting from the intersection of the points of intonation (pitch-units) with the points of attack (time-points) in a specified axis-system.

When the geometrical points of intersection do not coincide with the pitch-units of a scale, pitch-units nearest to the coincidence-points must be used.

Let us superimpose an Aeolian scale (2+1+2+2+1+2) on the axis-group illustrated in the preceding chapter. Let us assume  $a2P + bP + cP$ , i.e., a parallel PT correlation. And let  $P = 5$ , which in this case gives a symmetric distribution. Let further, pitch c be the primary axis. Then  $a2P$  extends from c to b<sup>b</sup>,  $bP$  from f to c and  $cP$  from g to c.

Here is the final construction of the axis group:

Scheme of the Points of Geometrical Intersection.





This diagram produces a slight deviation from the description given in the text, due to the fact that the scale is small enough to give deviations. However, this is not essential as further adjustments follow the scale.

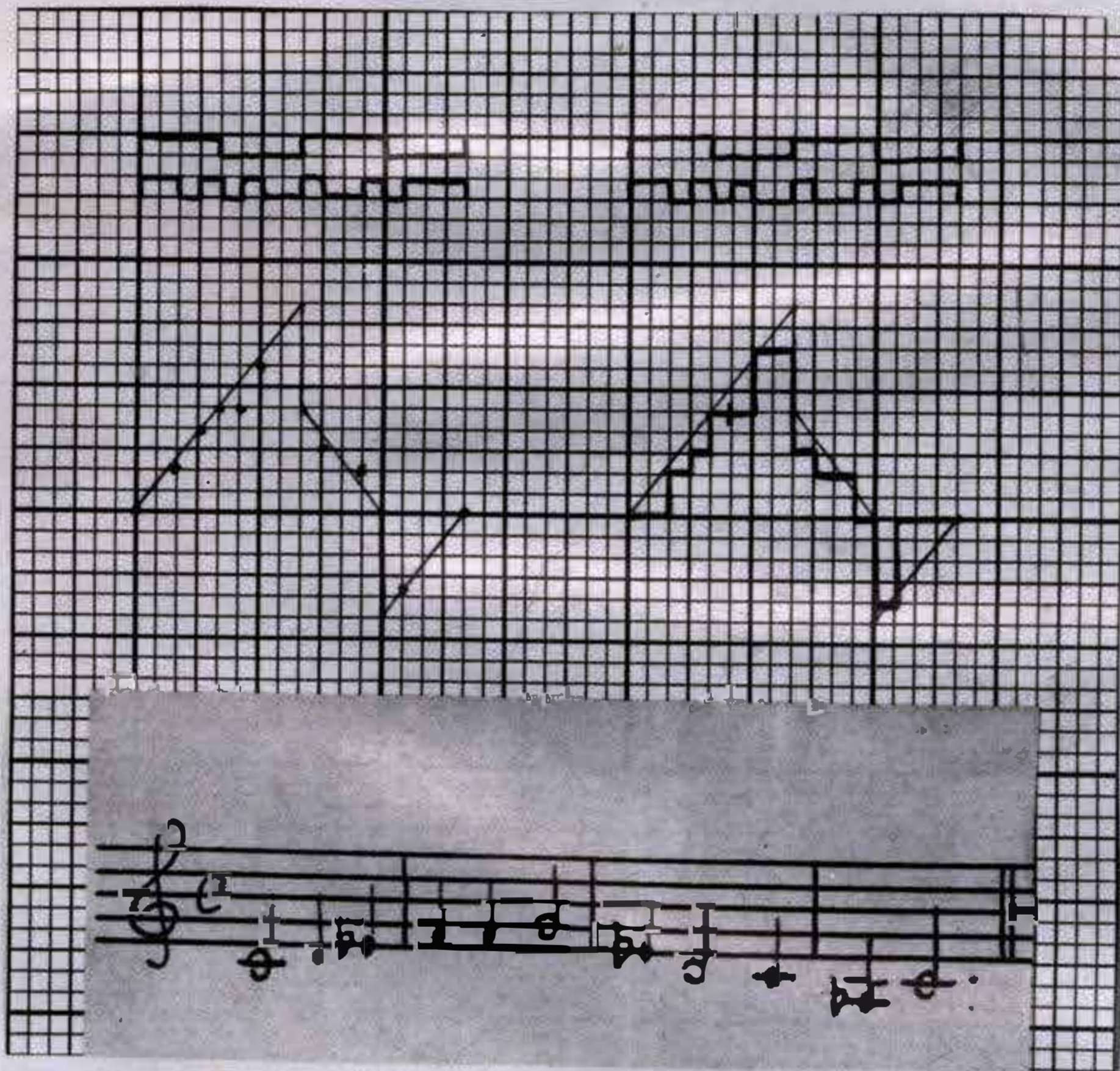
The next step is to adjust the points of intersection to the Aeolian scale. Let us analyze point by point.

If the first point of intersection is c, the nearest pitch-unit to the second point of intersection on the Aeolian scale is d. Next, we select e<sup>b</sup> as the nearest to the third intersection-point. The fourth falls exactly on f. The fifth falls on f<sup>#</sup> which is not in the scale. In this case either the repetition of f, or g is available. Next point is nearest to g. Through ascribed motion the entire axis a would start on d and end on b<sup>b</sup>.

As in inscribed motion, pitch-levels move toward the points of intersection; the first pitch-unit on b- axis will be either f or e<sup>b</sup>, as the geometrical intersection coincides with e<sup>b</sup>. The next intersection-point is nearer to d. In order to complete b- axis through inscribed motion it will be necessary to consider c as the last intersection point. C- axis through the inscribed motion gives its points of intersection at a<sup>b</sup> and c.



We shall reconstruct now the axis-group with respect to the Aeolian scale, as just described, and draw an inscribed trajectory. This trajectory is the most elementary form of an actual melody.



It would not be difficult to find all other versions, i.e., the ascribed trajectory and the trajectories where either axis may be realized in ascribed or inscribed motion.



Here is a chart of combinations:

Axes:	a	b	c
	ascribed	ascribed	ascribed
	ascribed	ascribed	inscribed
	ascribed	inscribed	ascribed
	inscribed	ascribed	ascribed
	inscribed	inscribed	inscribed
	inscribed	inscribed	ascribed
	inscribed	ascribed	inscribed
	ascribed	inscribed	inscribed

There are eight versions altogether.

After obtaining an actual melody, such melody becomes a subject to scale variation, tonal and geometrical expansions and inversions. For instance, the same melody in a "blue" scale would sound:



Or in a Chinese (2+3+2+2) scale (through the translation of the corresponding degrees):

(please see next page)







Here an allowance has to be made on the first note of the last bar, as the VI does not exist in the Chinese scale, (substituting it by the last degree of the scale, i.e., V, which is a).



Lesson LXII.IX. Forms of Trajectorial Motion

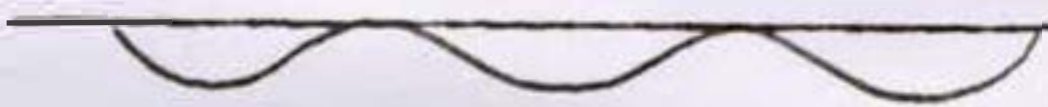
The trajectory obtained above was called "most elementary form of an actual melody" because its form of motion is simple harmonic, (i.e., scalewise) motion. According to Chapter VI, such melody cannot be too expressive or dramatic. In order to obtain an expressive melody it is necessary to build resistances. This cannot be realized without introducing more complex forms of motion.

We shall present now all the trajectorial forms with respect to the zero axis.

(1) Sin motion with constant amplitude:



(2) Cos motion with constant amplitude:

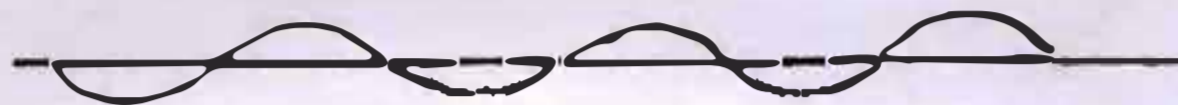


(3) Combined sin + cos motion with constant amplitude:

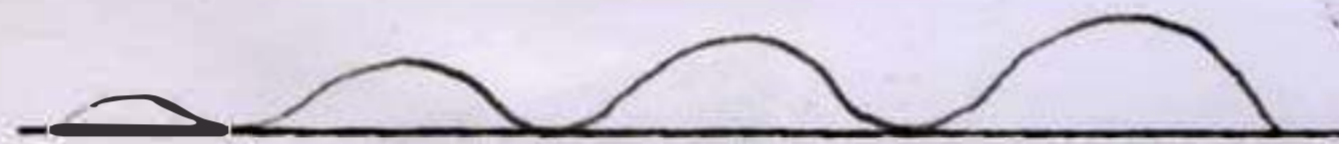




(4) Combined cos + sin motion with constant amplitude:



(5) Sin motion with increasing amplitude:



(6) Sin motion with decreasing amplitude:



(7) Sin motion with combined increasing-decreasing amplitude:





(8) Sin motion with combined decreasing-increasing amplitude:



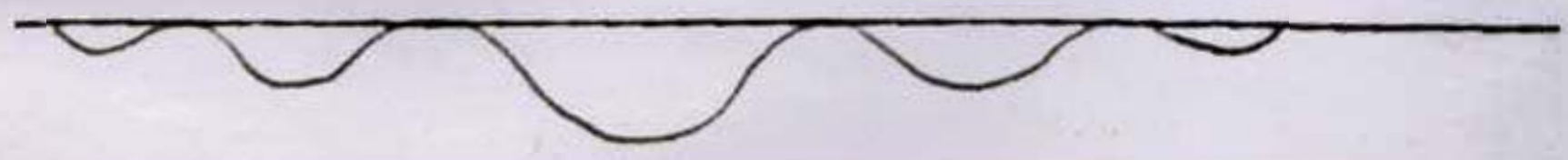
(9) Cos motion as (5):

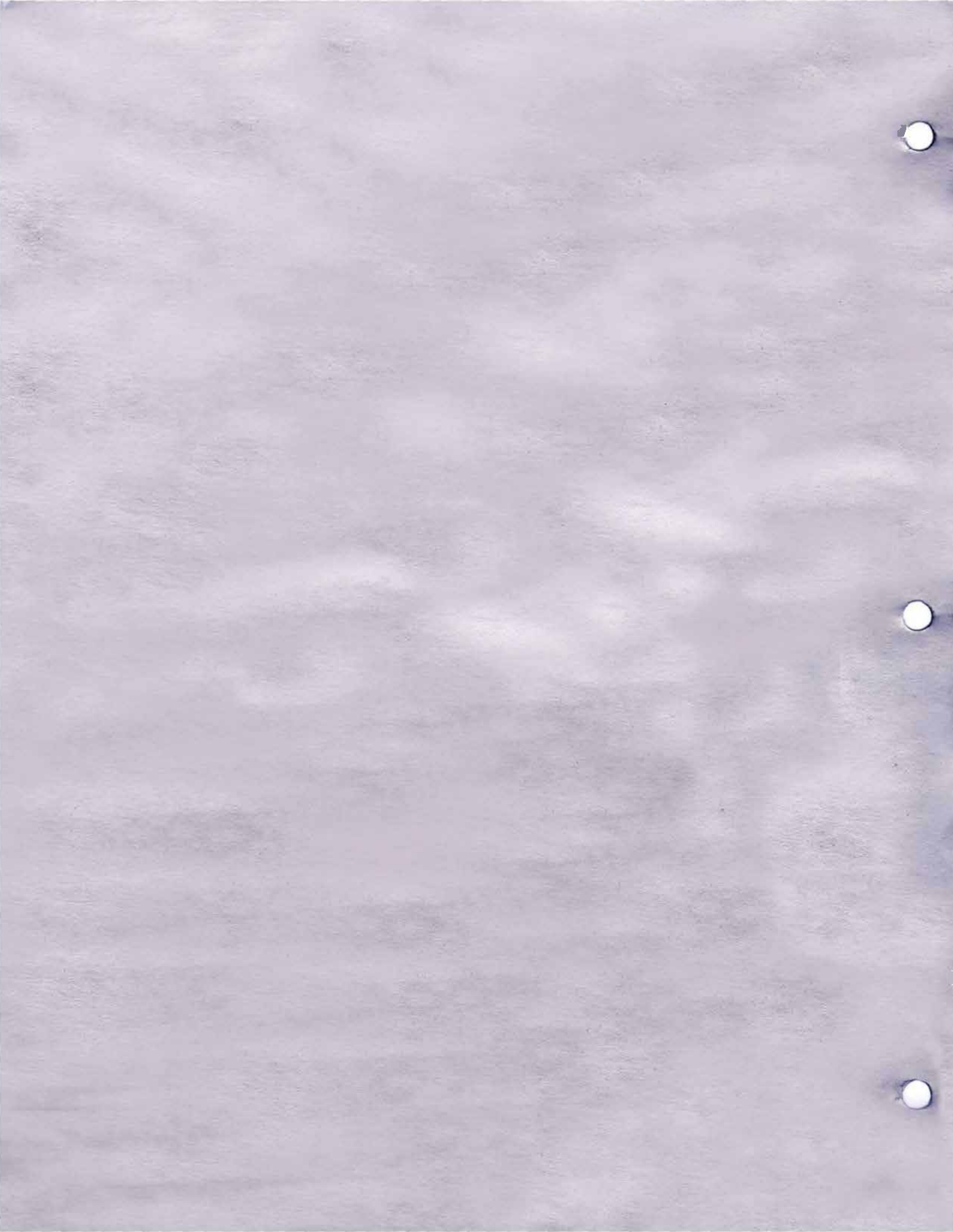


(10) Cos motion as (6):



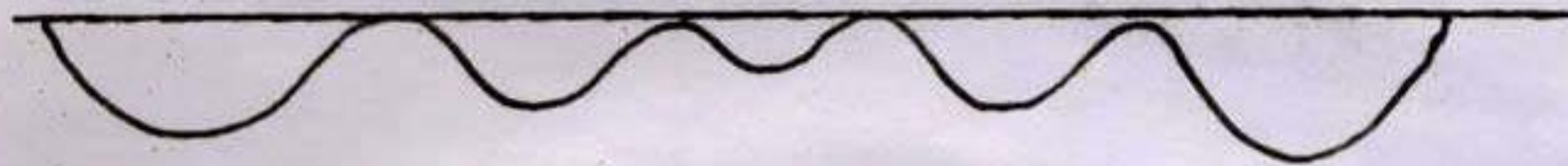
(11) Cos motion as (7):



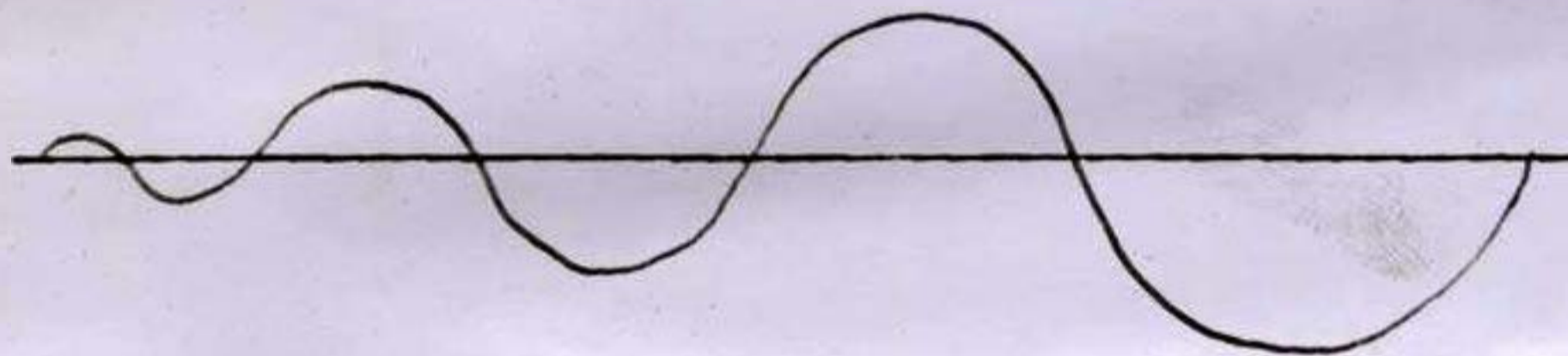




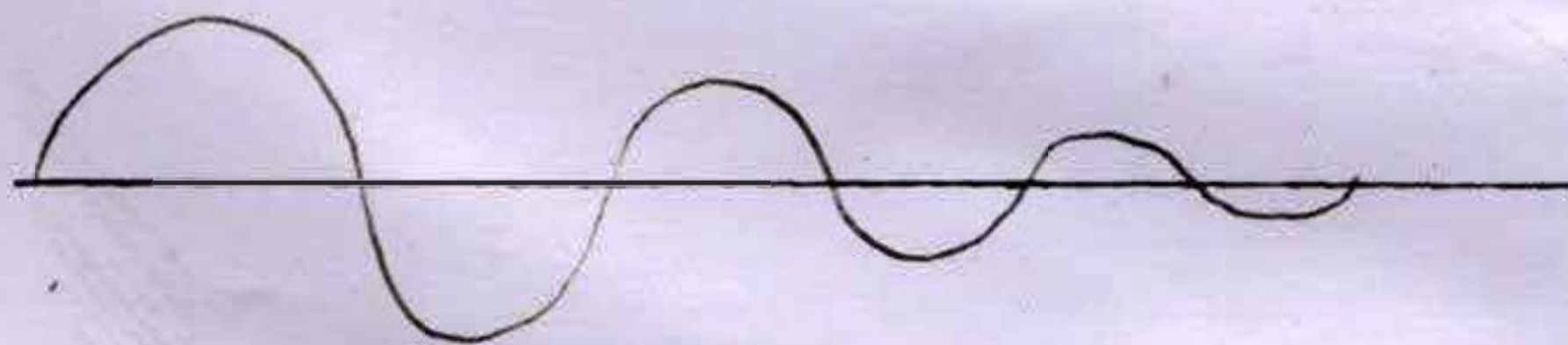
(12) Cos motion as (8):



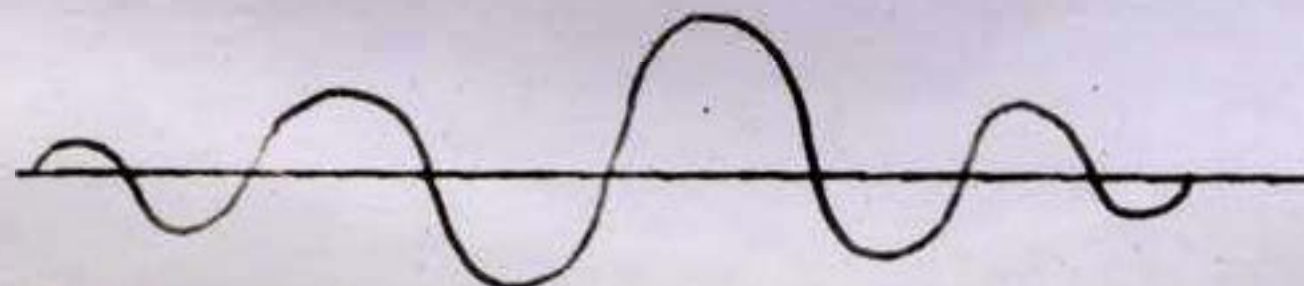
(13) Combined sin + cos motion with combined amplitude as (5):

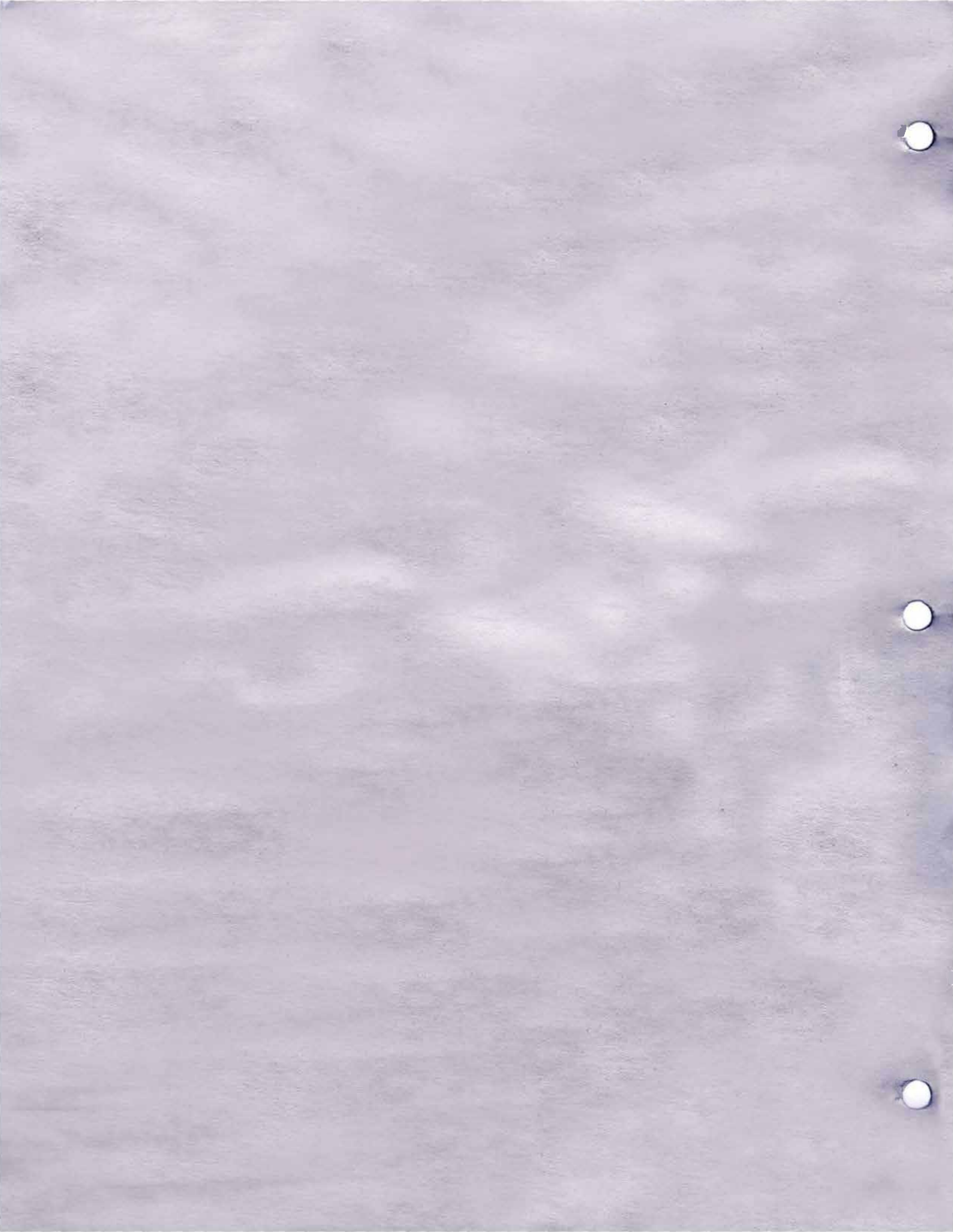


(14) Combined sin + cos motion with combined amplitude as (6):

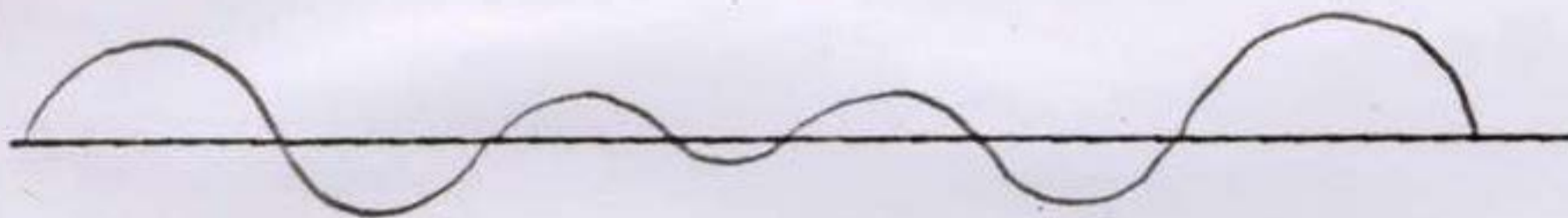


(15) Combined sin + cos motion with combined amplitude as (7):

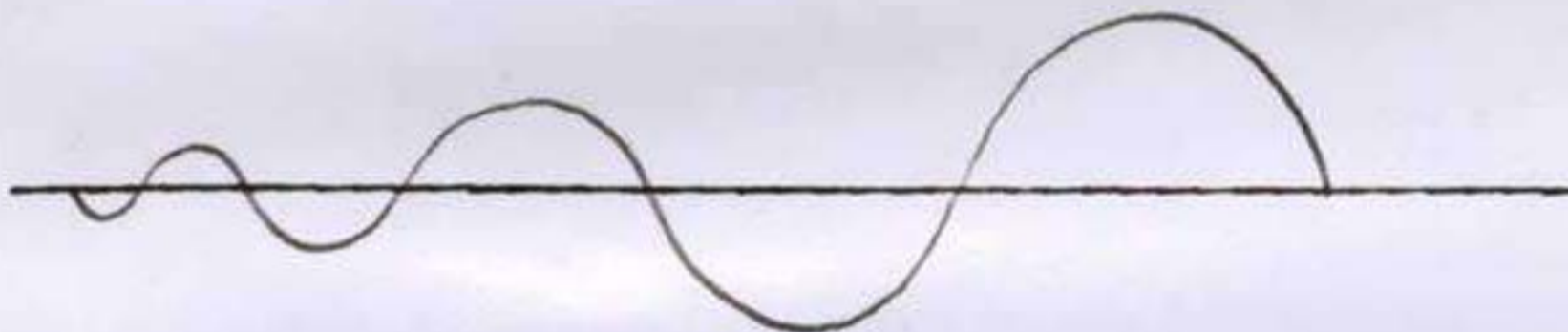




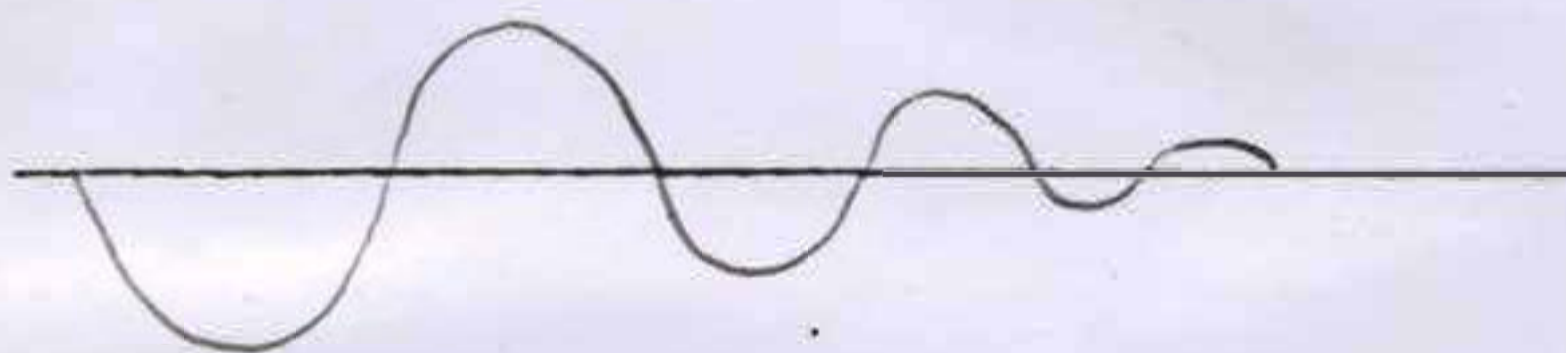
- (16) Combined sin + cos motion with combined amplitude as (8):



- (17) Combined cos + sin motion with combined amplitude as (13):

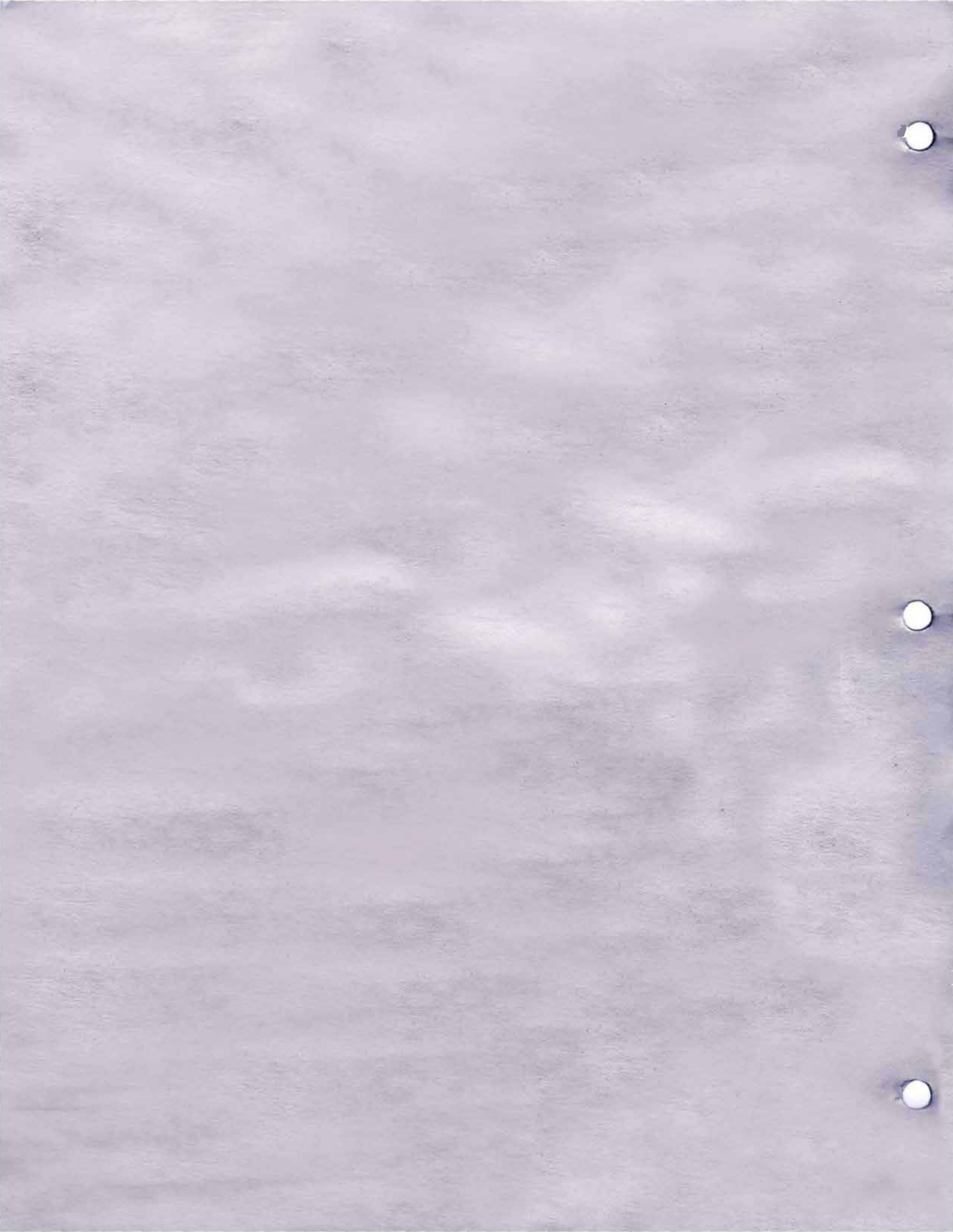


- (18) Combined cos + sin motion with combined amplitude as (14):



- (19) Combined cos + sin motion with combined amplitude as (15):





(20) Combined cos + sin motion with combined amplitude as (16):

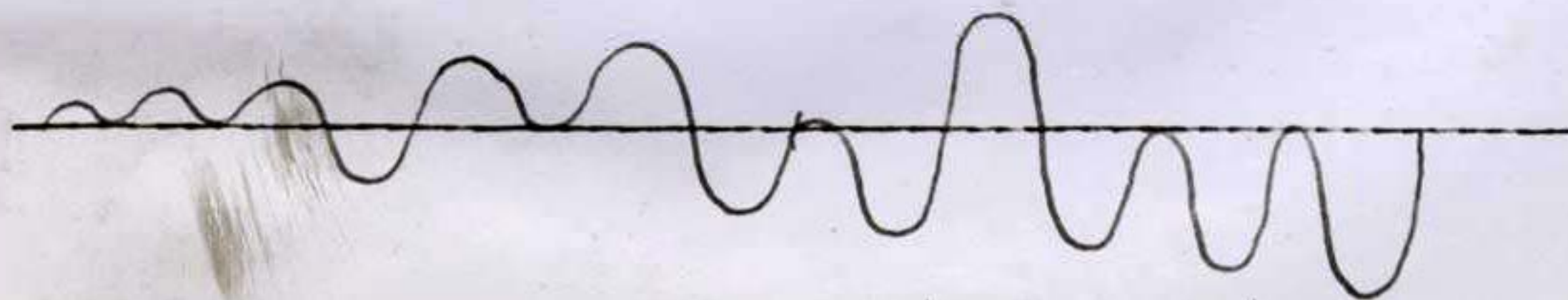


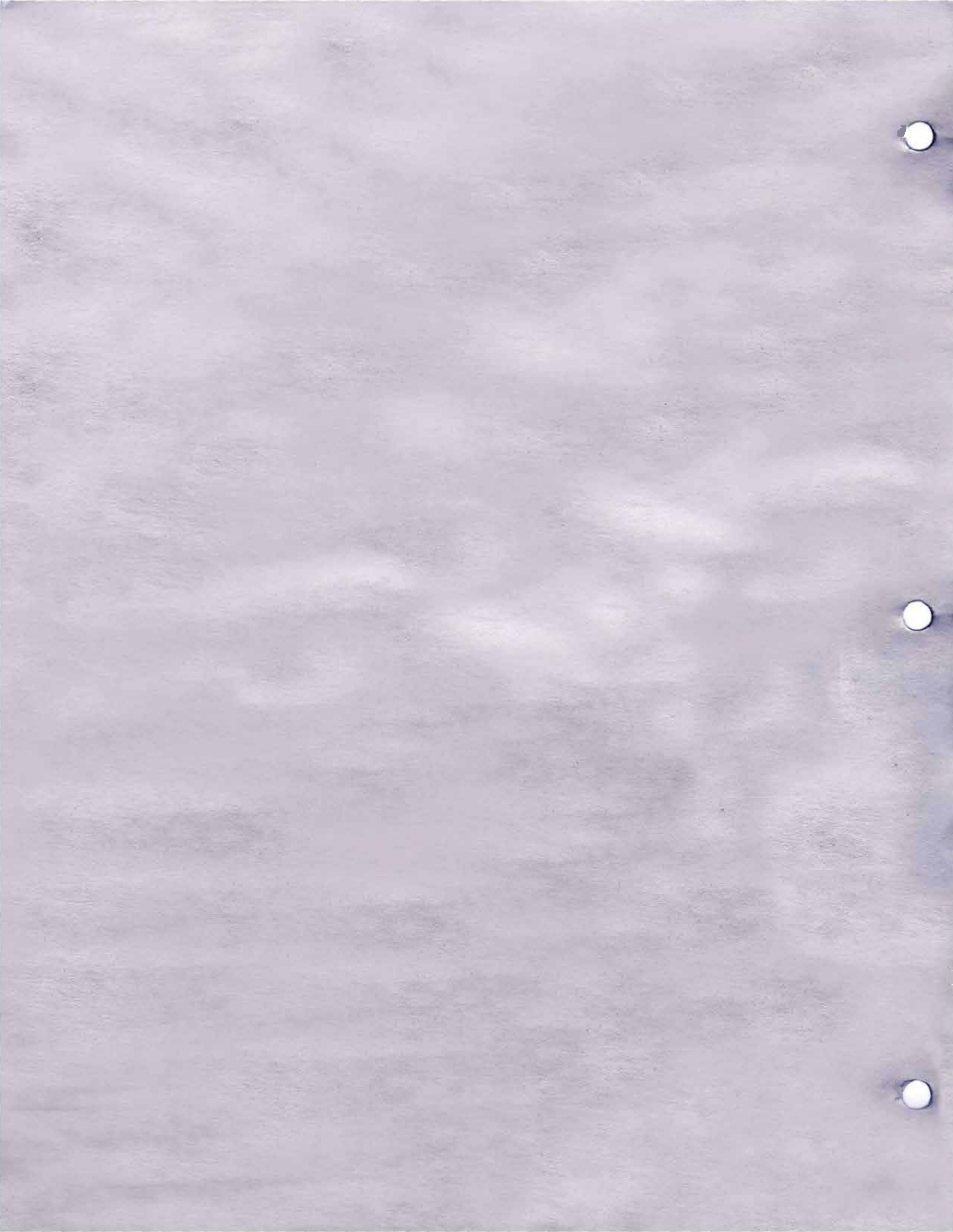
These twenty versions are merely variations of the two original forms, i.e., (1) and (5). Every cos is (d) of the sin and every decreasing amplitude is (b) of the increasing amplitude.

Further development of these trajectorial forms may be obtained through application of the coefficients of recurrence of the sin, the cos and the growth of amplitudes. For instance,  $3 \sin + \cos + 2 \sin + 2 \cos + \sin + 3 \cos$  on constant amplitude:



The same case on increasing amplitude:





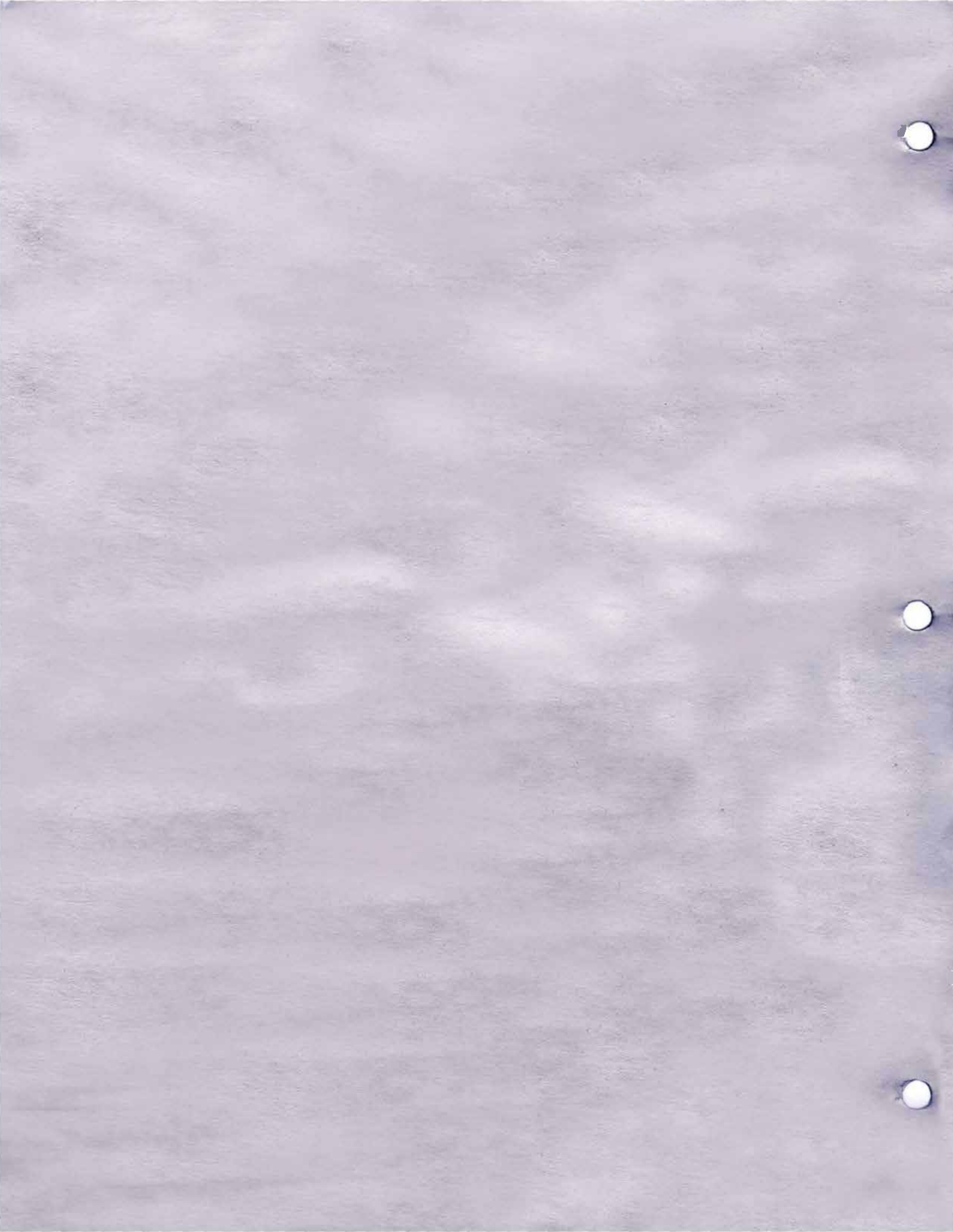
All these forms being transformed into rectangular trajectories, with respect to a definite intonation (tuning) system, become actual intonation-groups, i.e., melodic forms. For example, a groupetto is  $\sin + \cos$  with constant amplitude.

Including the zero of pitch variation, (absolute zero-axis trajectory), we have the following forms of trajectorial motion:

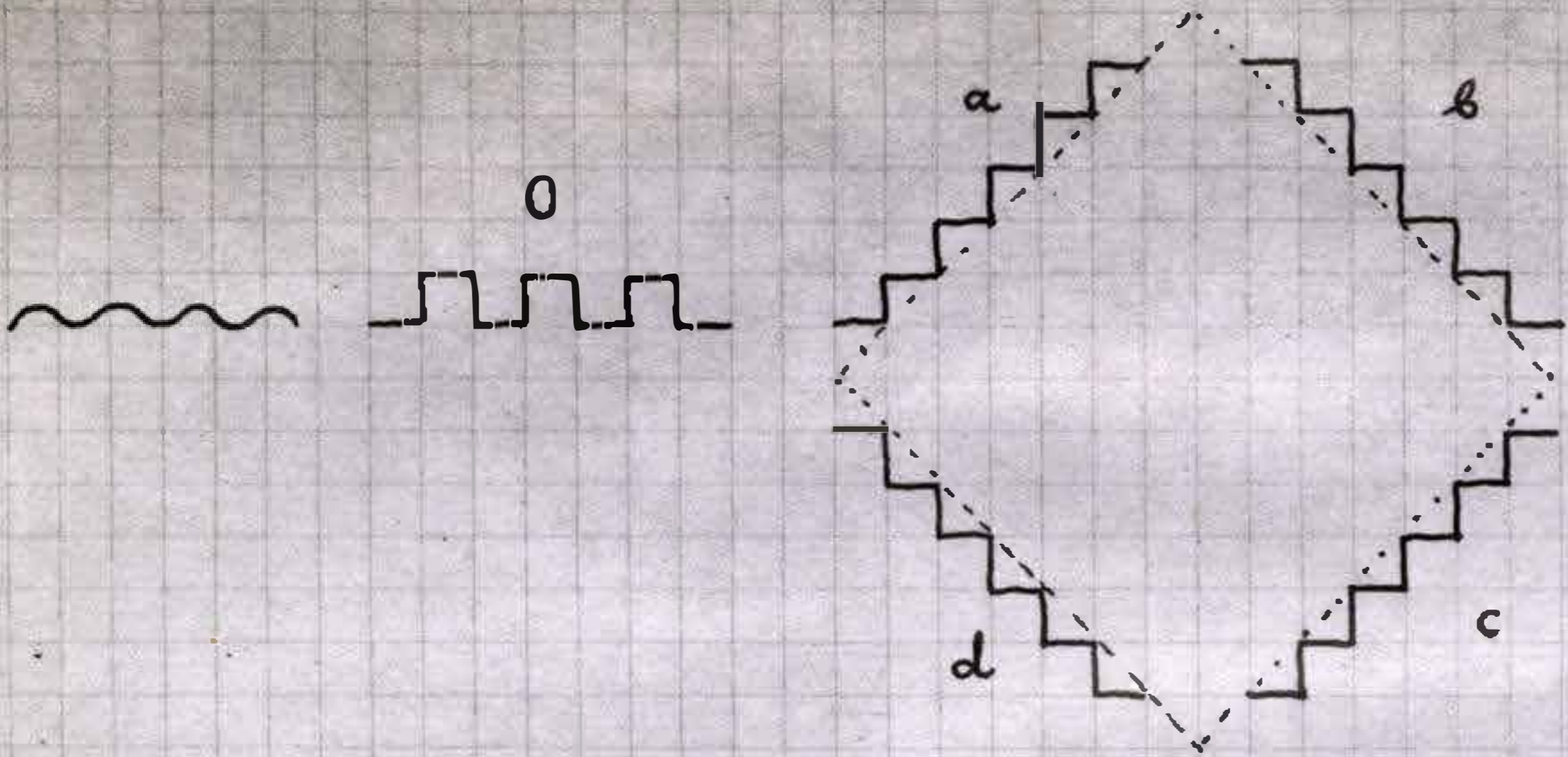
- (1) constant pitch trajectories (repetition on extension).
- (2)  $\sin$  or  $\cos$  trajectories (one phase motion).
- (3) combined trajectories (full period motion or rotation).

Application of various trajectorial forms to a, b, c and d axes gives the following correspondences. All the  $\sin$  of 0 remain  $\sin$  on all other axes. All the  $\cos$  of 0 remain  $\cos$  on all other axes. All the combined forms of 0 with respect to  $\sin$ ,  $\cos$  and the constancy of amplitude remain respectively the same on all other axes. Zero axis is the only one to be heard. The rest are merely hypothetical lines.

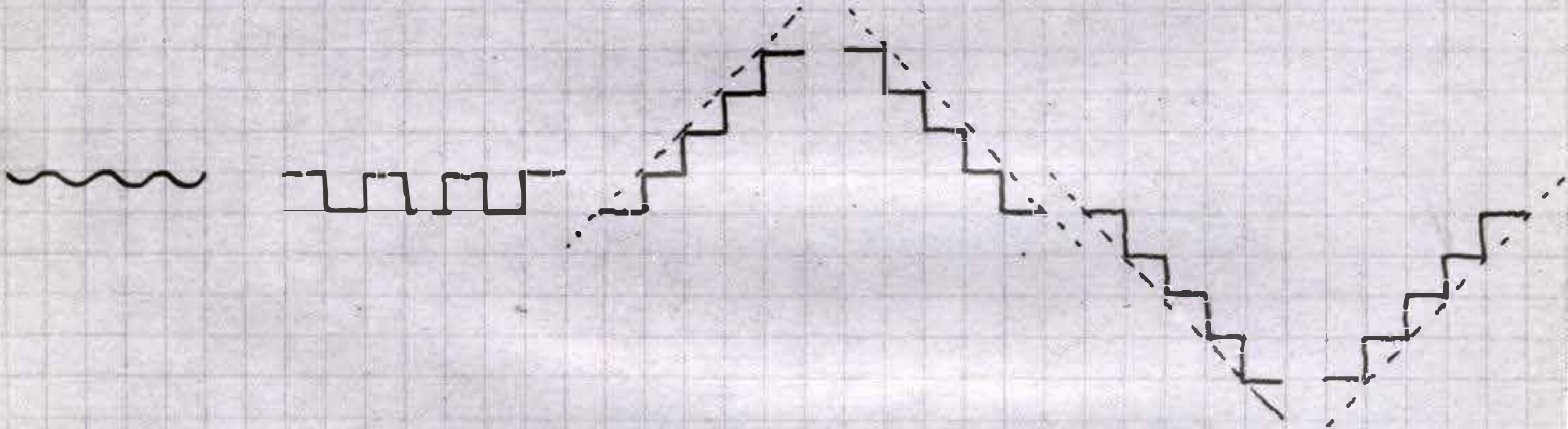
Here is an example of the corresponding translations of a curvilinear  $\sin$  trajectory into rectangular trajectories of the 0, a, b, c and d axes.





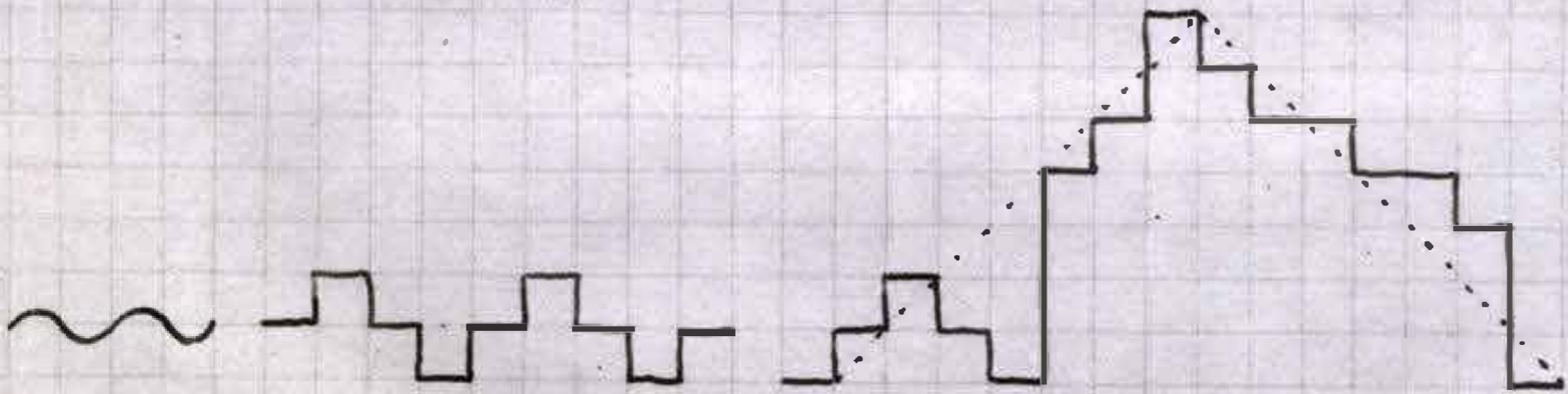


Translations of the cos trajectory:



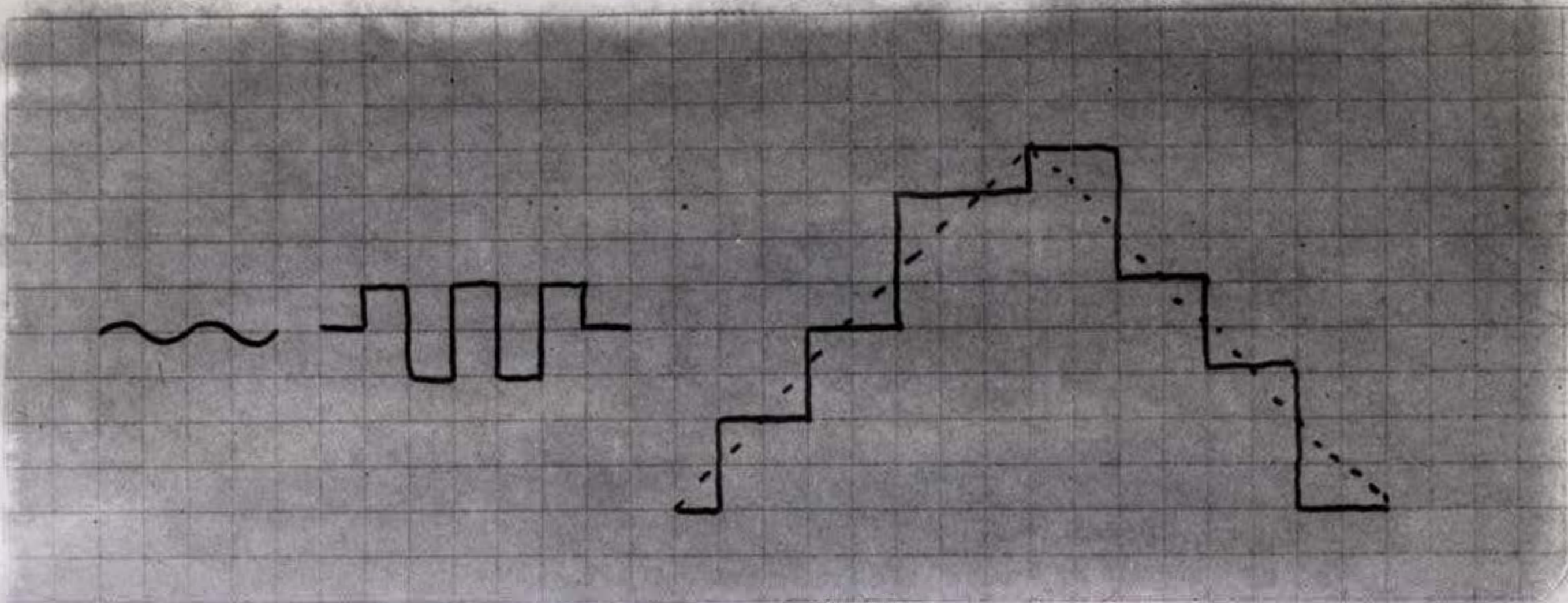
Translations of the combined trajectory:

(1) With continuous tangency:





(2) Without continuous tangency:



(1) may be called revolving trajectories.

(2) may be called crossing trajectories.

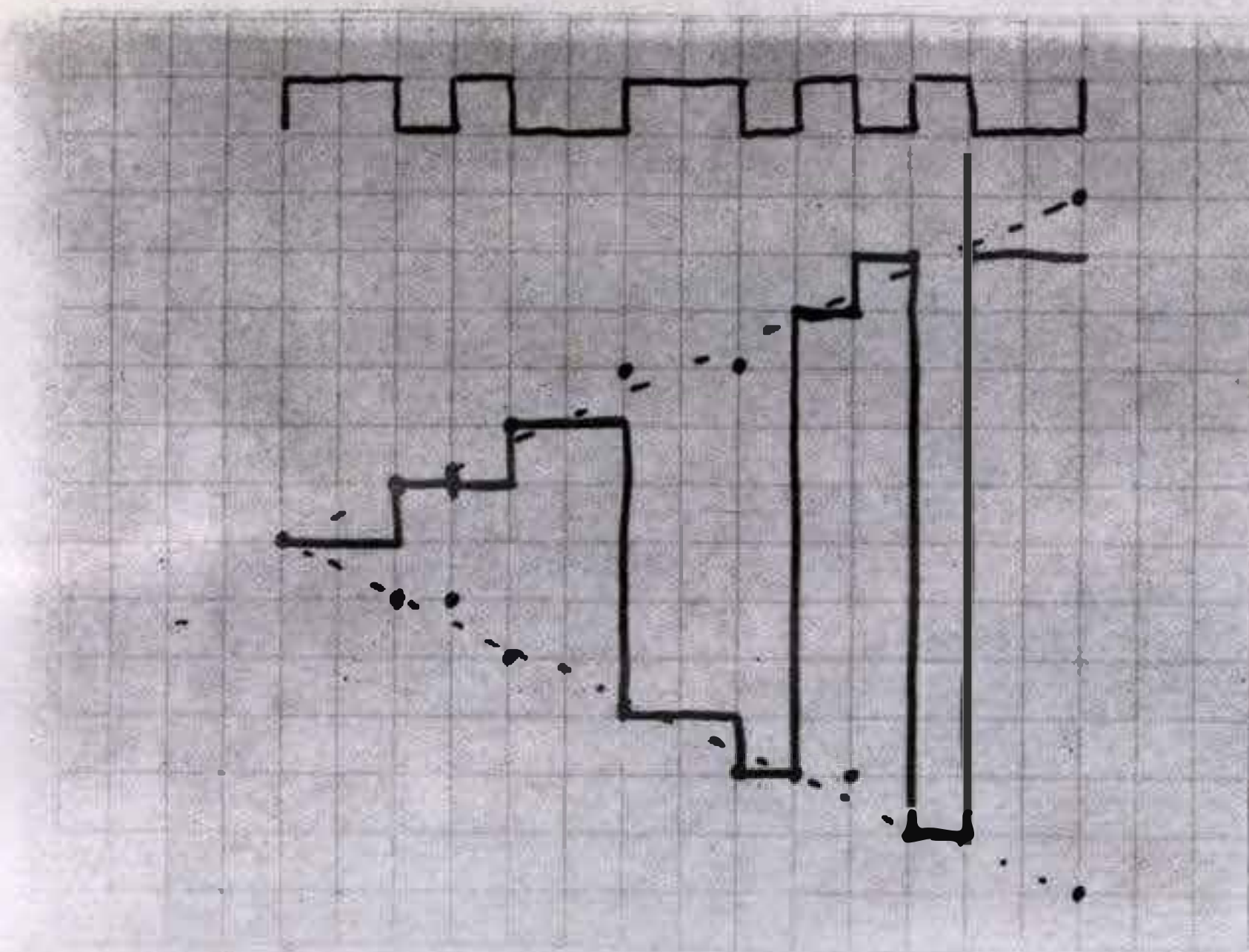
Deviation of a rectangular trajectory from its corresponding axis signifies inconsistency and lowers the esthetic value of a melody.

An esthetically efficient melody must display, besides consistency, a variety of the forms of motion.

When a trajectory is controlled by the two simultaneous axes (fundamental and complementary), the points of attack may fall on either axis according to the form of alternation.

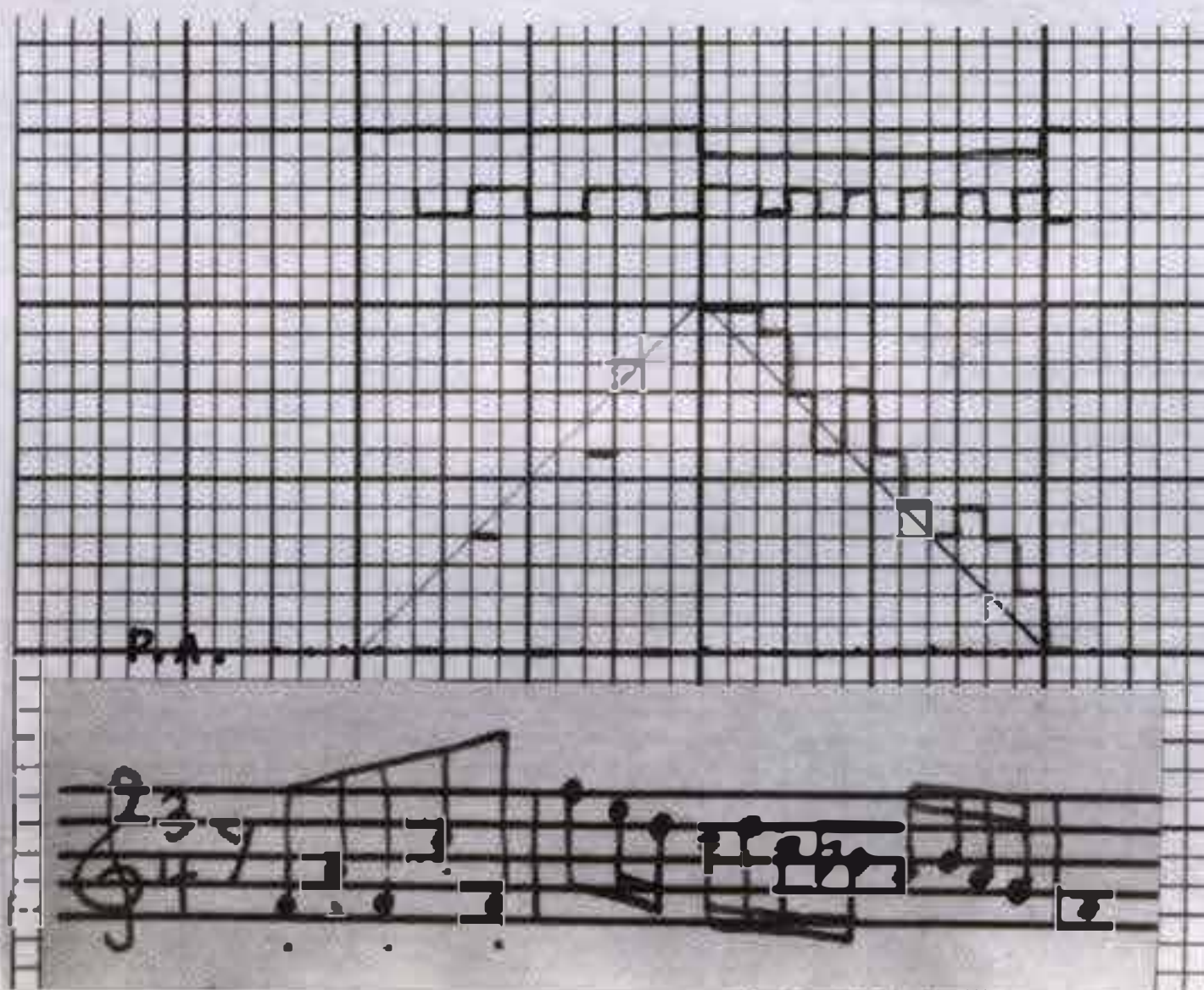
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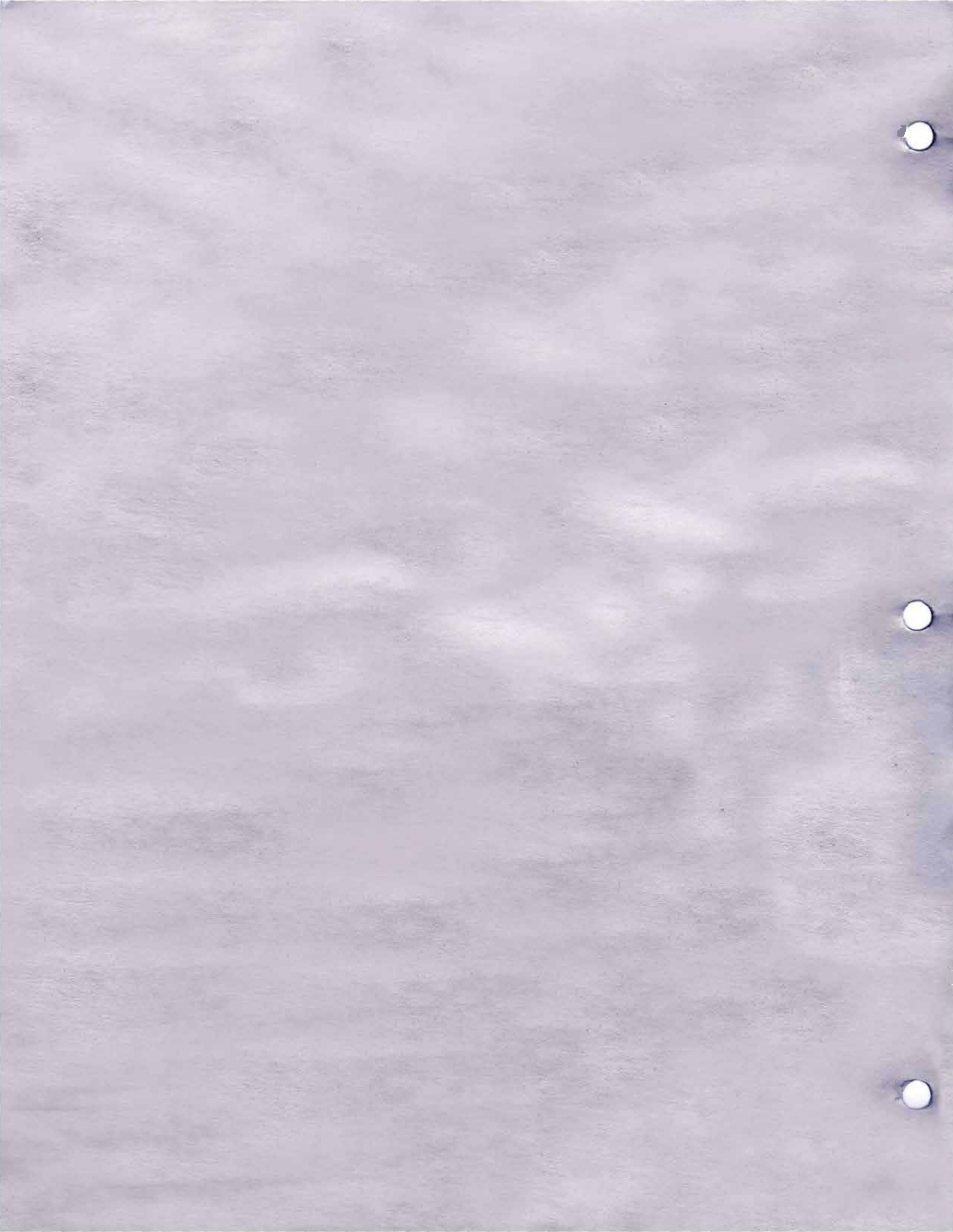


Example

The form of alternation is subject to distribution,  
i.e., rhythm.

An example of analysis of the trajectorial  
motion in J.S. Bach's Two-Part Invention, No. 8:





The staccato eighths are expressed on the graph as sixteenths.

This trajectory has a primary axis defined by its first, last and two intermediate attacks. The group of the secondary axes is:  $\frac{a}{o} + b$ . The pitch and time ratios are uniform, i.e.,  $\frac{a}{o}PT + bPT$ . The first attack of b is a climax. The form of motion on  $\frac{a}{o}$  is sin motion with increasing, (centrifugal), amplitude. The alternation of the points of attack on the two conjugated axes is uniform. The form of motion on b is combined (sin + cos) and has a constant amplitude. It is ascribed with respect to b. The effect of revolving due to the combined form produces a resistance and delays the balance. This melody would lose most of its esthetic value if the o-axis were eliminated (loss of resistance moving toward the climax), and the b-axis would have one-phase motion.

At this point it would be very advisable to make a thorough analysis of the outstanding as well as the deficient themes taken from the existing music. This procedure must follow all the nine chapters of the theory. A precise statement must be made on each item, (regarding the form and the measurement).

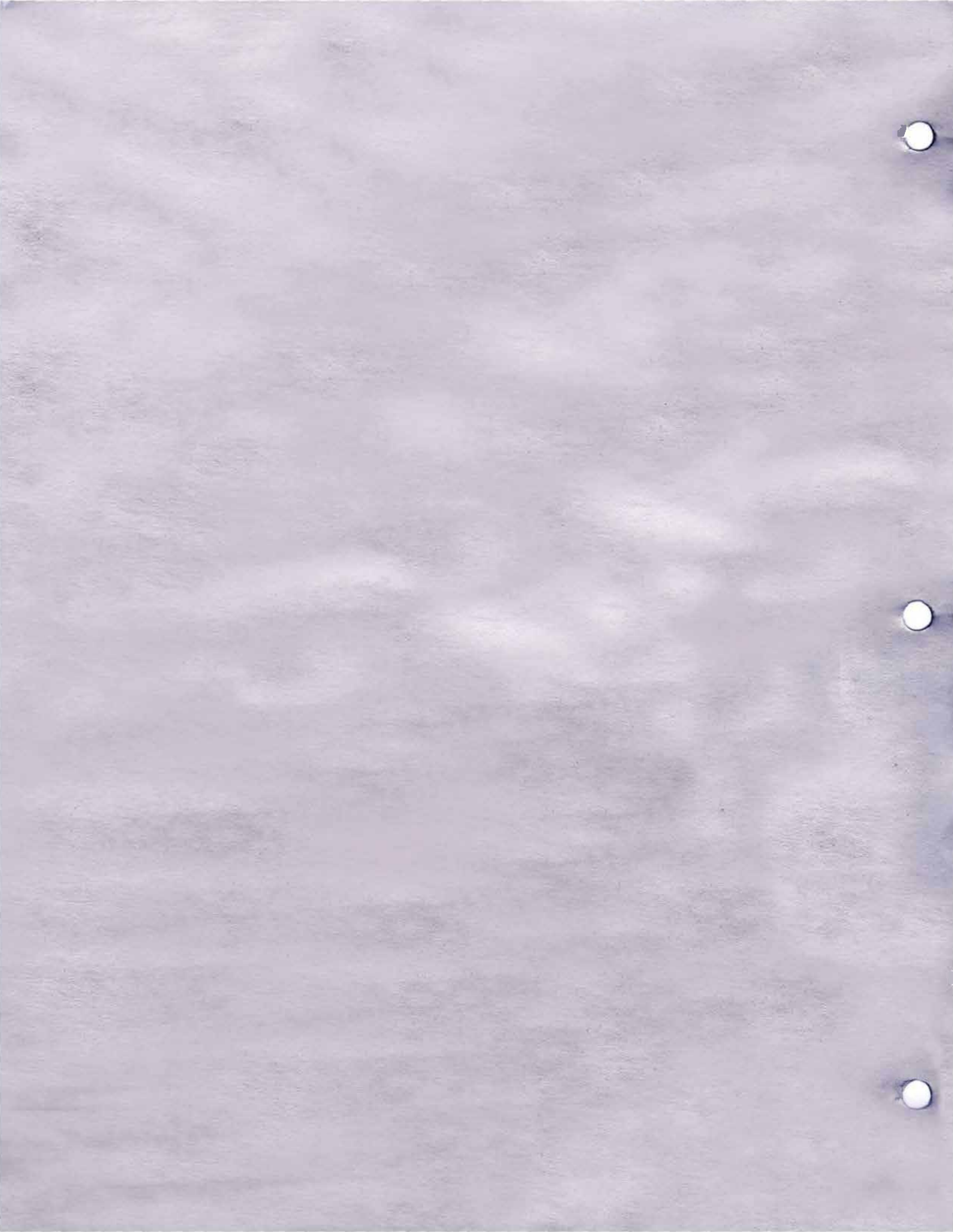
Though a theme of any dimensions (duration)





may be constructed to full satisfaction it is more practical in most cases to compose continuity out of a short original structure. Memory is very limited and the latter will produce an effect of greater unity.

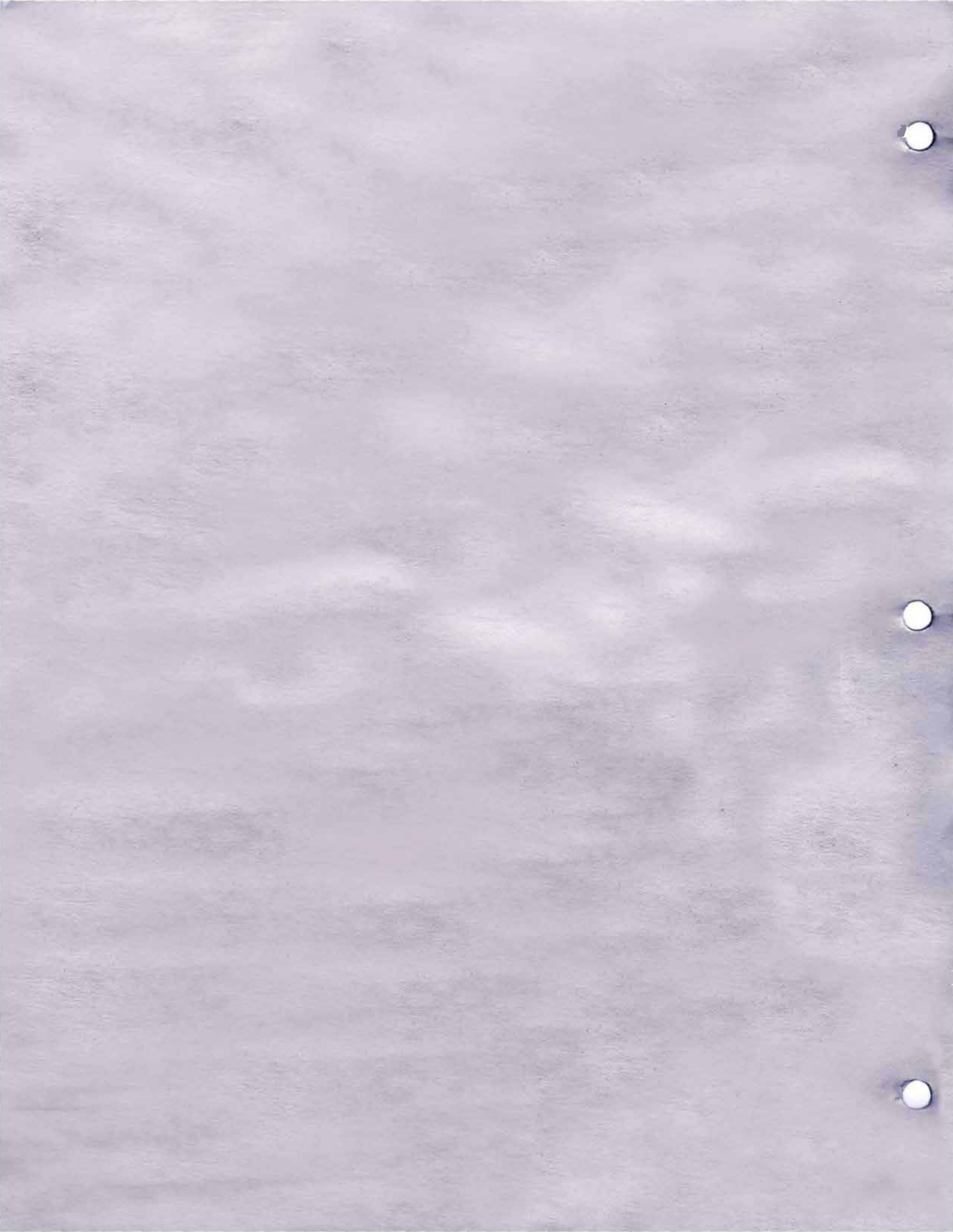
After having enough experience in analysis one may start composing melodies according to this theory. Success depends upon thorough knowledge of all the preceding material, and ability to think.



Lesson LXIII,X. Composition of Continuity.

Melody plotted according to this technique has the following properties:

1. Permutability of the secondary axes with their respective melodies in time continuity.
2. Permutability of the individual pitch-units (preferably through circular permutations) pertaining to one individual secondary axis.
3. Geometrical convertibility of the entire melody.
4. Geometrical convertibility of the portions of melody pertaining to the individual secondary axes or any groups thereof.
5. Tonal expansion of the entire melody.
6. Tonal expansion of the portions of melody pertaining to any individual secondary axis or portions thereof. In this case different axes may appear with different coefficients of expansion.
7. Combined variations of geometrical inversions and tonal expansions applied to the entire melody.
8. Combined application of geometrical inversions and tonal expansions applied to the portions of melody pertaining to individual secondary axes or any combinations thereof. In this case different coefficients of expansion may be combined with different geometrical inversions.



Continuity may be composed through any of the abovementioned forms of variation or any combination thereof.

Here is an example of the quantitative development of melodic continuity from the original theme. Let us take a trinomial axial combination, a, b, c. Each of the individual axes has four geometrical inversions. Thus, the number of combinations of the three axes being used in identical or different geometrical inversions equals  $4^3 = 64$ . This number refers to one constant E. If any of the individual axes appears in three forms of tonal expansion, the entire quantity will be  $64^3 = 262,144$ .

The following is a method of indicating a secondary axis where the geometrical positions and the coefficients of expansion are specified. For example, an axis a in position (C) in the second expansion ( $E_2$ ) may be expressed like this:

$$a \textcircled{C} E_2$$

A trinomial axial-combination consisting of a, b and c axes, with specified time and pitch ratios, and the geometrical positions and coefficients of expansion, assumes the following appearance:



Time ratios: 2 + 1 + 1

Pitch ratios: 1 + 2 + 3

Geometrical positions: (d), (a), (b)

Coefficients of expansion:  $E_0, E_2, E_1$

$aP2T_{(d)}E_0 + bT2P_{(a)}E_2 + cT3P_{(b)}E_1$

This method of indication emphasizes not the axial structure alone, but the pitch-units (intonation) as well. For example, a melody in its third displacement, on the axis a, in position (d), in the third expansion, may be expressed as follows:

$a_{(d)}E_3d_3$

When this method is systematically applied, the sequence of the different displacement phases, with regard to consecutive secondary axes, may assume different forms of distribution. For example, it may start with the first phase within the first axis, with the second phase within the second axis, with the third phase within the third axis, etc. It may follow a rhythm of any resultant or any of the series of growth:

$$(a) \quad d_3 + d_1 + d_2 + d_2 + d_1 + d_3$$

$$(b) \quad d_0 + d_1 + d_3 + d_6 + d_{11} + \dots$$

Naturally, the rhythm for such variations of motif depend upon the number of pitch-units within the motif.





The ability of producing expressive melodies (themes) does not make a great composer. The ability of producing an organic continuity out of original thematic material does.

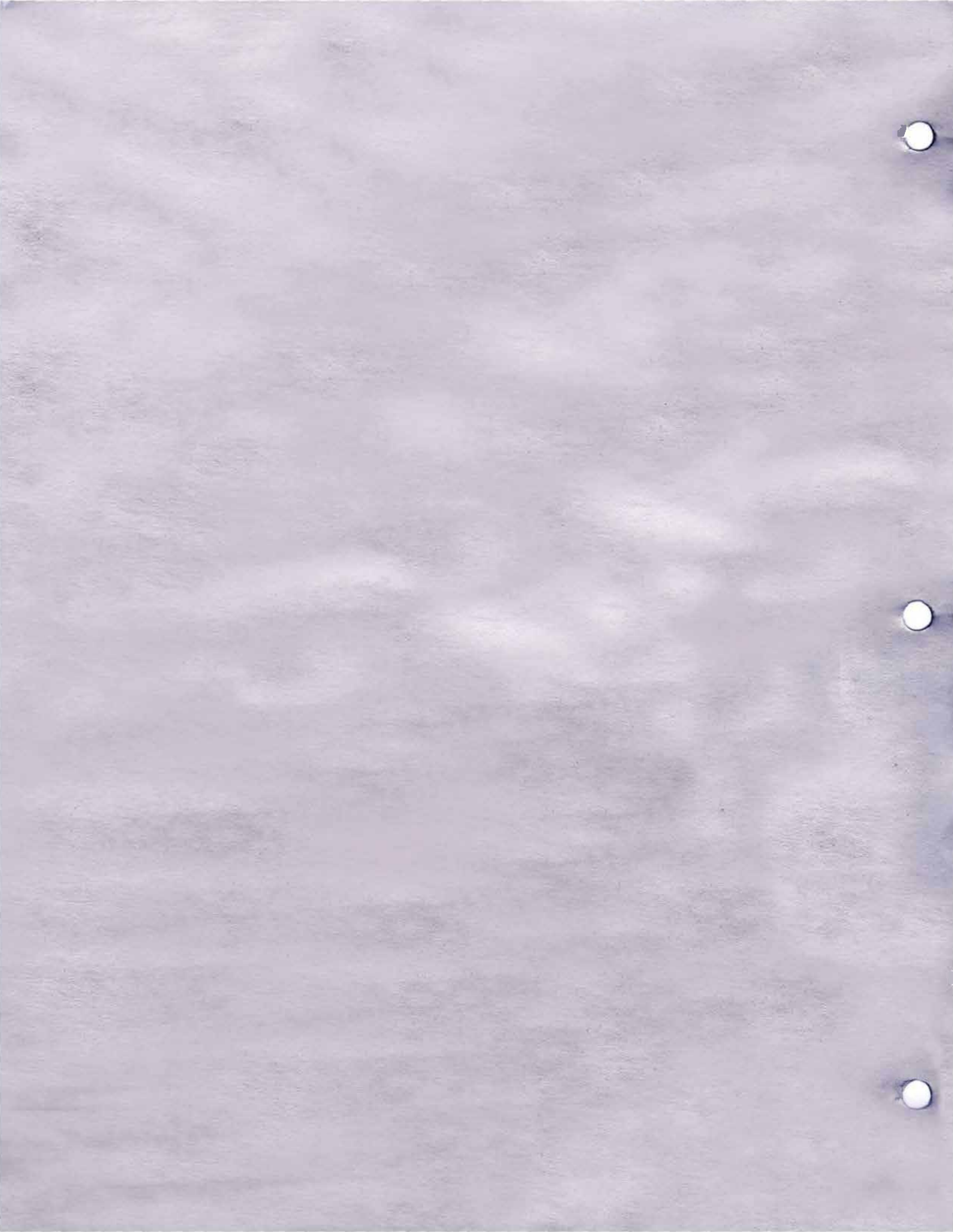
Going back as far as to the strict style of counterpoint written to a cantus firmus, we find that composition of continuity is based on uniform factorial periodicity. The theme regularly appears in different voices and that keeps the music moving.

In all elementary homophonic forms, continuity is based on composition of biners ( $a_1 + a_2$ ), usually similar structures with different endings, consisting of  $4 + 4$  or  $8 + 8$  bars. Next comes the method of terners, i.e.,  $a_1 + b + a_2$ , introduction of the new material in the center.

The most advanced forms in the past were offered first by J.S. Bach, who used a sequence of biners in contracting geometrical progressions (see Fugue V, Vol. II, Well Tempered Clavichord). In his case it meant that a greater overlapping, (stretto between the theme and the reply), occurred with each following announcement.

In Beethoven's case it meant a continuous breaking up of the original biner.

All these forms of continuity, (in the past), are rigidly attached to the  $\frac{2}{2}$  series.



Richard Wagner built his continuity according to the script, i.e., the operatic libretto. Though he wrote them himself, and was quite skilful at it, musical continuity greatly suffered from this syntactic dominance. Wagner's faults were adopted as virtues by Scriabine and others. Literary influence, together with linguistic logic and syntactic (propositional) technique were the factors that delayed, if not prevented, the sound development of the forms of musical continuity. (See the definition of program music in The Oxford History of Music, Vol. 6, Page III, which says: "Program music is a curious hybrid that is music posing as an unsatisfactory kind of poetry").

Forms of musical continuity are purely quantitative and pertain to motion. They are bio-mechanical, i.e., they are forms of growth. When they grow normally, they survive better. It is pure Darwinism: struggle for existence, the survival of the fittest. A star-fish is not "just a pretty pentagon" but an organic form evolved through the necessity of efficient functioning.

Many an unpretentious melody is appealing, i.e., esthetically efficient, due to the fact that within the eight-bar structure certain processes evolve in a very consistent manner. It happens quite



often that in some cases the efficiency of structure is greater in smaller portions and smaller in greater portions and vice-versa.

The bio-mechanical forms are primarily concerned with three basic factors:

- (1) Symmetric development, i.e., the axis-inversion.
- (2) The ratio of growth, such as summation.
- (3) Movement with respect to tension and release resulting in balance, i.e., an arithmetical or a geometrical mean.

Growth along the axis of symmetry (compare with a human body, with its growth along the spinal cord), is a continuity formed by geometrical inversions of the original structure or its portions, (melody), along the primary axis. The regularity of recurrence of the different inversions is subjected to rhythm. Pitch expansions (tonal and geometrical), combined with their geometrical inversions may be used as components of musical continuity.

The most fluent form of continuity results from the symmetric growth along the time-axis. This is the most complete form of continuity as it exemplifies birth, growth, maturity, decline and death, all in one process. To accomplish this it is necessary to split the original structure into a number of elements (such as bars or secondary axes), to show these elements in their gradual addition, and



then in their gradual subtraction.

Suppose we have a three-bar structure and split it into a, b, c elements. Gradual addition of the elements will give:  $a + ab + abc$ . Gradual subtraction of the elements will give:  $abc + bc + c$ . The combination of the two forms -- the time-axis on abc. The entire continuity will be this:

$$a + ab + abc + bc + c$$

### Examples

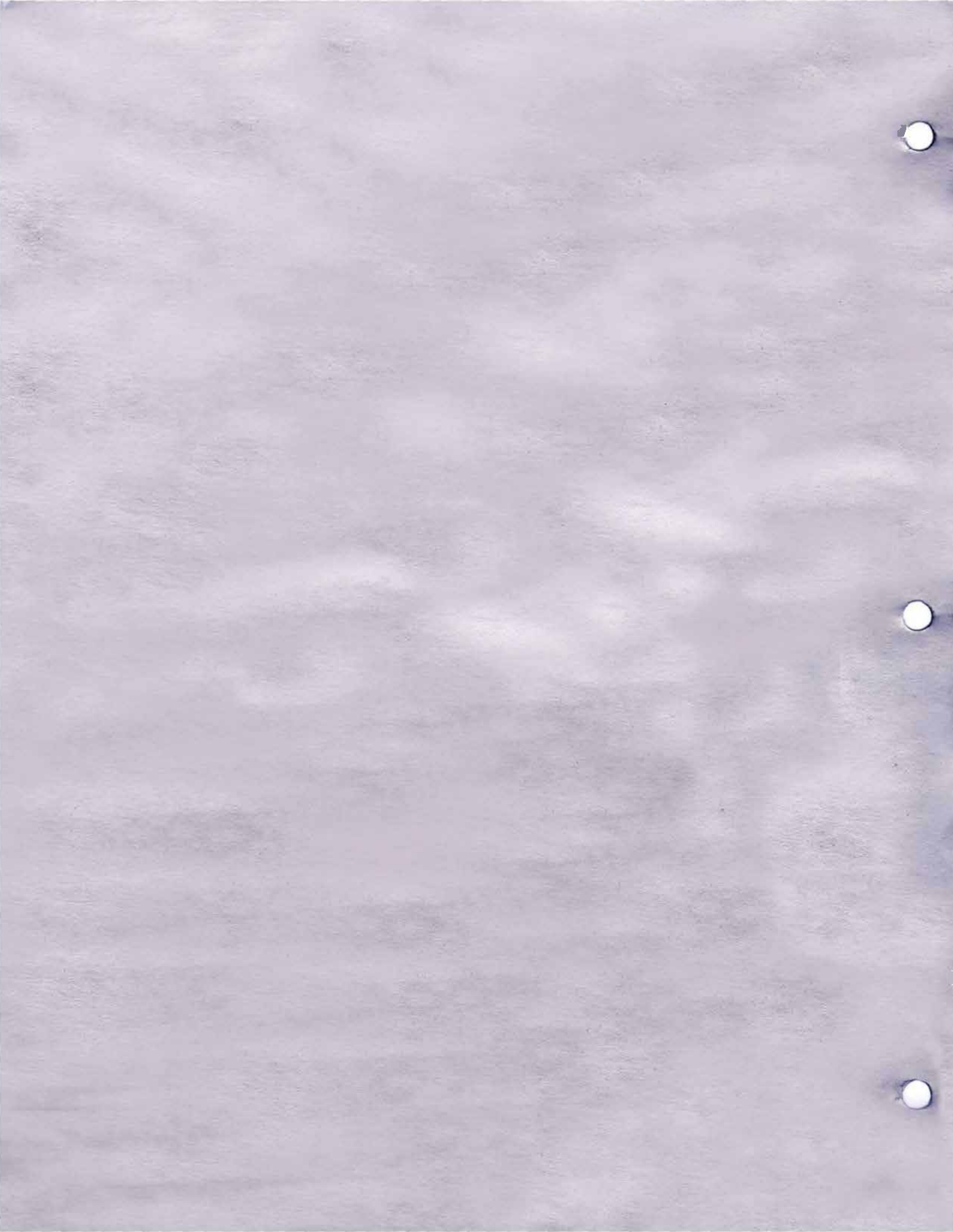
The original structure split into three elements:



Continuity composed through the time-axis.



The process of summation may pertain to the preceding procedure, as well as to factorial ratios of the secondary axes, or the number of





individual attacks.

An example of summation through the first summation series based on the time axis. Let us take an eight-bar structure and split it into a b c d e f g h elements. The continuity will have this form:

$a + ab + abc + abcde + abcdefgh + defgh + fgh + gh + h.$

The entire structure moves across itself through its own axis, while time goes on.

The next point is obvious. Using the same series for the T of the secondary axes, we obtain:

$T + 2T + 3T + 5T + 8T + 5T + 3T + 2T + T$

whatever axis (o, a, b, c or d) each term may represent.

Summation through the number of individual attacks may be found in many melodies. Take the popular song, "One-Two, Button Your Shoe", for instance. The first eight bars give the following summation of attacks:  $2 + 4 + 6 + 12$ , i.e.,  $2 + 4 = 6$ , and  $6 + 6 = 12$ . It means there are four distinct sub-structures, each containing the number of notes in this particular summation, carried out with an absolute precision.

It is important not to confuse the rhythm of attacks with the rhythm of durations.

The method of summation is very flexible, and with a little initiative one may accomplish a



great deal of variety.

In the song, "But I Only Have Eyes for You", you find the following scheme of attacks: 6 + 9 + 6 + 3. This is an incomplete form of 3 + 6 + 9 + 6 + 3, where the central term is the result of summation  $3 + 6 = 9$ . At the same time, the central term becomes an axis of time symmetry.

Movement, with respect to tension and release, resulting in balance may refer to factorial or fractional time-rhythm as well as to the rhythm of the number of individual attacks. Arithmetical mean is the most common device in this case.

An arithmetical mean is the quotient of the division of the sum, by the number of elements. With two elements, a and b for example, it equals  $\frac{a+b}{2}$ . Musical intuition has a certain amount of precision, and in some cases these means come out with a very good approximation. For example, in the first  $3\frac{1}{4}$  bar structure of "Stormy Weather", the first sub-structure has three attacks, the second -- seven, and the third -- four. The exact number for the last sub-structure would be  $\frac{3+7}{2} = 5$ , not 4. This is a very good amount of approximation, only 20 percent of error. Yet you get a greater satisfaction by adding one more attack. Try it by making a triplet out of



two eighths at the beginning of the third bar.

This procedure is adequate mechanically to: underbalancing - overbalancing - balancing, or overbalancing - underbalancing - balancing.



Lesson LXIV.

The following graphs and music serve as an example of composition of melodic continuity. Each example is a complete musical composition written for an unaccompanied instrument. This art has been greatly neglected today. In the XVII and XVIII Centuries, composers possessed enough technique to accomplish such tasks. J.S. Bach wrote many outstanding works, even sonatas, for violir or viola de gamba alone. Today only a very few high ranking composers like Paul Hindemith (Suite for Viola alone) or Wallingford Riegger, an American, (Suite for Flute alone, in seven movements) have dared to write a whole opus for an unaccompanied instrument.

The three compositions I offer here are constructed from the scales of the First Group. Each graph represents a theme originally plotted. Musical examples are complete compositions developed by means of variation.

The notation is as follows:

M -- the entire melody

a, b, c, d -- portions of melody pertaining  
to the respective axes





$$\frac{a'}{a}, \frac{b'}{b}, \frac{c'}{c}, \frac{d'}{d}$$

or

$$\frac{a}{a'}, \frac{b}{b'}, \frac{c}{c'}, \frac{d}{d'} \text{ -- parallel binary axes}$$

Ⓐ, Ⓑ, Ⓒ, Ⓓ -- geometrical positions of M  
or of the respective axes

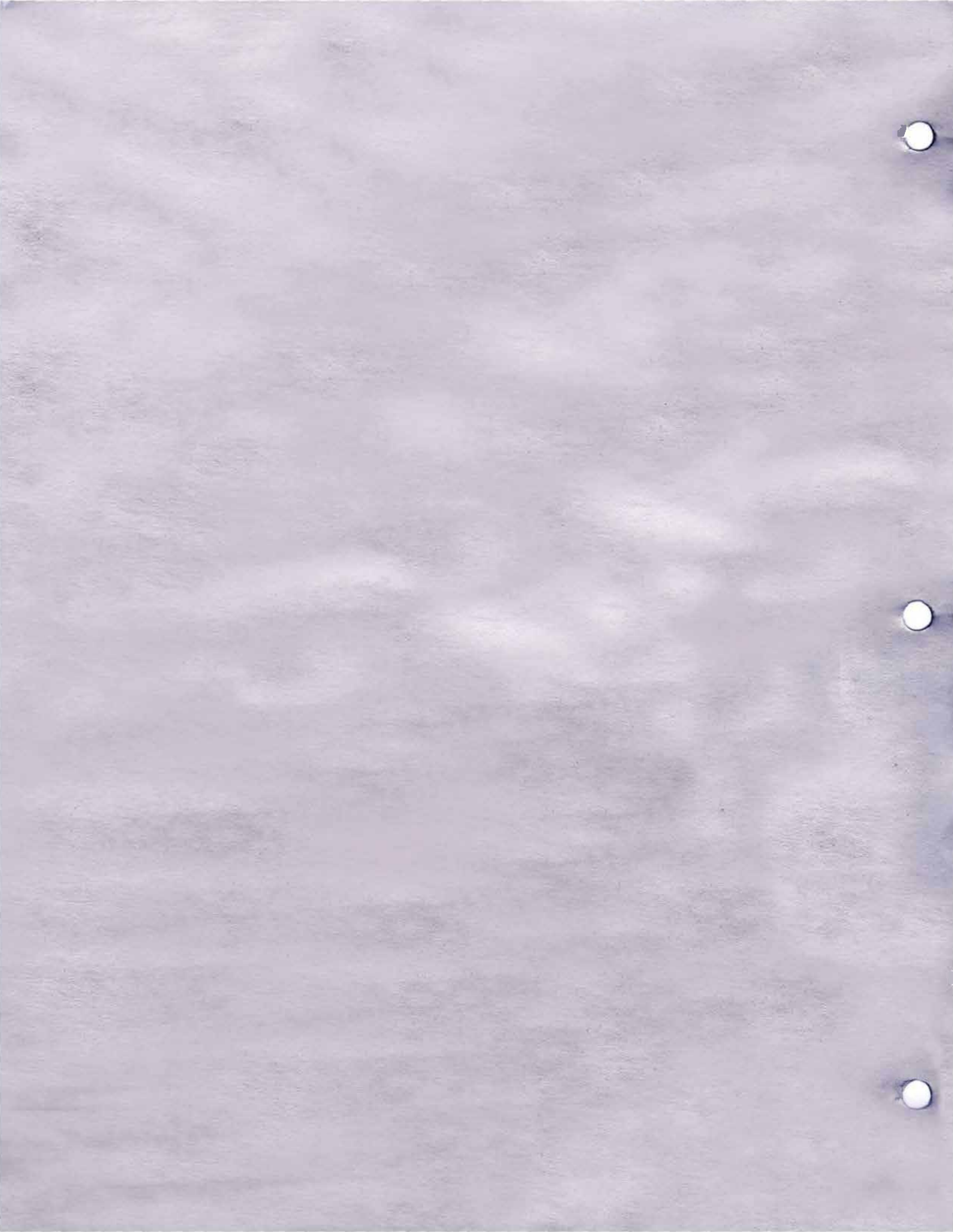
P<sub>0</sub>, P<sub>1</sub>, P<sub>2</sub>, ... -- permutations of pitch-units  
of M or of the respective axes

E<sub>0</sub>, E<sub>1</sub>, E<sub>2</sub>, ... -- tonal expansions of M or of  
the respective axes

In this form of notation each original melody (the theme of the composition) appears as MⒶP<sub>0</sub>E<sub>0</sub>.

It is advisable to be conservative in planning a complete melodic continuity, as application of too many variations at a time (i.e., p, E and the geometrical positions) increases the complexity of the entire composition beyond the listener's grasp.

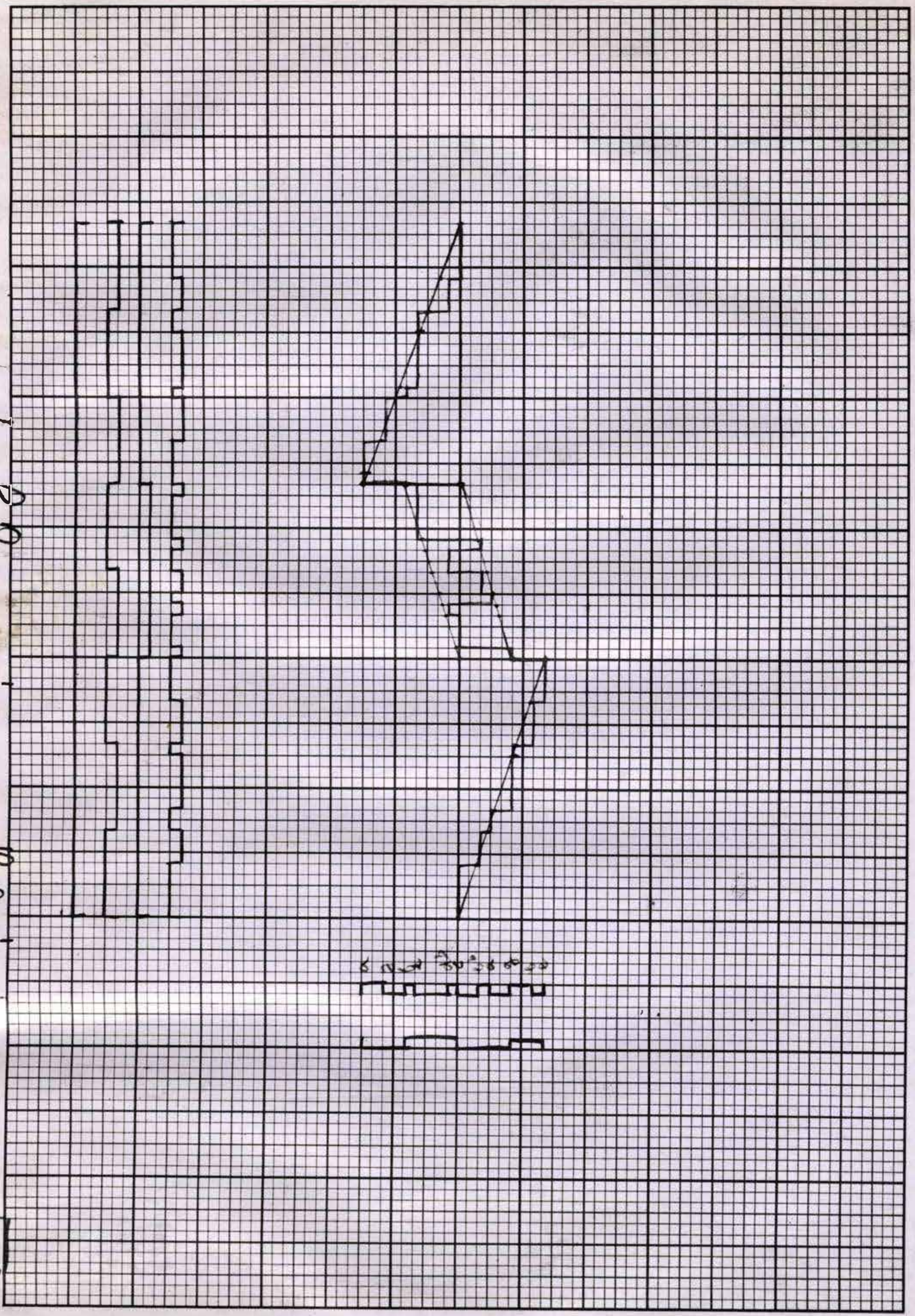
(please see following pages)

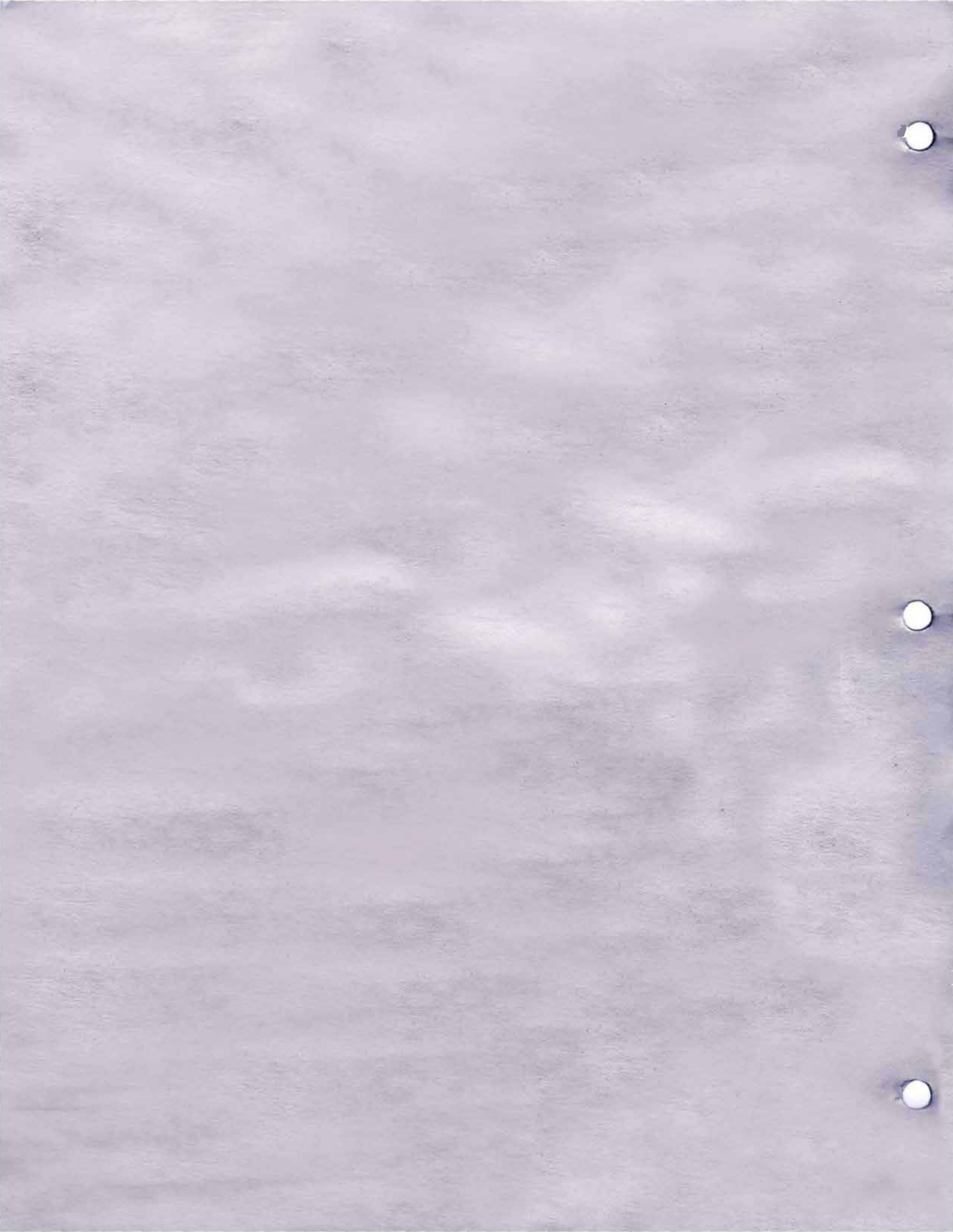


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c d b e f g a b b c d e f g a b b c

[2]

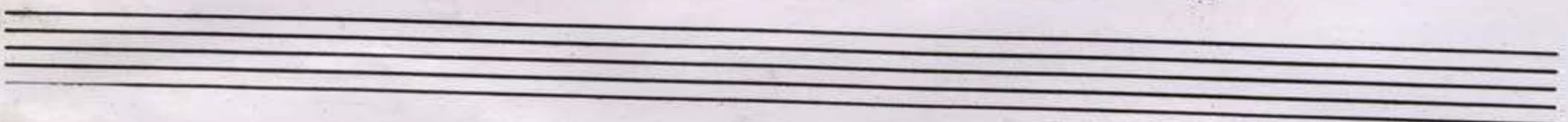


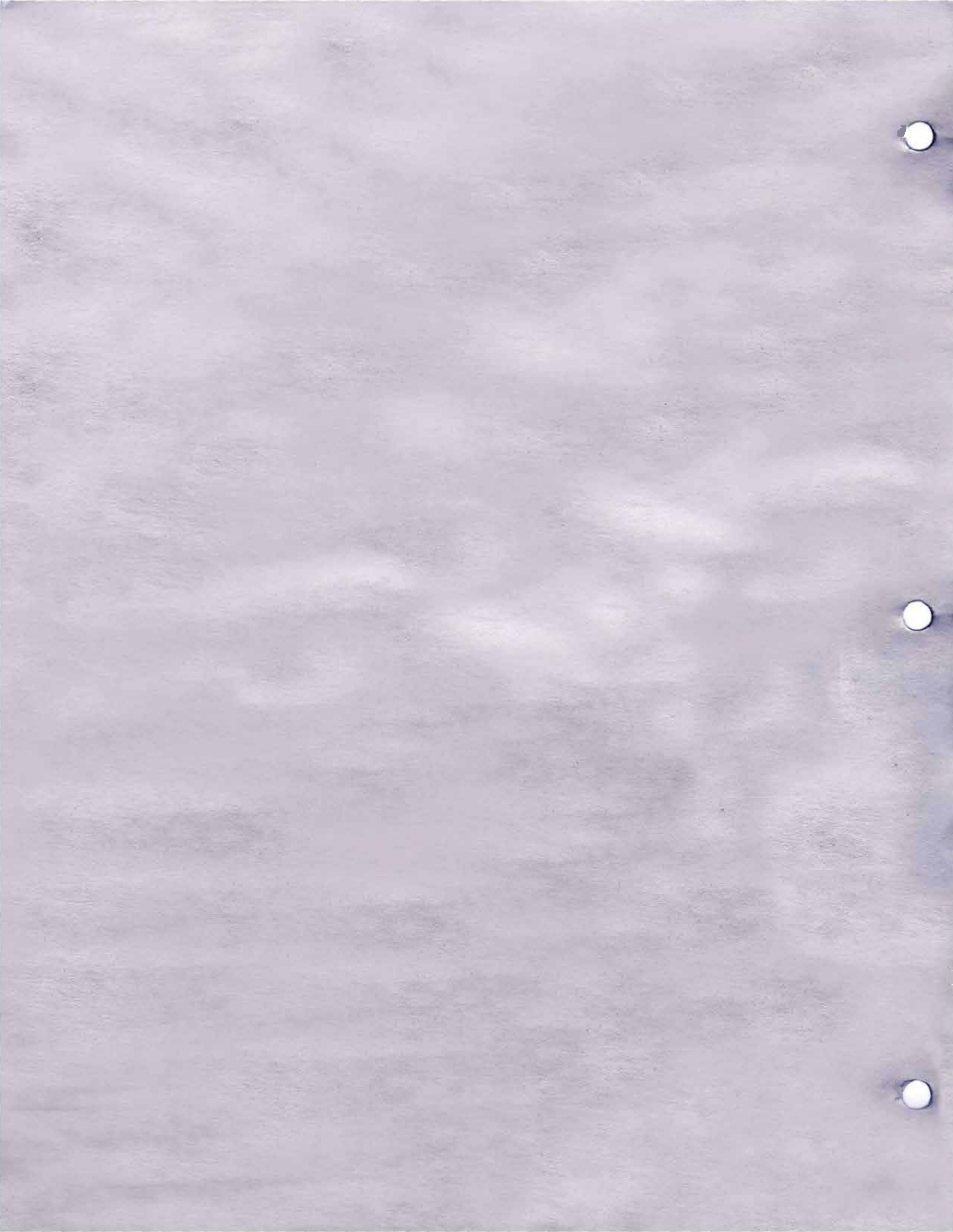


$[M \textcircled{a} E. p_2] + [C' \textcircled{a} E. p_0] + [D \textcircled{a} E. p_0] + [B \textcircled{a} E. p_0] + [D \textcircled{a} E. p_0] + [B \textcircled{a} E. p_2] +$   
 $[C' \textcircled{a} E. p_2] + [M \textcircled{a} E. p_0] + [E' \textcircled{a} E. p_4] + [D \textcircled{a} E. p_5] + [D \textcircled{a} E. p_0] + [M \textcircled{a} E. p_0]$



Handwritten musical score consisting of ten staves. The notation includes various note values (quarter, eighth, sixteenth notes), rests, and slurs. The first staff begins with a treble clef and a key signature of one flat (B-flat). The music is written in a single system across the ten staves, with a double bar line at the end of the tenth staff.

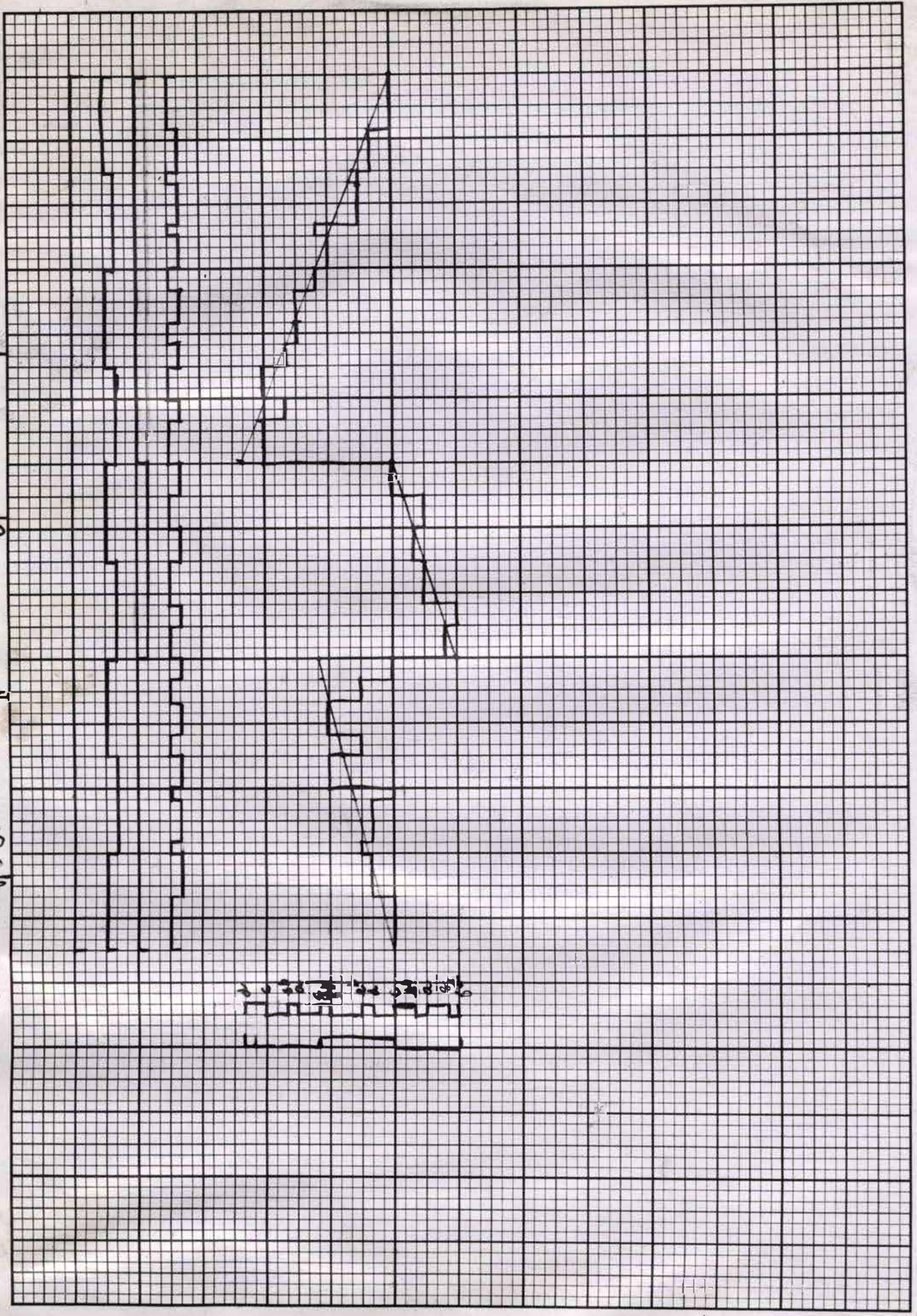




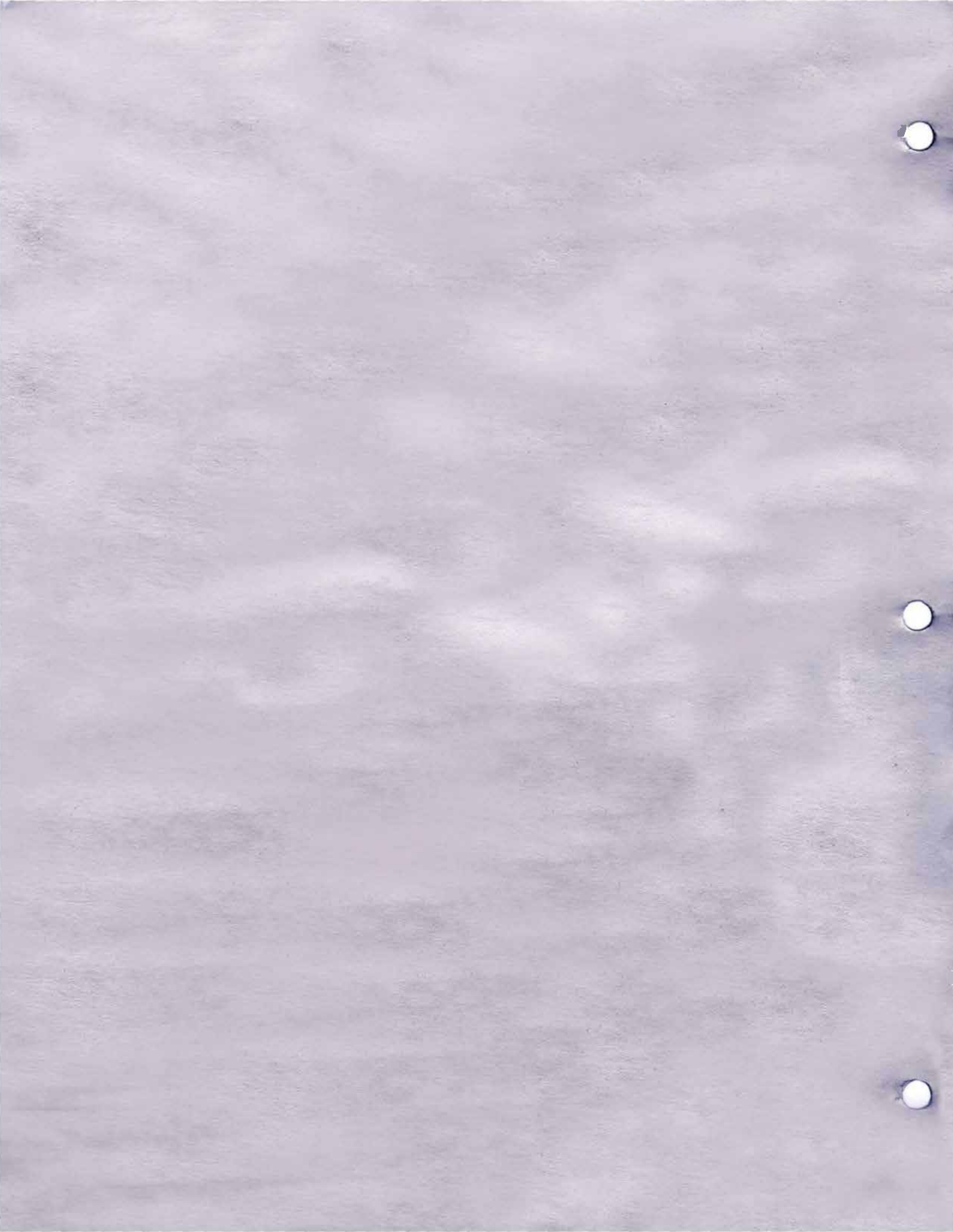
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5

c, d, e, f, g, a, b, c, d, e, f, g



Handwritten text and markings at the bottom of the grid, including a small square and some illegible characters.





$[M @ E_0 \flat.] + [M @ E_0 \flat.] + [C @ E_0 \flat.] + [B @ E_0 \flat.] + [A @ E_0 \flat.] + [B @ E_0 \flat.] +$   
 $[A @ E_0 \flat.] + [B @ E_0 \flat.]$

5

Handwritten musical score on ten staves. The first staff has a treble clef and a 9/8 time signature. The music consists of a single melodic line with various note values, rests, and accidentals (sharps, flats, naturals). Phrasing slurs are used throughout. The notation is dense and appears to be a study or exercise piece.

