JOSEPH SCHILLINGER

CORRESPONDENCE COURSE

With: Dr. Jerome Gross   Subject: Music

Lesson LXV.

SPECIAL THEORY OF HARMONY

Introduction

Special Theory of Harmony is confined to E_I of the First Group of Scales, which contain all musical names (c, d, e, f, g, a, b) and without repetition. There are 36 such scales in all. The total number of seven-unit scales equals 462.

The uses of E_I refer to both structures and progressions in the Diatonic System of Harmony. The latter can be defined as a system which borrows all its pitch units for both structures and progressions from any one of the 36 scales. While the structures are limited to the above scales, the progressions develop through all the semi-tonal relations of the Equal Temperament. The latter comprises all the Symmetric Systems of Pitch, i.e., the Third and the Fourth Group.

Chord-structures, contrary to common notion, do not derive from harmonics. If the evolution of chord-structures in musical harmony would parallel the evolution of harmonics, we would never acquire the developed forms of harmony we now possess.

To begin with, a group of harmonics, simultaneously produced at equal amplitudes, sounds like a saturated unison and not like a chord. In other words, a perfect harmony of frequencies and intensities does not result in musical harmony but in a unison. This means that through the use of harmonics, we would never have arrived at musical harmony. But we do get harmony, and exactly for the opposite reason. The relations of sounds we use in Equal Temperament are not simple ratios (harmonic ratios).

When acousticians and music theorists advocate "Just intonation", that is, the intonation of harmonic ratios, they are not aware of the actual situation. On the other hand, the ratios they give for certain trivial chords, like the major triad (4÷5÷6), the minor triad (5÷6÷15), the dominant seventh-chord (4÷5÷6÷7), do not correspond to the actual intonations of the Equal Temperament. Some of these ratios, like $\frac{7}{4}$, deviate so much from the nearest intonation, like the minor seventh, which we have adopted through habit, that it sounds to us out of tune. Habits in music, as well as in all manifestations of life, are more important than the natural phenomena. If the problem of chord-structures in harmony would be confined to the ratios nearest to Equal Temperament, we could have offered 16÷19÷24 for the minor triad for example,

as it approaches the tempered minor triad much better than 5÷6÷15. But this, if accepted, would discredit the approach commonly used in all textbooks on harmony, and for this reason. If such high harmonics as the 19th are necessary for the construction of a minor triad, what would chords of superior complexity, which are in use today, look like when expressed through ratios. When a violinist plays b as a leading tone to c and raises the pitch of b above the tempered b, his claims for higher acoustical perfection are nonsense, as the nearest harmonic in that region is the 135th.

Facing facts, we have to admit that all the acoustical explanations of chord-structures as being developed from the simple ratios, are pseudo-scientific attempts to rehabilitate musical harmony, to give the latter a greater prestige. Though the original reasoning in this field was caused by the honest spirit of investigation of Jean Philippe Rameau ("Generation Harmonique", Paris, 1737), his successors overlooked the development of acoustical science. Their inspiration was Rameau plus their own mental laziness and cowardice.

The whole misunderstanding in the field of musical harmony is due to two main factors:

(1) the underrating of habit;

(2) the confusion of the term "hermonic" in its mathematical connotation, i.e., pertaining to simple ratios with "harmony" in its musical connotation, i.e., simultaneous pitch-assemblages varied in time sequence.

Thus, musical harmony is not a natural phenomenon, but a highly conditioned and specialized field. It is a material of musical expression, for which we, in our civilization, have an inborn inclination and need. This need is cultivated and furthered by the existing trends in our music and musical education.

I. Diatonic System of Harmony.

Chord-structures and chord progressions in the Diatonic System of Harmony have a definite interdependence: chord-structures develop in the direction opposite to their progressions.

This statement brings about the practical classification of the Diatonic System into two forms: the positive and the negative.

As the term Diatonic implies, all pitch-units of a given scale constitute both structures and progressions, without the use of any other pitch-units (not existing in a given scale) whatsoever.

In the form which we shall call positive, all chord structures (S) are the component parts of the entire structure (Σ) emphasizing all pitch-units of a given scale in their first tonal expansion (E_I) and in position a. In the same form chord progressions derive from the same tonal expansion but in position b.

In the negative form of the Diatonic System, it works in the opposite manner. Chord-structures derive from the scale in E_I and in position b, while the progressions develop from E_I a.

According to the qualities we inherited and developed, the positive form produces upon us an effect of greater tonal stability. It is chronologically true that the negative form is an earlier one. It predominates in the works where the effect of tonality, as we know and feel it today, is, rather vague. Such is the XIV and XV Century Ecclesiastic music, developed on contrapuntal and not on harmonic foundations.

Many theorists confuse the negative form of the Diatonic System with "modal" harmony. As by Diatonic Tonality they mean, in most cases, Natural Major or Harmonic Minor scales moving in the positive form, they miss the tonal stability when harmony moves backwards. Losing tonal orientation they mistake such

progressions for modes, which are merely derivative scales, and may also have the positive, as well as the negative form. But as we have seen in the Theory of Pitch Scales, modes can be acquired from any original scale through the introduction of accidentals (sharps and flats).

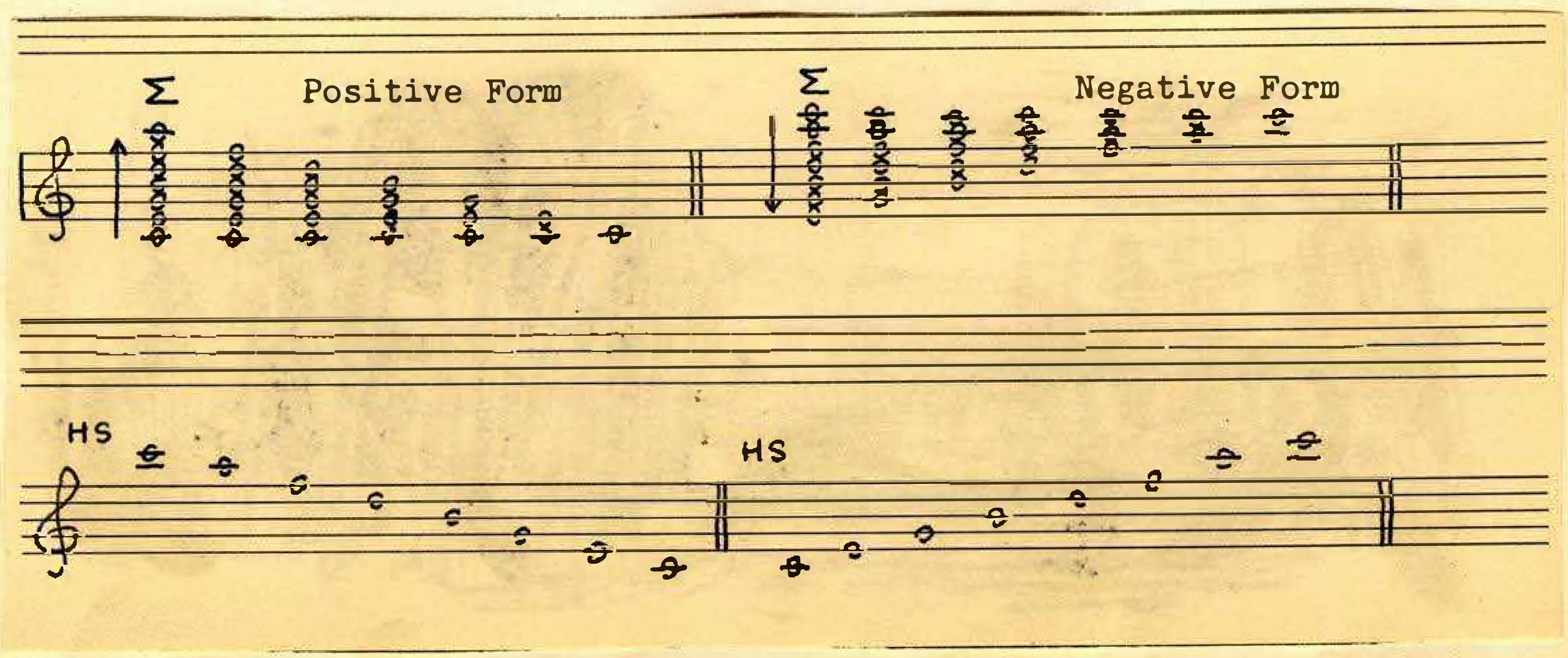
In the following table, MS represents "melody scale" (pitch-scale), and MH represents "harmony scale" (i.e., the fundamental sequence of chord progressions).

Diatonic System

Positive Form	Negative Form
$\Sigma = MS_{E,\,a}$	$\Sigma = MS_{E,\,b}$
$HS = MS_{E,\,b}$	$HS = MS_{E,\,a}$

Figure I.

Example (Natural Major)



In the positive form, chords are constructed upward, in the negative, on the contrary, downward. The matter is greatly simplified by the fact that any progression, originally written as positive, becomes negative, when read backward. All the principles of structures and motion involved are therefore reversible. No properly constructed harmonic continuity can be wrong in backward motion.

Some composers without training in harmony (for example, Modest Moussorgsky) as well as beginners, due to inadequate study, confuse the positive and the negative forms in writing their harmonic progressions. The resulting effect of such music is a vague tonality. The admirers of Moussorgsky consider such style a virtue (in Moussorgsky's case it is about half-and-half positive and negative), and do not realize that all the incompetent students of a harmony course incompetently taught possess full command over such style.

Lesson LXVI.

A. Diatonic Progressions (Positive Form)

Expansions of the original Harmony Scale produce the Derivative Harmony Scales. The original HS and its expansions form the Diatonic Cycles. Diatonic (or Tonal) Cycles represent all the fundamental chord progressions.

There are three Tonal Cycles in the Positive Form for the seven-unit scales. The First Cycle, or Cycle of the Third (C_3), corresponds to HS_{E0}; the Second Cycle, or Cycle of the Fifth (C_5), corresponds to HS_{E1}; the Third Cycle, or the Cycle of the Seventh (C_7), corresponds to HS_{E2}. Beyond these expansions of HS lies the Negative Form of Diatonic Progressions.

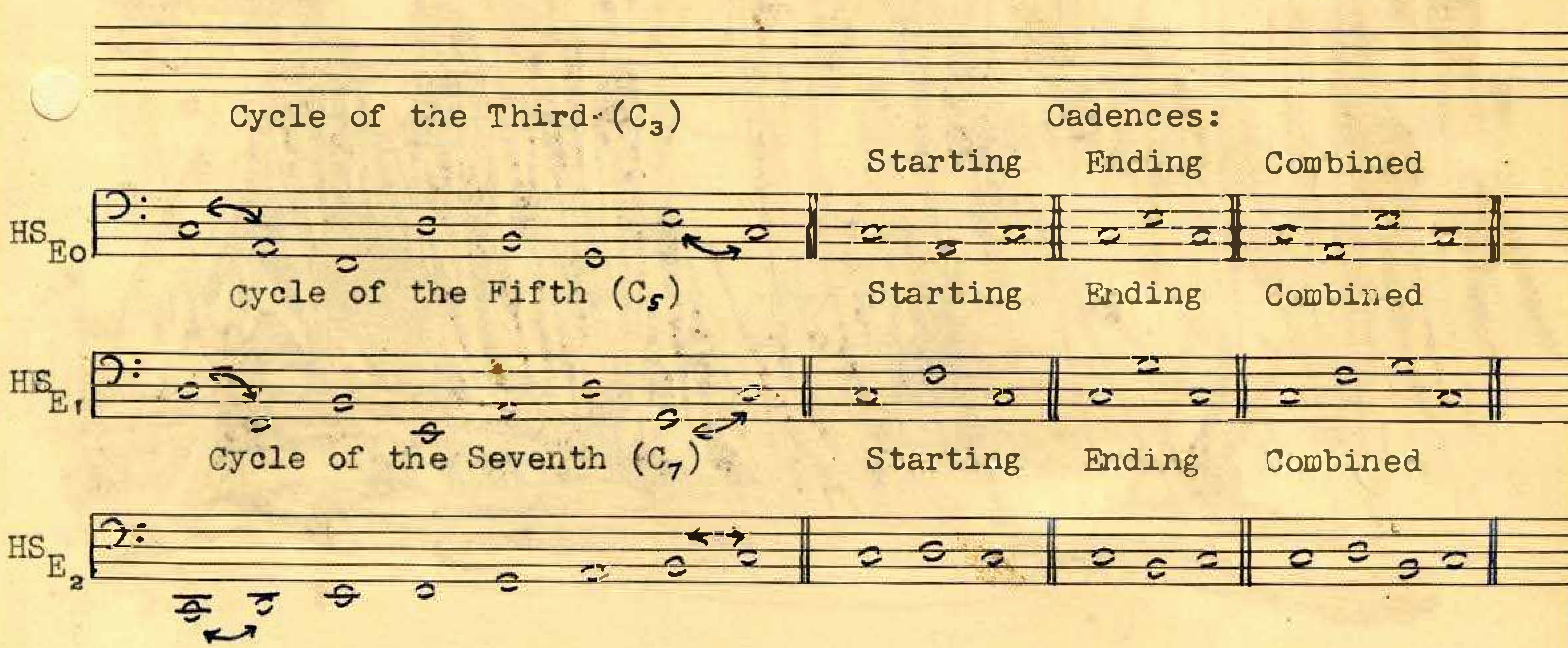
In addition to both forms of progressions, there may be changes in a chord pertaining to the same root (axis). Connections of an S with its modified S of the same root will be considered a Zero Cycle (C_0).

In the following table notes are used merely for convenience: they indicate the sequence of roots; their octave position was dictated by purely melodic considerations and by the necessity to moderate the range.

The respective intervals represening Cycles must be constructed downward for the Positive Form, regardless of their actual position on the musical staff.

Figure II.

Diatonic Cycles (Positive Form)



In the above table arrows indicate cadences of the respective cycles. Cadences consist of the axis-chord moving into its adjacent chord and back. It is interesting to note, that what is usually known as Plagal Cadences are the Starting Cadences and that Cadences known as Authentic are the Ending Cadences. The immediate sequence of Starting and

Ending Cadences produces <u>Combined Cadences</u> (the axis-chord is omitted in the middle).

Progressions of constant tonal Cycles (C_3, or C_5, or C_7 const.) produce a sequence of seven chords each appearing once and none repeating itself. The repetition of the axis-chord either completes the Cycle or starts a new one. The addition of Cadences to the Cycles is optional, as Cycles are self-sufficient.

Considering constant Cycles as a form of <u>Monomial Progressions,</u> we can devise <u>Binomial</u> and <u>Trinomial Progressions</u> by assigning a sequence of two or three Cycles at a time.

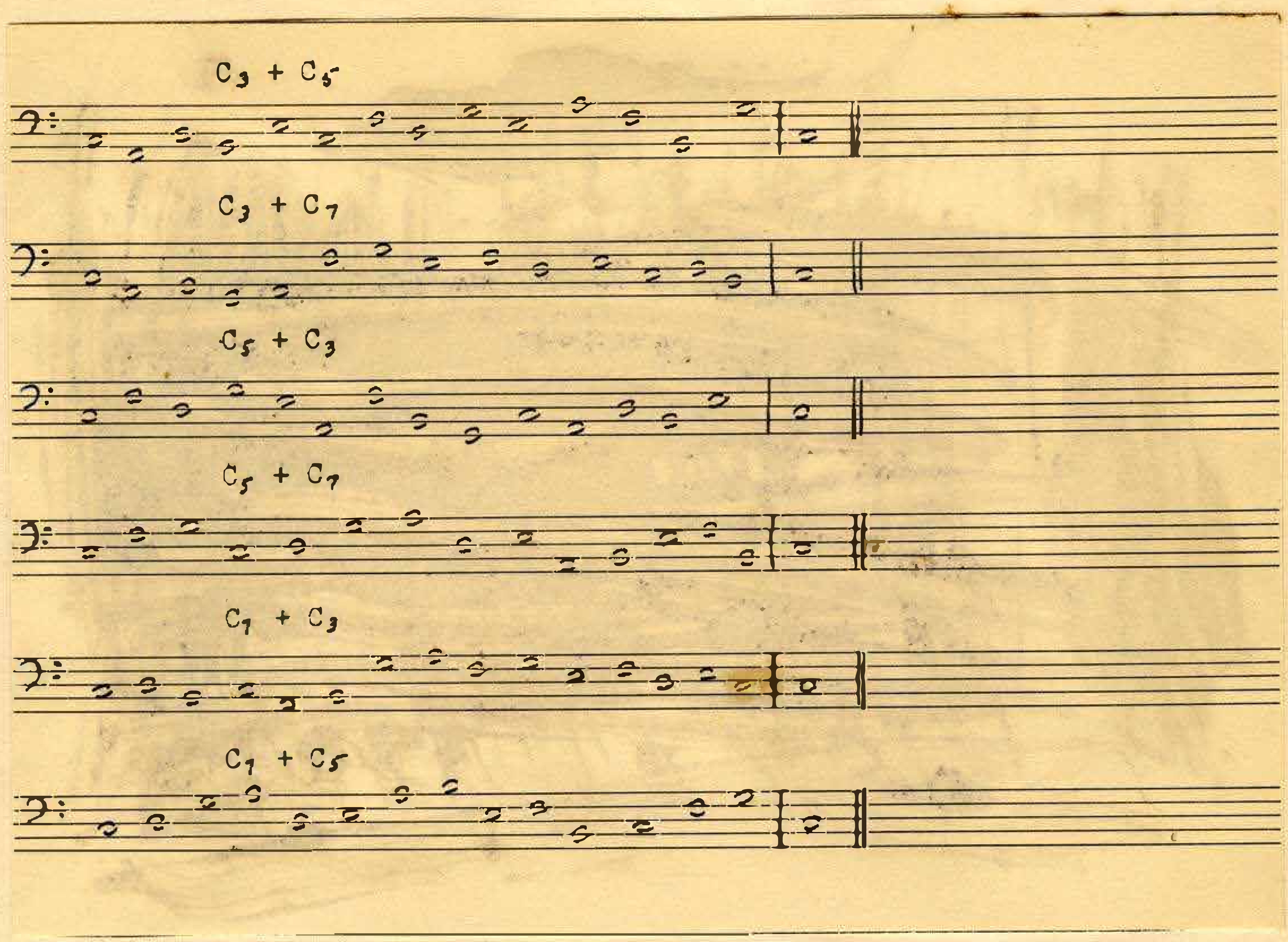
In Binomial Progressions <u>each chord</u> appears <u>twice</u> and in a different combination with the preceding and the following chord. Thus, a complete <u>Binomial Cycle</u> in a seven-unit scale consists of 2 x 7 = 14 chords.

<u>Figure III.</u>

<u>Binomial Cycles</u>

$C_3 + C_5$	$C_5 + C_3$	$C_7 + C_3$
$C_3 + C_7$	$C_5 + C_7$	$C_7 + C_5$

(please see next page)



In Trinomial Progressions <u>each chord</u> appears <u>three times</u> and in a different combination with the preceding and the following chord. Thus, a complete <u>Trinomial Cycle</u> in a seven-unit scale consists of 3 x 7 = 21 chord.

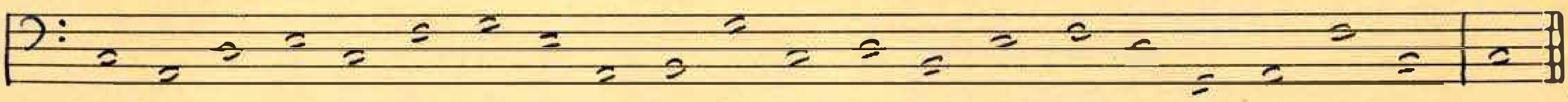
Figure IV.

Trinomial Cycles

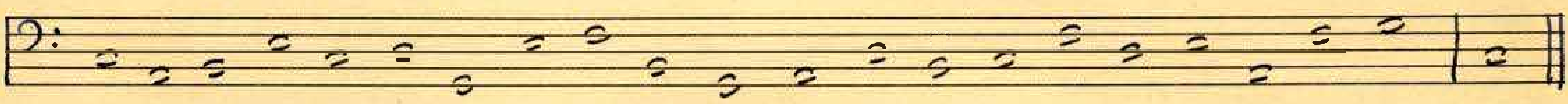
$C_3 + C_5 + C_7$ $C_7 + C_3 + C_5$ $C_5 + C_7 + C_3$

$C_3 + C_7 + C_5$ $C_5 + C_3 + C_7$ $C_7 + C_5 + C_3$

$C_3 + C_5 + C_7$

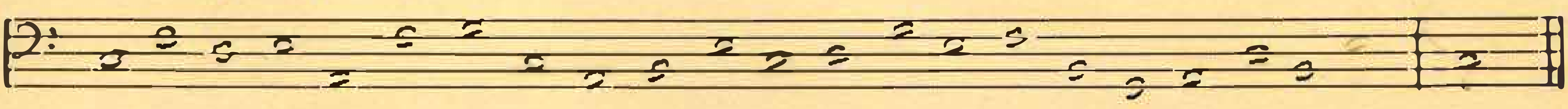


$C_3 + C_7 + C_5$

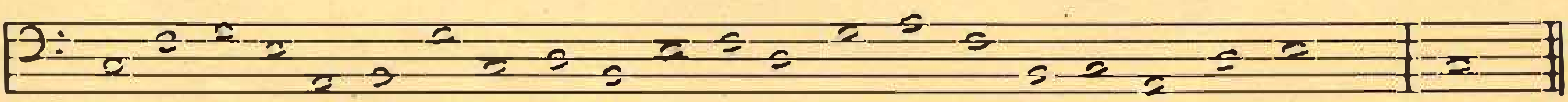


$C_7 + C_3 + C_5$

$C_5 + C_3 + C_7$



$C_5 + C_7 + C_3$



$C_7 + C_5 + C_3$

Lesson LXVII.

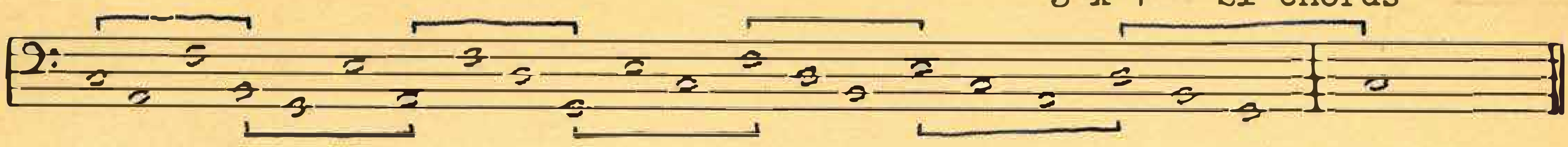
Both Binomial and Trinomial Cycles produce the ultimate variety combined with the absolute consistency of the character (style) of harmonic progressions. Being perfect in this respect they are of little use when a personal selection of character becomes a paramount factor.

In order to produce an individual style of harmonic progressions, it is necessary to use a selective continuity of Cycles. This can be accomplished by means of the Coefficients of Recurrence applied to a selected combination of Cycles. A combination of Cycles can be either a Binomial or a Trinomial. Groups producing coefficients of recurrence can be Binomial, Trinomial or Polynomial. The materials for these can be found in the Theory of Rhythm. Rhythmic resultants of different types and their variations provide various groups which can be used as coefficients of recurrence. Distributive Power-Groups as well as the different Series of Growth and Acceleration can be used for the same purpose.

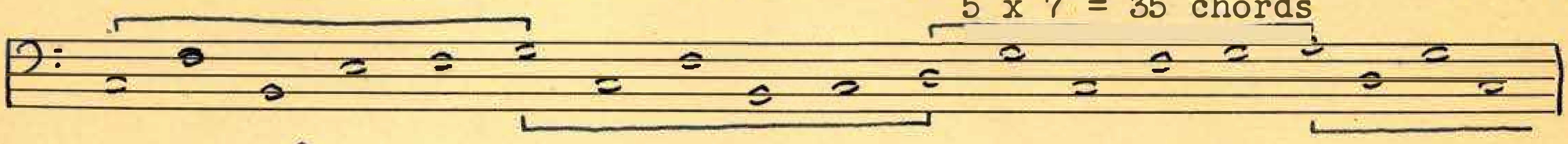
Figure V.

Binomial Cycles, Binomial Coefficients

Cycles: $C_3 + C_5$; Coefficients: 2+1 = 3t; Synchronized Cycles: $2C_3 + C_5$;
3 x 7 = 21 chords



Cycles: $C_5 + C_7$; Coefficients: 3+2 = 5t; Synchronized Cycles: $3C_5 + 2C_7$;
5 x 7 = 35 chords



Binomial Cycles, Coefficient-Groups with the number of terms divisible by 2.

Cycles: $C_7 + C_3$; Coefficients: $r_{4\div3}$ = 3+1+2+2+1+3 = 12 t

Synchronized Cycles: $3C_7 + C_3 + 2C_7 + 2C_3 + C_7 + 3C_3$; 12x7= 84 chords

Binomial Cycles, Coefficient-Groups producing interference with the Cycles (not divisible by 2)

Cycles: $C_5 + C_3$ Coefficients: $3 + 1 + 2 = 6t$

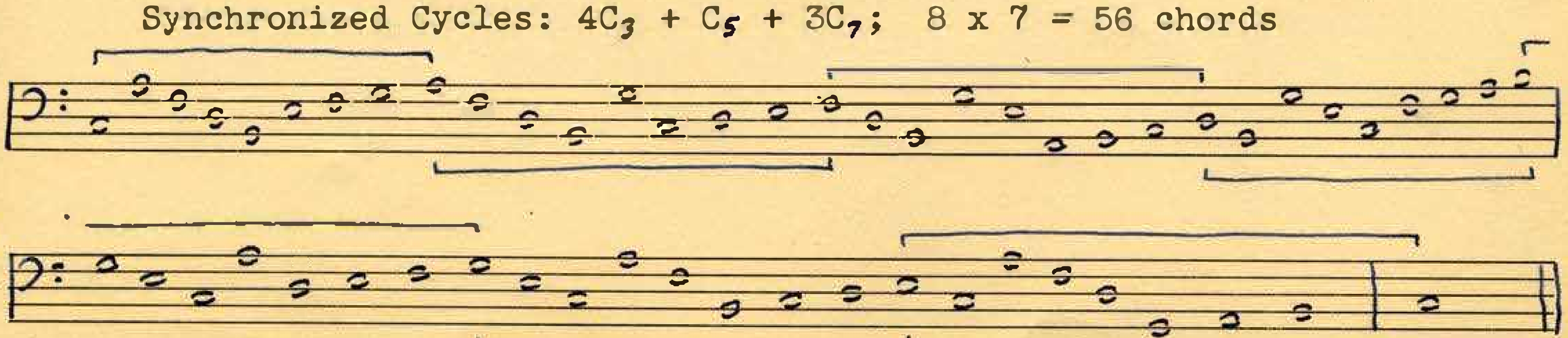
Synchronized Cycles: $3C_5 + C_3 + 2C_5 + 3C_3 + C_5 + 2C_3$

Synchronized coefficients: 6t x 2 = 12t; 12 x 7 = 84 chords

Trinomial Cycles, Trinomial Coefficients

Cycles: $C_3 + C_5 + C_7$ Coefficients: $4 + 1 + 3 = 8t$

Synchronized Cycles: $4C_3 + C_5 + 3C_7$; 8 x 7 = 56 chords



(Figure V, Cont.)

Trinomial Cycles, Coefficient-Groups with the number of terms divisible by 3.

Cycles: $C_7 + C_3 + C_5$; Coefficients: $r_{5\div 2} = 2+2+1+1+2+2 = 10t$

Synchronized Cycles: $2C_7 + 2C_3 + C_5 + C_7 + 2C_3 + 2C_5$; 10x7=70 chords

Trinomial Cycles, Coefficient-Groups producing interference with the Cycles (not divisible by 3).

Cycles: $C_7 + C_5 + C_3$; Coefficients: 3 + 1 = 4t

Synchronized Cycles: $3C_7 + C_5 + 3C_3 + C_7 + 3C_5 + C_3$

Synchronized Coefficients: 4t x 3 = 12t; 12 x 7 = 84 chords

The style of harmonic progressions depends entirely on the form of cycles employed. No composer confines himself to one definite cycle, yet it is the predominance of a certain cycle over others that makes his music immediately recognizable to the listener. In one case it may be that the beginning of a progression is expressed through the cadences of a certain cycle, in another case it may be a prominent coefficient group that makes such music sound distinctly different from the other. The style of harmonic progressions can be defined as a definite form of Selective Cycles. Both the combination of cycles (their sequence) and the coefficient group determining their recurrence are the factors of a style of harmonic progressions.

Lesson LXVIII.

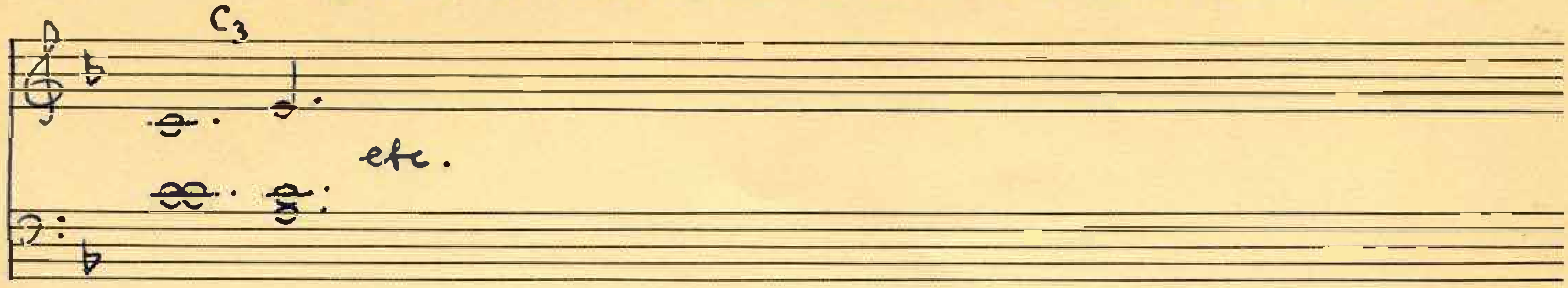
There is much to be said about the historical development of the cycles, as there are already some wrong notions established in this field.

Though the common belief is that the progressions from the tonic to the dominant and back to the tonic (ending cadence in C_5), is the foundation of diatonic harmony, historical evidence, as well as mathematical analysis prove to the contrary. During the course of centuries of European musical history, parallel to the development of counterpoint, there was an awakening of harmonic consciousness. The latter can be traced, in its apparent forms, back to XV Century A.D. At that time harmony meant concord, an agreeable, consonant, stabilized sonority of several voices simultaneously sustained. Concordant progressions could be accomplished therefore through consonant chords moving in consonant relations. Obviously such progressions require common tones, and the latter can be expressed as C_3. As the tonality, i.e., an organized progression of tonal cycles was at that time in the state of fermentation, it is natural to expect the cycle of the third to appear in both positive (C_3) and negative (C_{-3}) form.

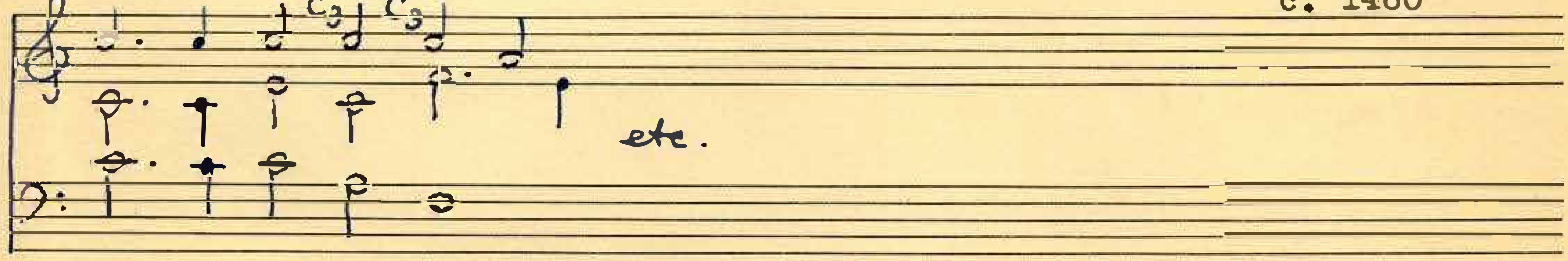
The following are a few illustrations taken from the music of XV and XVI Centuries.

Figure VI.

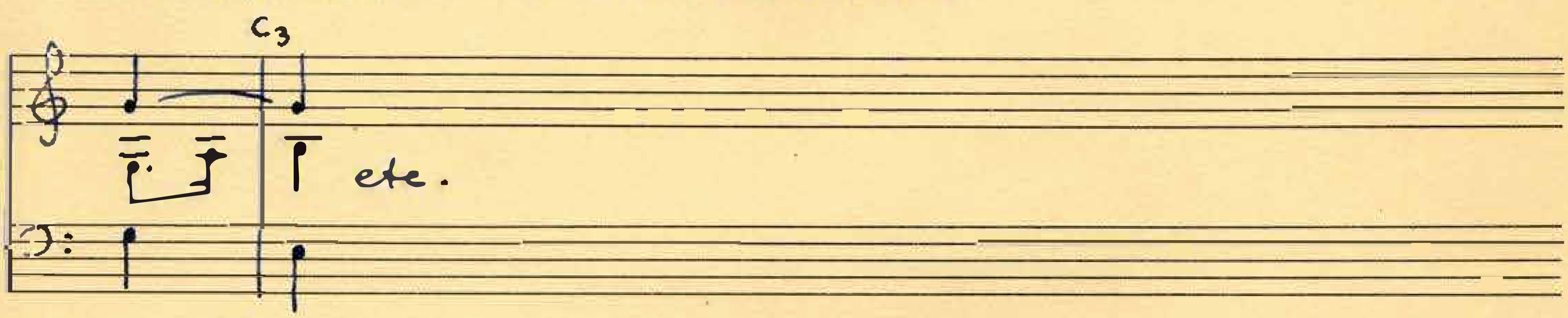
Opening of "Ave Regina Coelorum" - Leonel Power, c. 1460



"Benedicta Tu" MS. Pepysian 1236, Madrigal Collection, Cambridge, c. 1460



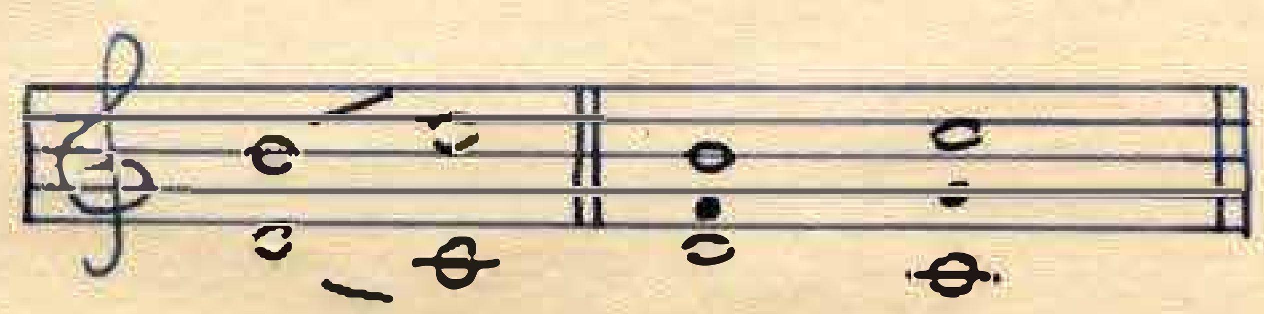
"Deutsches Lied" - Adam von Fulda (1470)



Julio Cacchini (1550-1618)

Cycle of the Seventh, on the other hand, has a purely contrapuntal derivation. When the two leading tones (the upper and the lower) move in a cadence into their respective tonics (like b→c and d→c) by means of contrary motion in two voices, we obtain the ending cadence of C_7. Further development of the third part was undoubtedly necessitated by the desire for fuller sonorities. This introduced an extra tone (f in a chord of b) with which the remaining tones form S(6) i.e., a third-sixth-chord or a sixth-chord, the first inversion of the root-chord: S(5).

Figure VII.



It is only natural to expect the predominance of the C_7 in contrapuntal music. Cadences as in Figure VII are most standardized in the XIII and XIV Century European music. See Guillaume de Machault (1300-1377) "Mass for the Coronation of Charles V" (phonograph recording published by the Gramophone Shop).

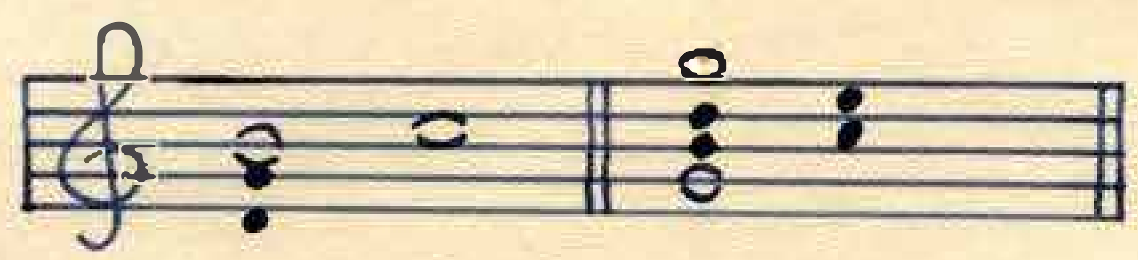
The appearance of the cycle of the fifth must be referred to a later date, when C_3 and C_7 were already in use. I offer the following hypothesis of the origin of C_5. The positive form might have occurred as a pedal point development, where by sustaining the tonic and changing the remaining two tones to their leading tones, the sequence would represent C_5. Another interpretation of the origin of C_5 is the one which this system of Harmony is based upon, i.e., omission of intermediate links in a series. This principle ties up musical harmony with harmonic structure of crystals, as used in crystallographic analysis.

Figure VIII.

The origin of the negative form of the cycle of the fifth (C-5) is due to the desire of acquiring a concord supporting a leading tone. Let b be a leading tone in the scale of c. The most concordant combination of tones in the pre-Bach time, i.e., in the mean temperament (the tuning system officially recognized in Europe before the advent of

equal temperament), harmonizing the tone b was the G-chord (g, b, d). But when moving from G-chord to C-chord the form of the cycle is positive. In reality both forms, the positive and the negative, are the beginning and the ending cadences. Compare Figure IX with Figure VIII.

Figure IX.



JOSEPH SCHILLINGER

CORRESPONDENCE COURSE

With: Dr. Jerome Gross Subject: Music

Lesson LXIX.

The development of harmonic progressions in the European music of the last three centuries can be easily traced back to their sources. The style of every composer is hybrid, yet the quantitative predominance of certain ingredients (like the cycles appearing with the different coefficients of recurrence) produces individual characteristics.

In the following exposition I will confine the concept of "style" to harmonic progressions in the diatonic system.

Richard Wagner was the greatest representative of C_3 in the XIX Century. This statement is backed by the statistical analysis of tonal cycles in his works as compared to his contemporaries and predecessors: C_5 was the universal vogue of a whole century preceding Wagner. In fact, it is not necessary to analyze all works of Wagner. The most characteristic progressions may be found at the beginnings of his preludes to musical dramas and also in the various cadences. The beginnings of major works of any

composers are important, for the reason that they cannot be casual: it is the "calling card" of a composer. The importance of cadences as determinants of harmonic styles was stressed upon by our contemporary, Alfredo Casella, in his paper, "Evolution of Harmony from the Authentic Cadence".

Wagner, being German and intentionally Germanic composer, undoubtedly has done some research of the earlier German music, as he intended to deal with the subjects of German mythology, in which he was well versed. The XV Century German music discloses such an abundance of C_3, that it is only natural to expect the influence of such an authentic source of Germanic music upon Wagner's creations. In his time, Wagner's harmonic progressions sounded revolutionary because many things were forgotten in four hundred years, and archaic acquired a flavor of modernistic. So far as the development of diatonic progressions in Wagner's music appears to the unbiased analyst, the whole mission of Wagner's life was to develop a consistent combined cadence in C_3.

Starting with an early work like "Tannhauser", we find that already the very beginning of the Overture is typical in this respect.

Figure X.

Later on we find more extended progressions of C_3, as in the Aria of Wolfram von Eschenbach (the scene of Minnesingers contest):

Figure XI.

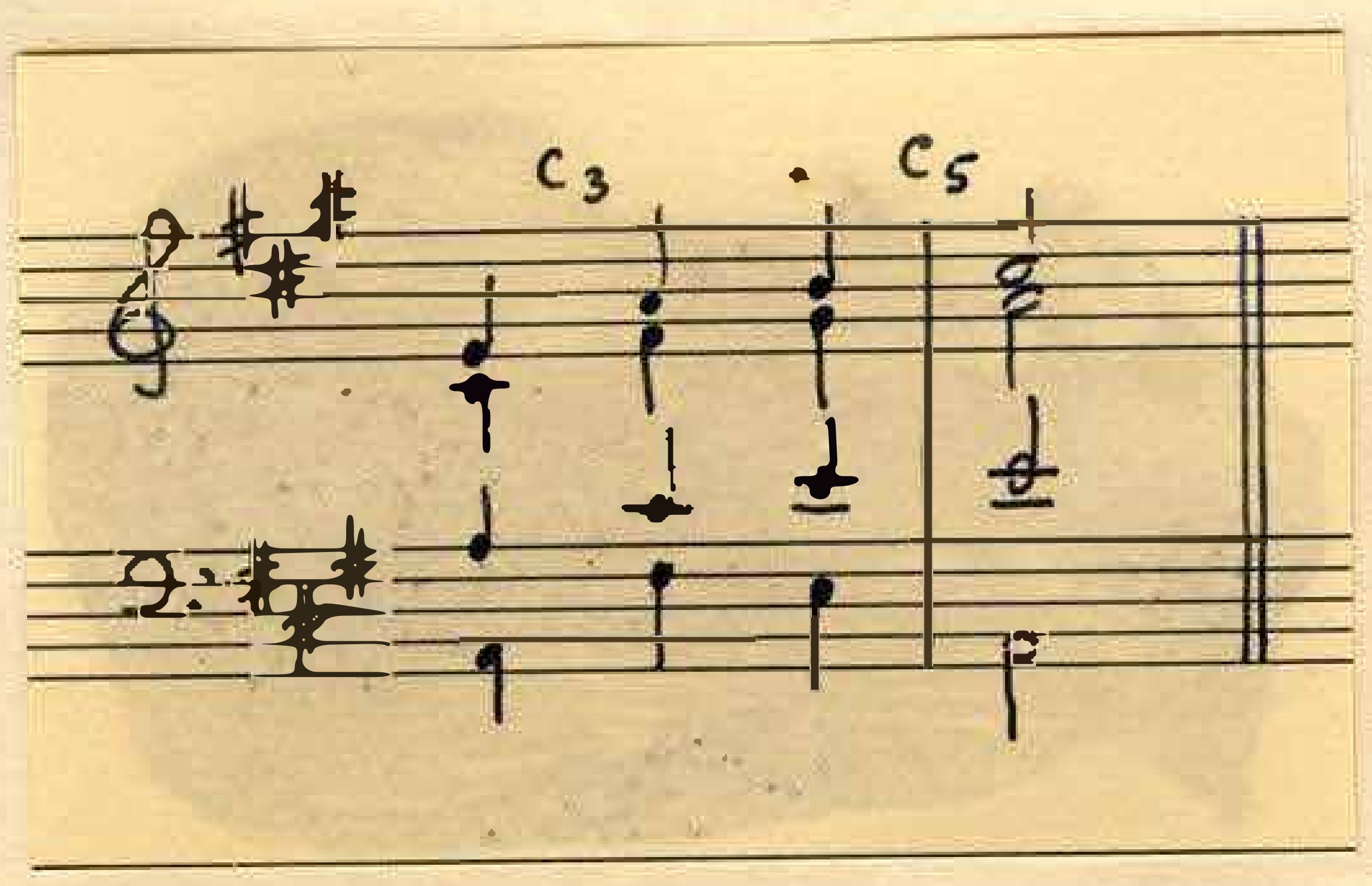
"Lohengrin" is even more abundant with C_3 than "Tannhauser". In "Farewell to Swan", as in many other places of the same opera, we find the

characteristic back-and-forth fluctuation: C_3+C-3.

Figure XII.

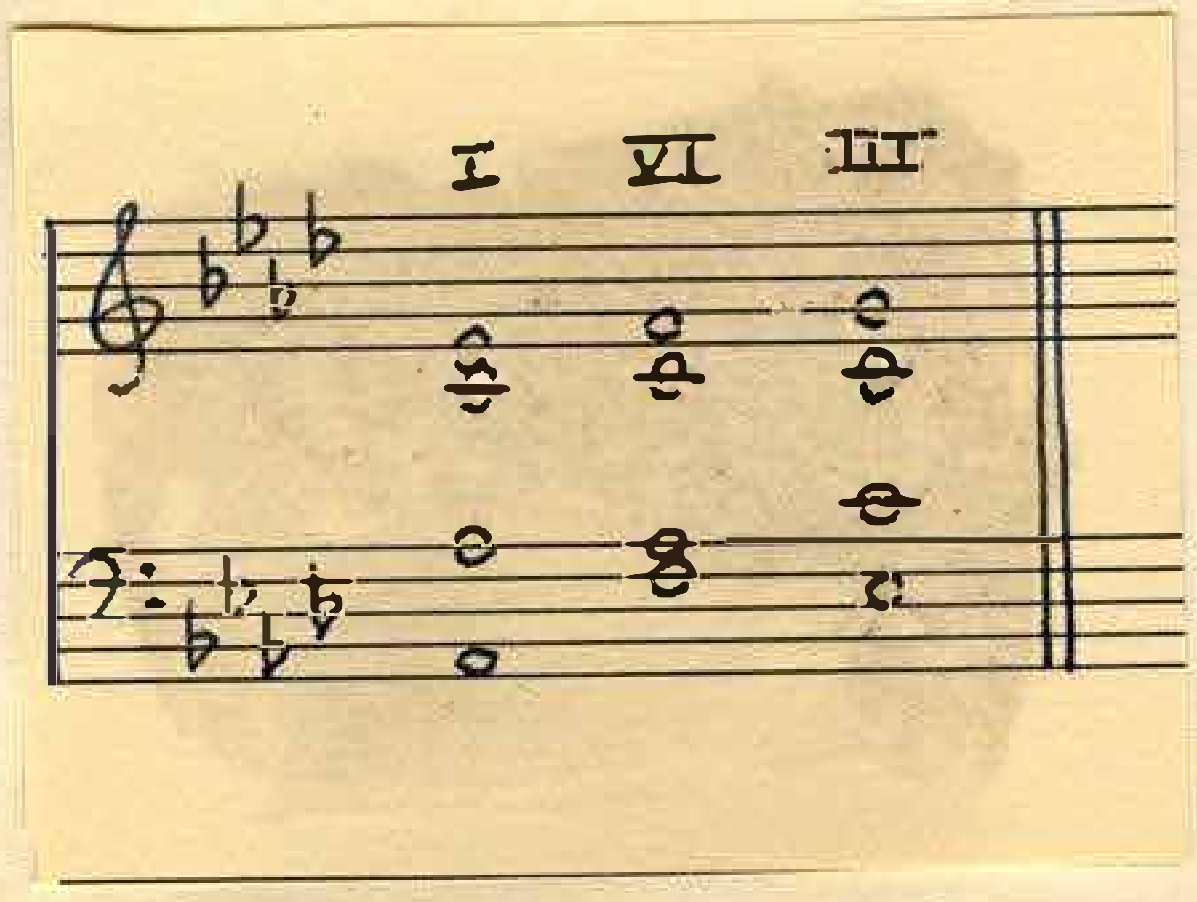
Forming his cadences, Wagner paid sometime his tributes to the dominating "dominant" of Beethoven (C_5). This produced combined hybrid cadences, which are characteristic of "Lohengrin". The first part of such a cadence is the beginning cadence in C_3, while the second part is the ending cadence in C_5: I - VI - V - I.

Figure XIII.



Dealing with other types of progressions than diatonic in the course of his career, Wagner came back to diatonic purity in its complete and consistent form in his last work "Parsifal". The beginning of the "Prelude to Act I" reveals that the composer came to the realization of the combined cadence of C_3: I - VI - III;

Figure XIV.



The more extensive sequences of C_3 are: I - VI - IV - II;

Figure XV.

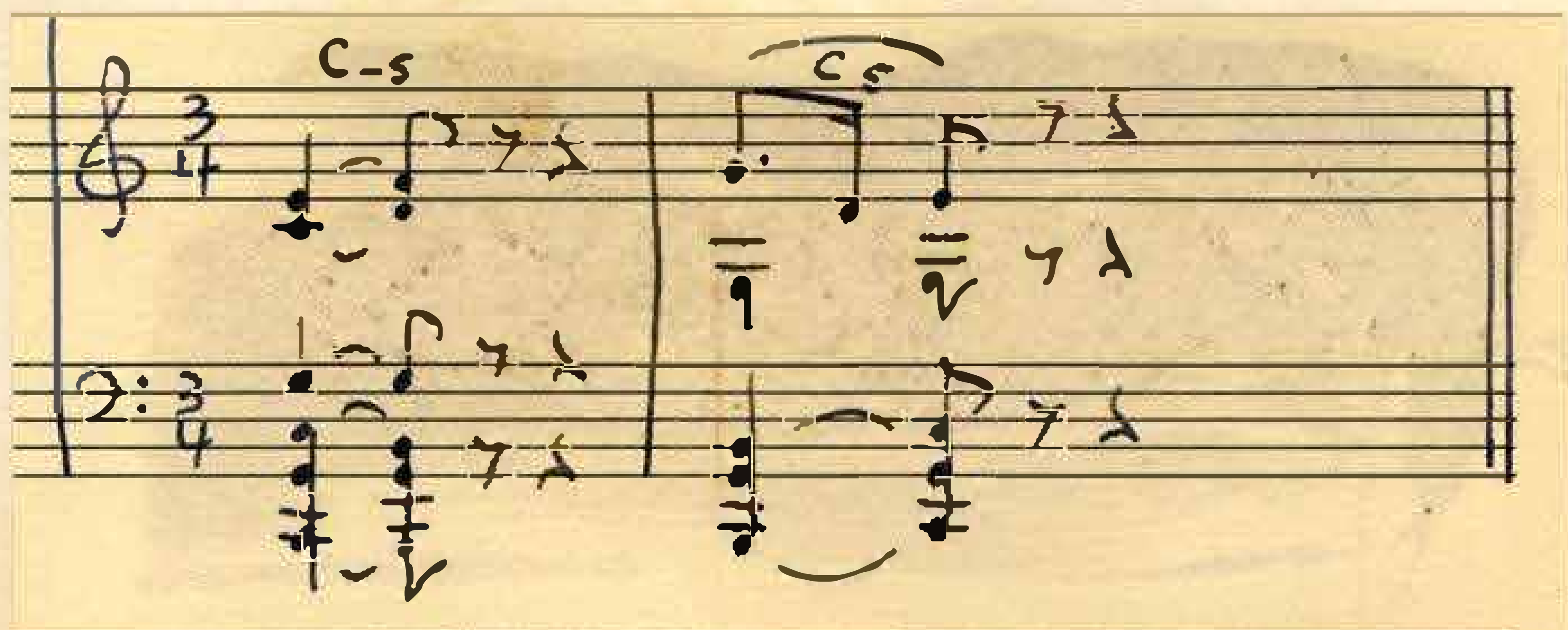
And the complete combined cadence ("Procession of the Noblemen of Graal"): I - VI - III - I.

Figure XVI.

The second half of the XVIII Century and the first half of the XIX Century cover the period of the hegemony of the dominant and C_5 in all its aspects in general. The latter are: continuous progressions of C_5; starting, ending and combined cadences (I - IV - I; I - V - I; I - IV - V - I). The main sources of music possessing these characteristics are: the Italian Opera and the Viennese School. To the first belong: Monteverdi, Scarlatti, Pergolesi, Rossini, Verdi. The second is represented by Dittersdorf, Haydn, Mozart, Beethoven, Schubert. Today this style disintegrated into the least imaginative creations in the field of popular music. Nevertheless it is the stronghold of harmony in the educational music institutions.

Here are a few illustrations of C_5 style in the early Sonatas for the Piano by Ludwig van Beethoven: Sonata Op. 7, Largo; Sonata Op. 13, Adagio Cantabile.

Figure XVII.

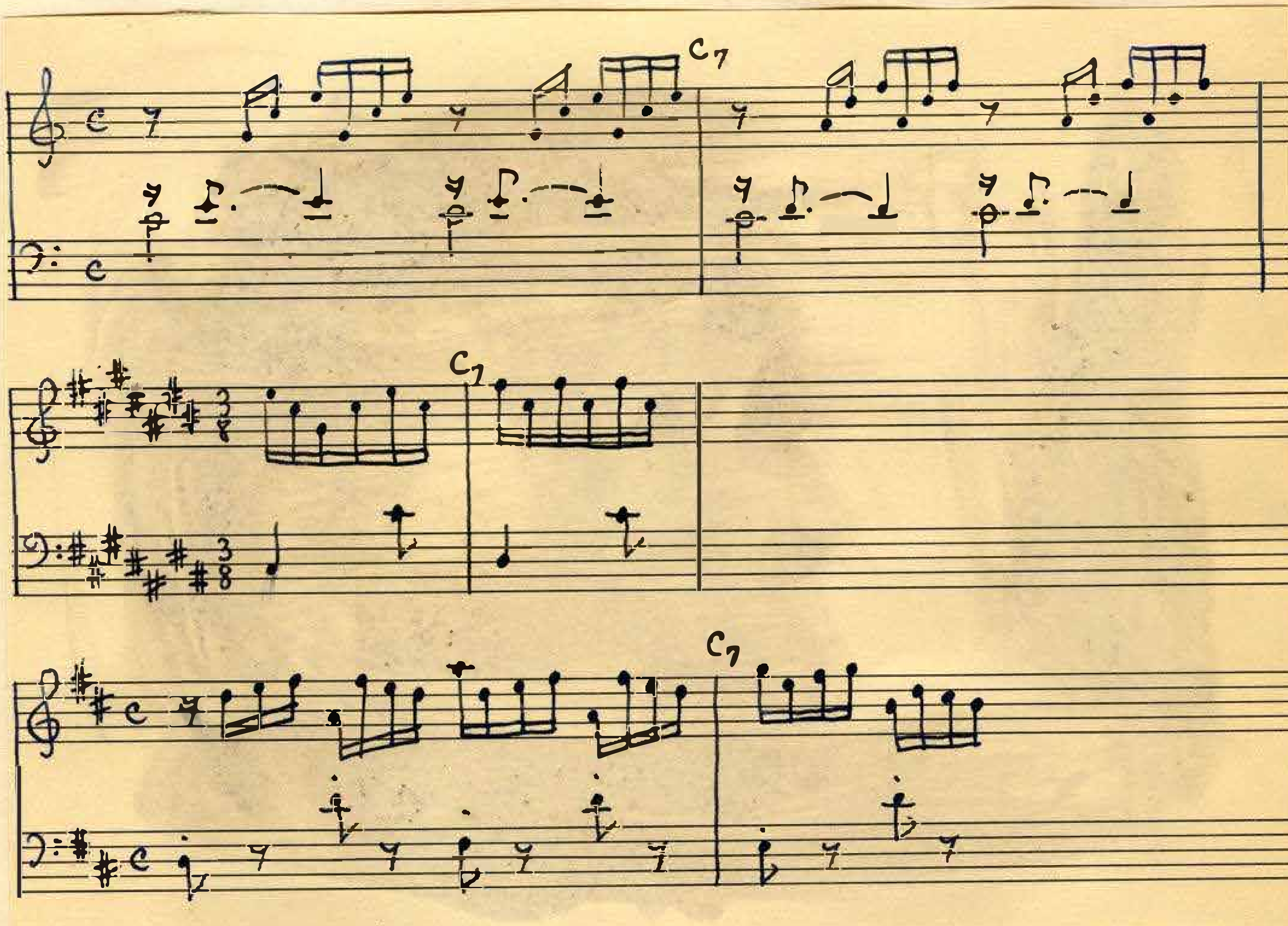


Any number of illustrations can be found in Mozart's and Beethoven's symphonies, particularly in the conclusive parts of the last movements.

Assuming that the historical origin of the cycle of the seventh can be traced back to contrapuntal cadences, it would be only logical to expect the evidence of C_7 in the works of the great contrapuntalists.

I choose for the illustration of C_7, as characteristic starting progressions, some of the well known Preludes to Fugues taken from the First Volume of "Well Tempered Clavichord" by Johann Sebastian Bach: Prelude I; Prelude III; Prelude V.

Figure XVIII.



Bach's famous "Chiaconna in D-minor" for Violin, discloses the same characteristics, as the first chord is d and the second chord is e, which makes C_7.

A consistent and ripe style of diatonic progressions corresponds to a consistent use of one form, either positive or negative and not to an indiscriminate mixture of both. Many theorists confuse the hybrid of positive and negative forms with <u>modal progressions,</u> which the theorists have never defined clearly. In reality, <u>modal progressions</u> are in no respect different from <u>tonal progressions,</u> except for the scale structure. Both types (tonal and modal) can be either positive, or negative, or hybrid. Modes can be obtained by the direct change of key signatures, as described in the "Theory of Pitch Scales" (transposition to one axis). Here is an example, typical of Moussorgsky, from "Boris Godounov" (opera):

Figure XIX.

In the above example the mode (scale) is Cd_5, the fifth derivative scale of the Natural Major in the key of C, known as Aeolian mode, while the progression of tonal cycles is a hybrid of positive and negative forms.

Lesson LXX.

B. Transformations of S(5).

In the traditional courses of harmony the problems of progressions and voice-leading are inseparable. Each pair of chords is described as sequence and a form of voice-leading. Thus each case becomes an individual case where the movement of voices is described in terms of melodic intervals (like: a fifth down, a second up, a leap in soprano, a sustained tone in alto, etc.). No person of normal mentality can ever memorize all the rules and exceptions offered in such courses. In addition to this unsatisfactory form of presentation of the subject of harmony, one finds out very soon that the abundance of rules covers a very limited material (mostly the harmony of the second-rate XVIII Century European composers).

The main defect of the existing theories of harmony is in the use of the descriptive method. Each case is analyzed apart from other cases and without any general underlying principles.

The mathematical treatment of this subject discloses the general properties of the positions and movements of the voices in terms of transformations of the chordal functions.

Any chord, no matter of what structure, from a mathematical standpoint, is an assemblage of

pitch units, or a group of conjugated functions (elements). These functions are the different pitch-units distributed in each group, assemblage or chord according to the different number of voices (parts) and the intervals between the latter.

In groups with three functions known as three-part structures (S = 3p) the functions are a, b and c. These functions behave through general forms of transformations and not through any musical specifications.

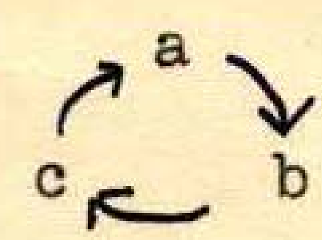
As in this branch we are dealing with so-called four-part harmony, we have to define the meaning of this expression more precisely.

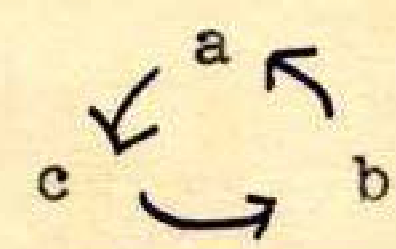
When an S(5) constitutes a chord-structure, the functions of the chord are: the root, the third and the fifth or 1, 3 and 5. In their general form they correspond to a, b and c, i.e., a = 1, b = 3, and c = 5. The bass of such harmony is a constant root-tone, i.e., const. 1 or const. a.

Thus the transformation of functions affects all parts except the bass. Here, therefore, we are dealing with the groups consisting of three functions.

Such groups have two fundamental transformations:

(1) clockwise (↻) and (2) counterclockwise (↺)

The clockwise transformation is: 

The counterclockwise transformation is: 

Each of these transformations has two meanings: the first to be read --

a is followed by b
b " " " c
c " " " a
for the ↻ and
a is followed by c
c " " " b
b " " " a
for the ↺

discloses the mechanism of the <u>positions of a chord;</u>

the second to be read --

a transforms into b
b " " c
c " " a
for the ↻ and
a transforms into c
c " " b
b " " a
for the ↺

constitutes the forms of voice-leading.

Positions.

The different positions of S(5) = 1, 3, 5 can be obtained by constructing the chordal functions downward from each phase of the transformations.

a	b	c		a	c	b
b	c	a	and	c	b	a
c	a	b		b	a	c

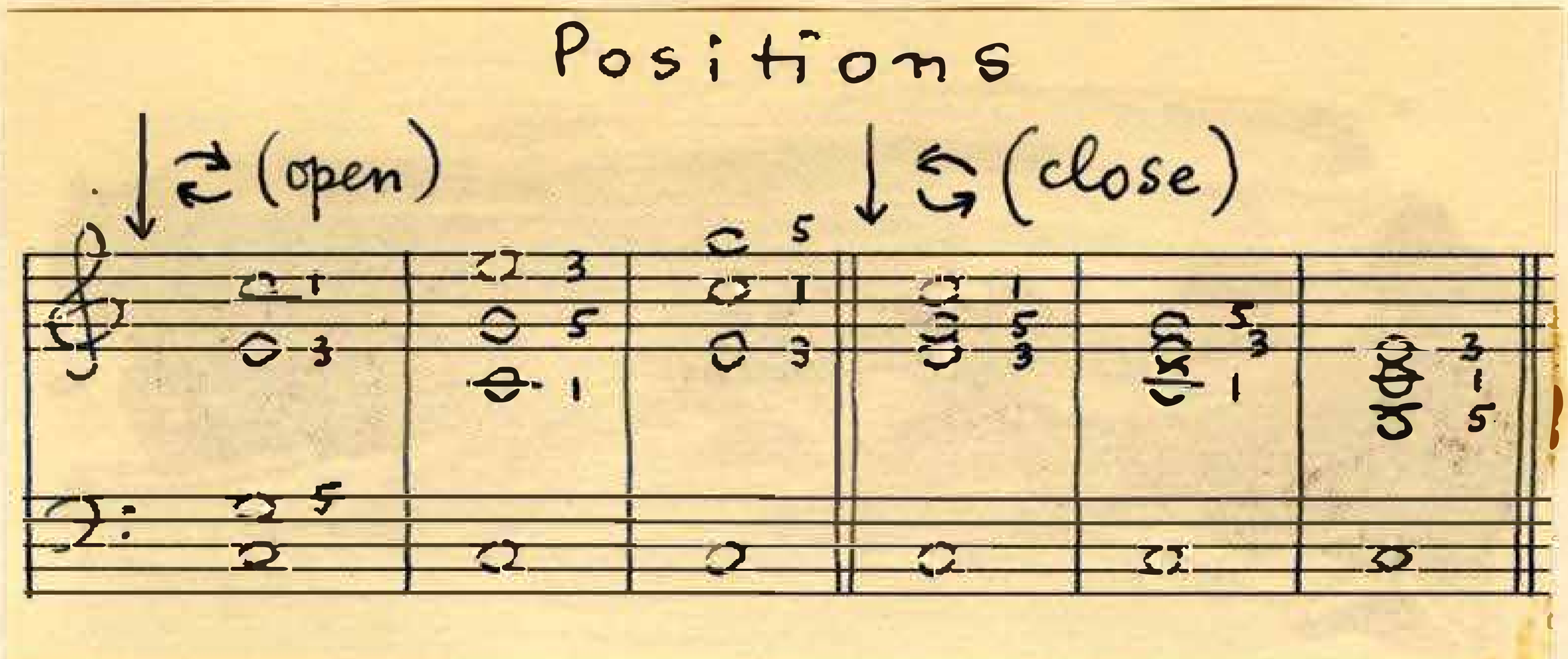
Substituting 1, 3, 5 for a, b, c, we obtain

1	3	5		1	5	3
3	5	1	and	5	3	1
5	1	3		3	1	5

The clockwise positions are commonly known as open, and the counterclockwise as close.

Here are the positions for S(5) = 4 + 3 = = c - e - g. Bass is added for the doubling of the root.

Figure XX.



Voice-Leading

The movement of the individual voices follows the groups of transformation in this form: a of the first chord transforms into b of the following chord; b of the first chord transforms into c of the following chord; c of the first chord transforms into a of the following chord. The above three forms constitute the clockwise voice-leading. For the counterclockwise voice-leading the reading must follow this order: a of the first chord transforms into c of the following chord; c of the first chord transforms into b of the following chord; b of the first chord transforms into a of the following chord.

$$\begin{array}{lcl} a \to b & & a \to c \\ b \to c & \text{and} & c \to b \\ c \to a & & b \to a \end{array}$$

Applying the above transformations to 1, 3, 5 of the S(5), we obtain:

$$\begin{array}{lcl} 1 \to 3 & & 1 \to 5 \\ 3 \to 5 & \text{and} & 5 \to 3 \\ 5 \to 1 & & 3 \to 1 \end{array}$$

Clockwise form:

The root of the first chord becomes the third of the next chord; the third of the first chord becomes the fifth of the next chord; the fifth of the first chord becomes the root of the next chord.

Counterclockwise form:

The root of the first chord becomes the fifth of the next chord; the fifth of the first chord becomes the third of the next chord; the third of the first chord becomes the root of the next chord.

Both forms apply to all tonal cycles.

Let us take C_3 in the natural major, for example. The first chord is C = c - e - g and the

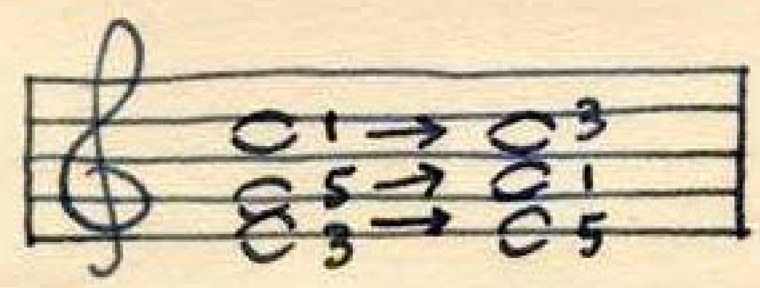
next chord is A = a - c - e.

Clockwise form gives the following reading:

c → c

e → c

g → a

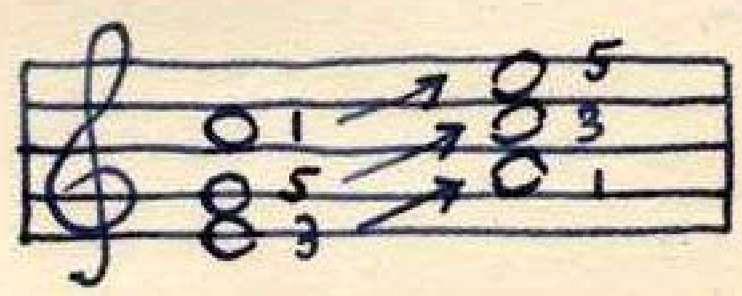


Counterclockwise form gives the following reading:

c → e

g → c

e → a

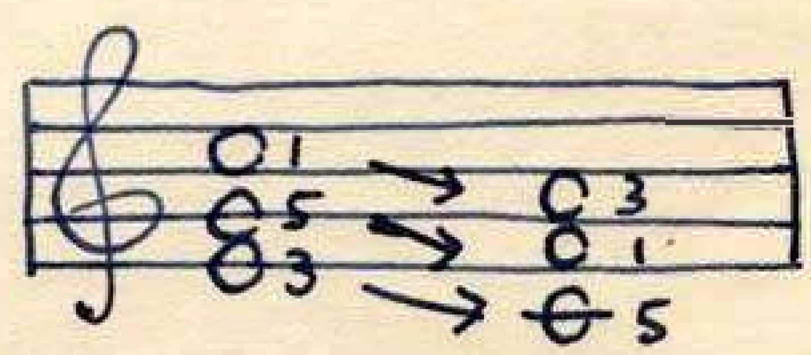


Let us take C_5 in the same scale. The chords are: C = c - e - g and F = f - a - c.

c → a

e → c

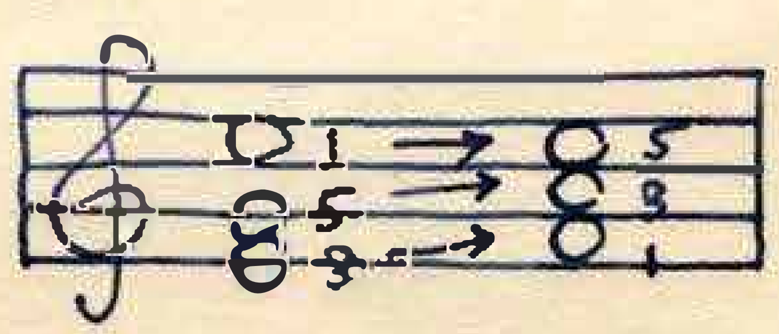
g → f



c → c

g → a

e → f

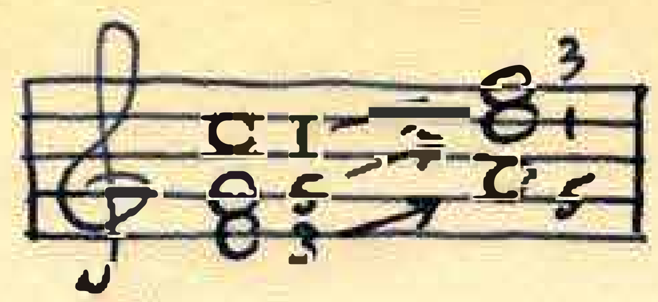


Let us take C_7 in the same scale. The chords are: C = c - e - g and D = d - f - a.

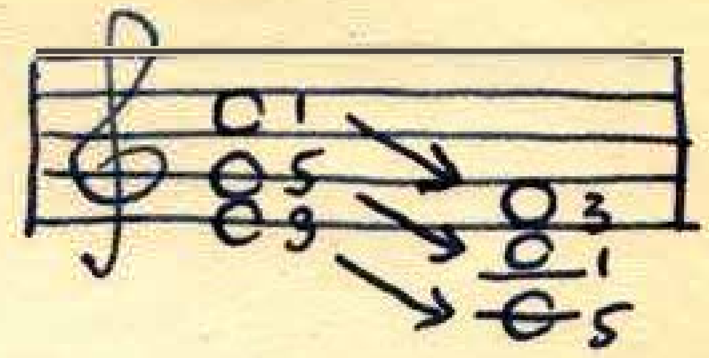
c → f

e → a

g → d



or



Both forms of are acceptable in this case, as the intervals in both directions are nearly equidistant.

c → a

g → f

e → d

Lesson LXXI.

Each tonal cycle permits a continuous progression through one form of transformation. In the following table const. 1 in the bass is added. Apostrophies indicate an octave variation when the extension of range becomes impractical.

In C_7 both directions are combined, offering the most practical form for the range.

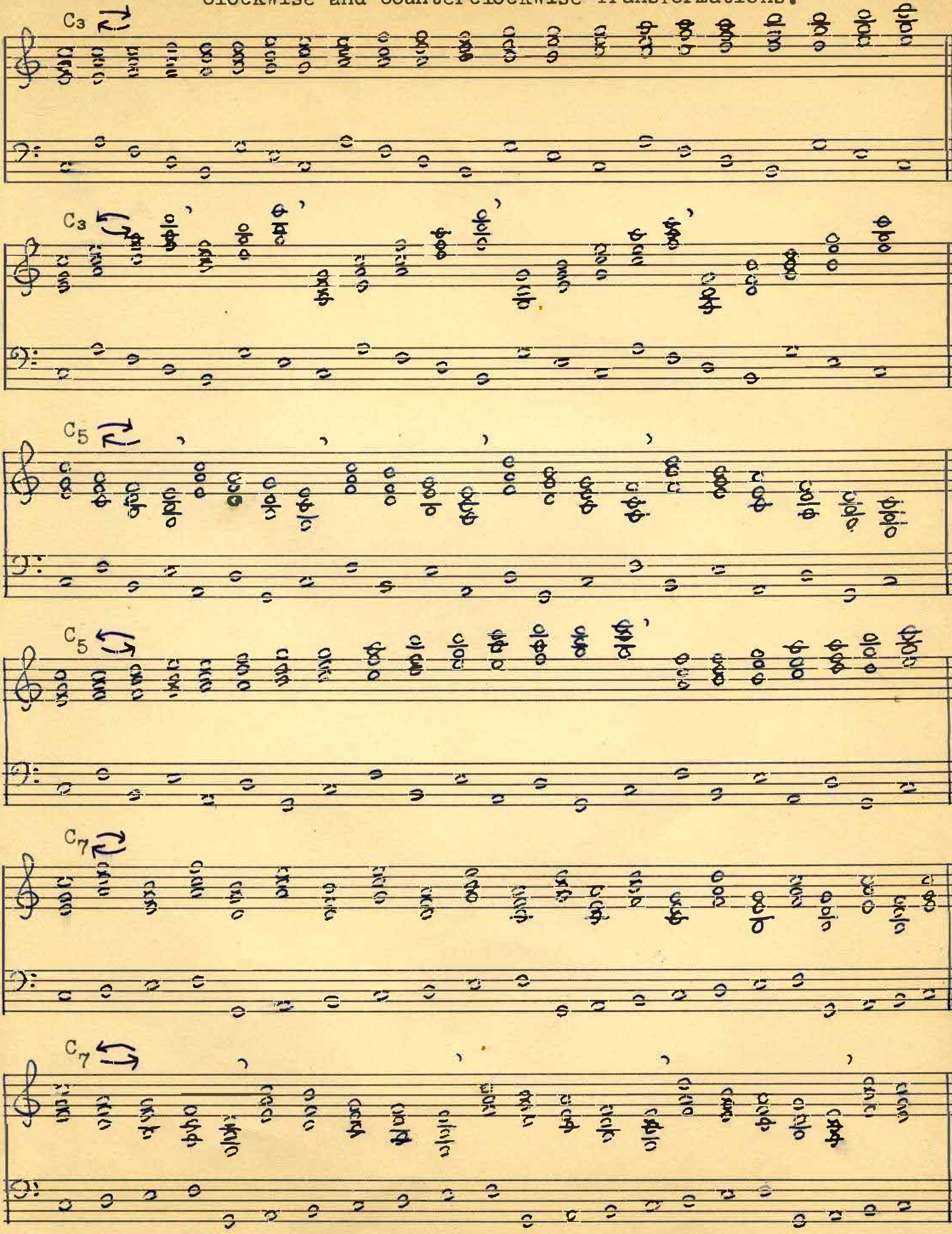
Figure XXI.

(please see following page)

Figure XXI.

Tonal Cycles

Clockwise and Counterclockwise Transformations.



The clockwise and the counterclockwise transformations are applicable to all positions for the starting chord. When the first chord is in the $\rightleftarrows$ (open) position, the entire progression remains automatically in such a position. When the first chord is in $\hookrightarrow$ (close) position, the entire progression remains in such a position.

The constancy of position (open or close) is not affected by the constancy of the tonal cycles, neither is it affected by the lack of their constancy.

The transition from close to open position and vice-versa can be accomplished through the use of the following formula:

Constant b transformation

	Const. 3
$a \rightarrow c$	$1 \rightarrow 5$
$b \rightarrow b$	$3 \rightarrow 3$
$c \rightarrow a$	$5 \rightarrow 1$

It is best to have 3 in the upper voice for such purposes, as in some positions voices cross otherwise. Function 3 from close to open position moves upward to the function 3 of the following chord. Reverse the procedure from open to close.

Figure XXII.

Const. 3 Transformation

Continuous application of const. 3 transformation produces a consistent variation of the ⇄ and the ⇆ positions, regardless of the sequence of tonal cycles.

The following table offers continuous progressions through const. cycles and const. 3 transformation.

Figure XXIII.

(please see next page)

Figure XXIII.

Constant 3 Transformations

C_3 Const. 3

C_5 Const. 3

C_7 Const. 3

J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross — Subject: Music

Lesson LXXII.

There are four forms of relationship between the cycles and the transformations with regard to the variability of both.

(1) const.-cycle, const.-transformation;

(2) const.-cycle, variable transformation;

(3) variable cycle, const.-transformation;

(4) variable cycle, variable transformation.

The forms of transformation produce their own periodic groups, which may be superimposed on the groups of cycles.

Monomial forms of transformations (const. transformations):

(1) $\rightleftarrows$ (2) $\leftrightarrows$ (3) const. 3

Binomial forms of transformations:

(1) $\rightleftarrows + \leftrightarrows$ (2) $\leftrightarrows + \rightleftarrows$

Here Const. 3 is excluded on account of the crossing of inner voices.

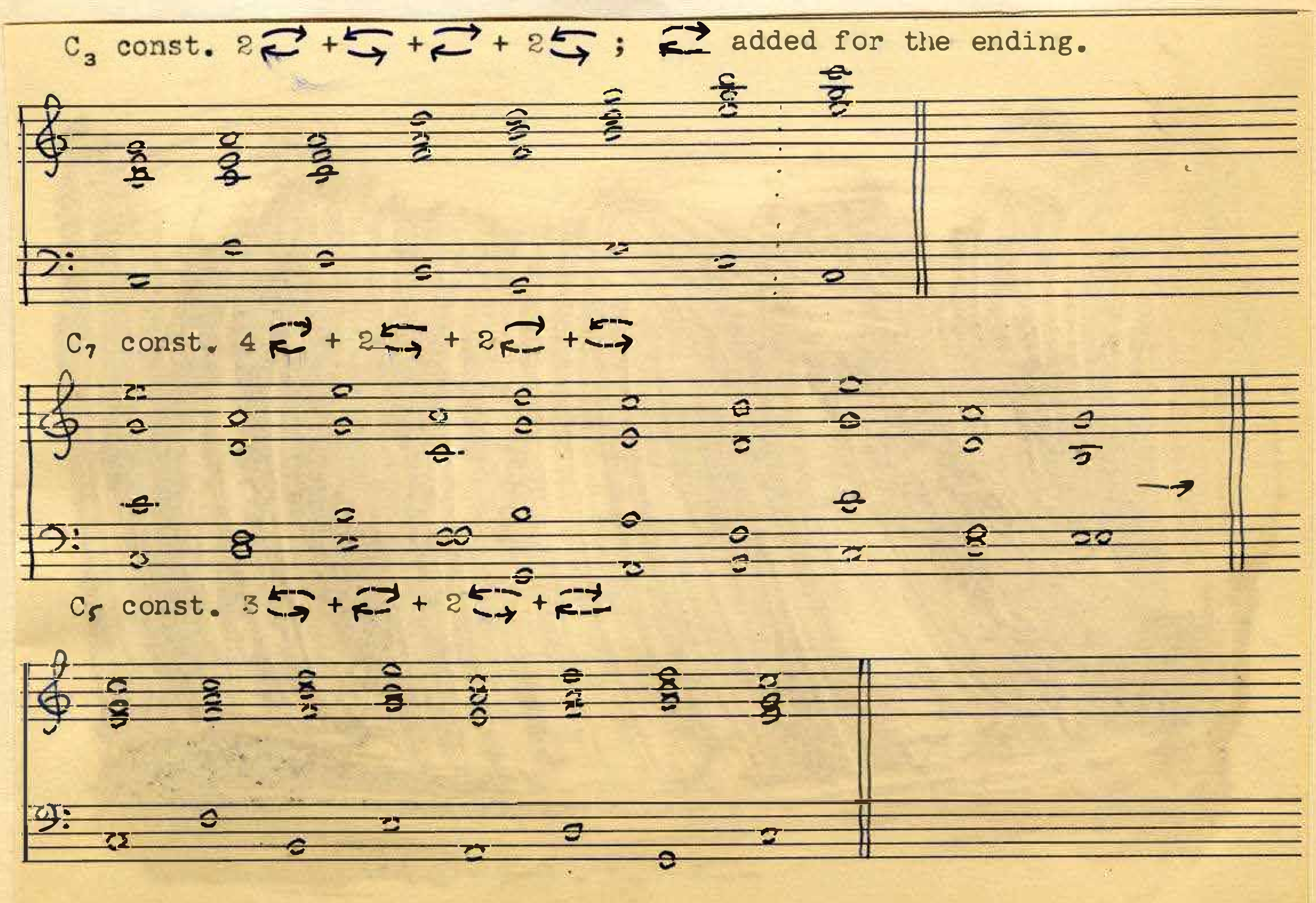
Coefficients of recurrence being applied to the forms of transformations produce <u>selective transformation-groups.</u>

For example: 2⇄ + ⇆ ; 3⇆ + 2⇄ ; 2⇄ + ⇆ + ⇄ + 2⇆ ; 4⇄ + ⇆ + 3⇄ + 2⇆ + + 2⇄ + 3⇆ + ⇄ + 4⇆ ; ⇆ + 2⇄ + 3⇆ + 5⇄ + + 8⇆ ; 4⇄ + 2⇆ + 2⇄ + ⇆ .

Though the groups of tonal cycles, as well as the forms of transformations, may be chosen freely with the writing of each subsequent chord, rhythmic planning of both guarantees a greater regularity and, therefore, greater unity of style.

Examples of variable transformations applied to constant tonal cycles.

Figure XXIV.



Examples of variable transformations applied
to variable tonal cycles.

Figure XXV.

$C_5 + C_7 + C_3$; 2 ⇆ + ⇄

$2C_7 + C_3 + 3C_5$; 4 ⇆ + 2 ⇄ + 2 ⇆ + ⇄

All forms of harmonic continuity, due to their property of redistribution, modal variability and convertibility, are subject to the following modifications:

(1) Placement of the voice representing constant function, and originally appearing in the bass, into any other voice, i.e., tenor, alto or soprano. There are four forms of such distribution:

S	S	S	S
A	A	A	A
T	T	T	T
B	B	B	B

Red letters represent the voice functioning as const. 1.

(2) General redistribution (vertical permutations) of all voices according to 24 variations of 4 elements.

(3) Geometrical inversions: a, b, c and d for any or all forms of distribution of the four voices.

(4) Modal variation by means of <u>modal transposition</u>, i.e., direct change of key signature, without replacing the notes on the staff.

Example of variations.

Figure XXVI.

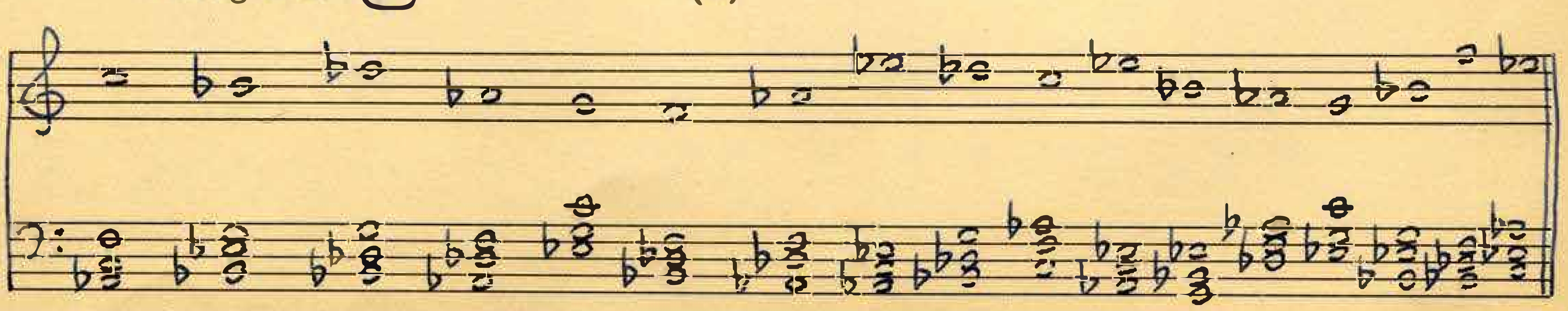
Original

(1)

(2)

Original: d

(3)



(4)

Original + 3♭

Original + (b♭ + f♯) = G mel. minor: d_3

Lesson LXXIII.

C. The Negative Form.

As it was previously defined, the negative form of harmony can be obtained by direct reading of the positive form in position (b) .

Here, for the sake of clarity in the entire matter, I am offering some technical details which explain the theoretical side of the negative form.

According to the definition given to the harmony scale in the negative form, we obtain the latter by means of further expansions of HS. In the positive form we have used: $HS_{E0} (= C_3)$, $HS_{E_1} (= C_5)$ and $HS_{E_2} (= C_7)$.

Now by further expanding HS, we acquire the cycles of the negative form: $HS_{E_3} (= C - 7)$, $HS_{E_4} (= C - 5)$, $HS_{E_5} (= C - 3)$.

Figure XXVII.

(please see next page)

As chord-structures are built downward from a given pitch unit, such a pitch unit becomes the root-tone of the negative structure: the negative root (- 1).

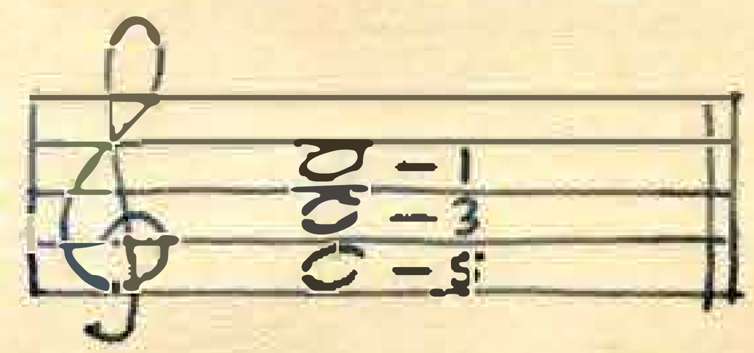
All chord-structures of the negative form, according to the previous definition, derive from HS b.

Thus in order to construct a negative S (5), it is necessary to take the next pitch-unit downward, which becomes the negative third (- 3) and the next unit downward from the latter, which becomes the negative fifth (- 5).

For example, starting from c as a - 1, we obtain a negative S (5), where a is - 3 and f is - 5.

Figure XXVIII.

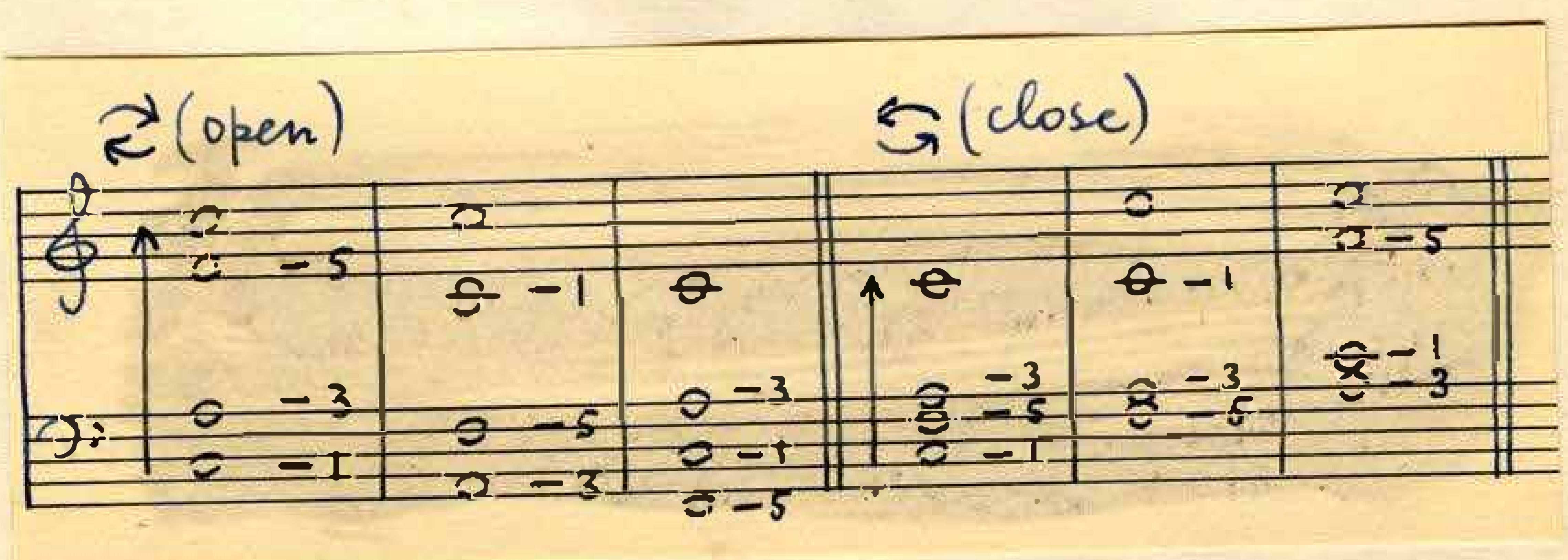
Natural C- Major.



Positions of chords, as they were expressed through transformations, remain identical in the negative form, providing they are constructed upward. In such a case, the addition of a const. 1 in the bass must be, strictly speaking, transferred to the soprano.

Here is how a negative CS (5) would appear in its four-part settings.

Figure XXIX.



If, under such conditions, the chord were constructed downward, the reversal of $\rightleftarrows$ and $\leftrightarrows$ reading would take place.

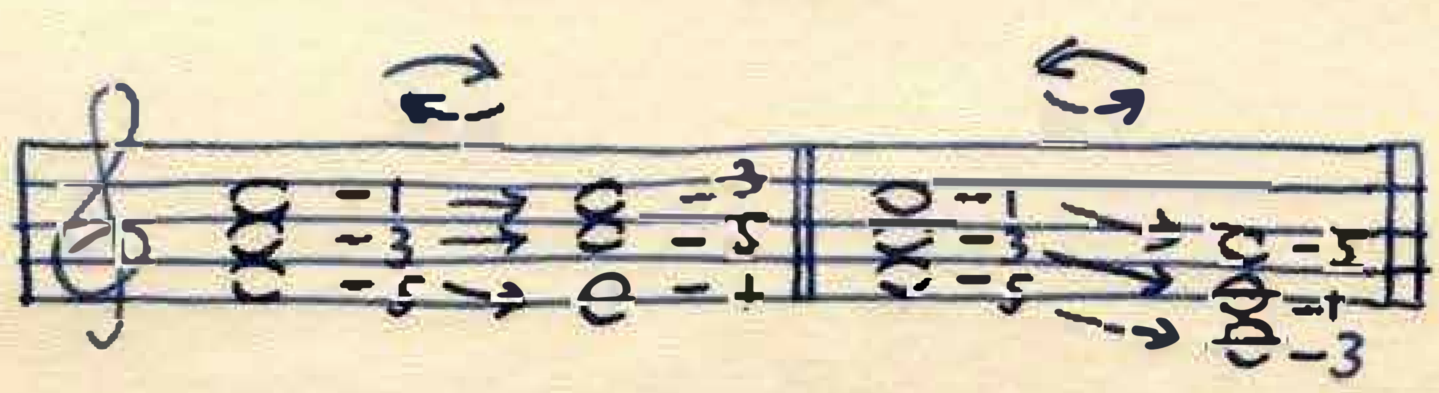
Transformations as applied to voice-leading possess the same reversibility: if everything is read downward, the $\rightleftarrows$ and the $\leftrightarrows$ transformations correspond to the positive form, while in the upward reading the $\rightleftarrows$ becomes the $\leftrightarrows$ and vice-versa.

Let us connect two chords in the negative cycle of the third: CS (5) + C_3 + ES (5).

CS (5) = -1, -3, -5 = c - a - f.

ES (5) = -1, -3, -5 = e - c - a.

Figure XXX.



It is easy to see that in the upward reading chord C corresponds to F, and chord E corresponds to A. Transposing this upward reading to C, we notice that this progression is $C \longrightarrow E$. This proves the reversibility of tonal cycles and the correctness of reading the positive form of progressions in position (b), when the negative form

is desired.

The mixture of positive and negative forms in continuity does not change the situation, but merely reverses the characteristics of voice-leading with regard to positive and negative forms. For example, C_3 in $\rightleftarrows$ in the positive system produces two sustained common tones. In order to obtain an analogous pattern of voice-leading in C_{-3}, it is necessary to reverse the transformation, i.e., to use the $\leftrightarrows$ form in this case.

Lesson LXXIV.

II. Symmetric System.

Diatonic harmony can be best defined as a system where chord-structures as well as chord-progressions derive from a given scale. Structural constitution of pitch assemblages, known as chords, as well as the actual intonation of the sequences of root-tones, known as tonal cycles, are entirely conditioned by the structural constitution of the scale, which is the source of intonation.

Symmetric harmony is a system of pre-selected chord-structures and pre-selected chord progressions, one independent from another. In the symmetric system of harmony scale is the result, the consequence of chords in motion. The selection of intonation for structures is independent from the selection of intonation for the progressions.

A. Structures of S(5).

In this course of harmony only such three-part structures will be used, which satisfy the definition of "special theory of harmony". The ingredients of chord-structures here are limited to 3 and 4 semitones. Under such limitations only four forms of S(5) are possible. It should be remembered, though, that the number of all possible

three-part structures would amount to 55, which is the general number of three-unit scales from one axis.

<u>Table of S(5)</u>

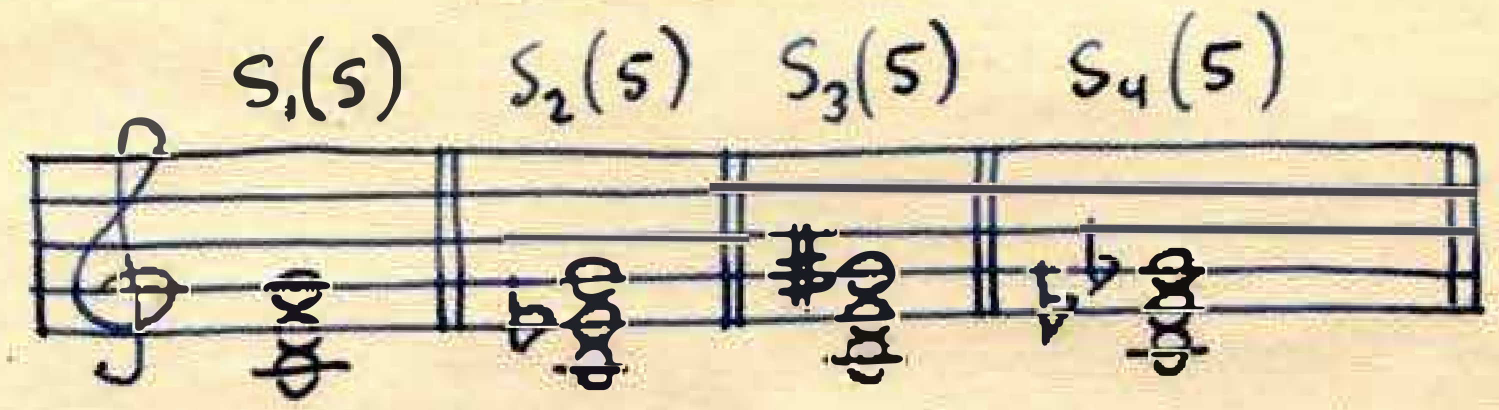
$S_1(5) = 4 + 3$, known as major triad;

$S_2(5) = 3 + 4$, known as minor triad;

$S_3(5) = 4 + 4$, known as augmented triad;

$S_4(5) = 3 + 3$, known as diminished triad.

<u>Figure XXXI.</u>



So long as S(5) will be the only structure for the present use, we shall simplify the above expressions to the following form:

S_1; S_2; S_3; S_4.

Whatever the chord-progression may be, structural constitution of chords appearing in such progression may be either constant or variable. Constant structures will be considered as monomial progressions of structures, while the variable structures will be considered as binomial, trinomial and polynomial structural groups.

Monomial forms of S(5)

$S_1 + \ldots.$

$S_2 + \ldots.$

$S_3 + \ldots.$

$S_4 + \ldots.$

Total: 4 forms

Binomial forms of S(5)

$S_1 + S_2$ $\quad S_2 + S_3$ $\quad S_3 + S_4$

$S_1 + S_3$ $\quad S_2 + S_4$

$S_1 + S_4$

6 combinations, 2 permutations each.

Total: 12 forms

Trinomial forms of S(5)

$S_1 + S_1 + S_2$ $\quad S_2 + S_2 + S_3$ $\quad S_3 + S_3 + S_4$

$S_1 + S_1 + S_3$ $\quad S_2 + S_2 + S_4$

$S_1 + S_1 + S_4$

$S_1 + S_2 + S_2$ $\quad S_2 + S_3 + S_3$ $\quad S_3 + S_4 + S_4$

$S_1 + S_3 + S_3$ $\quad S_2 + S_4 + S_4$

$S_1 + S_4 + S_4$

12 combinations, 3 permutations each.

Total: 36 forms

$S_1 + S_2 + S_3$ $S_2 + S_3 + S_4$

$S_1 + S_2 + S_4$

$S_1 + S_3 + S_4$

4 combinations, 6 permutations each.

Total: 24 forms.

The total of all trinomials: 36 + 24 = 60.

Quadrinomial forms of S(5).

$S_1 + S_1 + S_1 + S_2$ $S_2 + S_2 + S_2 + S_3$ $S_3 + S_3 + S_3 + S_4$

$S_1 + S_1 + S_1 + S_3$ $S_2 + S_2 + S_2 + S_4$

$S_1 + S_1 + S_1 + S_4$

$S_1 + S_2 + S_2 + S_2$ $S_2 + S_3 + S_3 + S_3$ $S_3 + S_4 + S_4 + S_4$

$S_1 + S_3 + S_3 + S_3$ $S_2 + S_4 + S_4 + S_4$

$S_1 + S_4 + S_4 + S_4$

12 combinations, 4 permutations each.

Total: 48 forms

$S_1 + S_1 + S_2 + S_2$ $S_2 + S_2 + S_3 + S_3$ $S_3 + S_3 + S_4 + S_4$

$S_1 + S_1 + S_3 + S_3$ $S_2 + S_2 + S_4 + S_4$

$S_1 + S_1 + S_4 + S_4$

6 combinations, 6 permutations each.

Total: 36 forms

$S_1 + S_1 + S_2 + S_3$ $\quad$ $S_2 + S_2 + S_3 + S_4$

$S_1 + S_1 + S_2 + S_4$

$S_1 + S_1 + S_3 + S_4$

$S_1 + S_2 + S_2 + S_3$ $\quad$ $S_2 + S_3 + S_3 + S_4$

$S_1 + S_2 + S_2 + S_4$

$S_1 + S_3 + S_3 + S_4$

$S_1 + S_2 + S_3 + S_3$ $\quad$ $S_2 + S_3 + S_4 + S_4$

$S_1 + S_2 + S_4 + S_4$

$S_1 + S_3 + S_4 + S_4$

12 combinations, 12 permutations each.

Total: 144 forms.

$S_1 + S_2 + S_3 + S_4$

1 combination, 24 permutations.

Total: 24 forms.

The total of all quadrinomials: 48 + 36 + 144 + 24 = 252.

In addition to all these fundamental forms of the groups of S(5), which represent a <u>neutral harmonic continuity of structures,</u> there are groups with coefficients of recurrence, which represent a <u>selective harmonic</u>

continuity of structures. The latter are subject to individual selection. Any rhythmic groups may be used as coefficients of recurrence.

Examples

(1) $2S_1 + S_3$

(2) $3S_3 + S_2$

(3) $3S_1 + 2S_3 + S_2$

(4) $2S_2 + S_1 + S_2 + 2S_1$

(5) $2S_1 + S_2 + S_3 + 2S_4$

(6) $3S_1 + S_2 + 2S_1 + 2S_2 + S_1 + 3S_2$

(7) $3S_1 + S_3 + 2S_2 + 2S_1 + S_3 + 3S_2$

(8) $4S_3 + 2S_2 + 2S_3 + S_2$

(9) $2S_1 + S_2 + S_1 + S_2 + S_1 + S_2 + 2S_1 + 2S_2 + S_1 + S_2 + S_1 +$
$+ S_2 + S_1 + 2S_2$

(10) $4S_1 + 2S_2 + 2S_4 + 2S_1 + S_2 + S_4 + 2S_1 + S_2 + S_4$

(11) $S_1 + 2S_2 + 3S_4 + 5S_3$

(12) $S_2 + 3S_1 + 4S_2 + 7S_1$

B. Symmetric Progressions.

Symmetric Zero Cycle (C_0)

A group of chords with a common root-tone but with variable positions and variable structures produces a symmetric zero cycle (C_0).

Such a group may be an independent form of harmonic continuity, as well as a portion of other symmetric forms of harmonic continuity.

Coefficients of recurrence in the groups of structures, when used in a continuity of C_0, acquire the following meaning: a structure with a coefficient greater than one changes its positions, until the next structure appears. The change of structure requires the preservation of the position of the chord.

This can be expressed as <u>a form of interdependence of structures and their positions in the C_0:</u>

S const. ———————— position var.

S var. ———————— position const.

For instance, in a case of $3S_1 + S_3 + 2S_2 = S_1 + S_1 + S_1 + S_3 + S_2 + S_2$, the constant and variable positions appear as follows:

$$\overset{\text{var.}}{S_1} + \overset{\text{var.}}{S_1} + \overset{\text{const.}}{S_1} + \overset{\text{const.}}{S_3} + \overset{\text{var.}}{S_2} + S_2$$

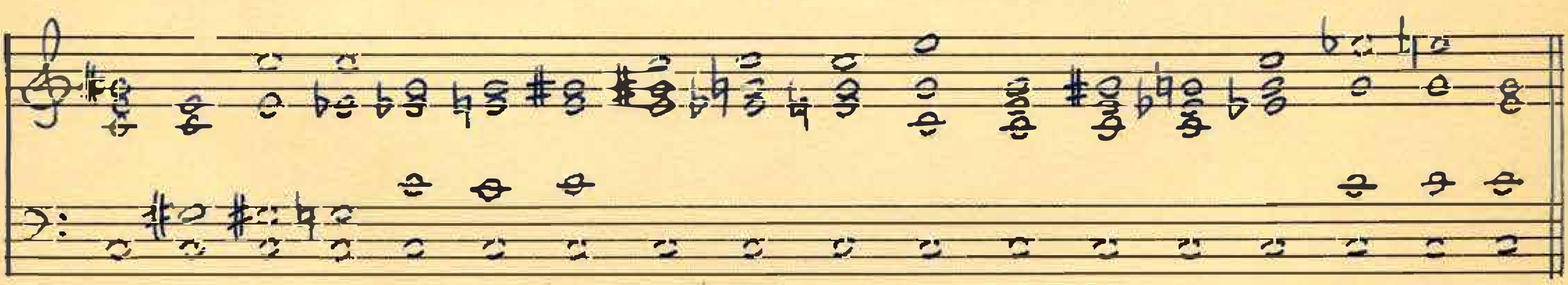
Examples of harmonic continuity in C_0.

Figure XXXII.

$2S_1 + S_2 + S_1 + 2S_2$ $\qquad$ $4S_2 + 2S_1 + 2S_2 + S_1$

$4S_1 + S_3 + 3S_2 + 2S_4 + 2S_3 + 3S_2 + S_4 + 4S_1$

$3S_3 + 2S_2 + S_1 + 2S_3 + S_2 + 3S_1 + S_3 + 3S_2 + 2S_1$



$S_4 + 3S_1 + 4S_3 + 7S_2$

Lesson LXXV.

Diatonic-Symmetric System of Harmony
(Type II).

Diatonic-Symmetric system of harmony must satisfy the following two requirements:

(1) all root-tones of the diatonic-symmetric system belong to one scale of the First Group;

(2) all chord structures must be pre-selected; they are not affected by the intonation of scale formed by the root-tones.

In this system of harmony structural groups must be superimposed upon the progressions of the root-tones belonging to one scale. This form of harmony has some advantages over the Diatonic System (to which I will refer as Type I). Like the diatonic system, the diatonic-symmetric system produces a united tonality, which is due to the structural unity of the scale. Unlike the diatonic system, the diatonic-symmetric system is not bound to use the structures which are considered defective in the Equal Temperament [like $S_4(5)$, for example], as the individual structures and the structural groups are a matter of free choice.

Unlike the diatonic system, the diatonic-

symmetric system has a greater variety of intonations, as the pre-selected structures unavoidably introduce new accidentals (alterations), which implies a modulatory character without destroying the unity of the tonality.

Examples of Harmony Type II.

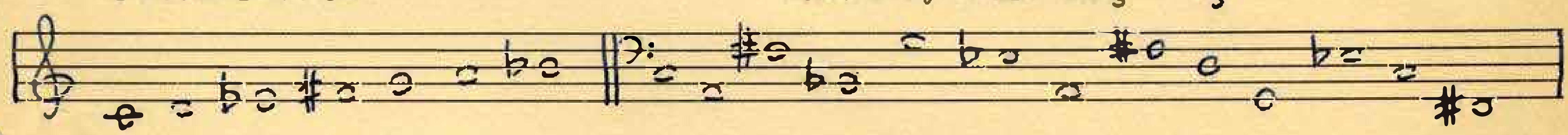
Figure XXXIII.

(please see following pages)

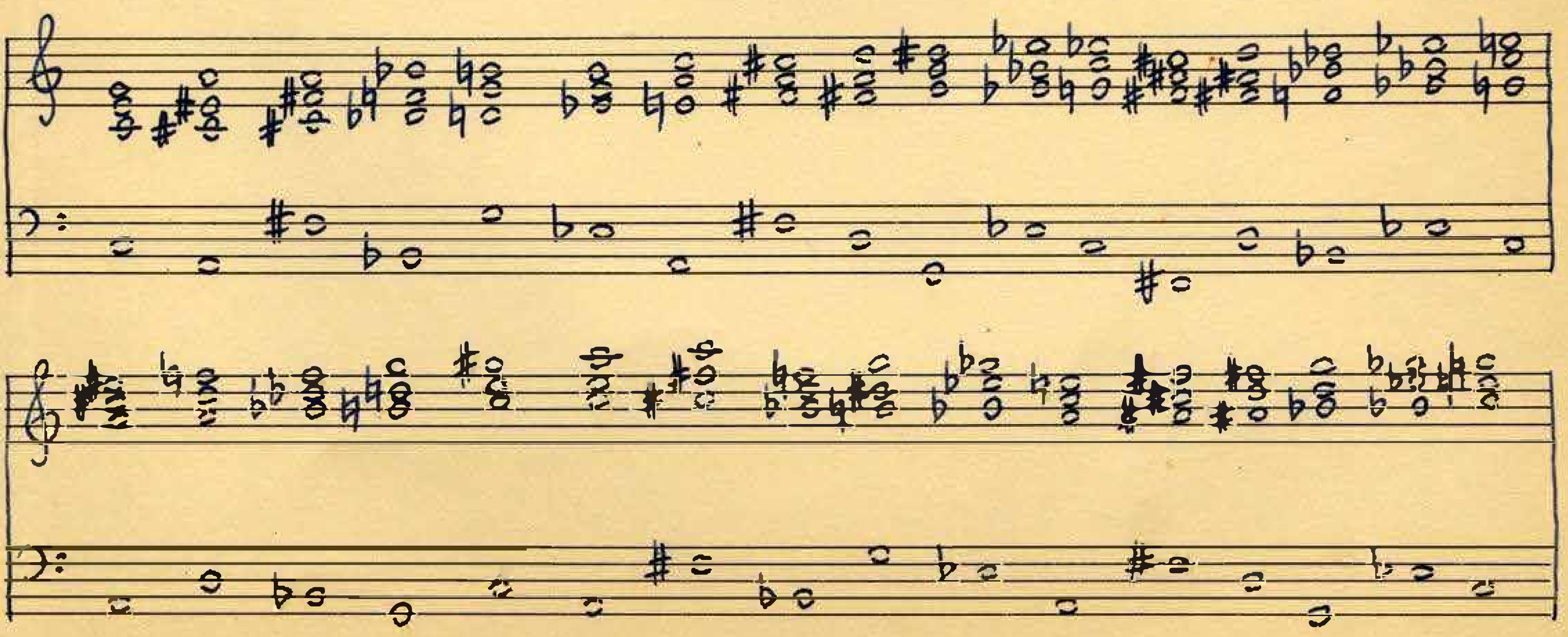
Pitch-scale: Tonal cycles: $2C_7 + C_3 + C_5$

Structural group: $S_1(5)$ const.

Pitch-scale: Tonal cycles: $2C_3 + C_5$



Structural group: $S_1 + S_3 + 2S_2$



J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross | Subject: Music

Lesson LXXVI.

Symmetric System of Harmony

(Type III)

Symmetric System of harmony must satisfy the following requirements:

(1) the root-tones and their progressions are the roots of two (i.e. $\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[4]{2}$, $\sqrt[6]{2}$, $\sqrt[12]{2}$), that is the points of symmetry of an octave.

(2) chord structures are pre-selected.

As a consequence of motion through symmetric roots, each voice of harmony produces one of the pitch-scales of the Third Group.

Symmetric C_0 represents one tonic;

$\sqrt{2}$ represents two tonics;

$\sqrt[3]{2}$ " three "

$\sqrt[4]{2}$ " four "

$\sqrt[6]{2}$ " six "

$\sqrt[12]{2}$ " twelve "

The correspondences of the tonal cycles and the symmetric roots are as follows:

One tonic: C ———————— C
C_0

Two tonics: C ——— F# ——— C
C_5 C_{-5}

Three tonics: C ——— A♭ ——— E ——— C
C_3 C_3 C_3

C ——— E ——— A♭ ——— C
C_{-3} C_{-3} C_{-3}

Four tonics: C ——— A ——— F# ——— E♭ ——— C
C_3 C_3 C_3 C_3

C ——— E♭ ——— F# ——— A ——— C
C_{-3} C_{-3} C_{-3} C_{-3}

Six tonics: C ——— D ——— E ——— F# ——— A♭ ——— B♭ ——— C
C_7 C_7 C_7 C_7 C_7 C_7

C ——— B♭ ——— A♭ ——— F# ——— E ——— D ——— C
C_{-7} C_{-7} C_{-7} C_{-7} C_{-7} C_{-7}

Twelve tonics: C ——— D♭ ——— D♮ ——— E♭ ——— E♮ ...
C_7 C_7 C_7 C_7

C ——— B ——— B♭ ——— A ——— A♭
C_{-7} C_{-7} C_{-7} C_{-7}

Transformations with regard to positions and voice-leading remain the same as in the diatonic system. In case of doubt cancel all the accidentals.

Two Tonics.

Two tonics break up an octave into two uniform intervals. The second tonic (T_2) being the $\sqrt{2}$ produces the center of an octave. This property makes the two-tonic system reversible. All points of intonation in the $\circlearrowright$ as well as in the $\circlearrowleft$ transformations are identical, i.e., both the clockwise and the

counterclockwise voice-leading produce the same pattern of motion. This is true only in the case of two tonics.

Two tonics form a continuous system, i.e., the recurring tonic does not appear in its original position. Two tonics produce a triple recurrence-cycle before the original position falls on the first tonic (T_1) for the and the .
Const. 3 produces a closed system.

Figure XXXIV.

The upper voice of harmony produces the following scale: c - $d^{\flat}$ - e - $f^{\#}$ - g - $a^{\#}$ - (c) =

= (1+3) + 2 + (1+3) + 2. All other voices of the above progression produce the same scale starting from its different phases.

It is easy to see that this scale belongs to the Third Group and is constructed on two tonics.

By selecting other structures and structural groups of S(5) one can get some other scales of the Third Group.

For example, the use of S_2 const. produces the following scale: $c - d^{\flat} - e^{\flat} - f^{\sharp} - g - a - (c) =$
$= (1+2) + 3 + (1+2) + 3$.

Structural groups may be used in two ways:

(1) S changes with each tonic;

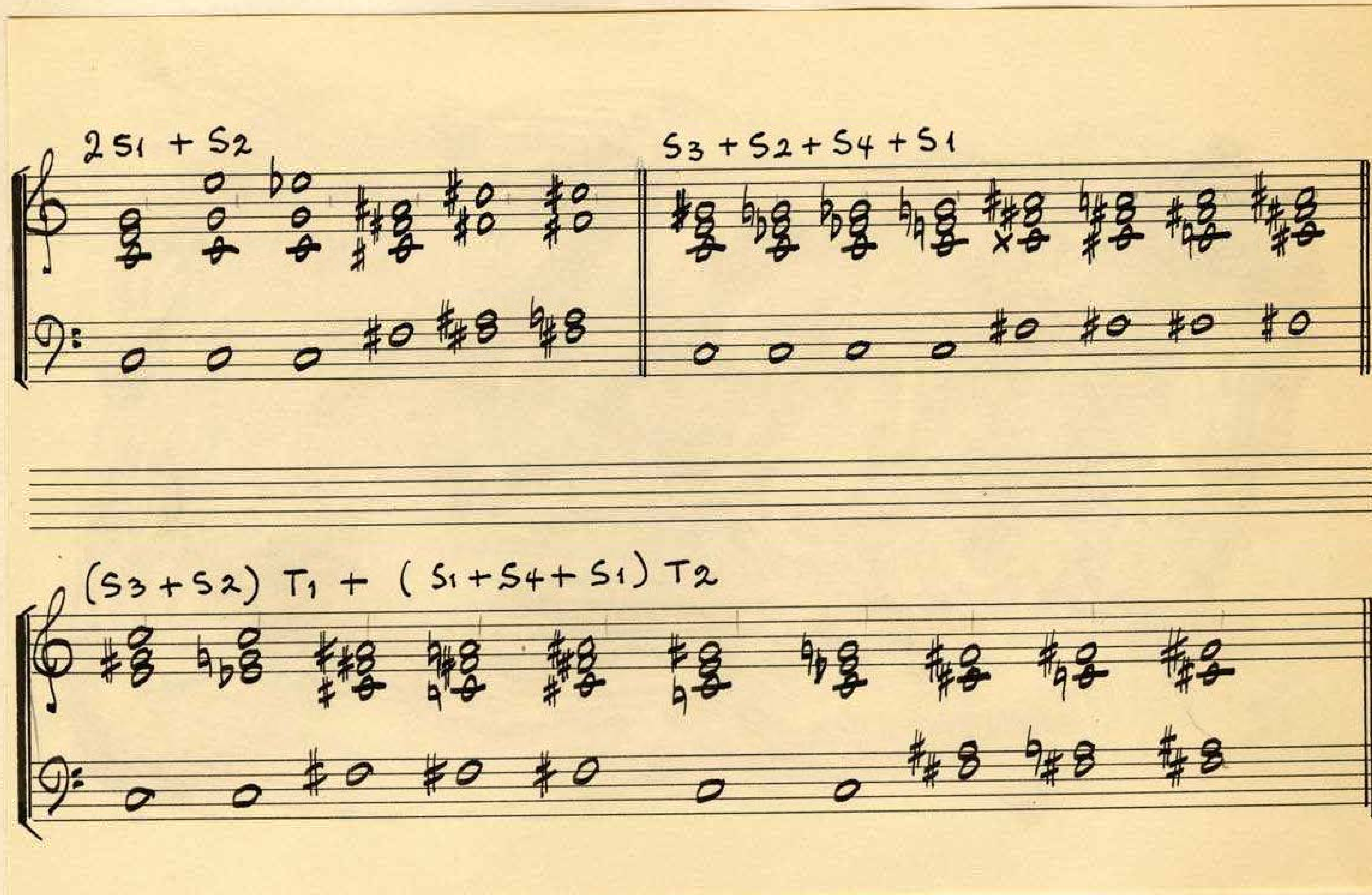
(2) the groups of S produce C_0 on each tonic.

<u>Illustrations of the first method</u>

<u>Figure XXXV.</u>

Illustrations of the second method

Figure XXXVI.



Combinations of the preceding two methods with regard to the structural selection for each tonic of one symmetric system are applicable to all symmetric systems.

Example

Figure XXXVII.

Longer progressions can be obtained through the use of longer structural groups, such as rhythmic resultants, power-groups, series of growth, etc.

In some cases the number of terms in the structural group produces interference against the number of tonics in the symmetric system.

Example

T_1, T_2; $2S_1 + S_2 + S_1 + S_2 + S_1 + S_2 + 2S_1$.

$(S_1T_1 + S_1T_2 + S_2T_1 + S_1T_2 + S_2T_1 + S_1T_2 + S_2T_1 +$

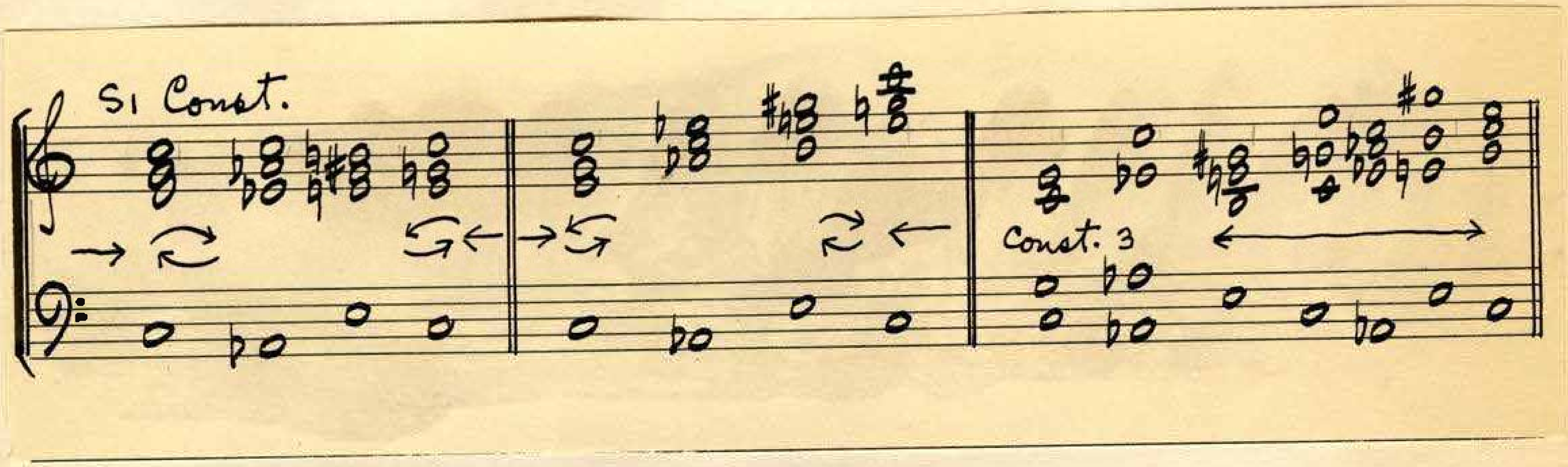
$+ S_1T_2 + S_1T_1) + (S_1T_2 + S_1T_1 + S_2T_2 + S_1T_1 + S_2T_2 +$

$+ S_1T_1 + S_2T_2 + S_1T_1 + S_1T_2)$.

Three Tonics.

Three tonics produce a closed system for and , and a continuous system (two recurrence-cycles) for const. 3.

Figure XXXVIII.



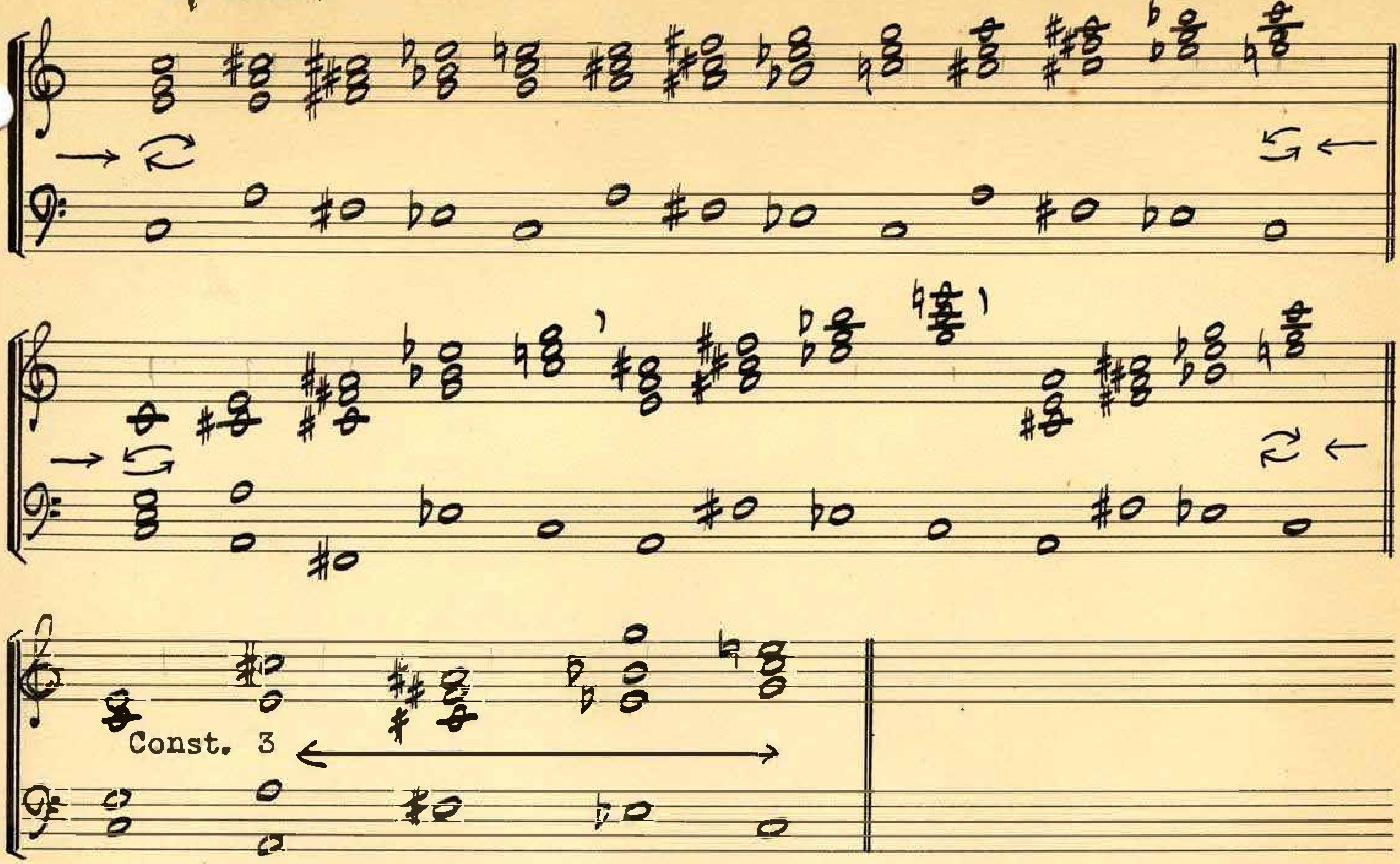
Four Tonics.

Four tonics produce a continuous system (three recurrence-cycles) for and , and a closed system for const. 3.

(please see next page)

Figure XXXIX.

S, const.

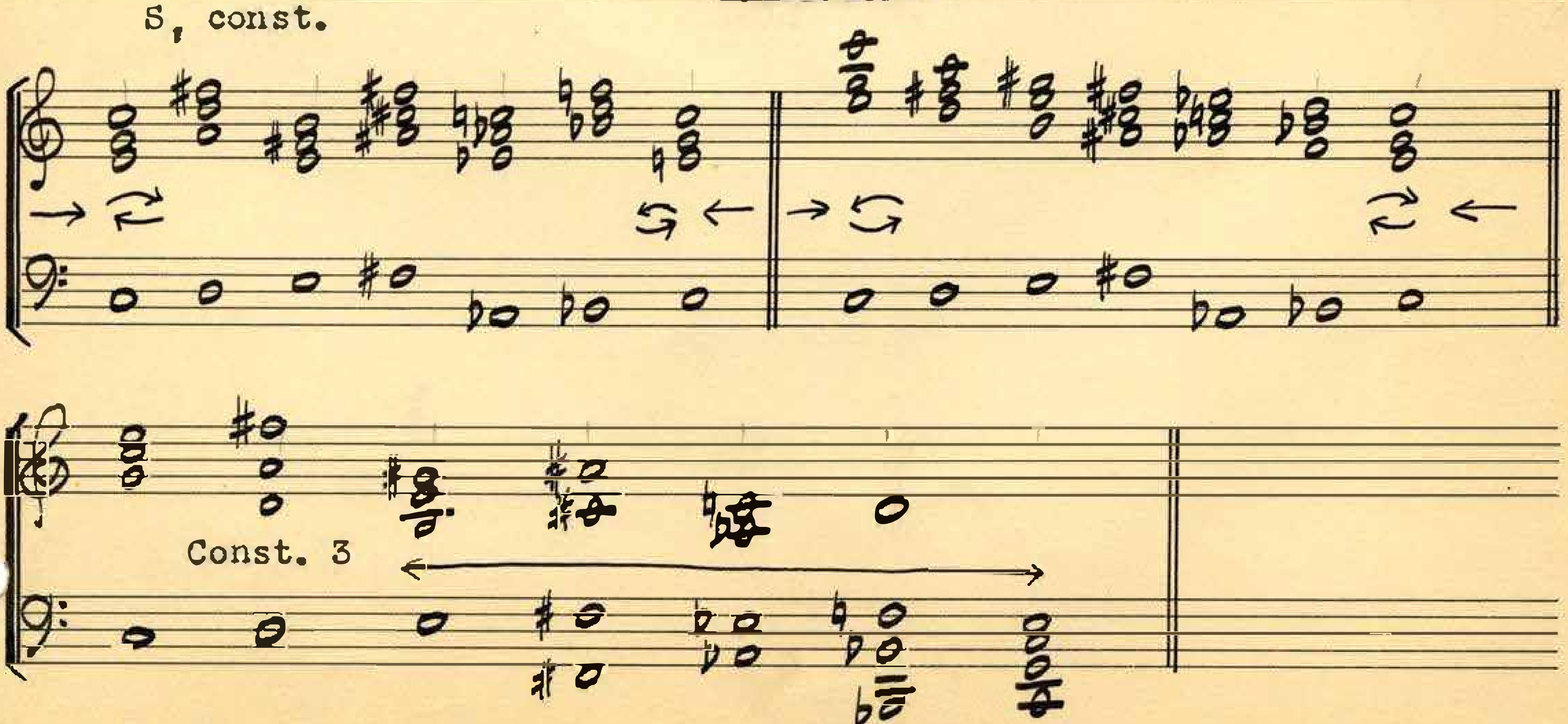


Six Tonics.

Six tonics produce a closed system for and , as well as for the const. 3.

Figure XL.

S, const.



Twelve Tonics.

Twelve tonics produce a closed system for and , as well as for the const. 3.

Figure XLI.

Lesson LXXVII.

Variable Doublings

Harmony, in many cases conceived as an accompaniment, may be given a self-sufficient character by means of variable doublings. This device attributes to chord progressions a greater versatility of sonority and voice-leading than the one usually observed.

Variable doublings comprise the three functions of S(5). Thus the root, the third or the fifth can be doubled. The corresponding notation to be used is: $S(5)^{1}$, $S(5)^{3}$ and $S(5)^{5}$.

As the root-tone remains in the bass, $S(5)^{1}$ is the only case of doubling where all three functions (1, 3, 5) appear in the upper three parts.

The following represents a comparative table of functions in the three upper parts under various forms of doubling.

$$S(5)^{1} = 1, 3, 5$$

$$S(5)^{3} = 3, 3, 5$$

$$S(5)^{5} = 3, 5, 5$$

Figure XLII.

In cases $S(5)^{3}$ and $S(5)^{5}$ only three positions are possible for each case. Black notes

represent variants where unison is substituted for an octave.

Positions

Figure XLIII.

Transformations

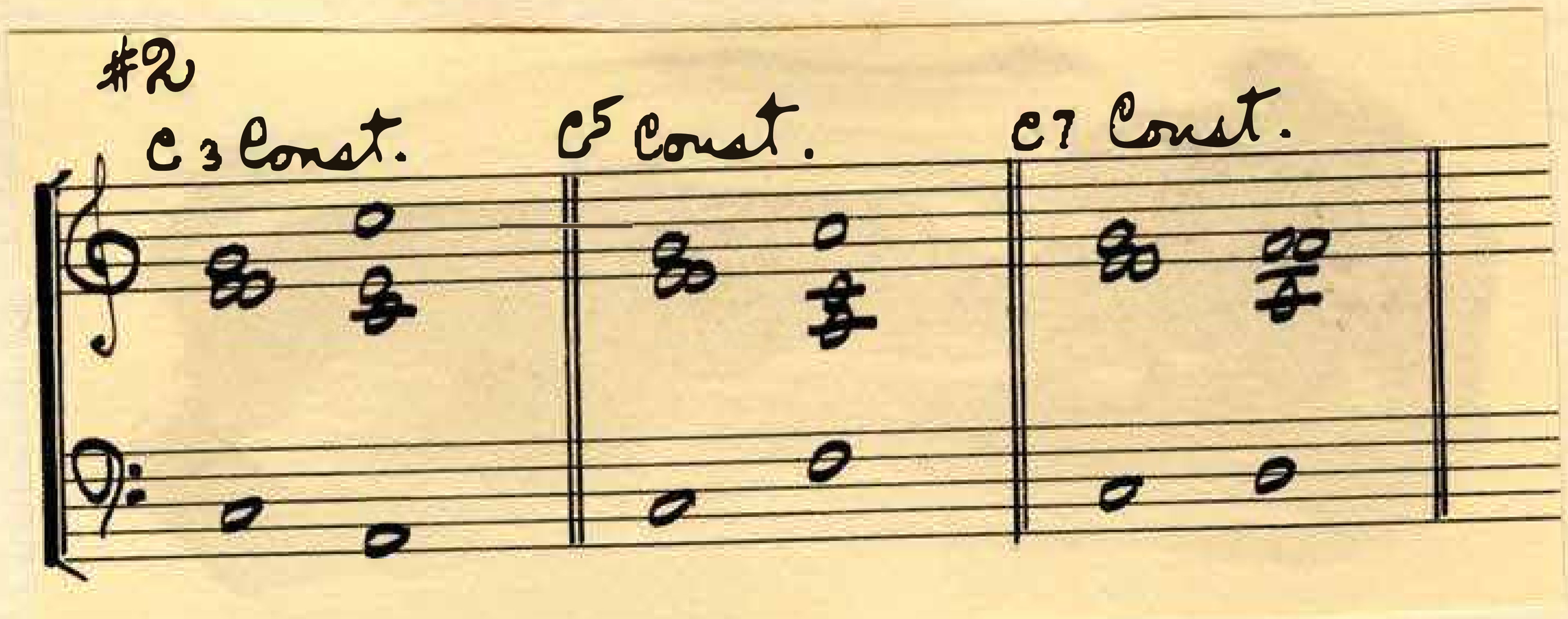
S(5) ①	⟷	S(5) ③
5⟷5	5⟷3	5⟷3
3⟷3	3⟷5	3⟷3
1⟷3	1⟷3	1⟷5

$S(5)^{1} \longleftrightarrow S(5)^{3}$

$5 \longleftrightarrow 3$

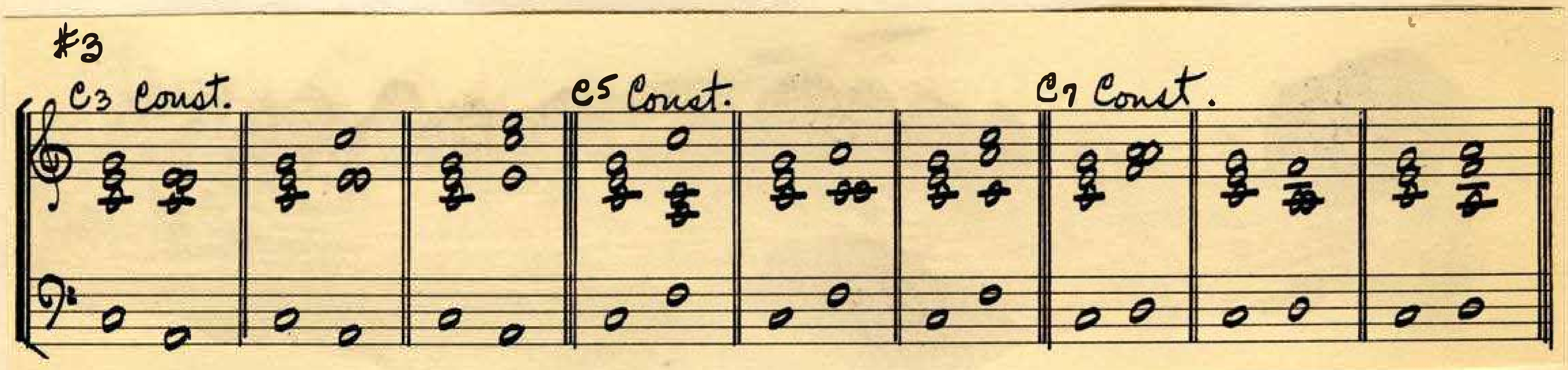
$3 \longleftrightarrow 5$

$3 \longleftrightarrow 3$



$S(5)^{1} \longleftrightarrow S(5)^{5}$

$5 \longleftrightarrow 5$	$5 \longleftrightarrow 3$	$5 \longleftrightarrow 5$
$3 \longleftrightarrow 5$	$3 \longleftrightarrow 5$	$3 \longleftrightarrow 3$
$1 \longleftrightarrow 3$	$1 \longleftrightarrow 5$	$1 \longleftrightarrow 5$



$S(5)^{5} \longleftrightarrow S(5)^{5}$

$5 \longleftrightarrow 3$

$5 \longleftrightarrow 5$

$3 \longleftrightarrow 5$

$S(5)^{3} \longleftrightarrow S(5)^{5}$

$5 \longleftrightarrow 5$

$3 \longleftrightarrow 5$

$3 \longleftrightarrow 3$

When reading these tables, consider identical directions of the arrows for the sequence of structures and for the corresponding transformations.

Notice that there always are three transformations when $S(5)^{1}$ participates and only one when it does not.

Musical tables in the above Figure are devised from the initial chord being in the same position. Similar tables can be constructed from all positions as well as in reverse sequence and also in the cycles of the negative form.

Variable doublings are subject to distributive arrangement and can be superimposed on any desirable cycle-group.

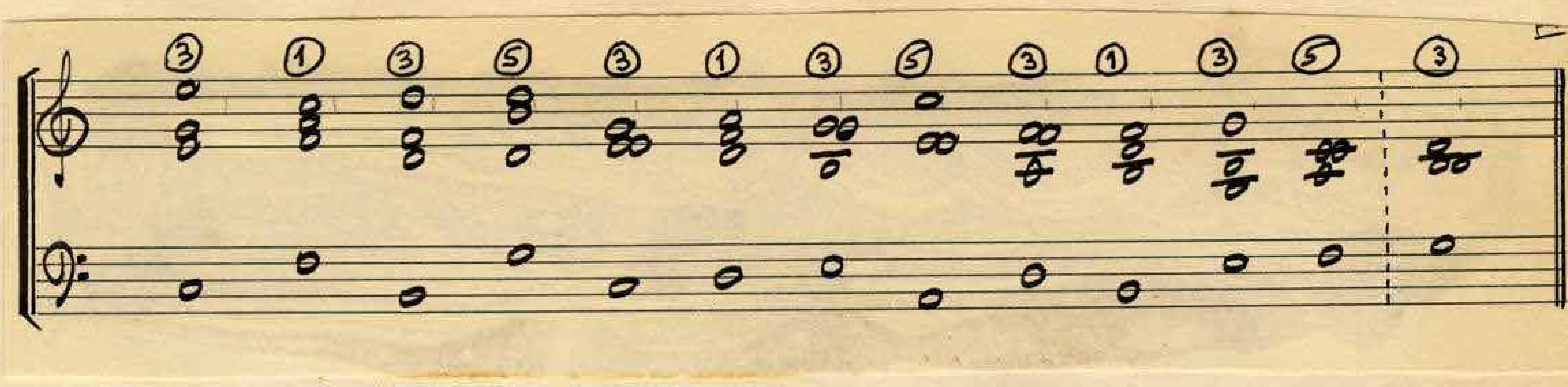
<u>Figure XLIV.</u>

Example: $2C_3 + C_5 + C_7;\quad S(5)^{1} + 2S(5)^{3} + S(5)^{5}.$

$$H^{\rightarrow} = S(5)^{1} + C_3 + S(5)^{3} + C_3 + S(5)^{3} + C_5 + S(5)^{5} + C_7 + S(5)^{1}.$$

Example: $2C_5 + C_3 + C_5 + 2C_7;\quad S(5)^{3} + S(5)^{1} + S(5)^{3} + S(5)^{5}.$

$$H^{\rightarrow} = S(5)^{3} + C_5 + S(5)^{1} + C_5 + S(5)^{3} + C_3 + S(5)^{5} + C_5 + S(5)^{3} + C_7 + S(5)^{1} + C_7 + S(5)^{3} + C_5 + S(5)^{5} + C_5 + S(5)^{3} + C_3 + S(5)^{1} + C_5 + S(5)^{3} + C_7 + S(5)^{5} + C_7 + S(5)^{3}.$$



Variable doublings are applicable to all types of harmonic progressions, thus including types II and III.

Figure XLV.

Type II (diatonic-symmetric).

$H^{\rightarrow}$ as in the preceding example.

$S^{\rightarrow} = 2S_2 + S_3 + S_1$

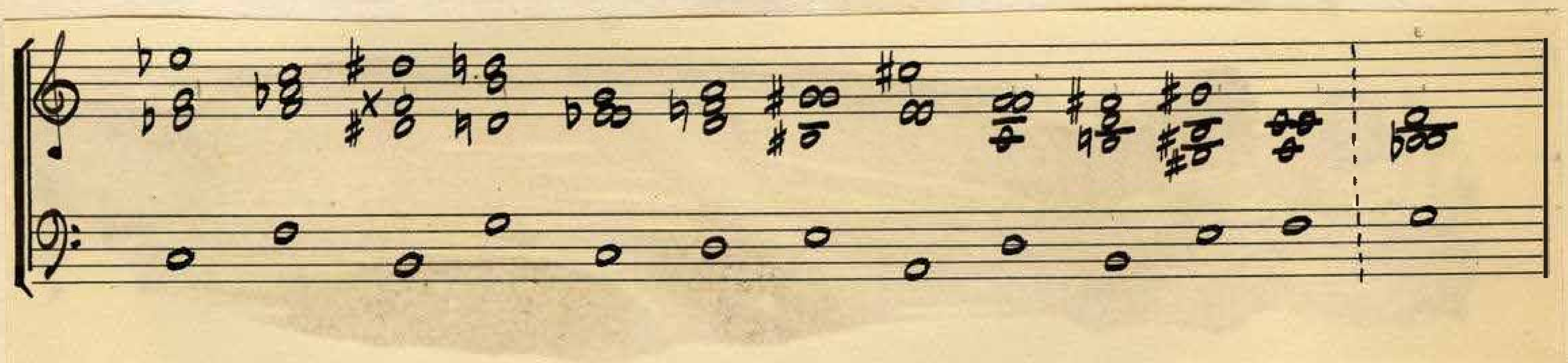
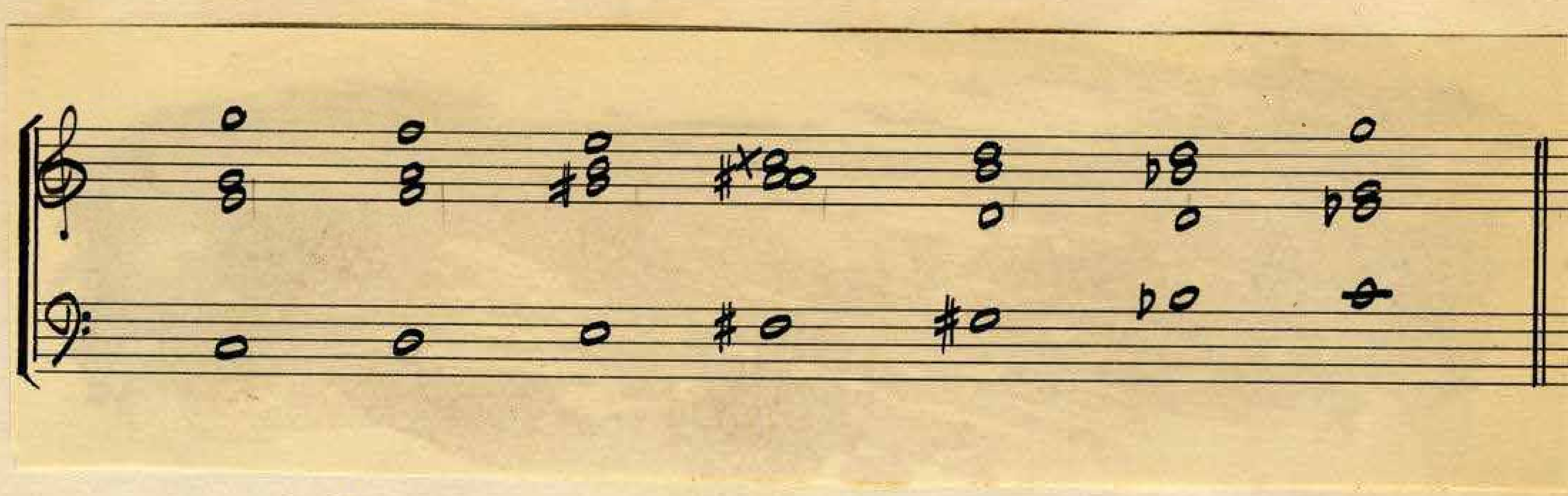


Figure XLVI.

Type III (symmetric).

$$H^{\rightarrow}(6T) = T_1 S_1^{(5)} + T_2 S_2^{(3)} + T_3 S_1^{(1)} + T_4 S_3^{(3)} + T_5 S_4^{(5)} + T_6 S_1^{(3)} + T_1 S_2^{(5)}.$$



Lesson LXXVIII.

Inversions of S(5)

The usual technique of inversions, strictly speaking, is unnecessary to a composer. The reason for this is, that by vertical permutations of the positions of parts in any harmonic continuity of S(5), the inversions appear automatically, as inner or upper parts become the bass parts under such conditions. This technique was fully described in my "Geometrical Projections of Music", in the branch dealing with the continuity of geometrical inversions.

For an analyst or a teacher, however, a thorough systematization of the classical technique of inversions is a necessity. There is no other branch of harmony I know of, where confusion is greater and the information less reliable.

The first inversion of S(5) is known as a "sixth-chord" or a "third-sixth-chord" and is expressed in this notation by the symbol S(6). The only condition under which S(5) becomes an S(6) is when the <u>third</u> (3) appears in the bass. The positions of the upper voices are not affected by such a change, the forms of doublings -- are. Which doublings are appropriate in each case, will be discussed later. Assuming that any S(6) may be either S(6)①, or S(6)③, or S(6)⑤, we obtain the following <u>Table of Positions:</u>

Figure XLVII.

It is easy to memorize the above table, as S(6)① and S(6)⑤ positions are systematized through the following characteristics: (1) the doubled function appears above the remaining function; (2) the doubled function surrounds the remaining function; (3) the doubled function appears below the remaining function.

S(6)③ is identical with S(5) positions, except for the bass having constant 3.

Harmonic progressions ($\overrightarrow{H}$) consisting of S(5) and S(6) are based on the following combinations by two:

(1) S(5) → S(5); (2) S(5) → S(6); (3) S(6) → S(5); (4) S(6) → S(6).

As the first case is covered by the previous technique, we are concerned, for the present, with the last three cases.

All the following transformations, being applied to voice-leading, are reversible, as in the case of Variable Doublings of S(5). Tonal cycles are always measured through root-tones.

Figure XLVIII.

S(5) ↔ S(6) 1

5 ↔ 5	5 ↔ 1	5 ↔ 1
3 ↔ 1	3 ↔ 5	3 ↔ 1
1 ↔ 1	1 ↔ 1	1 ↔ 5

S(5) ↔ S(6) 5

5 ↔ 1	5 ↔ 5	5 ↔ 5
3 ↔ 5	3 ↔ 1	3 ↔ 5
1 ↔ 5	1 ↔ 5	1 ↔ 1

S(5) ⟷	S(6) 3
5 ⟷ 3	5 ⟷ 1
3 ⟷ 1	3 ⟷ 3
1 ⟷ 5	1 ⟷ 5
⇆	Const. 3
⇄	Const. 3

S(6)① ⟷ S(6)①

5 ⟷ 1

1 ⟷ 5

1 ⟷ 1

S(6)① ⟷ S(6)⑤

5 ⟷ 1

1 ⟷ 5

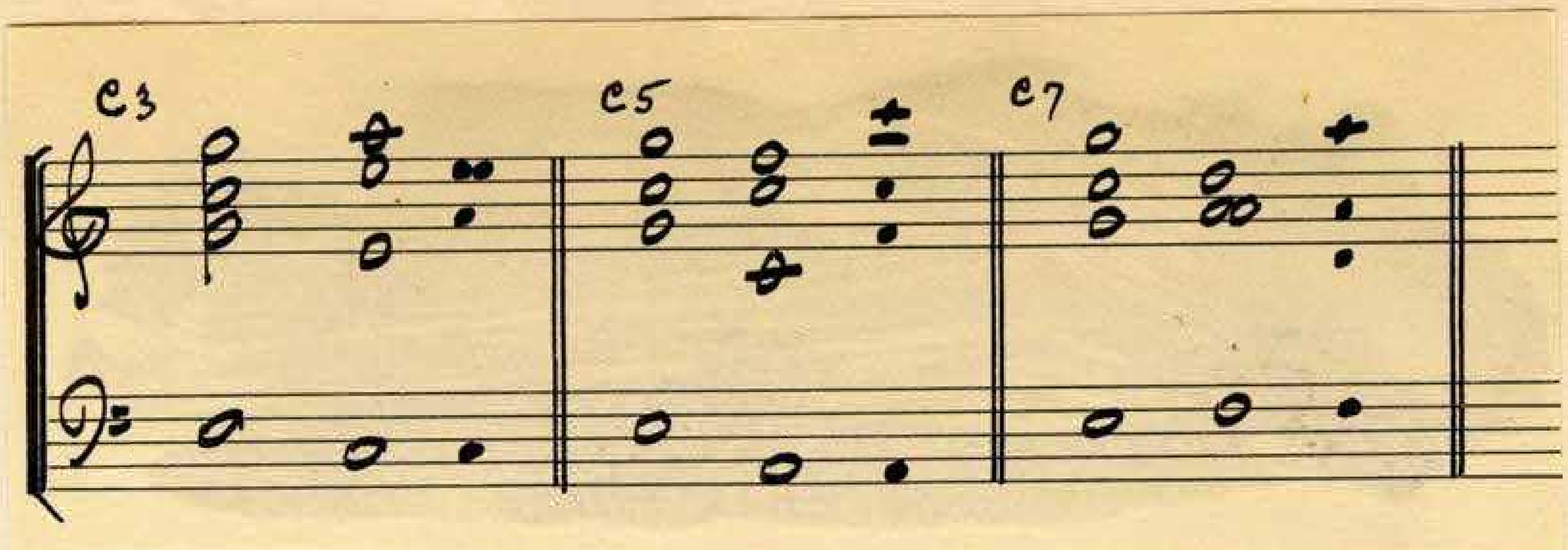
1 ⟷ 5

S(6)⑤ ⟷ S(6)⑤

5 ⟷ 1

5 ⟷ 5

1 ⟷ 5

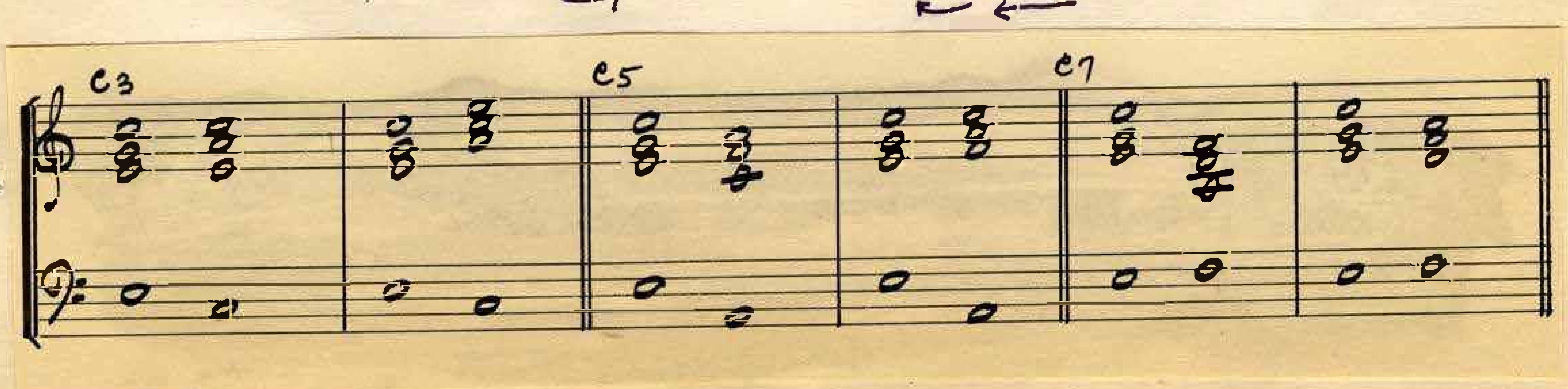


S(6) 1 ⟷ S(6) 3

5 ⟷ 5	5 ⟷ 1	5 ⟷ 3
1 ⟷ 3	1 ⟷ 5	1 ⟷ 1
1 ⟷ 1	1 ⟷ 3	1 ⟷ 5

S(6) 3 ⟷ S(6) 3

5 ⟷ 1	5 ⟷ 3
3 ⟷ 5	3 ⟷ 1
1 ⟷ 3	1 ⟷ 5



$S(6)^{5} \longleftrightarrow S(6)^{3}$

5 ⟷ 5	5 ⟷ 1	5 ⟷ 3
5 ⟷ 3	5 ⟷ 5	5 ⟷ 1
1 ⟷ 1	1 ⟷ 3	1 ⟷ 5

Any variants conformed to identical transformations (like the black notes in some of the preceding tables) are as acceptable as the ones in the tables.

Lesson LXXIX.

Doublings of S(6)

Musical habits are formed comparatively rapidly. Once they assume a form of natural reactions, they influence us more than the purely acoustical factors. This is particularly true in the case of doublings of S(6). The mere fact that identical doublings in the different musical contexts affect us in a different way, shows that our auditory reactions in music are not natural but conditioned.

The principles offered here are based on a comparative study of the respective forms of music.

There are two technical factors affecting the doubling in an S(6):

(1) the structure of the chord;

(2) the degree of the scale (on which the chord is constructed).

These two influences are ever-present regardless of the type to which the respective harmonic continuity belongs.

Yet, while in harmonic progressions of type II and III the structure of the chord is the most influential factor, in the diatonic progressions (type I) it is exactly the reverse. The influence of a constant pitch-scale is so overwhelming, that each

chord becomes associated with its definite position in the scale. Thus, one chord begins to sound to us as a dominant and another as a tonic, a mediant or a leading tone. This hierarchy of importance of the various chords calls for the different forms of doubling, particularly when the respective chords appear in the different inversions.

The following is most practical for use in diatonic progressions.

Figure XLIX.

Strong Factor			Weak Factor		
The degree of the scale	Regular Doubling	Irreg. Doubling	The structure of the chord	Regular Doubling	Irreg. Doubling
I, IV, V, VI	1, 5	3	$S_1(6)$	1, 5	3
II, III, VII	3	1, 5	$S_2(6)$	3	1, 5
			$S_3(6)$	3	—
			$S_4(6)$	1, 3, 5	—

Regular doublings are statistically predominant. Irregular doublings, in most cases are the result of melodic tendencies.

In reading the above table, give preference to the strong factor, except in the case of $S_3(6)$ and $S_4(6)$. It is customary to believe that an $S_1(6)$ must have doubled root or fifth. But in reality it seldom happens when such a chord belongs to II, III or VII.

Naturally, all our habits with regard to doublings are formed on more customary major and minor scales. The above table will work perfectly when applied to such scales. There will be no discrepancy when $S_3(6)$ and $S_4(6)$ will be compared with the data on the left side of the table, as such structures do not occur on the main degrees of the usual scales. When using less familiar scales, one or another type of doubling will not make as much difference. Yet in such cases the structure may become a more influential factor, though the sequence is diatonic.

In the types II and III the most practical forms of doublings are:

Figure L.

Structure	Regular Doubling	Irregular Doubling
$S_1(6)$	1, 5	3
$S_2(6)$	1, 5	3
$S_3(6)$	3	—
$S_4(6)$	1, 3, 5	—

Continuity of S(5) and S(6).

The comparative characteristic of S(5) is its stability, due to the presence of the root-tone in the bass. The absence of the root-tone in the bass of S(6) deprives this structure of such stability.

Composition of continuity consisting of S(5) and S(6) results in an interplay of stable and unstable units or groups. The following fundamental forms of continuity with utilization of the above-mentioned structures are possible:

(1) S(5) const. ———— stable

(2) S(6) const. ———— unstable

(3) [S(5) + S(6)] + . . . alternate

(4) 2S(5) + S(6) + S(5) + 2S(6)

3S(5) + S(6) + 2S(5) + 2S(6) + S(5) + 3S(6)

4S(5) + S(6) + 3S(5) + 2S(6) + 2S(5) + 3S(6) +

+ S(5) + 4S(6)

$\longrightarrow$ increasing instability

increasing stability $\longleftarrow$

(5) 4S(5) + 2S(6) + 2S(5) + S(6)

$\longrightarrow$ proportionately decreasing ratios

proportionately increasing ratios $\longleftarrow$

(6) S(5) + 2S(6) + 3S(5) + 5S(6) + 8S(5) + 13S(6)

$\longrightarrow$ progressive over-balancing of unstable elements

S(6) + 2S(5) + 3S(6) + 5S(5) + 8S(6) + 13S(5)

$\longrightarrow$ progressive over-balancing of stable elements

Many other forms of distribution of S(5) and S(6) may be devised on the basis of the "Theory of Rhythm".

Figure LI.

~~Diatonic~~

S(6) Const.; $2C_7 + 2C_5 + C_3 + C_5$

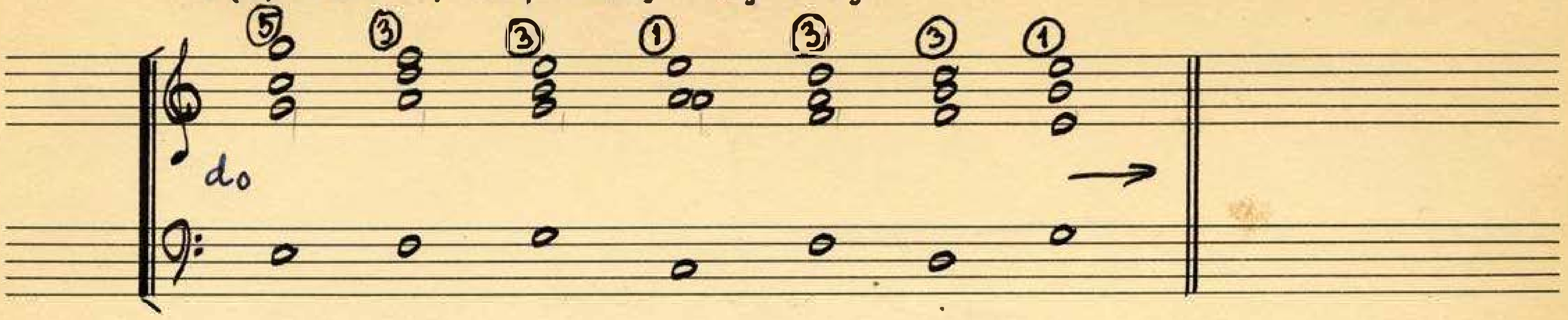


Figure LII.

Diatonic-Symmetric

$2S_2(6) + S_1(6) + S_3(6) + S_1(6) + S_4(6)$; $2C_5 + C_7 + C_5 + 2C_7$

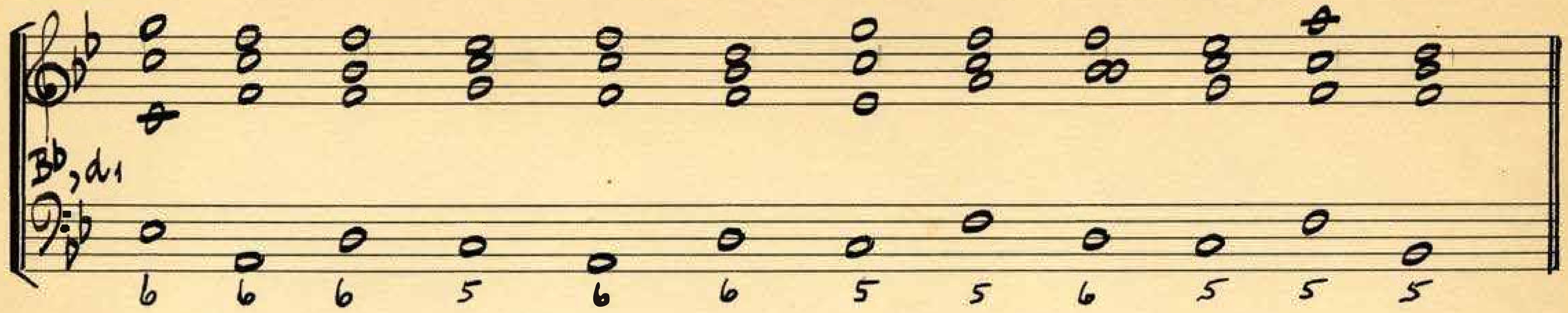
Symmetric

$S_3(6) + S_2(6) + S_4(6) + 2S_1(6)$; Six tonics

Figure LIII.

Diatonic

$3S(6) + S(5) + 2S(6) + 2S(5) + S(6) + 3S(5); \quad 2C_5 + C_7.$



Diatonic-Symmetric

$2S_2(6) + S_1(5) + S_1(6) + 2S_2(5); \quad 2C_7 + C_5$; Scale of roots: Aeolian

Symmetric

$\{[S_1(5) + S_2(6)]\ T_1 + [S_4(6) + S_1(5)]\ T_2\} + \ldots$ Four Tonics.

JOSEPH SCHILLINGER

CORRESPONDENCE COURSE

With: Dr. Jerome Gross   Subject: Music

Lesson LXXX.

Groups with Passing Chords

A. Passing Sixth-chords

A group with a passing S(6) is a pre-set combination of three chords, namely: S(5) + S(6) + + S(5). Every passing chord occupies the center of its group, appears on a weak beat and has a doubled bass. The complete expression for a group (G) with passing sixth-chord is:

$$G_6 = S(5) + S(6)^{2} + S(5).$$

This formula is not reversible in actual intonation. The relationship between the extreme chords of G_6 is C-5. This relationship remains constant in all cases of classical music. We shall extend this principle to all cycles. Under such conditions G_6 retains the following characteristics:

(1) The transformation between the extreme chords of the group is always clockwise for both the positive and the negative cycles

(2) The bass progression is: $1 \longrightarrow 3 \longrightarrow 1$, which necessitates the first condition.

In the classical form of G_6, bass moves by the thirds. Thus, 3 in the bass under S(6) is a third above its preceding position under the first S(5), and a third below its following position under the last S(5).

In order to obtain G_6, it is necessary to connect S(5) with the next S(5) through C-5 and add the intermediate third of the first chord in the bass, without moving the remaining voices.

Figure LIV.

$$G = S(5) + S(6)^3 + S(5)$$

C-5

There are three melodic forms for the bass movement.

Figure LV.

Combinations of these three forms in sequence produce a very flexible bass part and, being repeated with one G_6, make expressive cadences of Mozartian flavor.

Figure LVI.

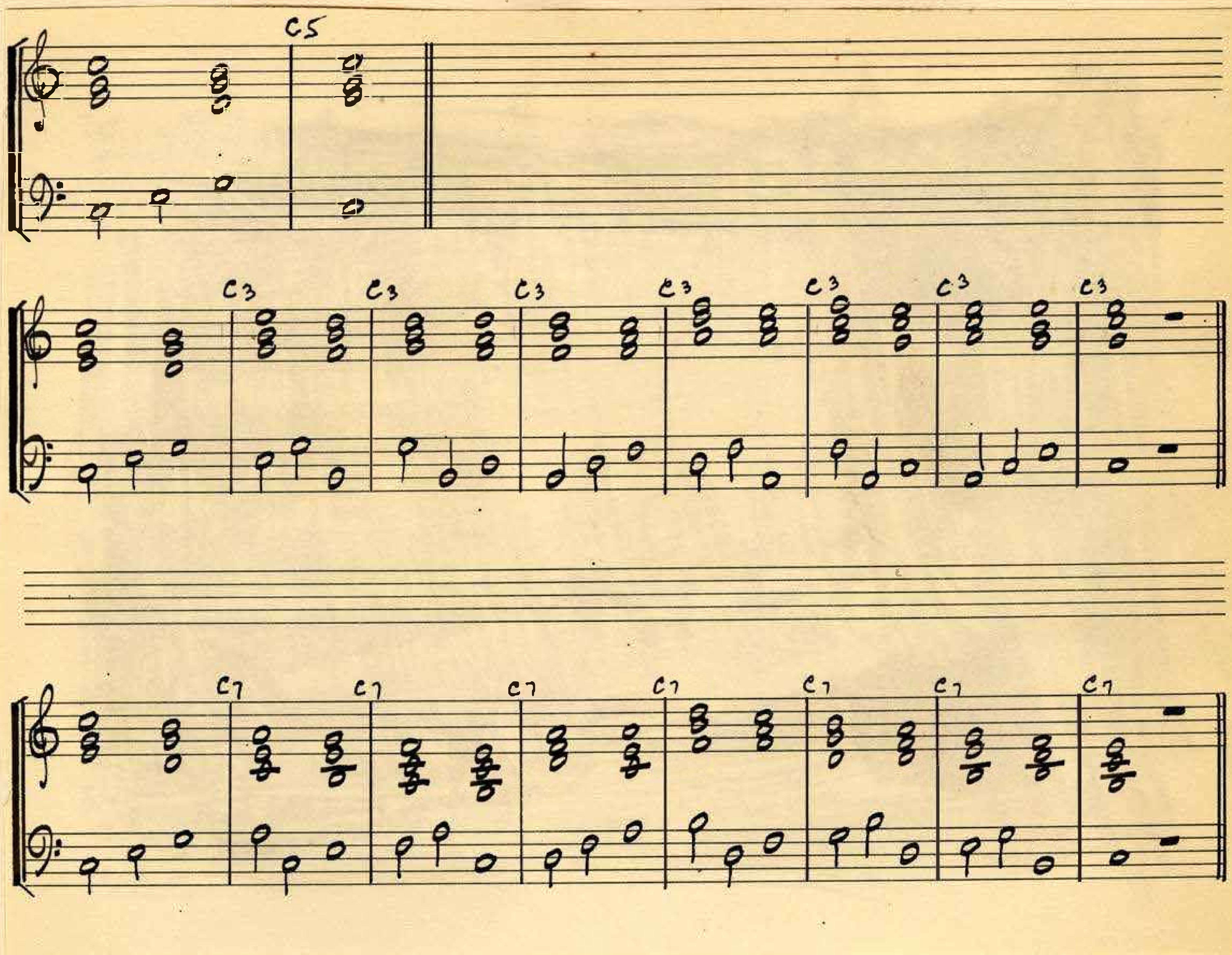
Continuity of G_6.

Continuity of such groups can be obtained by connecting them through the tonal cycles.

Connecting by C_5 closes the sequence, while C_3 and C_7 produce a progression of $7G_6$.

Figure LVII.

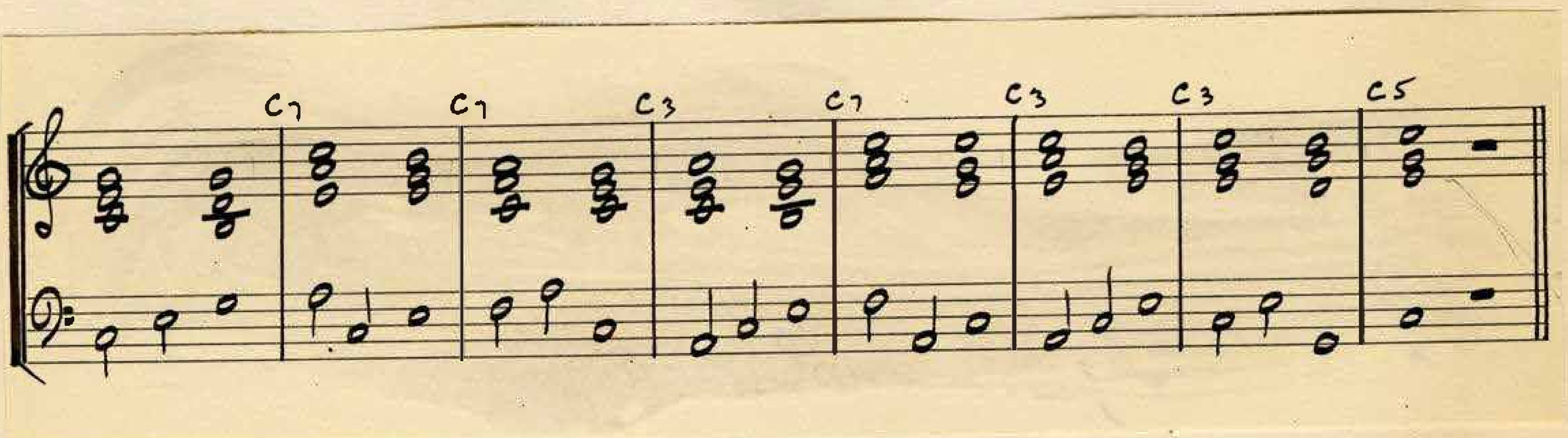
(please see next page)



Further versatility of G_6 progressions can be achieved by varying the cycles between the groups. Any time a decisive cadence is desirable C_5 must be introduced, as this cycle closes the progression.

Figure LVIII.

$$H^{\rightarrow} = G_6 + C_7 + G_6 + C_7 + G_6 + C_3 + G_6 + C_7 + G_6 +$$
$$+ \ C_3 + G_6 + C_3 + G_6 + C_5$$



Generalization of G_6

In addition to the classical form of G_6, other forms can be developed through the use of other than C-5 cycles within the group. Of course, each cycle produces its own characteristic bass pattern.

Figure LIX.

Various forms of G_6:

The respective variations of the bass pattern will be as follows:

Figure LX.

Continuity of the generalized G_6

Such a continuity can be developed through the selective progressions of the various forms of G_6 combined with the various cycle connections between the groups.

Example:

Figure LXI.

$H^{\rightarrow} = G_6(C-5) + C_7 + G_6\ (C_3) + C_5 + G(C-7) + C_3 +$

$+ G(C-5) + C_5 + G(C_7) + C_5$

Generalization of the Passing Third

It follows from the technique of groups with a passing sixth-chord, that the first two chords, i.e., S(5) and S(6)3, belong to C_0, and that as the position of the three upper parts does not change until the last chord of the group appears. This last chord, S(5), can be in any relation but C_0 with the preceding chord. If we think about the appearance of the third in the bass during S(6)3 merely as a passing third, it is easy to see that this entire technique can be generalized. The passing 3 can be used after any S(5), providing the transformation between the latter and

the following S(5) is clockwise for all the cycles. Such a device can be applied to any progressions of S(5) with the root-tones in the bass.

Figure LXII.

Example:

The effect of such harmonic continuity is one of overlapping groups of G_6, as marked in the above Figure.

Lesson LXXXI.

Applications of G_6 to Diatonic-Symmetric (Type II) and Symmetric (Type III) Progressions.

The use of structures of S(5) and S(6) 3 in the groups with a passing sixth-chord must satisfy the following requirement: the adjacent S(5) and S(6) 2 of one group must have identical structures.

This requirement does not affect the form of the last S(5) of a group; neither does it influence the selection of the forms of S(5) in the adjacent groups.

As each G_6 consists of three places, two of which are identical, the number of structural combinations for the individual groups equals $4^2 = 16$.

$S_1 + S_1$	$S_2 + S_1$	$S_3 + S_1$	$S_4 + S_1$
$S_1 + S_2$	$S_2 + S_2$	$S_3 + S_2$	$S_4 + S_2$
$S_1 + S_3$	$S_2 + S_3$	$S_3 + S_3$	$S_4 + S_3$
$S_1 + S_4$	$S_2 + S_4$	$S_3 + S_4$	$S_4 + S_4$

Thus we obtain 16 forms of G_6 with the following distribution of structural combinations.

$$G_6 = S_1(5) + S_1(6)^{3} + S_1(5)$$

$$G_6 = S_1(5) + S_1(6)^{3} + S_2(5)$$

$$G_6 = S_1(5) + S_1(6)^{3} + S_3(5)$$

$$G_6 = S_1(5) + S_1(6)^{3} + S_4(5)$$

$$G_6 = S_2(5) + S_2(6)^{3} + S_1(5)$$

$$G_6 = S_2(5) + S_2(6)^{3} + S_2(5)$$

$$G_6 = S_2(5) + S_2(6)^{3} + S_3(5)$$

$$G_6 = S_2(5) + S_2(6)^{3} + S_4(5)$$

$$G_6 = S_3(5) + S_3(6)^{3} + S_1(5)$$

$$G_6 = S_3(5) + S_3(6)^{3} + S_2(5)$$

$$G_6 = S_3(5) + S_3(6)^{3} + S_3(5)$$

$$G_6 = S_3(5) + S_3(6)^{3} + S_4(5)$$

$$G_6 = S_4(5) + S_4(6)^{3} + S_1(5)$$

$$G_6 = S_4(5) + S_4(6)^{3} + S_2(5)$$

$$G_6 = S_4(5) + S_4(6)^{3} + S_3(5)$$

$$G_6 = S_4(5) + S_4(6)^{3} + S_4(5)$$

As the melodic interval in the bass, while moving from the root (1) in S(5) to the third (3) in

S(6)③ is identical for the forms S_1 and S_3, as well as S_2 and S_4, the total quantity of intonations in the bass part for one type of G_6 is $\frac{4}{2} = 2$.

$$S_1 + S_1$$

$$S_1 + S_2$$

$$S_2 + S_1$$

$$S_2 + S_2$$

As each intonation has 3 melodic forms and there are two different intonations, the total number of intonations combined with melodic forms in the bass part is 2 x 3 = 6.

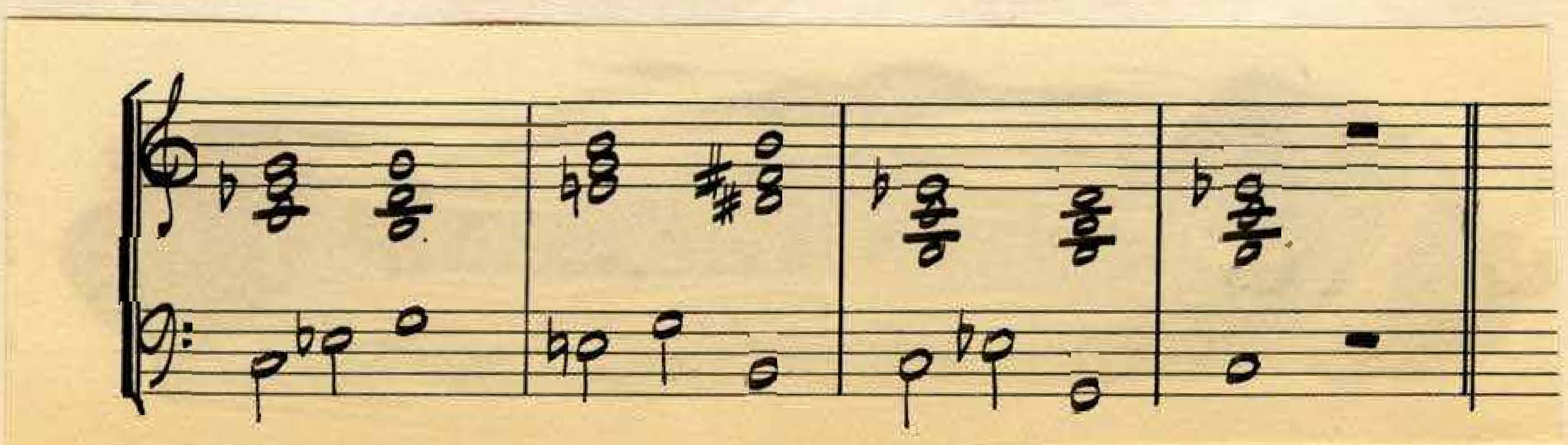
Progressions of the type II.

Figure LXIII.

Example:

Forms of S: $S_2(5) + S_2(6)$③ $+ S_1(5)$

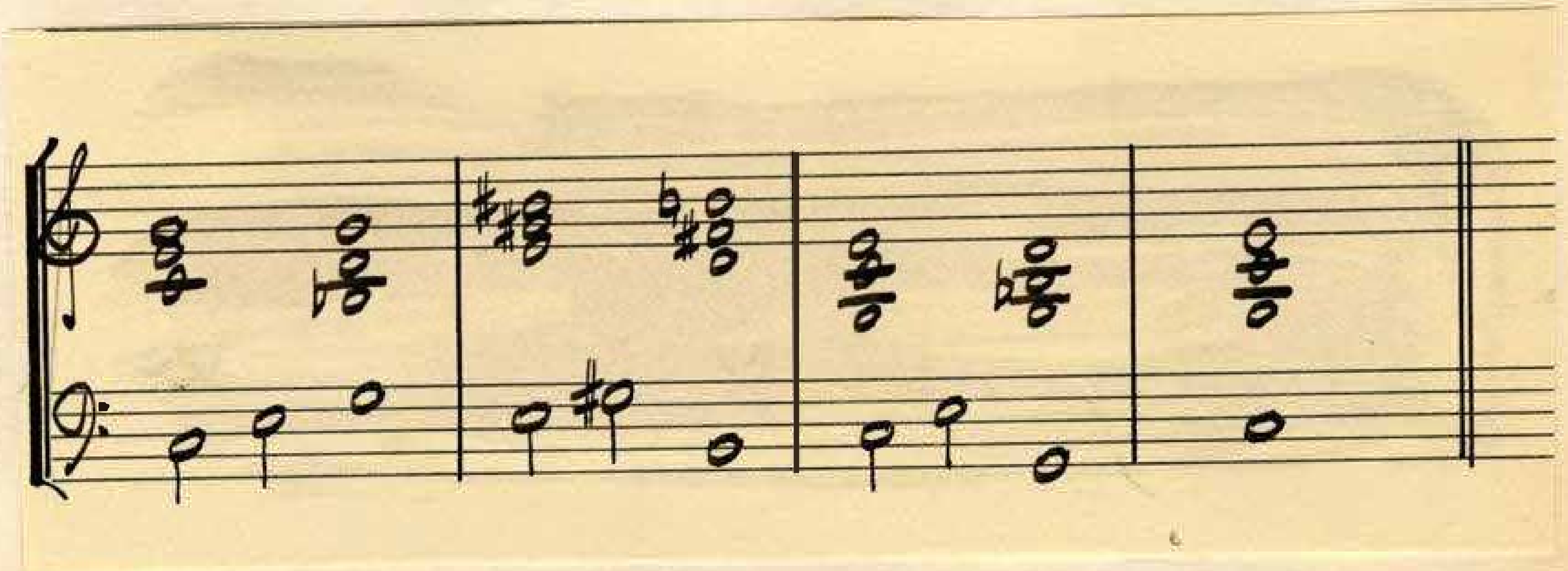
$H^{\rightarrow} = G_6(C-5) + C_3 + G_6(C-5) + C_7 + G_6(C-5) + C_5$.



Example:

Forms of S: $[S_1(5) + S_1(6)③ + S_2(5)] + [S_3(5) +$
$+ S_3(6)③ + S_2(5)]$

$H^{\rightarrow}$ = as in the preceding example.



Example:

Forms of S: $S_2(5) + S_2(6)③ + S_2(5)$

$H^{\rightarrow}$ = as in Figure LXI.

Generalization of the passing third is applicable to this type of harmonic progressions as well. The following is an application of the structural group $2S_1 + S_2 + 2S_1 + S_2 + 2S_1$ to the Figure LXII.

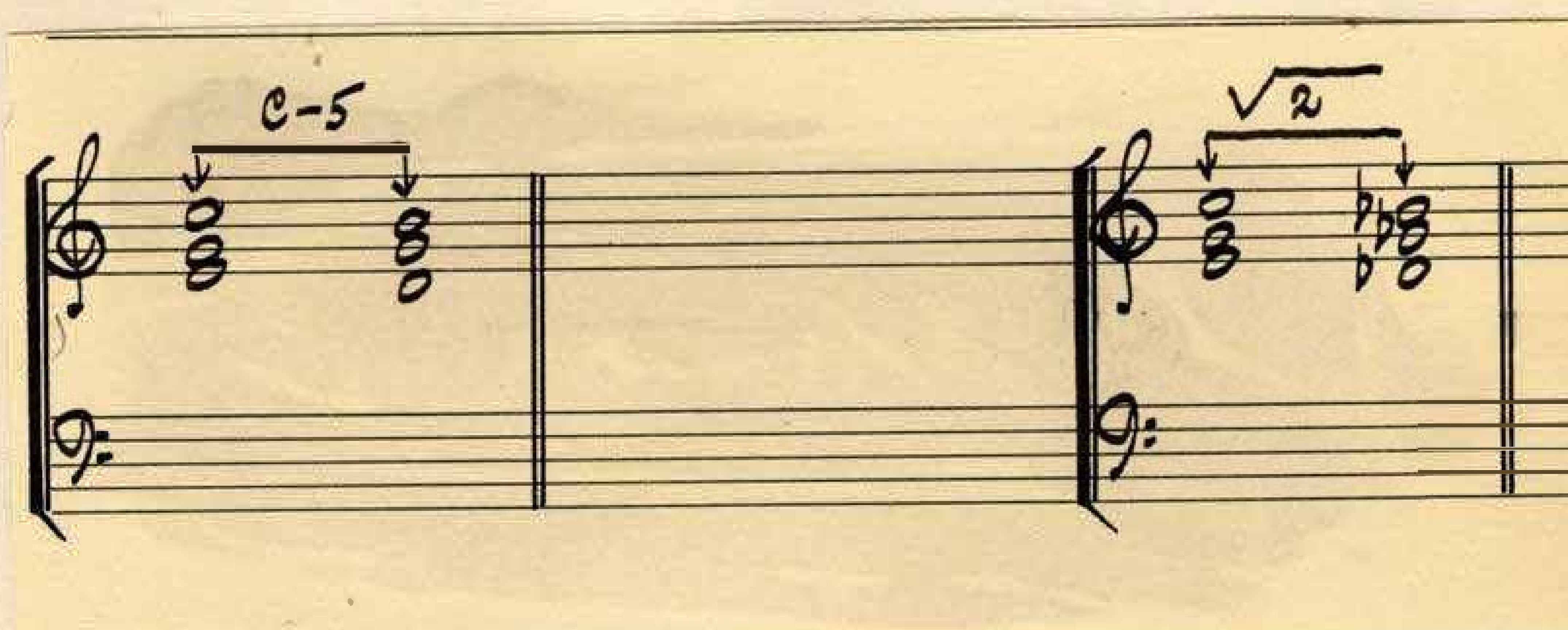
Figure LXIV.

Lesson LXXXII.

Progressions of the type III.

Applications of G_6 to symmetrical systems of tonics disclose many unexplored possibilities, among which the two-tonic system deserves a particular attention. As intervals forming the two tonics are equidistant, the passing tones of S(6)③, which in turn may also be equidistant from T_1 and T_2, thus produce, in the bass movement, diminished seventh-chords in symmetric harmonization, a device heretofore unknown.

The justification for the use of G_6 in the symmetrical systems of tonics is based on the following deduction from the original classical form, i.e., G_6(C-5).



(Diatonic) (Symmetric)

The abovementioned equidistancy of the two tonics permits to obtain $H^{\rightarrow} = 3G_6$ until the cycle

closes. Selecting S_1 for the entire G_6, we obtain:

Figure LXV.

The overlapping of groups, indicated by the brackets in the above Figure, is an invariant of the symmetrical systems. Thus, the passing third can be considered a general device for progressions of the type III.

The number of bass patterns for the cycle of the two tonics equals: $2^2 = 4$.

The number of intonations in each cycle of the two tonics equals: $2^2 = 4$. The latter is due to the use of the different forms of S(5). The interval between 1 and 3 equals 4 and is identical for $S_1(5)$ and $S_3(5)$. The interval between 1 and 3 equals 3 and is identical for $S_2(5)$ and $S_4(5)$. Thus, by

distributing the different structures through two tonics, we obtain the following combinations:

$S_1(T_1) + S_1(T_2)$

$S_1(T_1) + S_3(T_2)$ — identical intonations in the bass part

$S_3(T_1) + S_1(T_2)$

$S_3(T_1) + S_3(T_2)$

$S_2(T_1) + S_2(T_2)$

$S_2(T_1) + S_4(T_2)$ — identical intonations in the bass part

$S_4(T_1) + S_2(T_2)$

$S_4(T_1) + S_4(T_2)$

$S_1(T_1) + S_2(T_2)$

$S_1(T_1) + S_4(T_2)$ — identical intonations in the bass part

$S_3(T_1) + S_2(T_2)$

$S_3(T_1) + S_4(T_2)$

$S_2(T_1) + S_1(T_2)$

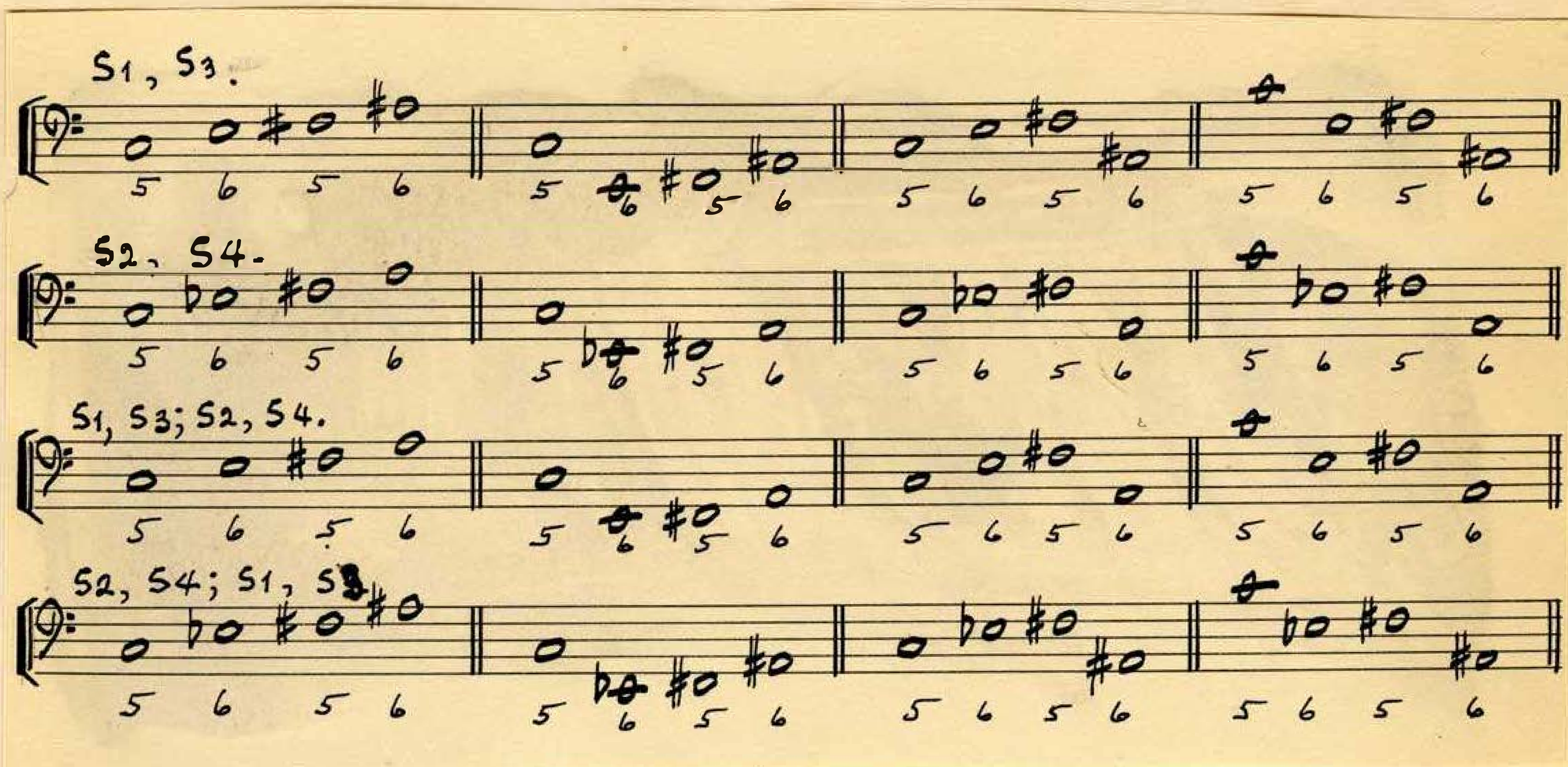
$S_2(T_1) + S_3(T_2)$

$S_4(T_1) + S_1(T_1)$ — identical intonations in the bass part

$S_4(T_1) + S_3(T_2)$

The following is a table of intonations and melodic forms in the bass part on two tonics. Total: $4^2 = 16$.

Figure LXVI.



The above combinations can be incorporated into a versatile continuity of G_6 on two tonics.

Figure LXVII.

Example:

Application of G_6 to three tonics produces 8 melodic forms in the bass part: $2^3 = 8$.

Figure LXVIII.

$T_1a_2 + T_2a_2 + T_3a_2$

$T_1b_2 + T_2a_2 + T_3a_2$

$T_1a_2 + T_2b_2 + T_3a_2$

$T_1a_2 + T_2a_2 + T_3b_2$

$T_1b_2 + T_2b_2 + T_3a_2$

$T_1b_2 + T_2a_2 + T_3b_2$

$T_1a_2 + T_2b_2 + T_3b_2$

$T_1b_2 + T_2b_2 + T_3b_2$

Figure LXVIII (cont.)

The number of distributions of the different S through three tonics is $4^3 = 64$, while the number of non-identical intonations is $2^3 = 8$.

Non-identical intonations:

$S_1(T_1) + S_1(T_2) + S_1(T_3)$

$S_1(T_1) + S_1(T_2) + S_2(T_3)$

$S_1(T_1) + S_2(T_2) + S_1(T_3)$

$S_2(T_1) + S_1(T_2) + S_1(T_3)$

$$S_2(T_1) + S_2(T_2) + S_1(T_3)$$

$$S_2(T_1) + S_1(T_2) + S_2(T_3)$$

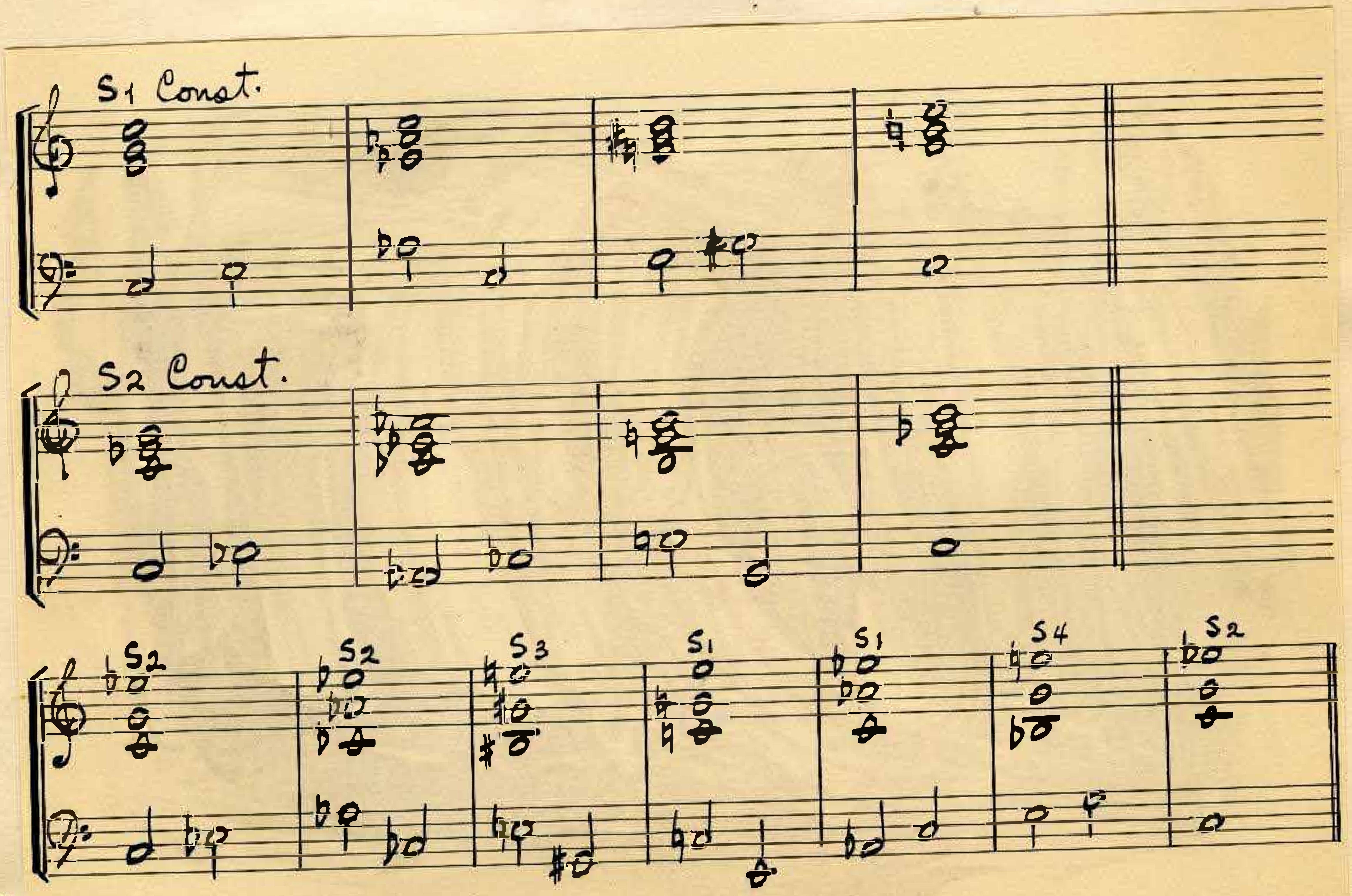
$$S_1(T_1) + S_2(T_2) + S_2(T_3)$$

$$S_2(T_1) + S_2(T_2) + S_2(T_3)$$

The total number of different intonations and melodic forms in the bass part is $8^2 = 64$.

Examples of continuity of G_6 on three tonics

Figure LXIX.



Application of G_6 to <u>four tonics</u> produces $2^4 = 16$ melodic forms in the bass part.

The number of distributions of the four forms of S through four tonics produces $4^4 = 256$ intonations.

The number of intonations in the bass part is limited to $2^4 = 16$.

Thus the total number of intonations and melodic forms in the bass part is $16^2 = 256$.

<u>Examples of continuity of G_6 on four tonics.</u>

<u>Figure LXX.</u>

(please see next page)

Figure LXX.

Application of G_6 to six tonics produces $2^6 = 64$ melodic forms in the bass part.

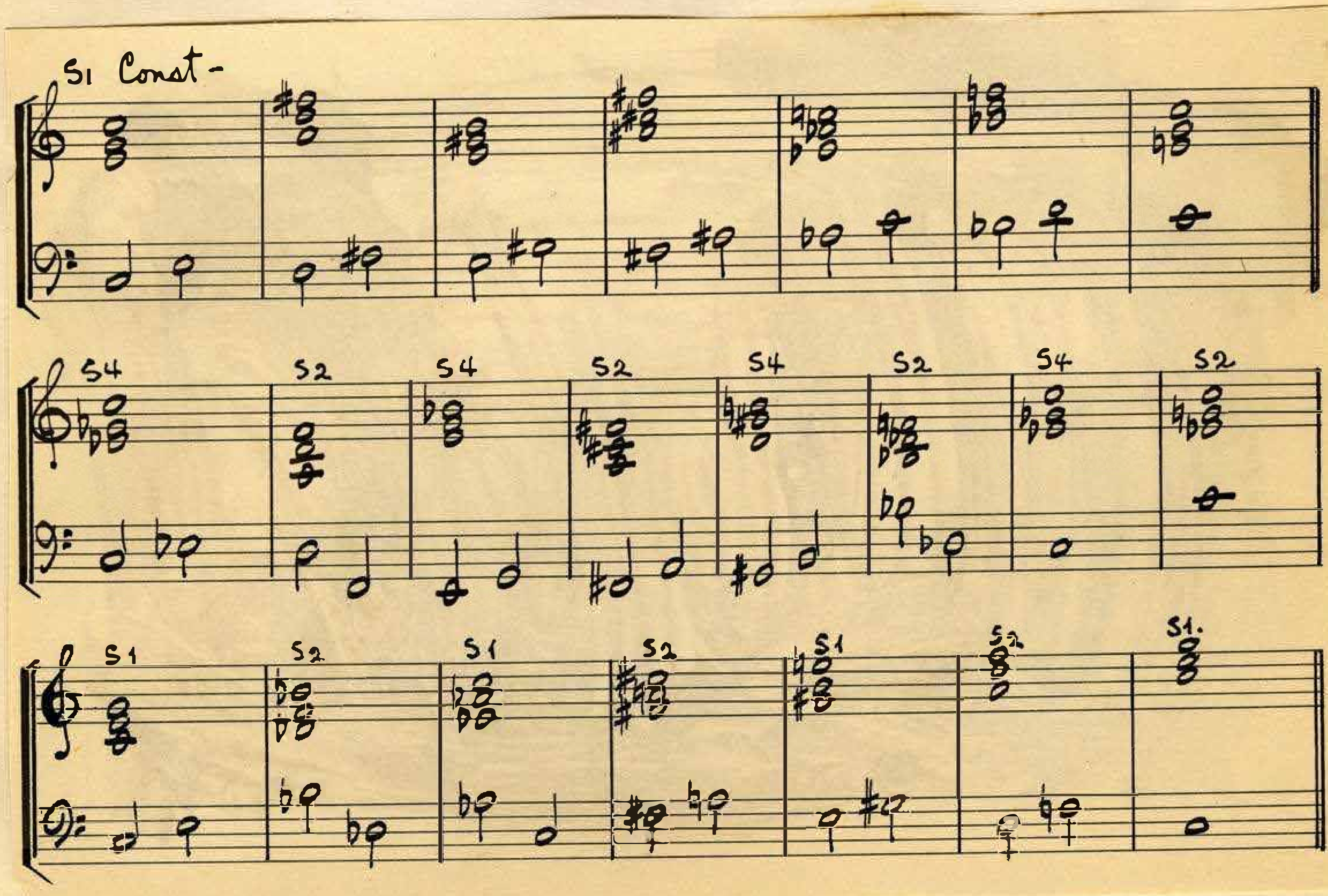
The number of distributions of the four forms of S through four tonics produces $4^6 = 4096$ intonations.

The number of intonations in the bass part is $2^6 = 64$.

The total number of intonations and melodic forms in the bass part is $64^2 = 4096$.

Examples of continuity of G_6 on six tonics.

Figure LXXI.



Application of G_6 to twelve tonics produces $2^{12} = 4096$ melodic forms in the bass part.

The number of distributions of the four forms of S through four tonics produces $4^{12} = 16,777,216$.

The number of intonations in the bass part is $2^{12} = 4096$.

The total number of intonations and melodic forms in the bass part is $4096^2 = 16,777,216$.

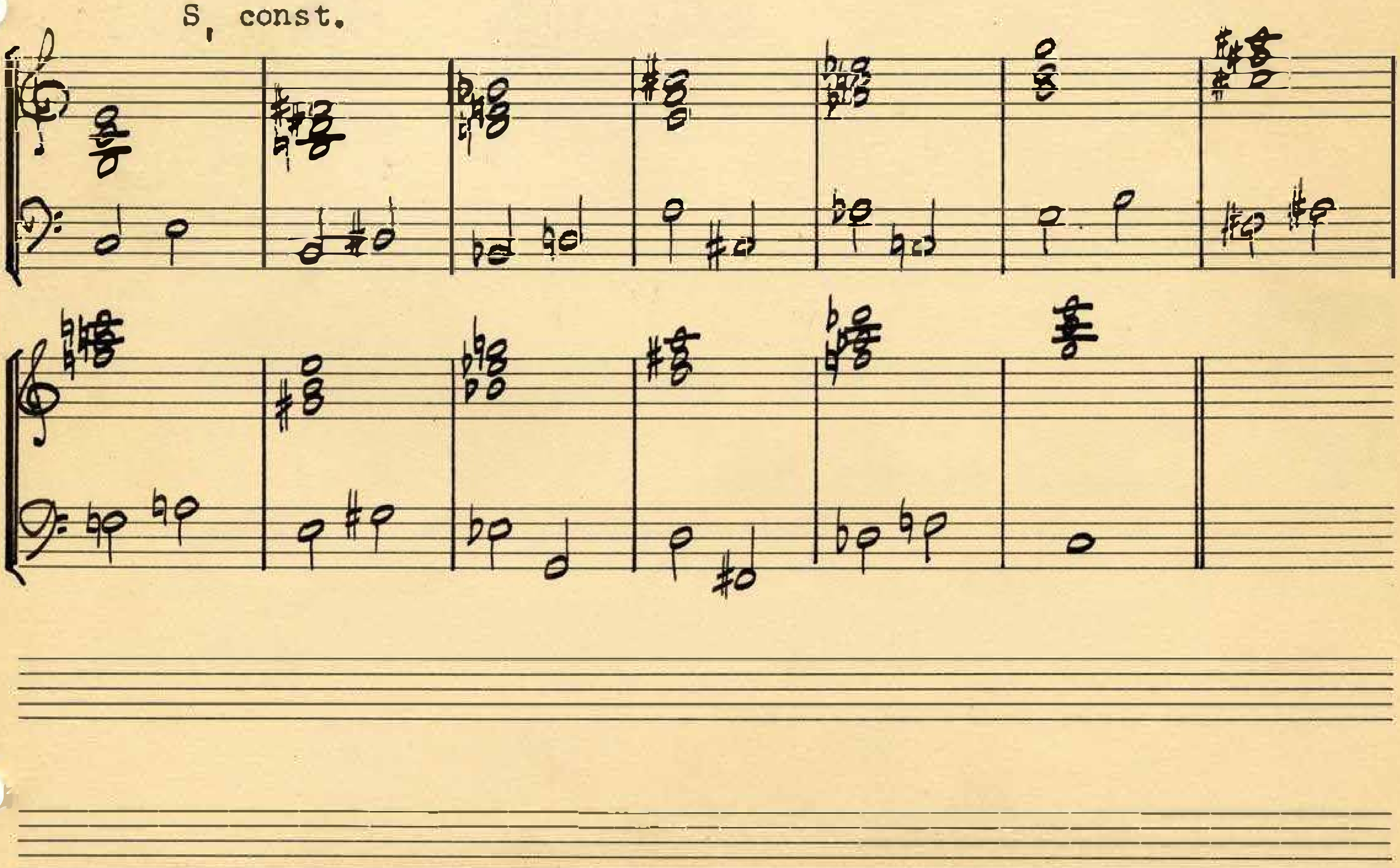
Examples of continuity of G_6 on twelve tonics.

Figure LXXII.

(please see next page)

S_1 const.

S_1 const.



Lesson LXXXIII.

B. Passing Fourth-Sixth Chords: $S(^{6}_{4})$.

The second inversion of S(5) is a fourth-sixth chord: $S(^{6}_{4})$. This name derives from the old basso continuo or generalbass, where intervals were measured from the bass.

$S(^{6}_{4})$ has a fifth (5) in the bass, while the three upper parts have the six usual arrangements.

The use of $S(^{6}_{4})$ in classical music is a very peculiar one. This chord appears only in definite pre-set combinations. One of them is the group with a passing fourth-sixth chord: G^{6}_{4}.

As in the case of G_6, the passing chord itself appears on a weak beat, being surrounded by the two other chords, and has a doubled fifth: $S^{6\;5}_{4}$.
The two other chords of G^{6}_{4} are: S(5) and S(6). The latter can have two forms of doubling (regardless of the chord-structure): $S(6)^{1}$ and $S(6)^{5}$.

The group with a passing fourth-sixth chord, contrary to G_6, is reversible.

$$G^6_4 = S(5) + S(^6_4) + S(6).$$

This property being combined with the choice of two possible doublings produces four variants.

$$G^6_4\uparrow^{1} = S(5) + S(^6_4) + S(6)^{1}$$

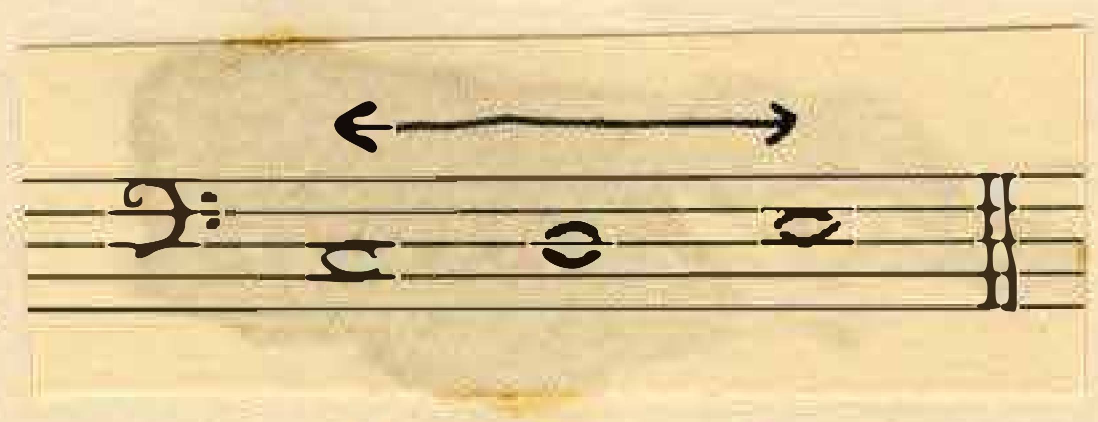
$$G^6_4\downarrow^{1} = S(6)^{1} + S(^6_4) + S(5)$$

$$G^6_4\uparrow^{5} = S(5) + S(^6_4) + S(6)^{5}$$

$$G^6_4\downarrow^{5} = S(6)^{5} + S(^6_4) + S(5)$$

The arrows in the above formulae specify the directions of the bass pattern which is always scalewise, and therefore can be either ascending or descending.

The bass pattern is developed on <u>three adjacent pitch-units,</u> which correspond to the three chords of G^6_4.

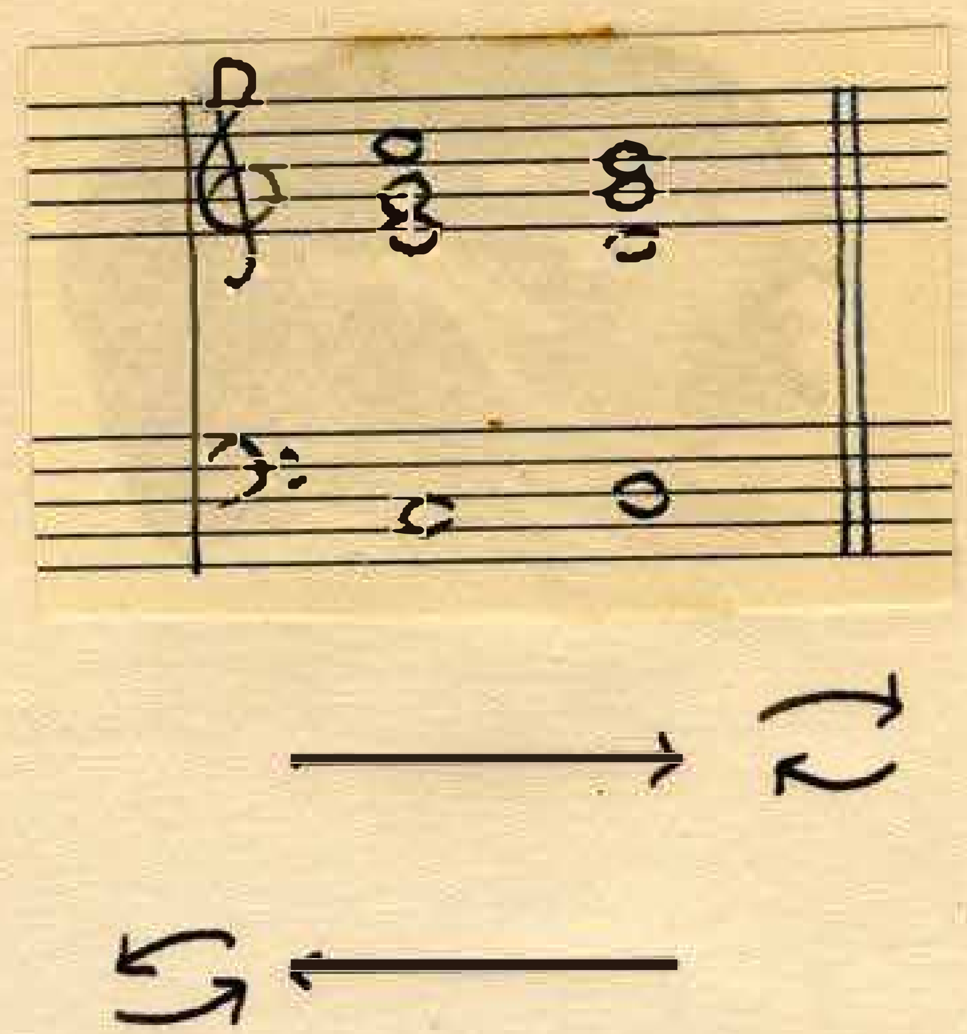


1 5 3

$S(5)$ $S(^6_4)$ $S(6)$

Arabic numberals represent the respective chordal functions.

Transformations between $S(5)$ and $S\binom{6}{4}$ in the G^6_4: as the bass moves from 1 to 5, when read in upward motion, the three upper voices must move clockwise, in order to get the transformation of 1 into 3.



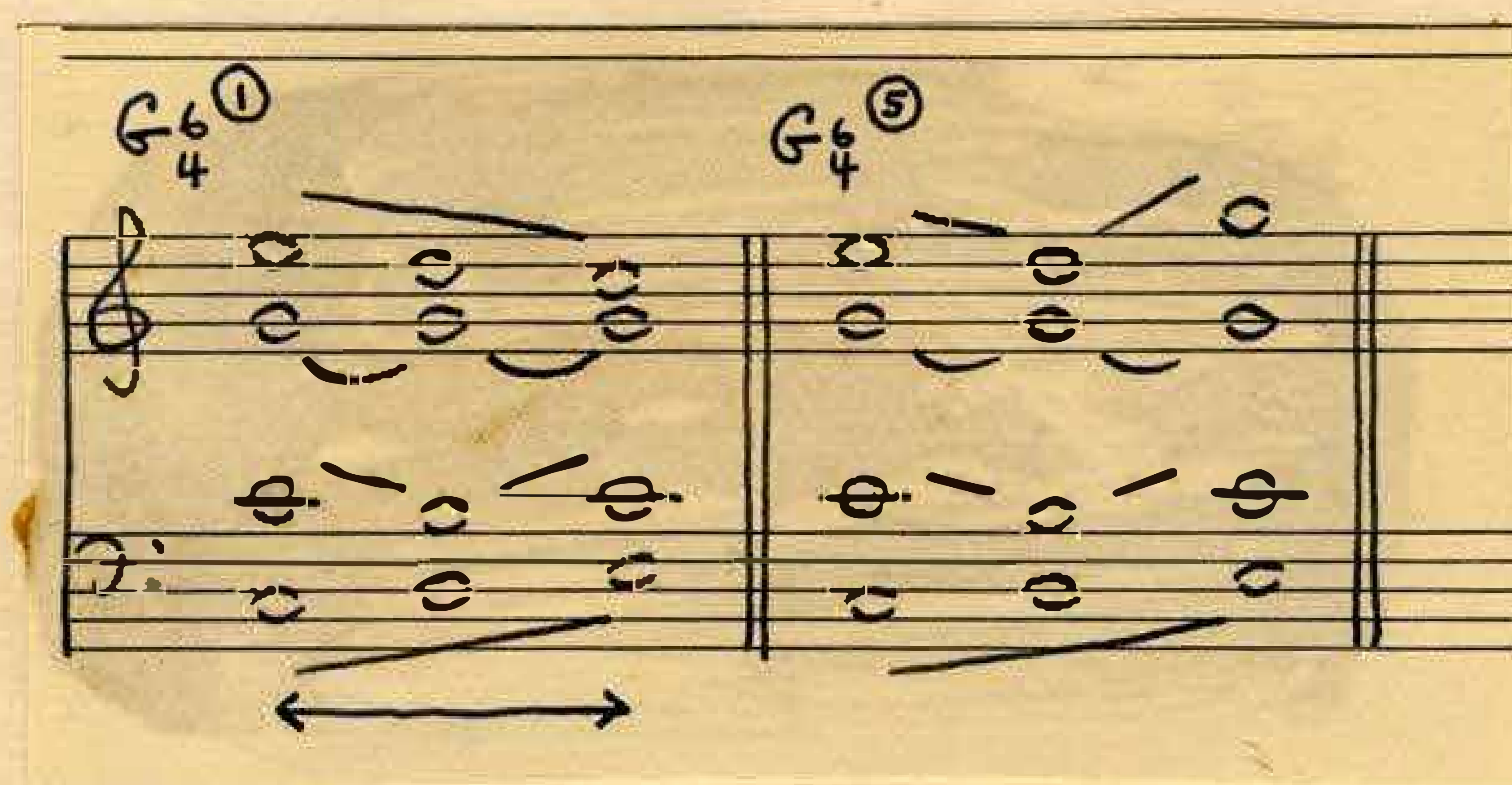
The transition from $S\binom{6}{4}$ into $S(6)^{1}$ or $S(6)^{5}$ follows the forms of transformations, where two identical functions participate, as in the cases of $S(5) \longleftrightarrow S(6)^{1}$ and $S(5) \longleftrightarrow S(6)^{5}$.

However, classical technique adopted definite routines concerning this transition:

(1) one part must carry out a melodic form reciprocal to the bass (i.e., position b of the bass melody);

(2) it is this reciprocal part that deviates from its path in order to supply the doubling of the fifth in an $S(6)$.

Under such conditions G_4^6 acquires the following appearances:



The following sequence of operations is recommended:

(1) bass

(2) part reciprocating the bass

(3) common tone

(4) part supplying the third for $S\binom{6}{4}$

The relations between the chords of G_4^6 are as follows:

$$\overbrace{S(5) + C\text{-}5 + S\binom{6}{4} + C_5 + S(6)}^{Co}$$

$$\overbrace{S(6) + C\text{-}5 + S\binom{6}{4} + C_5 + S(5)}^{Co}$$

Each group can be carried out in 6 positions which depend on the starting position.

The following is the table of all four forms of G_4^6 in one position.

Figure LXXIII.

The different forms of G_4^6 can be connected by means of tonal cycles and their coefficients of recurrence can be specified.

It is desirable to make the following tables:

(1) $G_4^6\uparrow$ 1 const.; C_3 const., C_5 const., C_7 const.

(2) $G_4^6\downarrow$ 1 const.; " " " " " "

(3) $G_4^6\uparrow$ 5 const.; " " " " " "

(4) $G_4^6\downarrow$ 5 const.; " " " " " "

(5) $G^6_4\uparrow^{1}$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$

(6) $G^6_4\downarrow^{1}$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$

(7) $G^6_4\uparrow^{5}$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$

(8) $G^6_4\downarrow^{5}$ const.; $C^{\rightarrow} = C_3 + C_5 + C_7$

(9) $G^6_4\uparrow^{1} + G^6_4\downarrow^{5} + G^6_4\uparrow^{5} + G^6_4\downarrow^{1}$; C_3 const.

(10) " " " " ; C_5 const.

(11) " " " " ; C_7 const.

(12) " " " " ; $C^{\rightarrow} = C_3 + C_5 + C_7$

$C^{\rightarrow}$ is the symbol of a group of cycles (cycle continuity).

Continuity of G^6_4, when connected through a constant tonal cycle, consists of seven cycles: $C^{\rightarrow} = 7C$.

Figure LXXIV.

Example: $G^6_4\uparrow^{1}$ const. $C^{\rightarrow} = C_3$ const.

Continuity of G_4^6 of different forms and connection through different cycle-groups can be applied in its present form to Diatonic progressions.

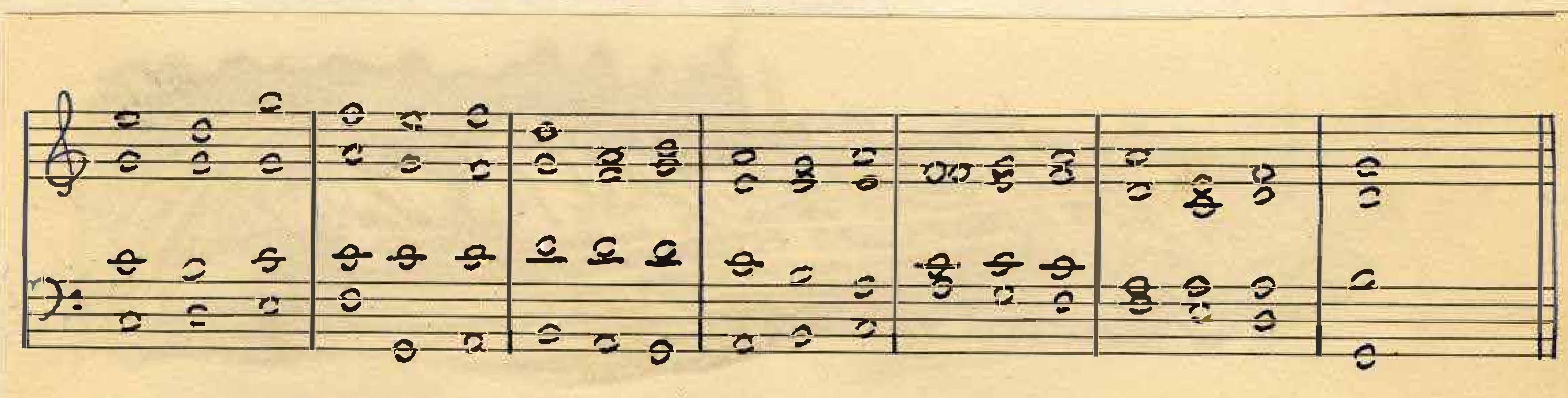
G_4^6 in symmetric progressions of the types II and III require <u>identical structures for the two extreme chords of one group.</u> This requirement does not affect the middle chord of the group, i.e., $S(^6_4)$, nor does it influence the selection of structures for the following groups.

<u>Examples of continuity with G_4^6</u>

<u>in progressions of the types I and II.</u>

<u>Figure LXXV.</u>

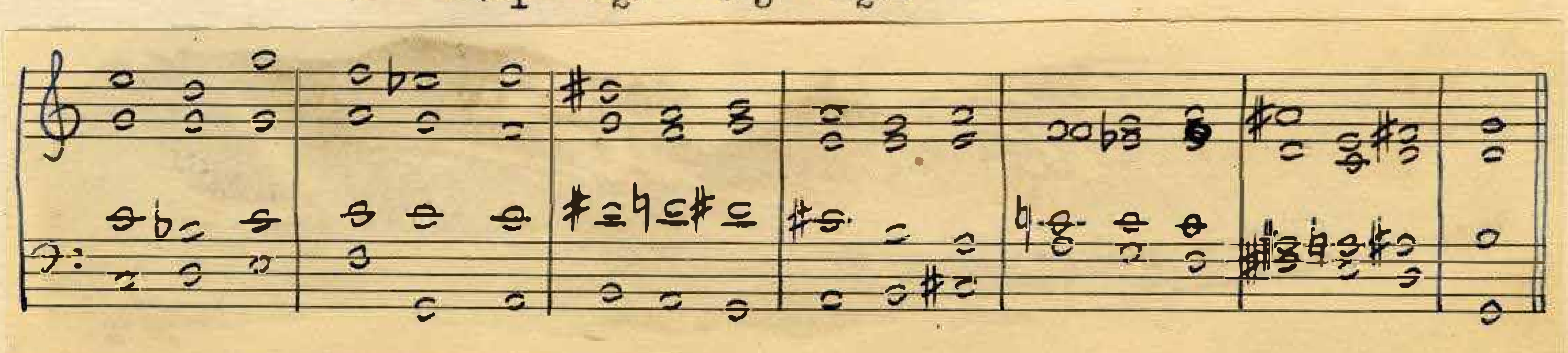
$H^{\rightarrow} = 2G_4^6\uparrow + G_4^6\downarrow + G_4^6\uparrow + 2G_4^6\downarrow$; $C^{\rightarrow} = C_5 + 2C_7 + 2C_3 + C_5$.



<u>Figure LXXVI.</u>

$H^{\rightarrow}$ and $C^{\rightarrow}$ as in the preceding example.

$S^{\rightarrow} = 2(S_1 + S_2) + (S_3 + S_2)$.



J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross Subject: Music

Lesson LXXXIV.

Application of G_4^6 to symmetric systems requires the following sequence of tonics:

$$GH^{\rightarrow} = (T_1 + T_2 + T_1) + (T_2 + T_3 + T_2) +$$

$$+ (T_3 + T_4 + T_3) + \ldots$$

For example, the three-tonic system must be distributed as follows:

$$GH^{\rightarrow} = (T_1 + T_2 + T_1) + (T_2 + T_3 + T_2) +$$

$$+ (T_3 + T_1 + T_3).$$

The quantity of tonics in the respective system specifies the cycle. Each group may begin with either S(5) or S(6).

Each group acquires the following distribution of inversions:

$$G_4^6 = T_1 S(5) + T_2 S\binom{6}{4} + T_1 S(6)$$

$$\longrightarrow$$

$$\longleftarrow$$

Under such conditions, each tonic appears in all the three inversions.

Table of G_4^6 applied to all symmetric systems

Figure LXXVII.

Two Tonics.

Three Tonics.

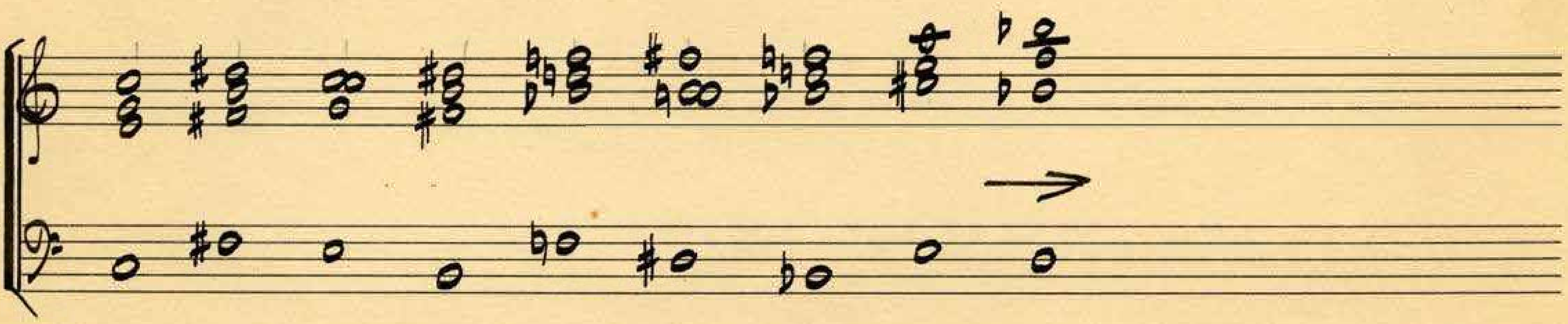
Four Tonics.

Six Tonics

TWELVE TONICS.

SIX TONICS: NEGATIVE FORM.

TWELVE TONICS: NEGATIVE FORM.



Other negative forms are not as practical: inversions weaken tonality.

Example of variation of structures

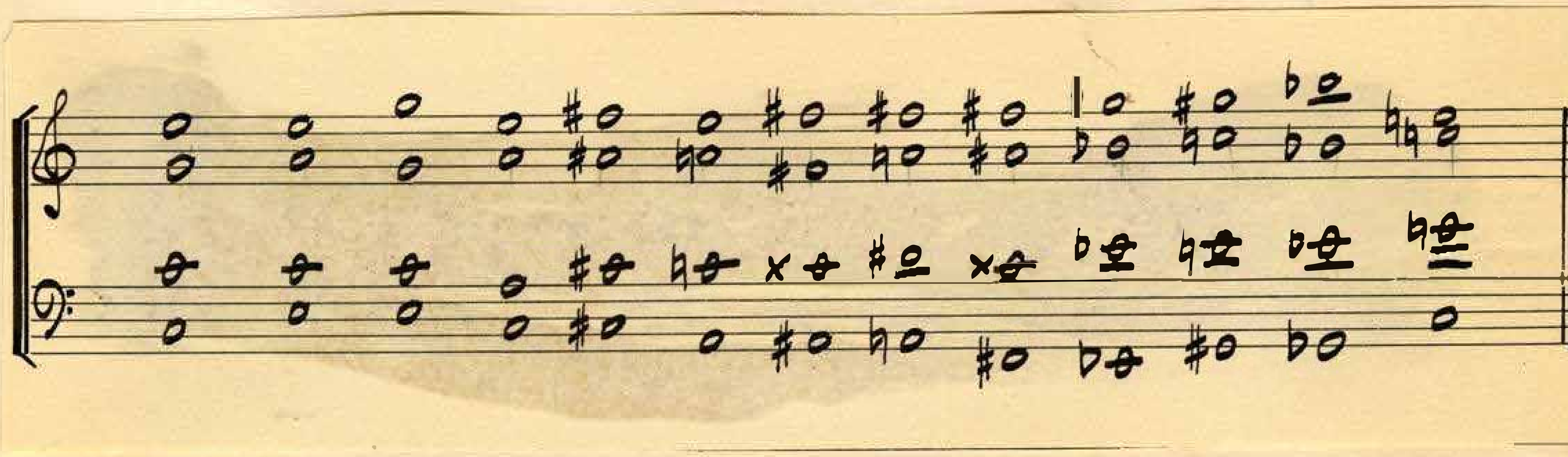
and directions.

Figure LXXVIII.

Four Tonics.

$$GH^{\curvearrowright} = [S_1(5) + S_2\binom{6}{4} + S_1(6)] + [S_2(6) + S_1\binom{6}{4} + S_2(5)] +$$

$$+ [S_3(6) + S_4\binom{6}{4} + S_3(5)] + [S_2(5) + S_3\binom{6}{4} + S_2(6)]$$



C. Cycles and Groups Mixed.

Tonal cycles can be introduced into the continuity of groups, as well as groups can be introduced into the continuity of cycles.

It is convenient to plan the mixed form of cycle-group continuity by the bars (T).

Bars of cycles and bars of groups can be

assigned to have different coefficients of recurrence.

When planning such a continuity in advance, it is important to consider that there is always a cycle-connection between the bars.

Examples:

Figure LXXIX.

$$H^{\rightarrow} = 2TC + TG + TC + 2TG = (C_5 + C_3) + C_7 + (C_3 + C_7) +$$

$$+ C_5 + G_6 + C_7 + (C_3 + C_3) + C_5 + G^6_4\uparrow① + C_7 + G^6_4\downarrow⑤ + C_3.$$