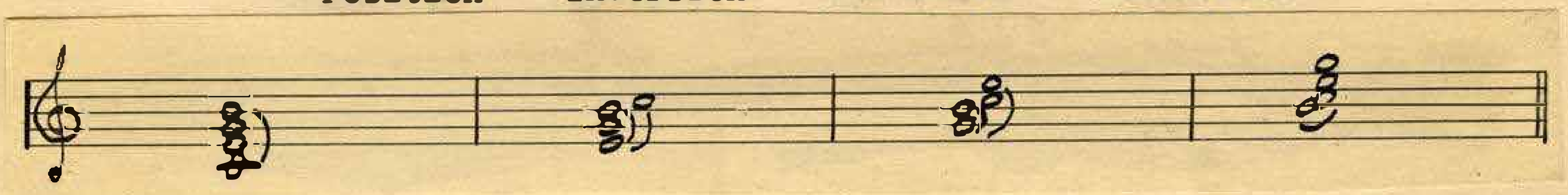
Lesson LXXXV.

SEVENTH-CHORDS. S(7)

Diatonic System

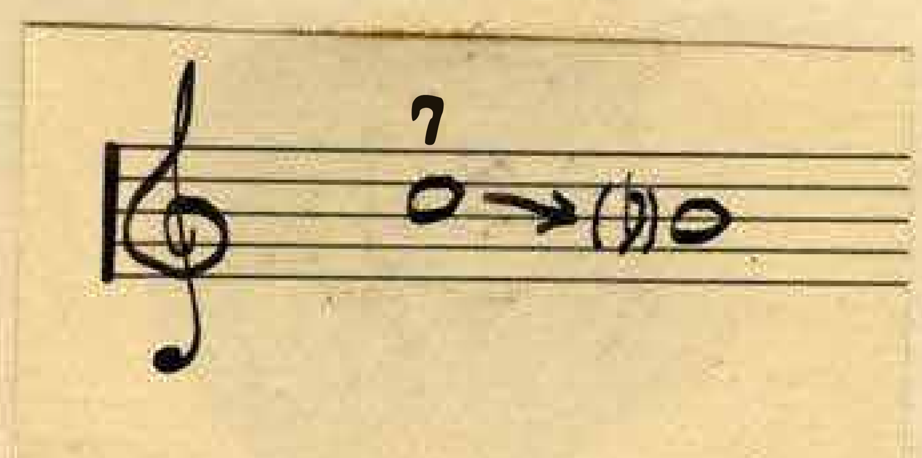
Fundamental Position	The First Inversion	The Second Inversion	The Third Inversion



S(7) Seventh-Chord	$S\binom{6}{5}$ Fifth-Sixth Chord	$S\binom{4}{3}$ Third-Fourth Chord	S(2) Second Chord

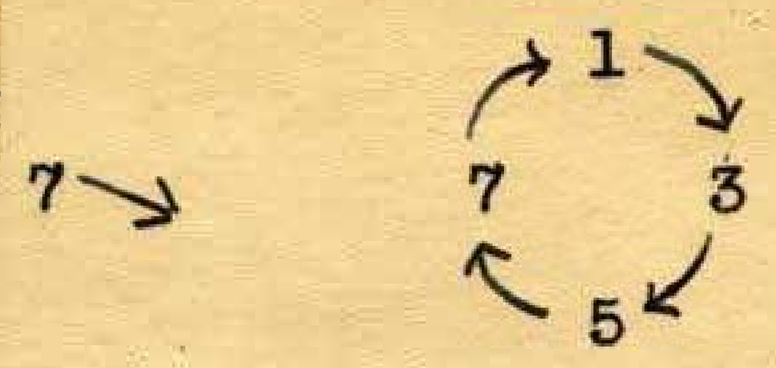
A seventh-chord including all inversions has 24 positions altogether.

The classical system of harmony is based on the postulate of resolving seventh: seventh moves one step down.

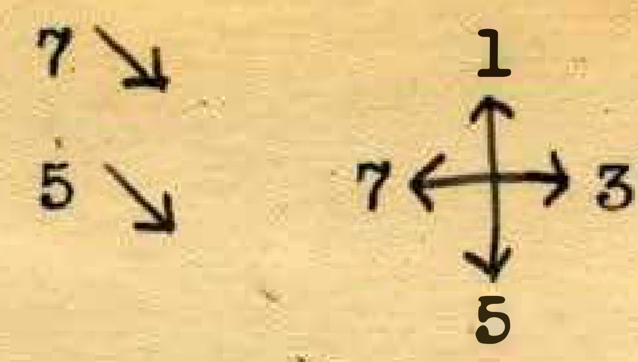


This postulate provides a medium for continuous progression of S(7) as well as establishes the entire system of diatonic continuity (cycles).

One movement is required to produce C_3: the movement of the seventh alone. It results in a clockwise transformation.

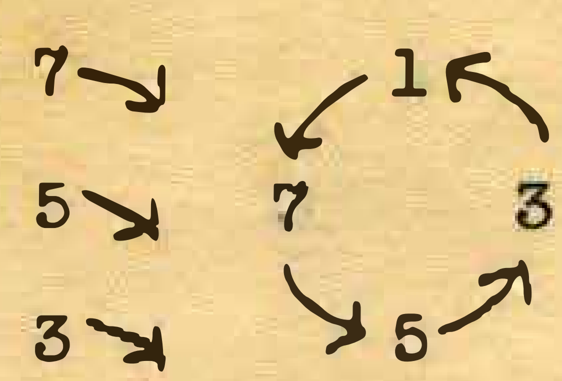


<u>Two movements</u> are required to produce C_5: the movement of the seventh and of the fifth one step down. It results in a crosswise transformation.



<u>Three movements</u> are required to produce C_7: the movement of the seventh, of the fifth and of the third one step down. It results in a counter-clockwise transformation.

Taking the chords over two from C_3 we obtain:



This type of music may be found among contrapuntalists of XVII - XVIII Centuries. Palestrina, Bach, Haendel obtained similar results by means of suspensions.

Assigning a system of cycles we can produce

a continuity of S(7). The starting chord may be taken in any position.

Example: $C_5 + C_3 + C_5 + C_7 + C_3 + C_3 + C_5$

This continuity being entirely satisfactory harmonically may prove, in some cases, unsatisfactory melodically on account of continuous descending in all voices. This form, when desirable, may be eliminated by means of the two devices:

(1) exchange of the common tones

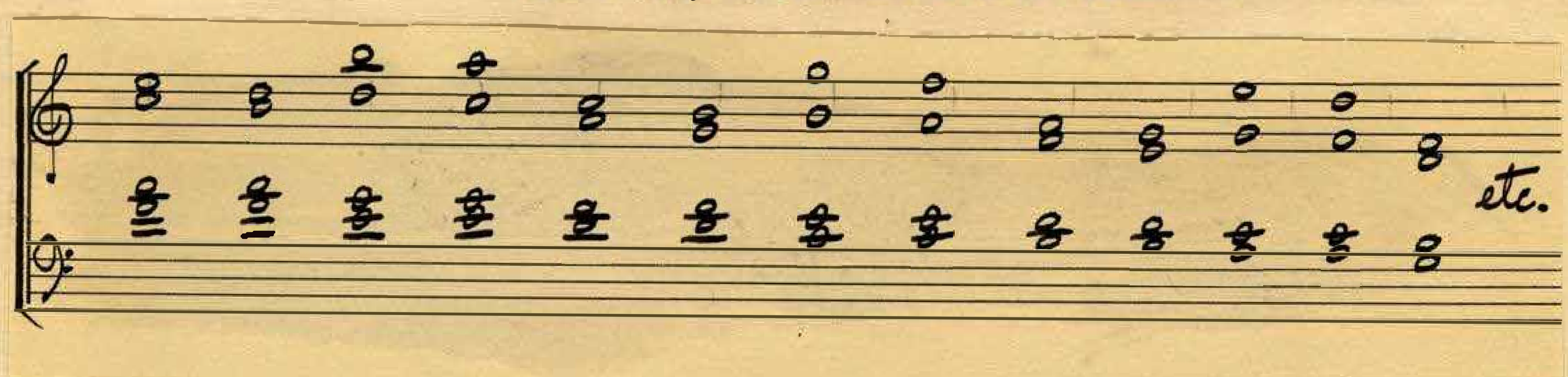
(2) octave inversion of the common tones

The same continuity of cycles assumes the following form:

Obviously C_7 does not provide common tones, thus excluding the above devices.

As the continuity of the second type offers better melodic forms for all voices, it may be desirable to pre-set certain melodic forms in advance. For example, it is possible to obtain, by means of continuous C_5, the following form of descending through two parallel axes (b) or (d), as in the music of Frederick Chopin.

This may be harmonized as follows:



Diatonic C_0 becomes a necessity in order to avoid the excess of saturation typical of the continuity of S(7) with variable cycles.

The principle of moving continuously through C_0 is based on the exchange and inversion of common tones.

The exchange and inversion of adjacent functions brings the utmost satisfaction. Neverthe-less it is not desirable to use the two extreme functions for such purpose as they cause a certain amount of harshness.

An example of continuity of the C_o:

The final form of continuity of S(7) consists of the mixtures of all cycles (including C_o) based on a rhythmic composition of the coefficients of recurrence.

Example: $2C_5 + C_0 + 2C_3 + C_0 + 2C_7 + C_0$

Lesson LXXXVI.

Resolution of S(7)

Resolution of an S(7) into an S(5) in all positions and inversions may be defined as a transition from four functions to three functions.

S(5) in the four-part harmony and with a normal doubling (doubled root) consists of:

1, 1, 3, 5

S(7) consists of:

1, 3, 5, 7

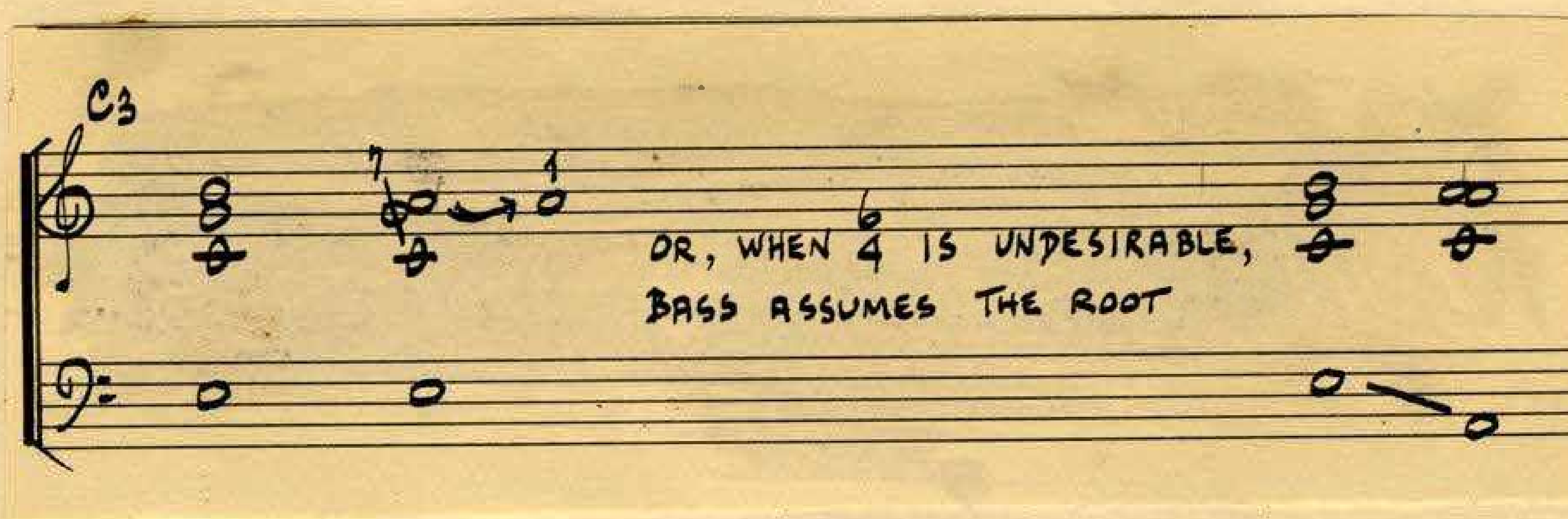
When a transition occurs, obviously the root takes the place of the seventh. Therefore the resolution is provided through the motion of S(7) → S(7) and the substitution of one for the seven, i.e., the function which would become a seventh in the continuity of seventh-chords becomes a root-tone when a resolution is desired.

Example:

7 → 1
5 → 7 → 1
3 → 5
1 → 3

Note: Do not move S(7) → S(5) in the C_0

Resolutions in the Diatonic Cycles.



This case provides an explanation why a tonic triad acquires a tripled root and loses its fifth.

Preparation of S(7)

There are three methods of preparation of S(7), i.e., of transition from S(5) to S(7):

(1) suspending

(2) descending

(3) ascending

The first method is the only one producing the <u>positive</u> (C_3, C_5, C_7) cycles.

The methods (2) and (3) are the outcome of the intrusion of melodic factors into harmony. They are obviously in conflict with the nature of harmony (like the groups with passing chords) as they produce the negative cycles, which in turn contradict the postulate of the resolving seventh universally observed in classical music.

The technique of preparation of the seventh consists of assigning a certain consonant function (1, 3, 5) to become a dissonant function (7) and to either sustain the assigned function of the S(5) over the bar line or to move it one step downward or upward.

The last two forms of a seventh must occur on a weak beat.

Exercise in different positions, inversions and cycles the S(5)$\longrightarrow$S(7) transition.

(1) Suspending:

1 7	3 7	5 7
C_7	C_5	C_3

(2) Descending:

$\left| 1 \searrow 7 \right|$ $\qquad$ $\left| 3 \searrow 7 \right|$ $\qquad$ $\left| 5 \searrow 7 \right|$

C_0 $\qquad$ C_{-3} $\qquad$ C_{-5}

(3) Ascending:

$\left| 1 \nearrow 7 \right|$ $\qquad$ $\left| 3 \nearrow 7 \right|$ $\qquad$ $\left| 5 \nearrow 7 \right|$

C_{-3} $\qquad$ C_{-5} $\qquad$ C_{-7}

(Please see next page)

Preparation of S(7)

(1) Suspending

(2) Descending

(3) Ascending

The mixture of the zero, positive and negative cycles provides the final form of continuity based on S(5) and S(7).

For more efficient planning of such continuity use bar lines for the layout. The preparation of S(7) may be either positive or negative; the resolution - always positive.

<u>Example:</u>

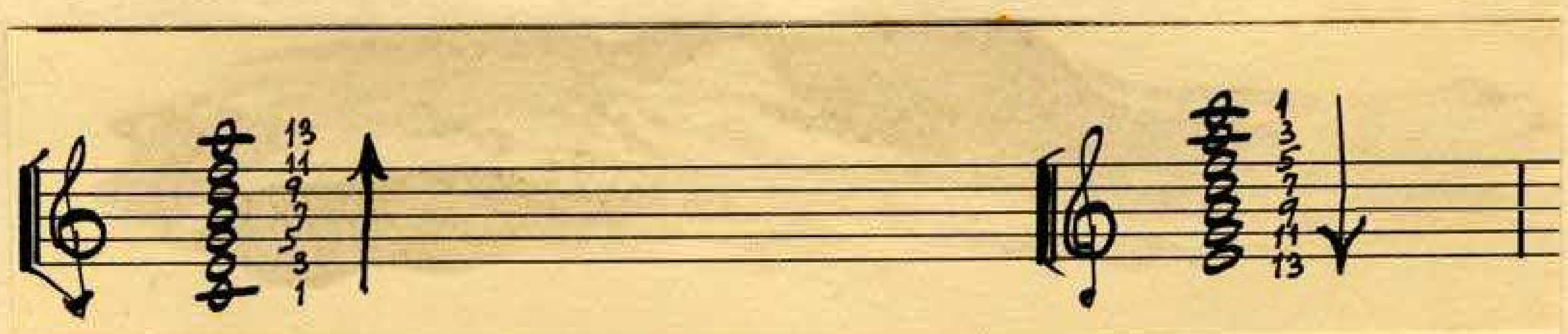
Lesson LXXXVII.

The negative system of tonal cycles may be used as an independent system. The negative system is in reality a geometrical inversion of the positive system. Every principle, rule or regulation of the positive system becomes its own converse in the negative.

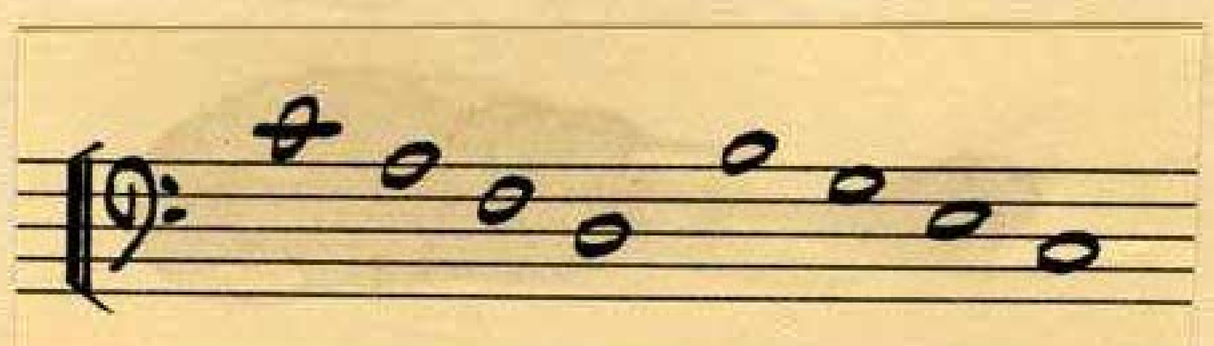
Chord structures become E_{b} of the original scale. Chord progressions are based on E_{a} which forms the C_{-3}. Clockwise transformations become counterclockwise and vice versa.

Chord Structures

Positive Negative



Tonal Cycles:



+ Positive

Negative -

Transformations:

+

-

The postulate of resolving seventh for the negative system must be read: <u>the negative seventh moves one step up.</u> The C_{-5} requires the negative seventh and negative fifth to move one step up. The C_{-7} requires all the tones except the root to move up. This system may be of great advantage in building up climaxes.

Positive:

C_3

Negative:

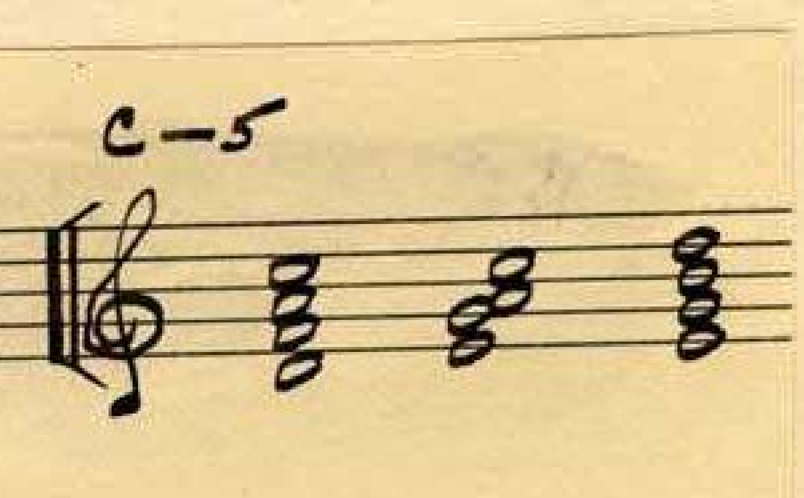
C_{-3}

The root-tone of the negative system is the seventh of the positive and vice-versa.

It is easy to see how the other cycles would operate.

C_5

C_{-5}



C_7

C_{-7}

If one wishes to read the negative system as if it were positive, the rules must be changed as follows:

The C_{-3} requires the ascending of 1

The C_{-5} " " " " 1 and 3

The C_{-7} " " " " 1, 3 and 5

Special Applications of S(7)

A. Groups

S(7) finds its application in G^{6}_{4}, either as the first or the last chord of the group.

The following forms are possible:

$S(5) + S\binom{6}{4} + S\binom{6}{5}$

$S(5) + S\binom{6}{4} + S\binom{4}{3}$

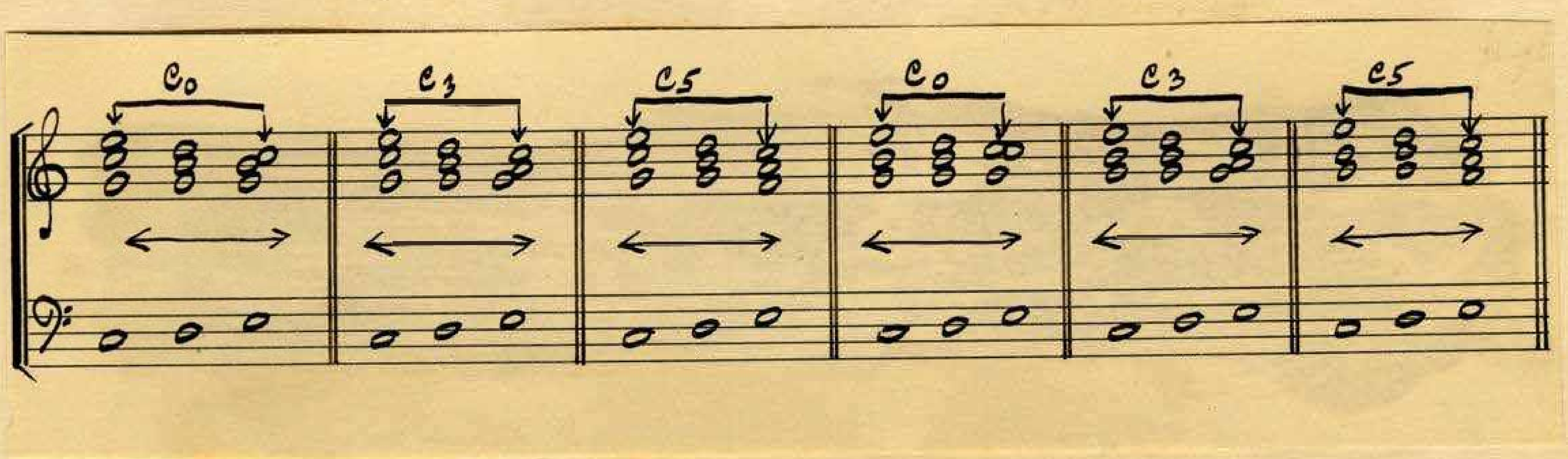
$S(5) + S\binom{6}{4} + S(2)$

$S(7) + S\binom{6}{4} + S(6)$

$S(7) + S\binom{6}{4} + S\binom{4}{3}$

$S(7) + S\binom{6}{4} + S(2)$

The cycle between the extreme chords of G^{6}_{4} may be either C_0, or C_3, or C_5.



Besides G^{6}_{4} there is a special group where $S(\frac{4}{3})$ is used as a passing chord. There are two forms of this group.

(A) $G^{4}_{3(5)} = S(5)^{(3)} + S(\frac{4}{3}) + S(7)$ or $S(5)$

(B) $G^{4}_{3(6)} = S(6)^{(5)} + S(\frac{4}{3}) + S(7)$ or $S(5)$

These two forms may be used in one direction only.

All positions are available.

Rule of voice-leading: bass and one of the voices of doubling move stepwise down. Common tones sustained.

The cycle between the extreme chords in the first form is C_3; in the second form it is C_0.

B. Cadences.

The following applications of S(7) are commonly known:

(1)	IV_7	$I_{6 \atop 4}$ - V_7	I_5
(2)	$IV_{6 \atop 5}$	" "	"
(3)	$II_{6 \atop 5}$	" "	"
(4)	$II_{4 \atop 3}$	" "	"

In addition to this the following forms may be offered:

(5)	Any of the previous forms	$I_{6 \atop 4}$ - $III_{6 \atop 5}$	I_5
(6)	"	$I_{6 \atop 4}$ - $VII_{6 \atop 5}$	I_5

Besides these there are two ecclesiastic forms:

(1) I_5 - $IV^{5}_{(II_{6 \atop 5})}$ I_5

(2) I_5 - $IV^{5}_{(II_{4 \atop 3})}$ I_5

(please see next page)


(1)
(2)
(3)
(4)
etc.
etc.
(5)
(1,2,3,4)
(6)
(1,2,3,4)
(1) 5
(2)

JOSEPH SCHILLINGER

CORRESPONDENCE COURSE

With: Dr. Jerome Gross Subject: Music

Lesson LXXXVIII.

Symmetric Zero Cycle (C_0)

Symmetric C_0 offers an extraordinary versatility on S(7) as seven structures of the latter have been in use.

If evolving of the forms of S(7) would have been devised scientifically, they would be obtained in the following order.

Taking c - e - g - b♭ (4 + 3 + 3) as the most common form and producing variations thereof, we obtain two other forms:

c - e♭ - g - b♭ (3 + 4 + 3)

and c - e♭ - g♭ - b♭ (3 + 3 + 4)

Taking another form, c - e - g - b (4 + 3 + 4), we obtain two other forms:

c - e - g♯ - b (4 + 4 + 3)

and c - e♭ - g - b (3 + 4 + 4)

These two groups of three are distinctly different but as music has made the use of them for quite some time our ear does not find it objectionable any longer to mix all of them in one harmonic continuity.

Besides these six forms there is a $c - e^{\flat} - g^{\flat} - b^{\flat\flat}$ (3 + 3 + 3 + 3) and might have been $c - e - g^{\sharp} - b^{\sharp}$ (4 + 4 + 4 + 4) if there would not be an objection to the fact that $c - b^{\sharp}$ is an enharmonic octave.

A continuity on symmetric C_0 of all seven structures offers 5040 permutations. Thus a c - chord alone can move (without changing its position and without coefficients of recurrence being applied) for 5040 x 7 = 35,280 chords.

A method of selecting the best of the available progressions must be based on the following principle: the best progressions on symmetric C_0 are due to <u>identity of steps</u> or to <u>contrary motion.</u>

<u>Example</u>

(1) Identity of Steps:

all semitones

(2) Contrary motion:

The principle of variation of the chord-structures and their positions remains the same as in S(5):

Structure		Position
Constant	———————	Variable
Variable	———————	Constant

S(7) in the following table has a dual system of indications: letter symbols and adjectives. The adjectives are chosen so that they do not adhere to the degrees of any scale but to structure alone. Thus, such a common adjective as "dominant" had to be sacrificed.

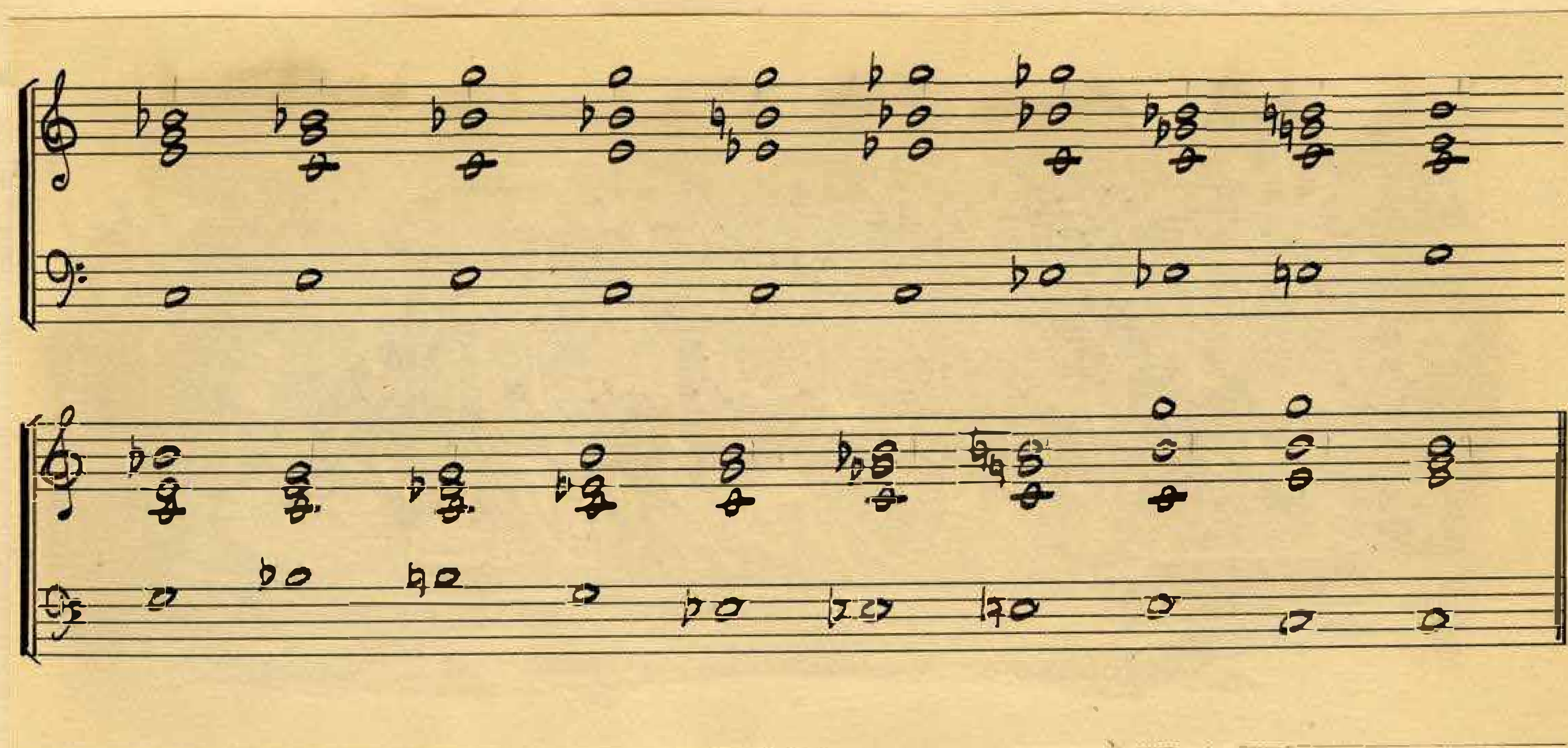
S(7) Table of Structures

An Example of Continuity in C_0:

Structures: $S_3 + S_7 + S_4 + S_1$

Coefficients $(r_{5 \div 4})$: $4S_3 + S_7 + 3S_4 + 2S_1 + 2S_3 +$

$+ 3S_7 + S_4 + 4S_1$



As in S(5), any combination of the forms of S(7) by 2, 3, 4, 5, 6 and 7 may be used.

Type III (Symmetric).

As in the previous cases when dealing with symmetrical tonics C_0 may be applied either to any of the tonics or as a continuous change of chord structures occurring with each tonic.

When structures of S(5) and S(7) have to be specified in one continuity, they must have full indications:

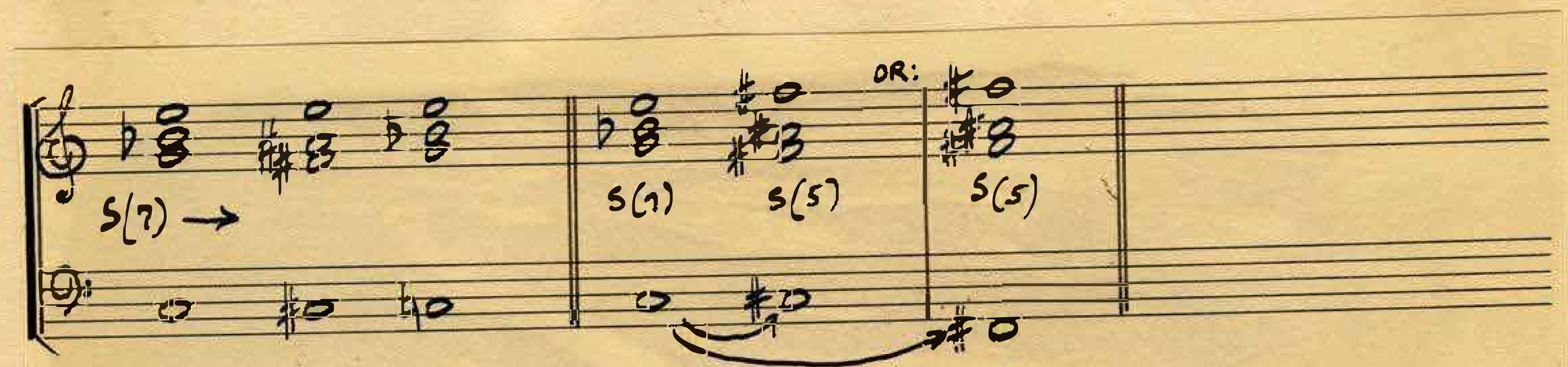
$S_1(5)$; $S_2(5)$; $S_3(5)$; $S_4(5)$ and

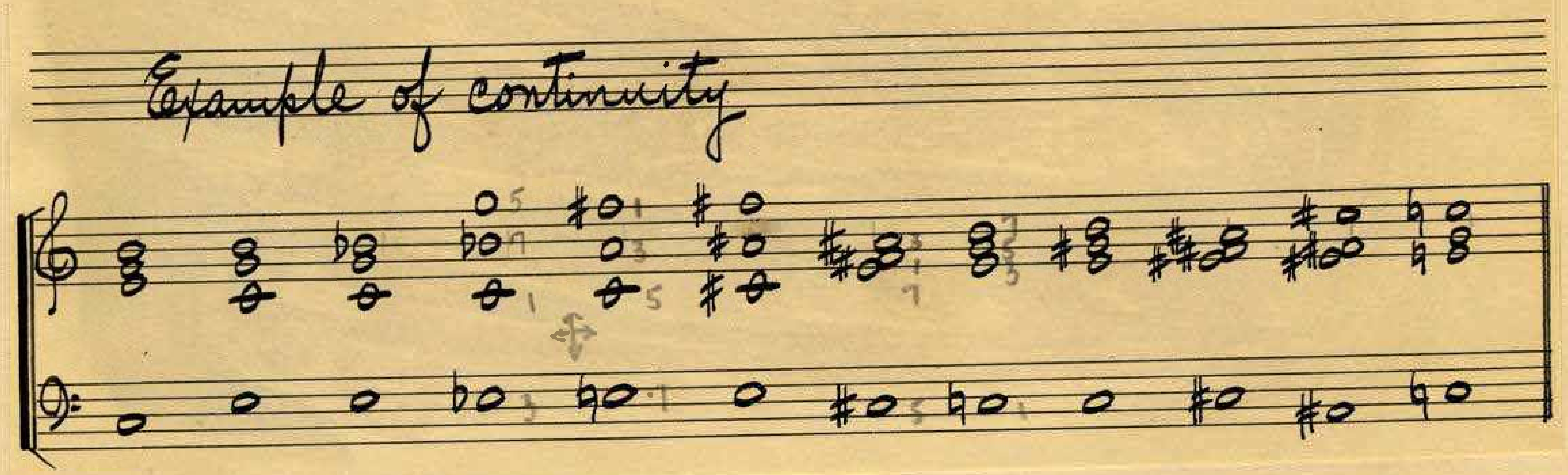
$S_1(7)$; $S_2(7)$; $S_3(7)$; $S_4(7)$; $S_5(7)$; $S_6(7)$; $S_7(7)$

<u>Two Tonics</u> ($\sqrt{2}$)

As the $\sqrt{2}$ forms the center of the octave the progression $1 \longrightarrow \sqrt{2}$ ($C \longrightarrow F^{\#}$) is positive and $\sqrt{2} \longrightarrow 2$ ($F^{\#} \longrightarrow C$) is negative.

The system of Two Tonics which was continuous on S(5) becomes closed on S(7). Transformations correspond to C_5.



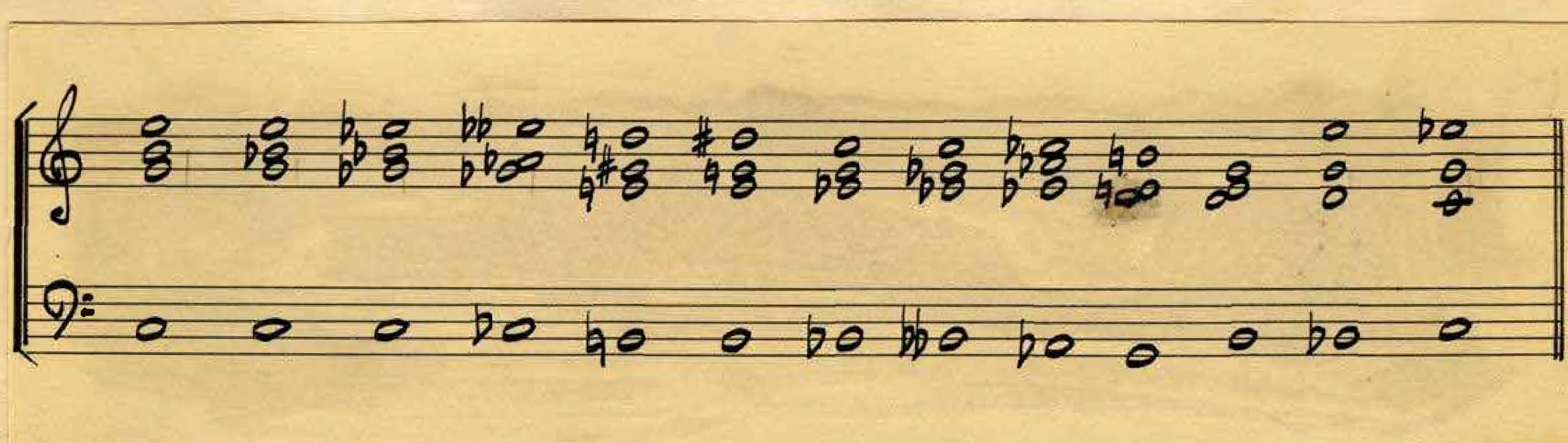


<u>Three Tonics</u> ($\sqrt[3]{2}$)

Continuous system: moves four times. Transformations correspond to C_3. To obtain S(7)

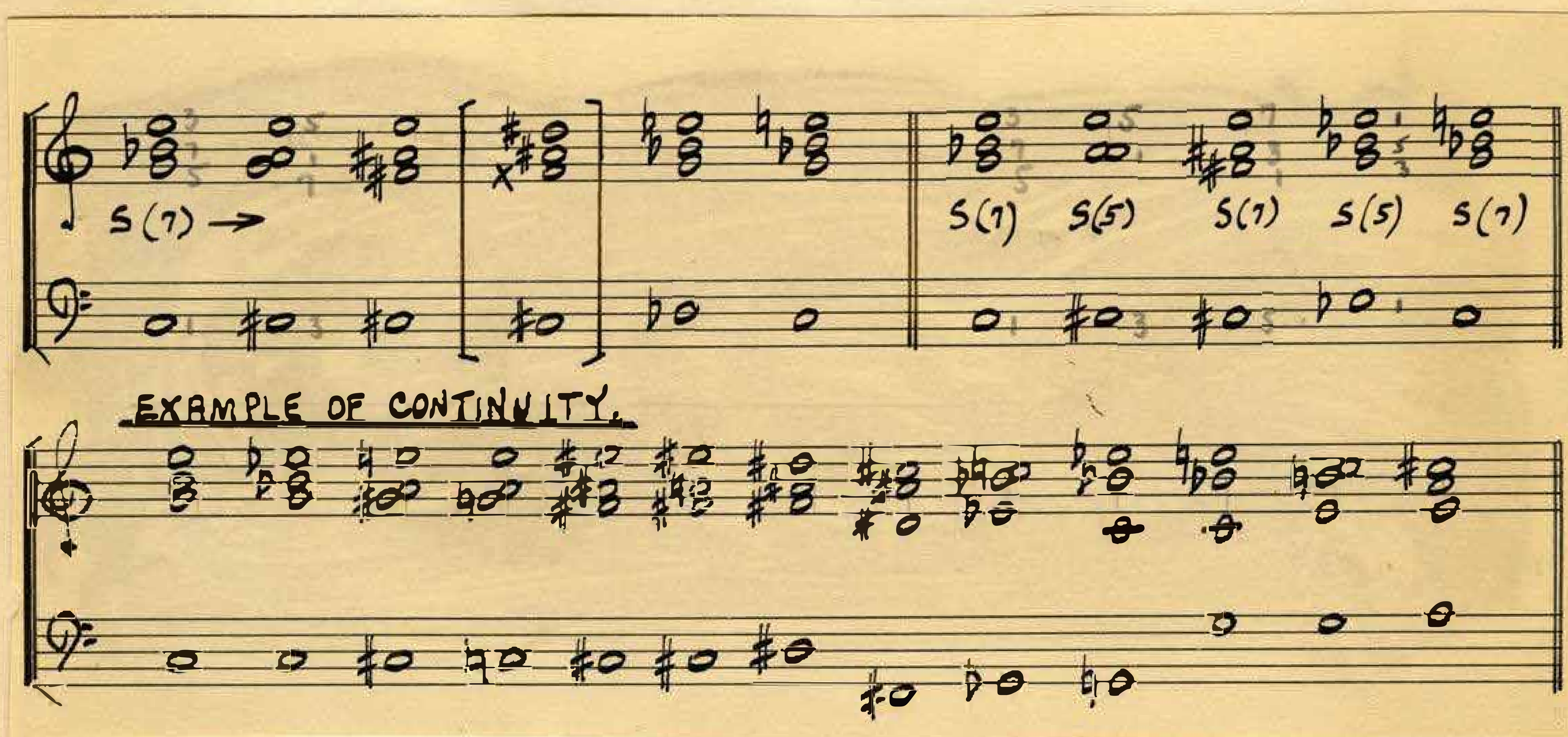
after an S(5) use the position which would correspond to continuous progression of S(7).

Example of Continuity:



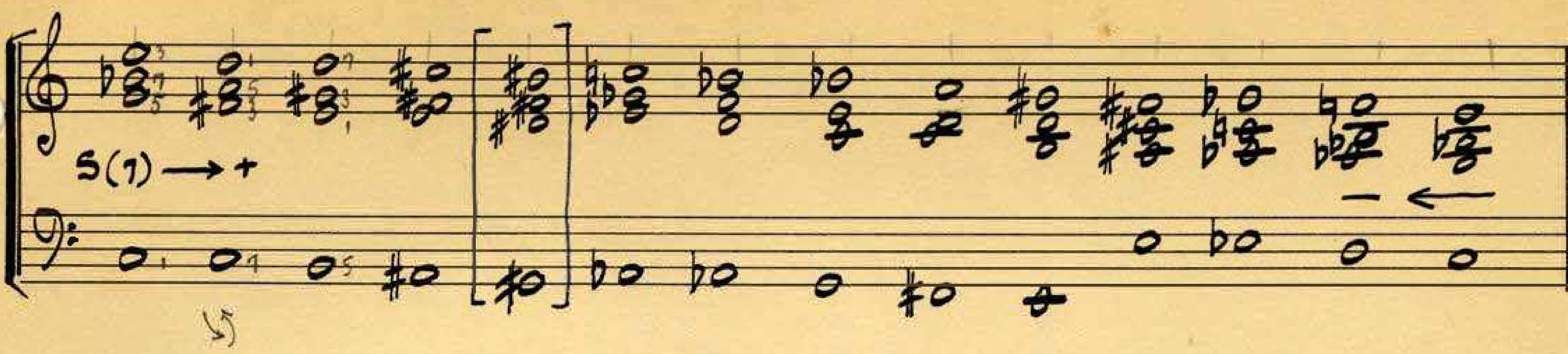
Four Tonics ($\sqrt[4]{2}$)

Closed system. Transformations correspond to C_3. S(7) after S(5) as in Three Tonics.



Six Tonics ($\sqrt[6]{2}$)

Continuous system: moves two times. Transformations correspond to C_7. S(7) after S(5) as in previous cases. Both positive and negative progressions are fully satisfactory. To obtain the negative progressions read the positive backwards.

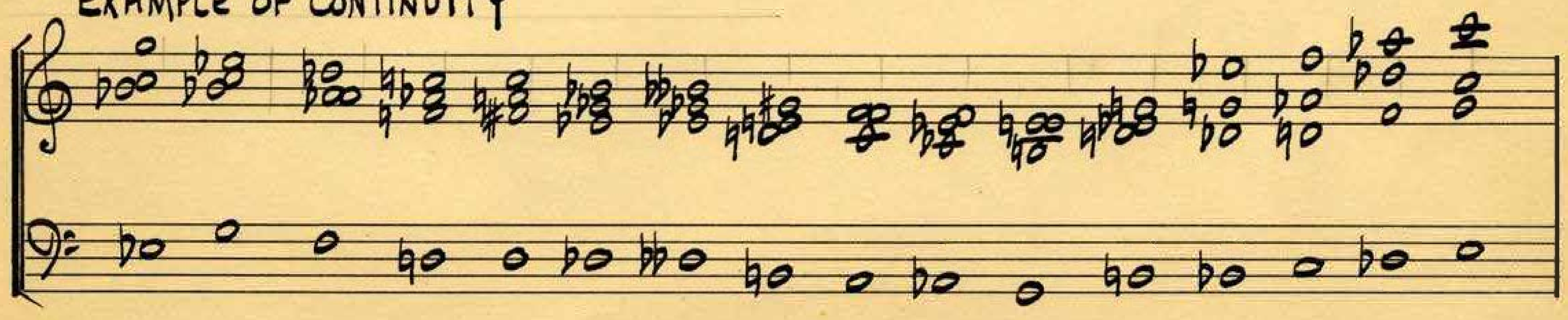


EXAMPLE OF CONTINUITY.

Twelve Tonics ($\sqrt[12]{2}$)

Closed system. All specifications and applications as in Six Tonics.

EXAMPLE OF CONTINUITY



Lesson LXXXIX.

Hybrid Five-Part Harmony

The technique of continuous S(7) makes it possible to evolve a hybrid five-part harmony, where bass is a constant root tone and the four upper functions assume variable forms of S(7) with respect to bass.

By placing an S(7) either on root, or third, or fifth, or seventh of the bass root we obtain all forms of S in five-part harmony. An S(5) has to be represented with the addition of 13^{th} (the so-called "added sixth").

Forms of Chords in Hybrid Five-Part (4 + 1) Harmony

The 4 Upper Parts.	5	7	9	11	13
	3	5	7	9	11
	1	3	5	7	9
S	13	1	3	5	7
The Bass	1	1	1	1	1
The Forms of Tension	S(5)	S(7)	S(9)	S(11)	S(13)

It is possible to move continuously either form or any of the combinations of forms in any

rhythmic form of continuity. It is important to realize that the tonal cycles do not correspond in the upper four parts to the tonal cycles in the bass when the forms of tension are variable. For example, f - a - c - e may be 3 - 5 - 7 - 9 in a DS(9) as well as 7 - 9 - 11 - 13 in a GS(13). In such a case a progression C_5 for the bass with S(9) ⟶ S(13) produces C_0 for the upper four parts.

The principle of exchange and octave-inversion of the common tones holds true.

Three forms of harmonic continuity will be used in the following illustrations (these forms of continuity are applicable in the four-part harmony as well). When chord structures acquire greater tension and also when the compensation for the diatonic deficiency is required, it is often desirable to use preselected forms of chord-structures yet <u>moving diatonically</u>. Such system has a bass belonging to one definite diatonic scale, while the chord structures acquire various accidentals in order to produce a definite sonority. In the general classification of the harmonic progressions the latter type is known as <u>diatonic-symmetric</u>.

<u>Three Types of Harmonic Progressions</u>

I. Diatonic

II. Diatonic-Symmetric

III. Symmetric

The following examples will be carried out in all three types of harmonic continuity. Constant and variable forms of tension will be offered.

In order to select a desirable form of structures for the forms of different tension it is advisable to select a scale first, as such a scale offers all forms of tension. For example, if the scale selected is $c - d - e - f^{\#} - g - a - b^{\flat}$,
$S(5) = c - e - g - a$; $S(7) = c - e - g - b^{\flat}$;
$S(9) = c - e - g - b^{\flat} - d$; $S(11) = c - g - b^{\flat} - d - f^{\#}$;
$S(13) = c - b^{\flat} - d - f^{\#} - a$.

Though the same scale would be <u>ideal</u> for the progression, it is not impossible and not very undesirable to use any other scale for the chord-progressions.

(please see following pages)

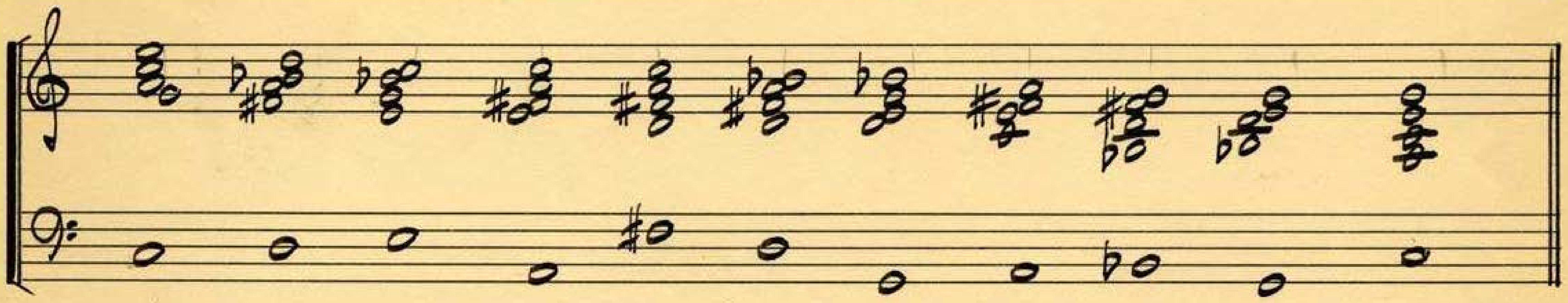
Hybrid Five-Part Harmony

(Tables and Examples)

(1) Continuity of S(5) [monomials]

Scale: c - d - e - f$^{\#}$ - g - a - b^{b}

Type I.



Type II.

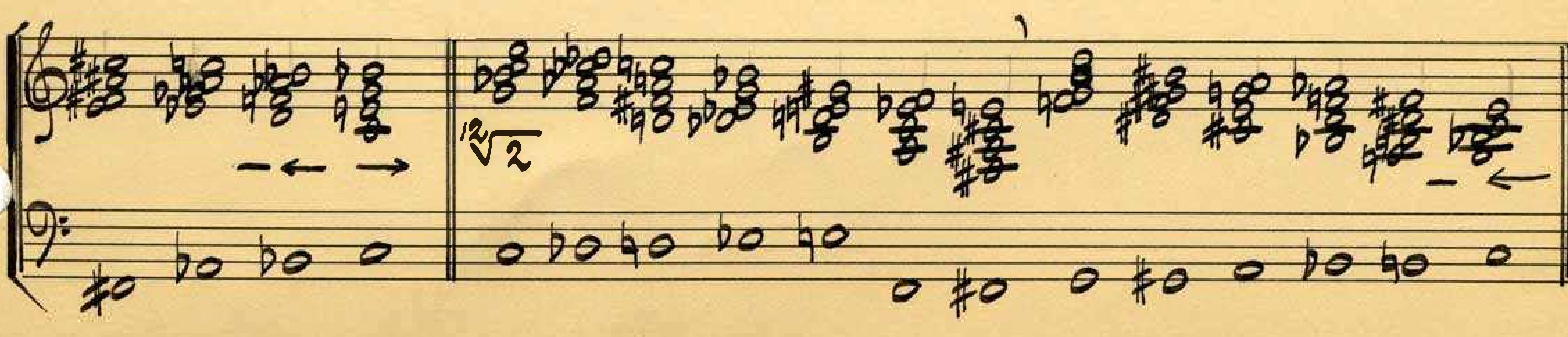
Type III.

(2) Continuity of S(7) [monomials]

Type I.

Type II.

Type III.

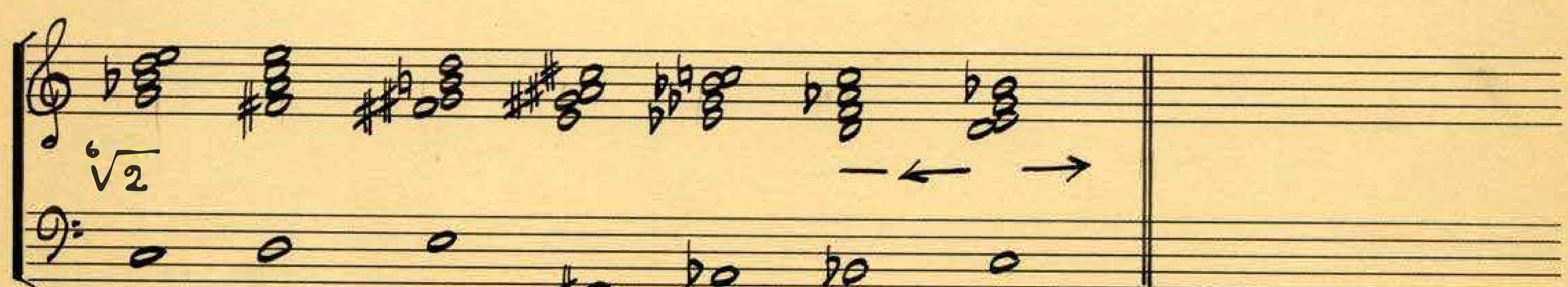


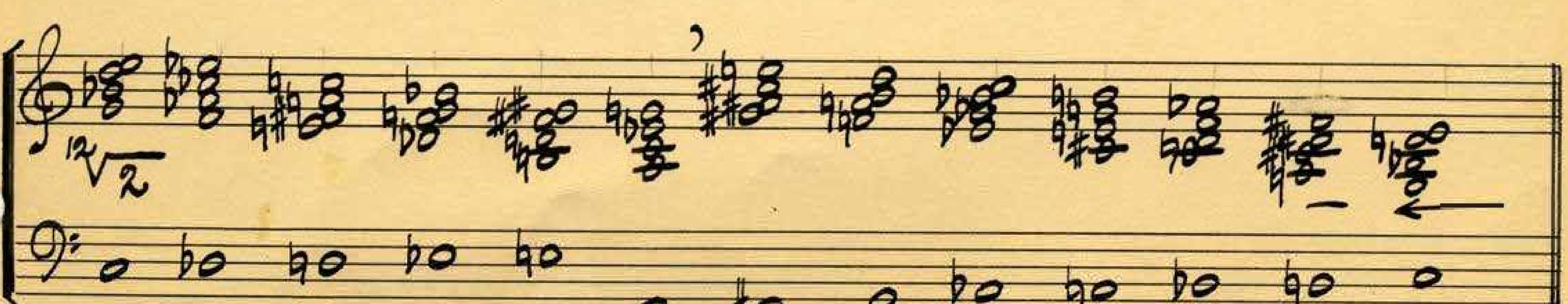
(3) Continuity of S(9) [monomials]

Type I.

Type II.

Type III.



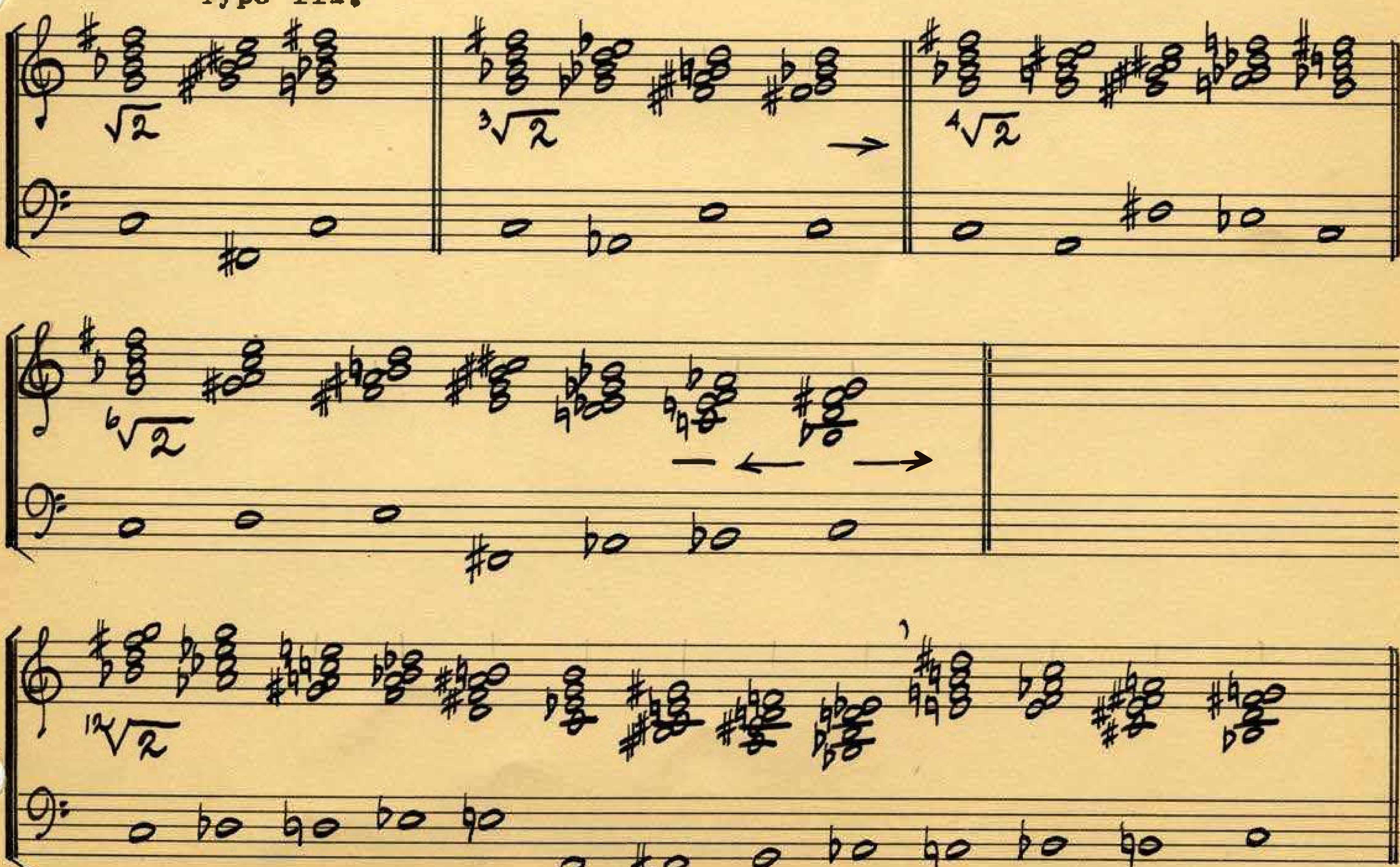


(4) Continuity of S(11) [monomials]

Type I.

Type II.

Type III.

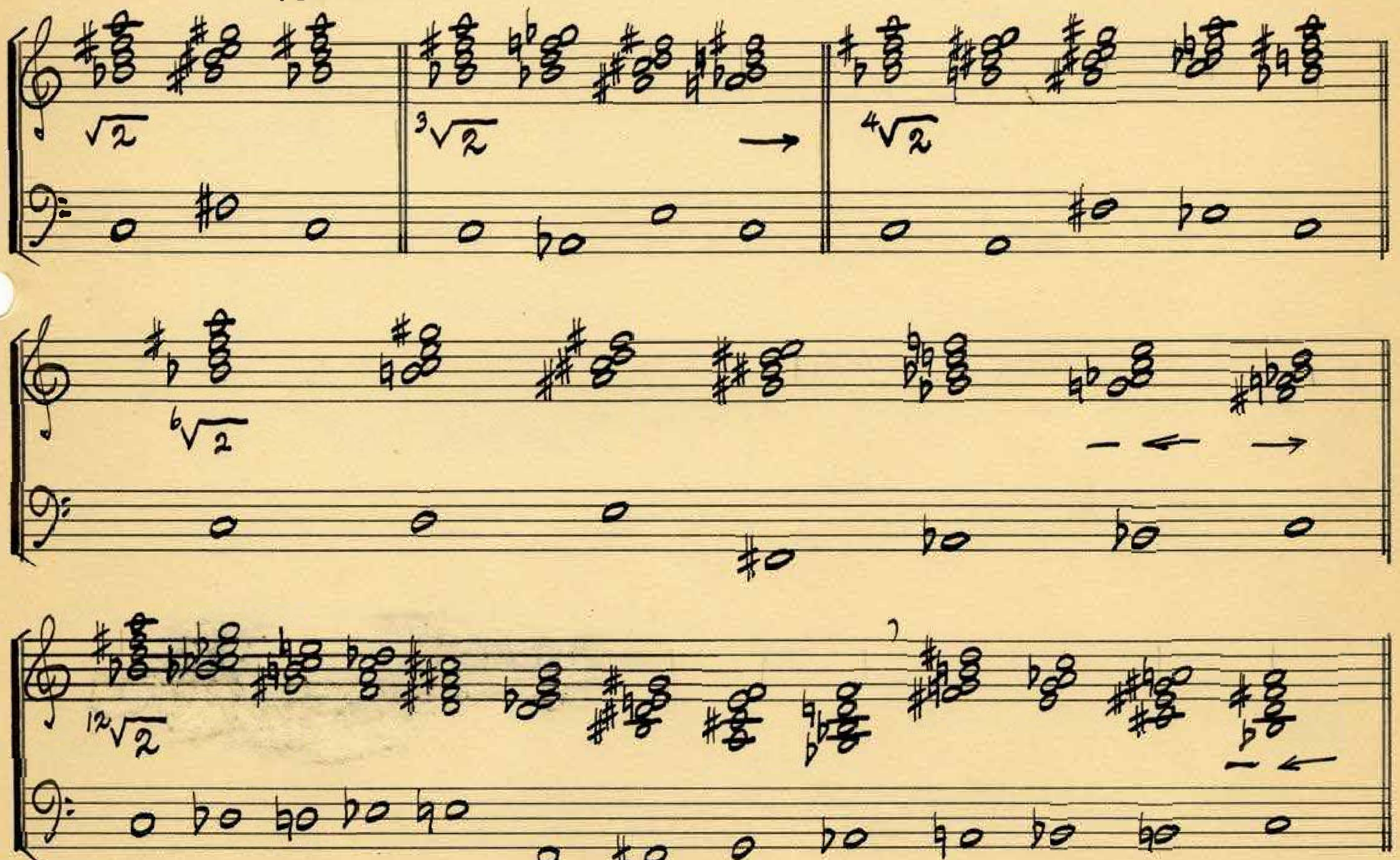


(5) Continuity of S(13) [monomials]

Type I.

Type II.

Type III.



Combinations by two (binomials), three (trinomials), four (quadrinomials) and five (quintinomials) may be devised in a similar way.

Table of Combinations

Arabic numbers in the following tables represent

Chord Structures (S)

Combinations by 2

5 + 7	7 + 9	9 + 11	11 + 13
5 + 9	7 + 11	9 + 13	
5 + 11	7 + 13		
5 + 13			

10 combinations, 2 permutations each

Total: 10 x 2 = 20

Combinations by 3

5 + 7 + 9	7 + 9 + 11	9 + 11 + 13
5 + 7 + 11	7 + 9 + 13	
5 + 7 + 13	7 + 11 + 13	
5 + 9 + 11		
5 + 9 + 13		
5 + 11 + 13		

10 combinations, 6 permutations each

Total: 10 x 6 = 60

<u>Combinations by 4</u>

5 + 7 + 9 + 11

5 + 7 + 9 + 13

5 + 7 + 11 + 13

5 + 9 + 11 + 13

7 + 9 + 11 + 13

5 combinations, 24 permutations each

Total: 5 x 24 = 120

<u>Combinations by 5</u>

5 + 7 + 9 + 11 + 13

1 combination, 120 permutations

Total: 1 x 120 = 120

All other cases of trinomial, quadrinomial, quintinomial and bigger combinations are treated as coefficients of recurrence.

<u>Example:</u> $S^{\rightarrow} = 2S(5) + S(7) + 2S(9) =$

$= S(5) + S(5) + S(7) + S(9) + S(9)$,

i.e., a quintinomial with two identical pairs.

Coefficients of recurrence may be applied to the composition of continuity consisting of the forms of variable tension.

Examples

Type I. 2S(5) + S(9) + S(13) + 2S(7)

Type II.

Type III. $\sqrt[3]{2}$

Lesson XC.

Ninth-Chords. S(9)

Diatonic System

Ninth-chords in four-part harmony are used with the root-tone in the bass only, thus forming a hybrid four-part harmony [like S(5) with the doubled root]. The three upper parts consist of 3, 7 and 9. The seventh and the ninth are subject to resolution through the stepwise downward motion. If one function resolves at a time it is always the higher one (the ninth). A resolution of one function at a time produces C_0. Other cycles derive from the simultaneous resolutions of both functions (the ninth and the seventh). No consecutive S(9) are possible through this system [they alternate with S(7) and S(5)].

The reason for resolving the 9th and not the 7th first in C_0 is the latter results in a chord-structure alien to the usual seven-unit diatonic scales (the intervals in the three upper voices are fourths).

Positions of S(9)

As bass remains constant, the three upper voices are subject to 6 permutations resulting in the corresponding distributions.

Table of Positions of S(9)

Resolutions of S(9)

Resolutions (except C_0) produce positive cycles only. C_3 is characteristic of Mozart, Clementi and others of the same period. C_5 (the second resolution) is the most commonly known, especially with b♭

in the first chord (making a dominant chord of F-major of it). C_7 is characteristic of Bach and contrapuntalists. They achieved such progression through the idea of two pairs of voices moving in thirds in contrary motion. Read the last bar with $b^{\flat}$ and $f^{\#}$ and add S(5) g-minor. All these cases of resolution were known to the classics through melodic manipulations (contrapuntal heritage) and not through the idea of independent structures we call S(9).

Preparation of S(9) bears a great similarity with the preparation of S(7). There is even an absolute correspondence in the cycles with respect to technical procedures.

The same three methods constitute the technique of preparation (suspending, descending, ascending).

Table of Preparations.

(1) Suspending:

3 9	5 9	7 9
1 7	3 7	5 7
C_7	C_5	C_3

(2) Descending:

3 → 9	5 → 9	7 → 9
1 → 7	3 → 7	5 → 7
C_0	C_{-3}	C_{-5}

(3) **Ascending:**

3 ⟶ 9	5 ⟶ 9	7 ⟶ 9
1 ⟶ 7	3 ⟶ 7	5 ⟶ 7
C-3	C-5	C-7

Preparations of S(9)

It follows from the above chart that some of the preparations of S(9) require an S(5), some - S(7) and some allow both. It is practical to have S(5) or S(7) preparing S(9) with the root in the bass.

The first form of preparation was known to the classics as double suspension.

Example:

Similar cadence was used in major.

Another example of a characteristic classical cadence:

Example of Continuity Containing S(9):

Homework:

(1) Make complete tables of preparations and resolutions from all positions.

(2) Write diatonic continuity containing S(9).

(3) Make some modal transpositions of the examples thus obtained.

(4) Write continuity containing S(9) in the second type (diatonic-symmetric) of harmony. Select chord-structures from the examples of hybrid five-part harmony.

Lesson XCI.

Ninth-Chords, S(9)

Symmetric System.

The above described classical (preparation-resolution) technique commonly used in the diatonic system is applicable to the symmetric system as well. Symmetric roots correspond to the respective cycles: C_5 - to $\sqrt{2}$; C_3 - to $\sqrt[3]{2}$ and $\sqrt[4]{2}$; C_7 - to $\sqrt[6]{2}$ and $\sqrt[12]{2}$. With this in view, continuity consisting of S(5), S(7) and S(9) and operated through the classical technique may be offered.

Symmetric C_0 is quite fruitless when S(9) alone is used, as the upper three functions (3, 7, 9) produce an incomplete seventh-chord, the permutations of which ($3 \leftrightarrow 7$, $3 \leftrightarrow 9$) sound awkward with the exception of one: $7 \leftrightarrow 9$.

As S(9) in the hybrid four-part harmony is an incomplete structure (5 is omitted), the adjectives may be applied only with a certain allowance for the 5th.

There are two distinctly different families of S(9) not to be mixed except when in C_0:

(1) <u>The minor seventh family</u>

(2) <u>The major seventh family</u>

The minor 7th family includes the following structures:

You may attribute to them the following adjectives in their respective order:

$7^{\flat}S_1$ - large

$7^{\flat}S_2$ - diminished

$7^{\flat}S_3$ - minor

$7^{\flat}S_4$ - small

The major 7th family includes the following structures:

Their respective adjectives are:

$7^{\natural}S_1$ - major

$7^{\natural}S_2$ - augmented I

$7^{\natural}S_3$ - augmented II

These are the only possible forms.

It seems that all combinations of the two families, except the ones producing consecutive seventh (${}_7\flat S_4 \leftrightarrow {}_7\natural S_1$; ${}_7\natural S_2 \leftrightarrow {}_7\natural S_3$; ${}_7\flat S_3 \leftrightarrow {}_7\natural S_2$; ${}_7\flat S_1 \leftrightarrow {}_7\flat S_4$), are satisfactory when in C_0. On the different roots the forms of S(9) must belong to one family.

Example of C_0 Continuity:

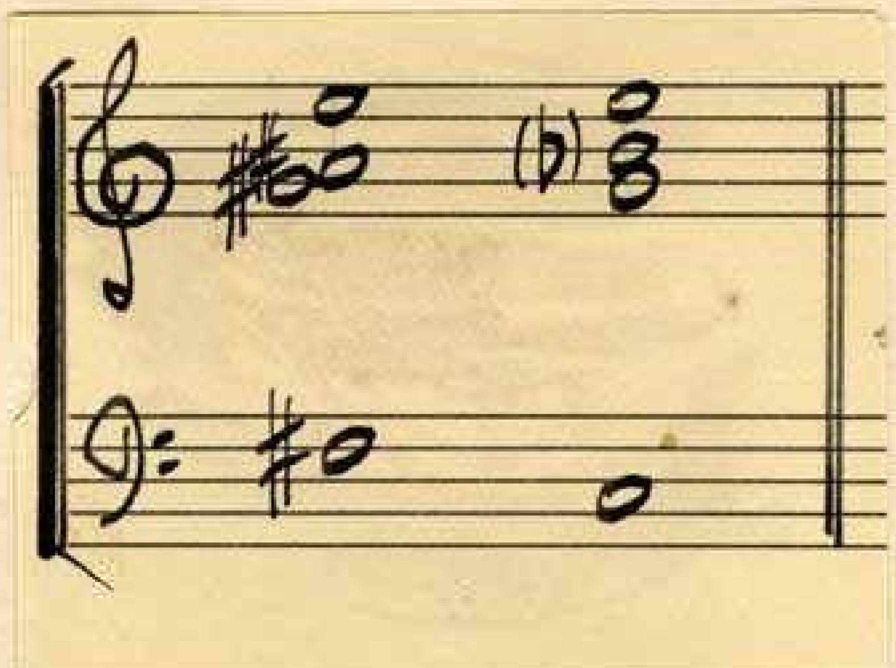
Full indication for S(9) when used in combinations with S(5) and S(7):

${}_7\flat S_1(9)$; ${}_7\flat S_2(9)$; ${}_7\flat S_3(9)$; ${}_7\flat S_4(9)$

${}_7\natural S_1(9)$; ${}_7\natural S_2(9)$; ${}_7\natural S_3(9)$.

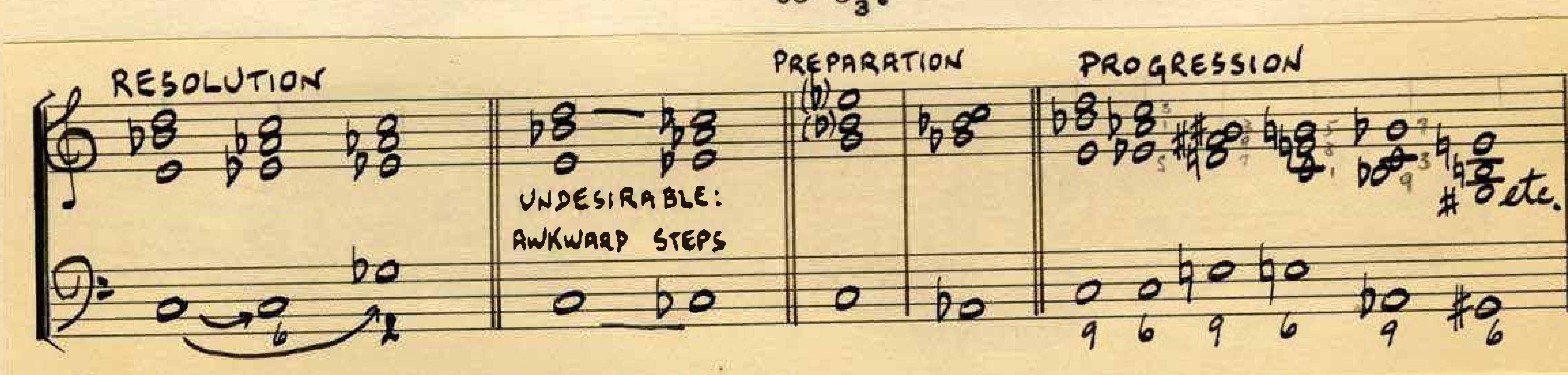
Two Tonics ($\sqrt{2}$). The technique corresponds to C_5.

To resolve the last chord of the preceding table use position (b) of the resolution technique.



Example of Continuity:

Three Tonics ($\sqrt[3]{2}$). The technique corresponds to C_3.



In order to acquire a complete understanding of the voice-leading in the preceding table of progressions (9 - 6 - 9 - 6 etc.), reconstruct mentally an S(7) instead of an S(6). Then the first two chords will appear in the following positions:

It is clear now that $d^{\sharp}$ and $f^{\times}$ are the necessary 7 and 9 of the following chord.

Example of Continuity:

Four Tonics ($\sqrt[4]{2}$). The technique corresponds to C_3.

Resolution Preparation Progression

Example of Continuity:

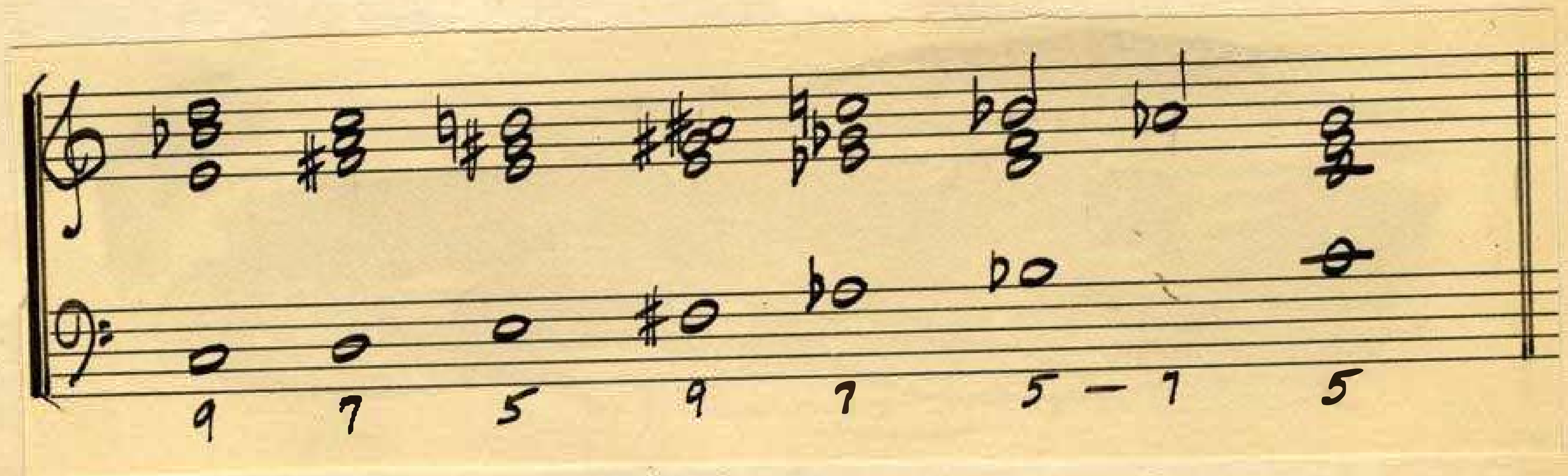
Six Tonics ($\sqrt[6]{2}$). The technique corresponds to C_7.

Resolution Preparation Progression

The above consecutive sevenths are unavoidable with this technique.

The position of every S(9) is based on the assumption that the preceding chord was S(5) and not S(7).

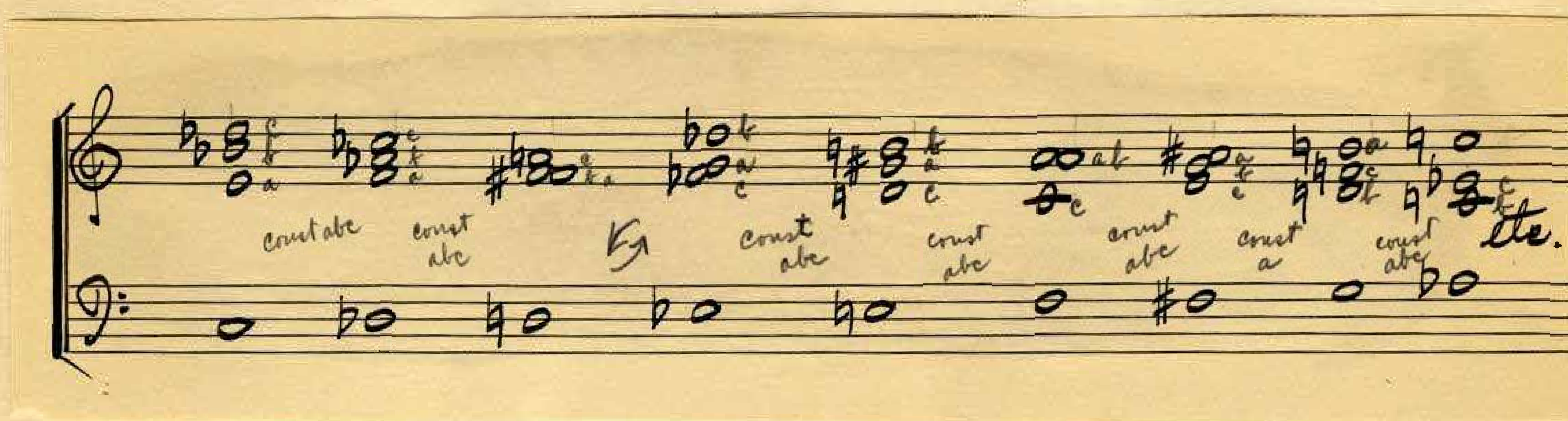
Continuity: S(9) + S(7) + S(5)



The negative system which may be obtained by reading the above tables in position b is not as desirable with these media as the positive. The same concerns the following $\sqrt[12]{2}$. More plastic devices (general forms of transformations) will be offered later.

Twelve Tonics ($\sqrt[12]{2}$). The technique corresponds to C_7.

Continuity: S(9) + S(7) + S(5)



Homework: Exercises in the different symmetric systems containing S(5), S(7) and S(9) with application of different structures and the C_0 between the roots.

JOSEPH SCHILLINGER

CORRESPONDENCE COURSE

With: Dr. Jerome Gross   Subject: Music

Lesson XCII.

Four-Part Harmony (Continuation)

Eleventh-Chords. S(11)

Diatonic System.

Eleventh-chords in four-part harmony are used with root-tone in the bass only, thus forming a hybrid four-part harmony [like S(5) with the doubled root]. The three upper parts consist of 7, 9, 11. An S(11) has an advantage over S(9) as the upper functions form a complete S(5). All three upper functions are subject to resolution through the stepwise downward motion. Resolutions of less than three upper functions produce C_0.

No consecutive S(11) are possible through this system. They alternate with the other structures.

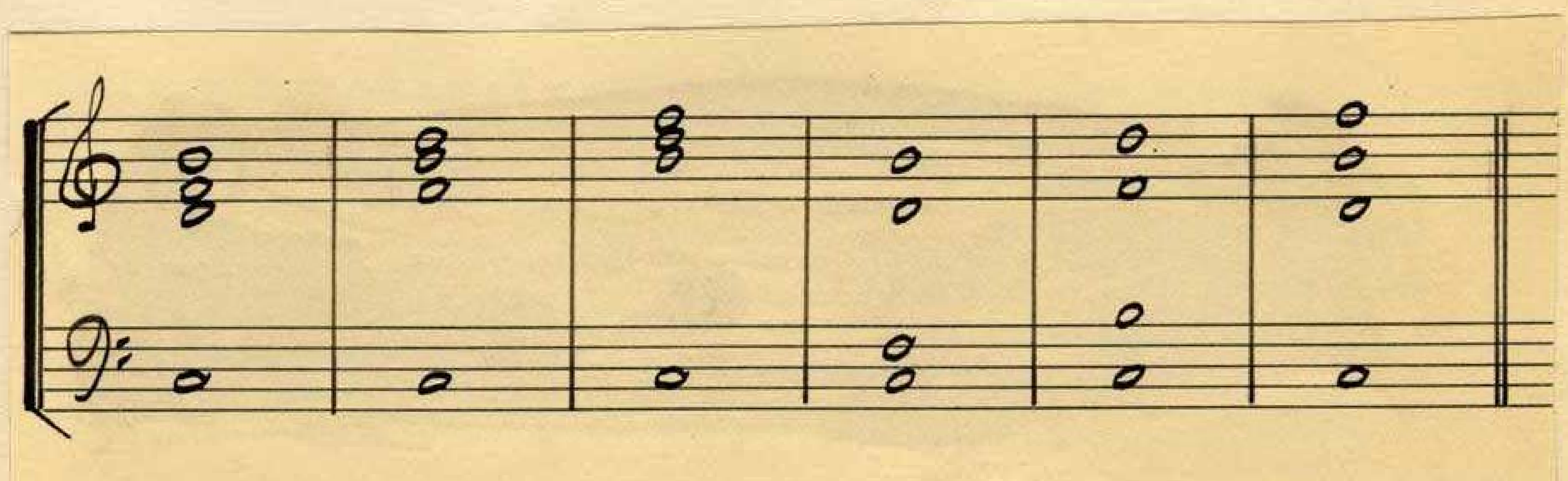
For the reasons explained in the previous chapter the C_0 resolutions must follow in the direction of the decreasing functions: first 11 must be resolved, then 9, then 7. When two functions resolve simultaneously they are 11 and 9. An S(11) allows a continuous chain of resolutions.

S(11) 11 $\rightarrow$ S(9) 9 $\rightarrow$ S(7) 7 $\rightarrow$ S(6) 3

An eleventh-chord through resolution of the eleventh becomes a ninth-chord; a ninth-chord through resolution of the ninth becomes an incomplete seventh-chord (without a fifth), or a complete $S\left(\begin{smallmatrix}4\\3\end{smallmatrix}\right)$ as in the corresponding resolutions of S(9); an incomplete seventh-chord through resolution of the seventh becomes a sixth-chord with the doubled third.

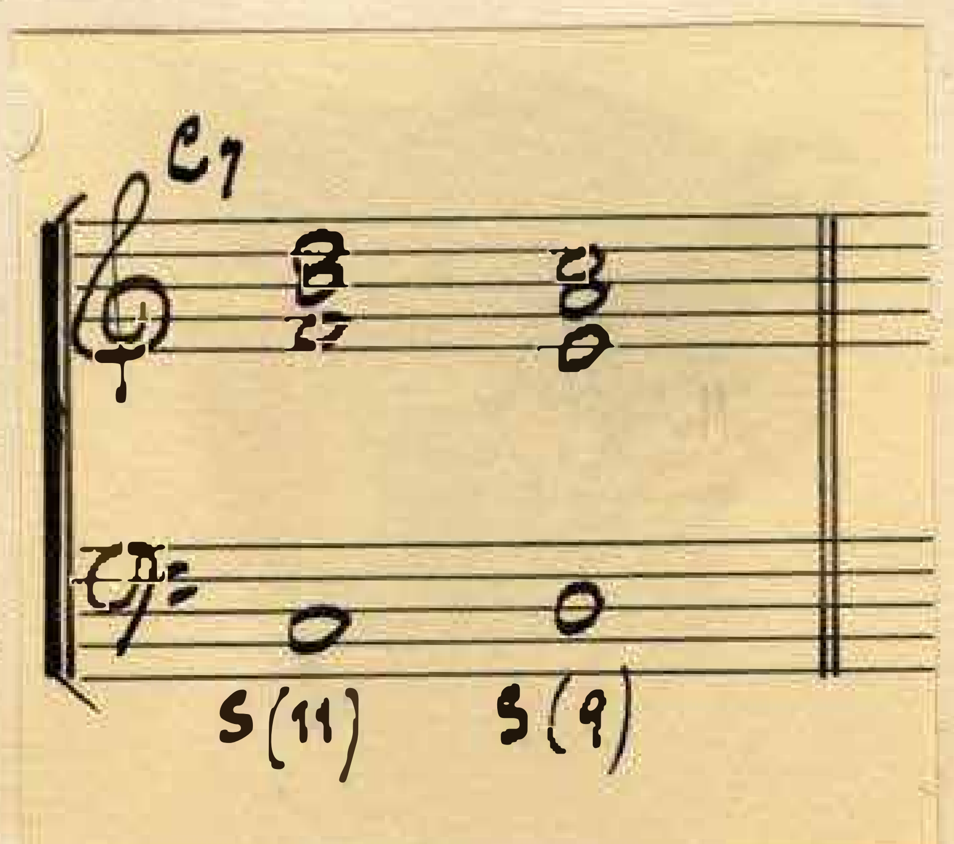
Positions of S(11).

As bass remains constant, the three upper voices are subject to 6 permutations. Seventh, ninth and eleventh form a triad corresponding to a root, a third and a fifth while the bass is placed one degree higher. A c S(11) has an appearance of b S(5) with a bass raised one step.



Resolutions of S(11).

As it follows from the above table, when S(11) resolves into S(9) in C_0, S(9) has its proper structural constitution (i.e., 1, 3, 7, 9). For the same reason the C_7 resolution does not appear on this table, as the structural constitution of S(9), into which S(11) would resolve, is 1, 5, 7, 9 and this does not sound satisfactory according to our musical habits.



The above resolutions correspond to the classical resolutions of the triple suspensions.

Preparation of S(11) in the positive cycles has the cyclic correspondence with the preparation of S(7) and S(9) through suspensions. Nevertheless the manner of reasoning is somewhat different in this case.

As S(11) has an appearance of an S(5) with a bass placed one step higher, the most logical assumption is: take S(5), move its bass one step up and this will produce an S(11) of a proper structural constitution. In such a case the relation of the three stationary upper functions is C_0. Being common tones they may be inverted or exchanged.

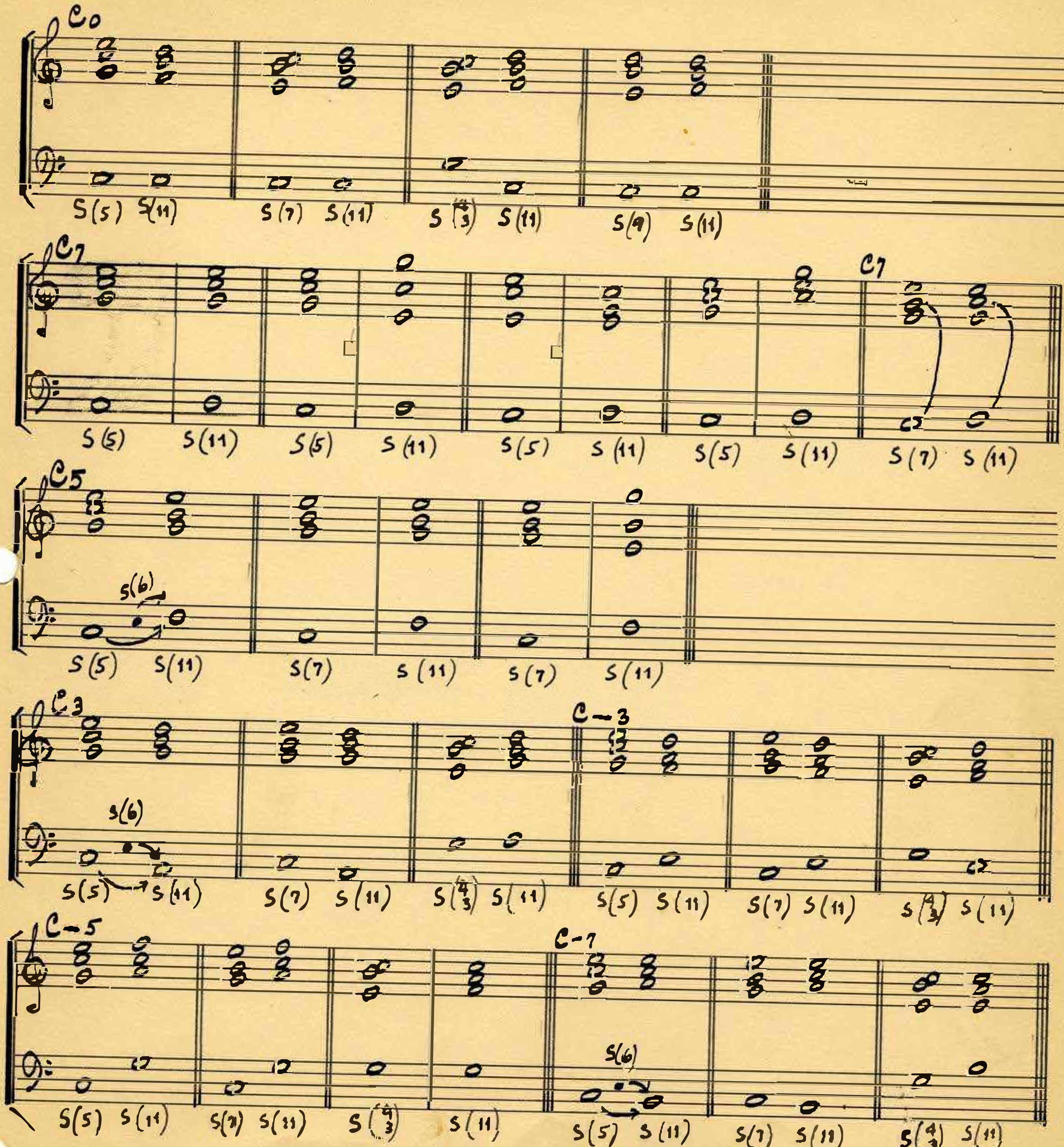
The first case gives a clue to the preparation of other cycles (positive and negative as well).

The method of preparation implies merely the most gradual transformation ($\rightleftarrows$ or $\leftrightarrows$) for the three upper functions.

To prepare S(11) after an S(5) in C_0 move all upper functions down scalewise and leave the bass stationary (which is the converse of the first proposition).

(please see next page)

Preparations of S(11)



When all tones are in common in the three upper parts it is advisable to use the suspension (over the bar) method.

When some of the upper parts move and some remain stationary either the within the bar or the over the bar preparation may be used.

Characteristic progressions and cadences where all forms of tension [from S(5) to S(11)] are applied:

(please see next page)

Example of Continuity Containing S(11).

Lesson XCIII.

Eleventh-Chords. S(11)

Symmetric System.

The above described technique of diatonic progressions containing S(11) is applicable to the symmetric system as well. The cyclic correspondence previously used remains the same. Thus preparations of S(11) are possible in all systems of the symmetric roots, while resolutions can be performed only when the acting cycle is C_3 ($\sqrt[3]{2}$ and $\sqrt[4]{2}$) and C_5 ($\sqrt{2}$). There is no difficulty with any preparation of S(11) after a resolution, as the latter always consists of 1, 3, 5 and therefore may be connected with the following chord through the usual transformations.

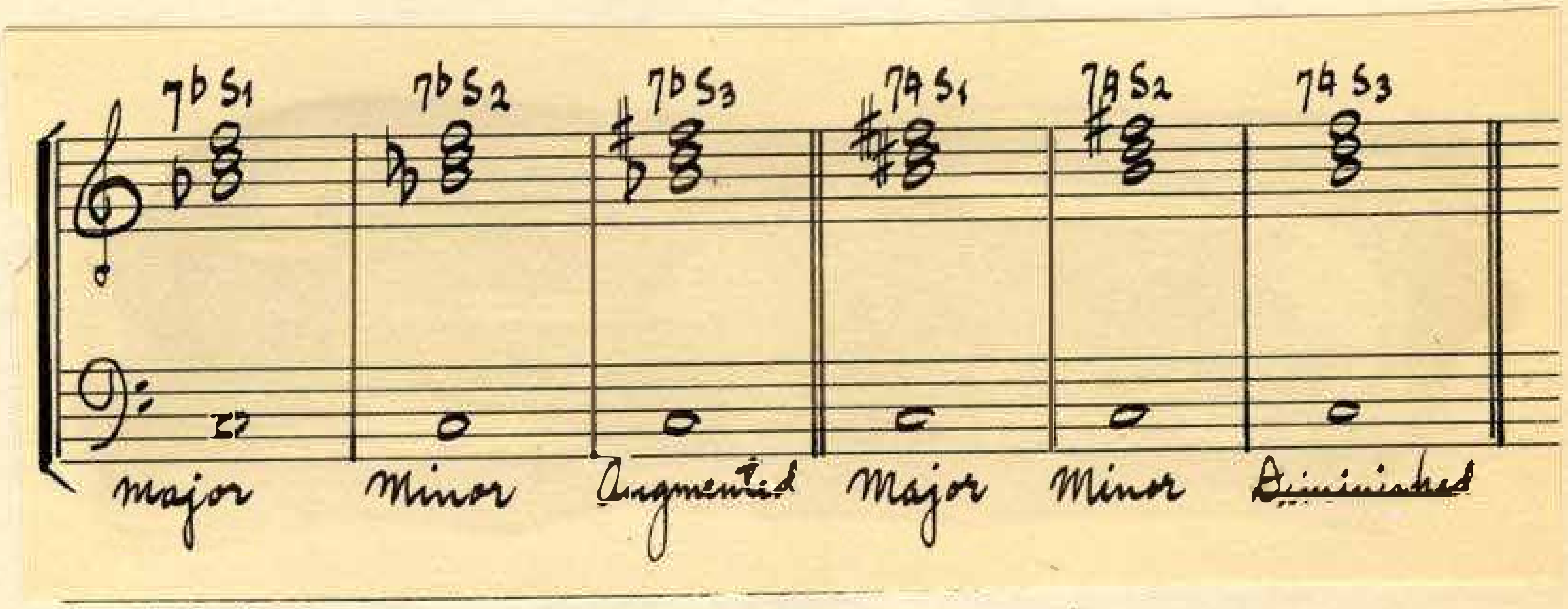
Contrary to S(9), S(11) produces a highly satisfactory C_0, due to the presence of all functions without gaps in the three upper parts.

As in the ninth-chords, there are two distinctly different families of S(11) not to be mixed except when in C_0. The distinction becomes even greater than before and the danger of mixing more dangerous.

The structural constitution of S(11) permits the classification of such structures as S(5) with regard to their three upper functions.

Forms of S(11).

The Minor Seventh Family — The Major Seventh Family



These are the only possible forms as the diminished in the first group equals (enharmonically) a diminished S(9) and the augmented in the second group equals (enharmonically) the second augmented S(9) with a fifth and without a third.

The selection of better progressions in C_0 for the continuity of S(11) must be analagous to the selection of forms for S(5). Consecutive seventh shall not be used.

Example of C_0 Continuity.

Full indications for S(11) when used in combinations with other structures:

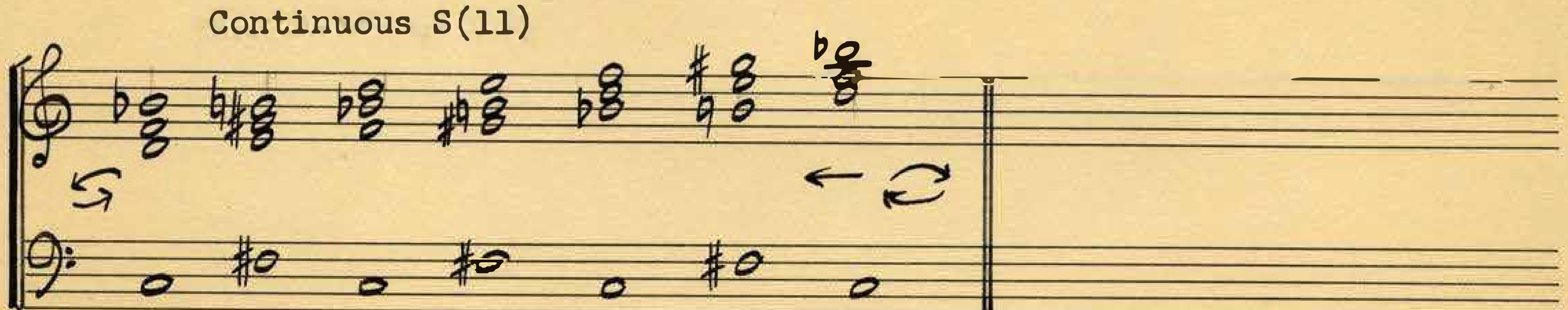
$7^{\flat}S_1(11)$; $7^{\flat}S_2(11)$; $7^{\flat}S_3(11)$

~~$7^{\natural}S_1(11)$; $7^{\natural}S_2(11)$; $7^{\natural}S_3(11)$~~

Two Tonics ($\sqrt{2}$). The technique corresponds to C_5. Clockwise or counterclockwise transformations for continuous S(11).

Resolution Preparation Progression

Continuous S(11)



You may consider the upper three parts either as 7, 9, 11 in ⇄ and ⇆ transformations or as 1, 3, 5 with a displaced bass.

Example of Continuity.

Three Tonics ($\sqrt[3]{2}$). The technique corresponds to C_3 or to the ⇄ and ⇆ transformations.

Resolution Preparation Progression

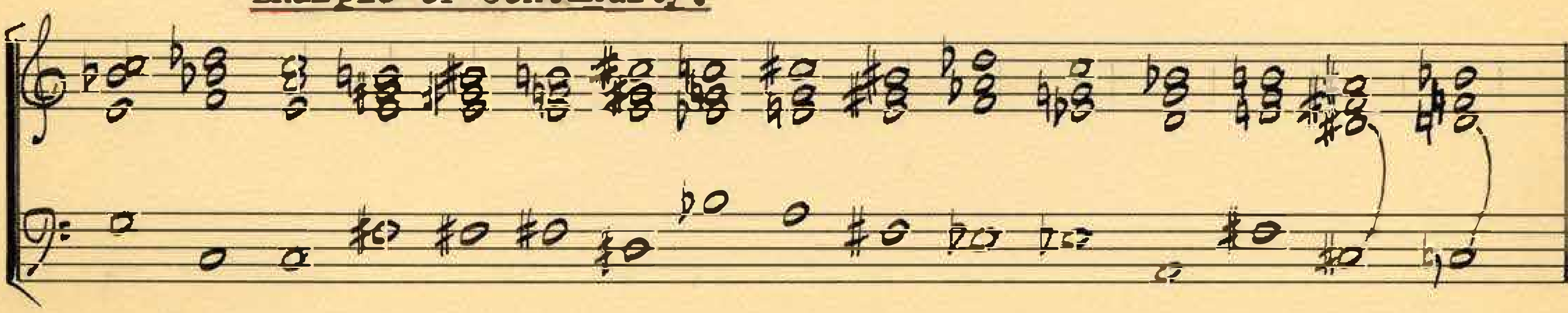
Progression

Continuous S(11)

Example of Continuity.

Four Tonics ($\sqrt[4]{2}$). The technique corresponds to C_3 or to the ⇄ and ⟲ transformations.

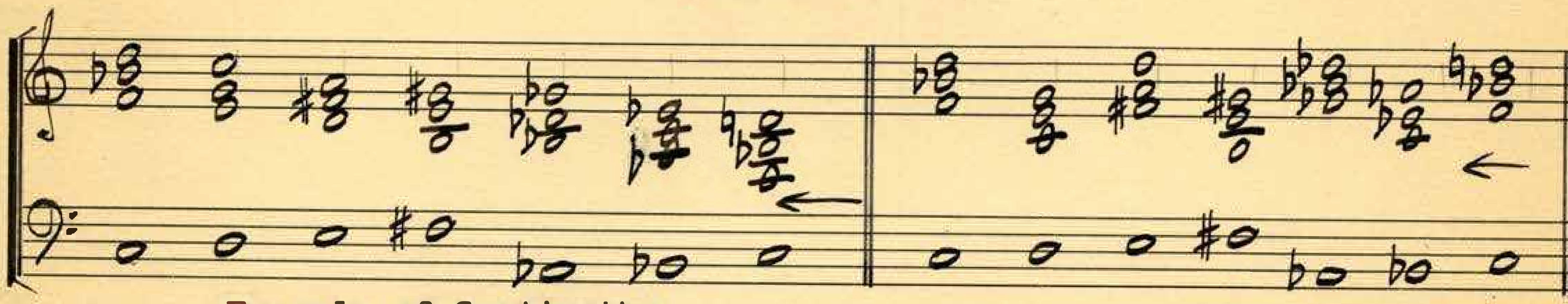
Example of Continuity.



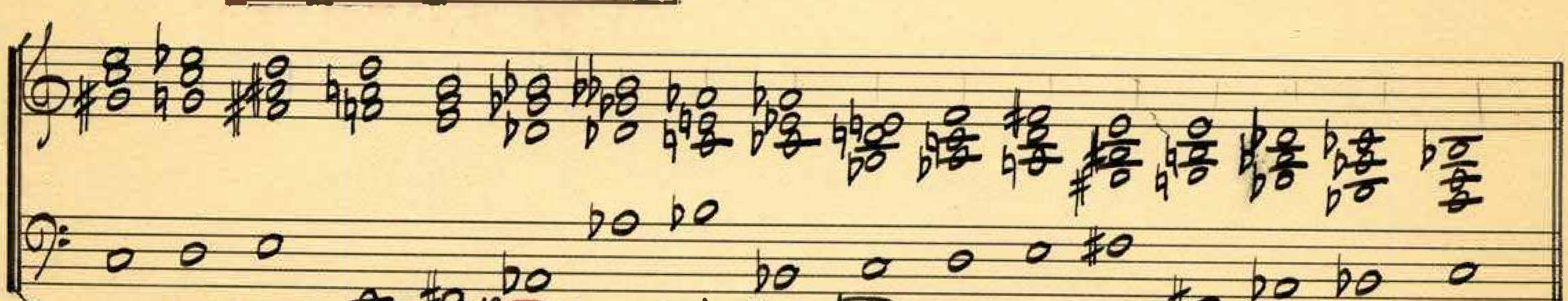
With the complexity of the harmony above, the consecutive ninths (if they both are major and move on a whole tone) are perfectly admissible.

Six Tonics ($\sqrt[6]{2}$). and transformations only.

Continuous S(11)



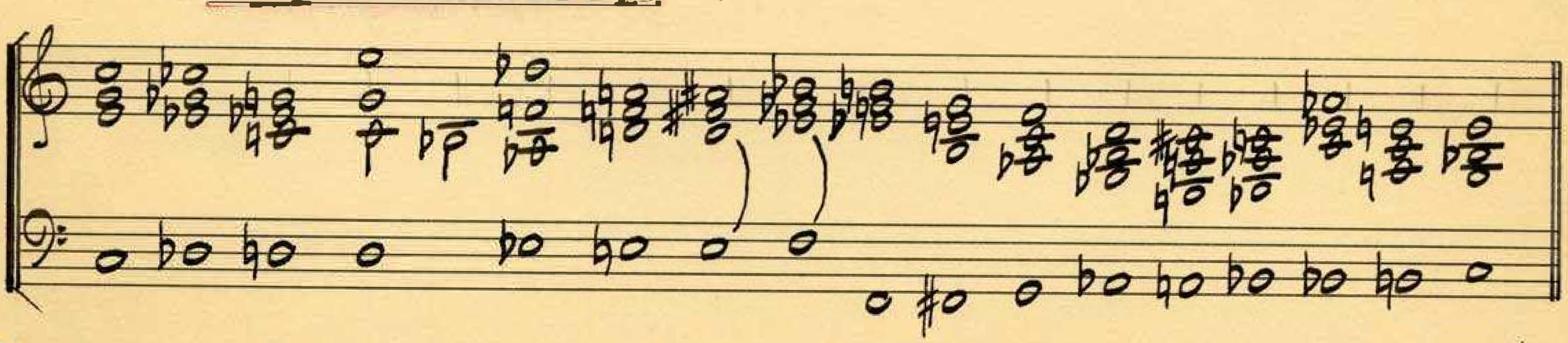
Example of Continuity.



Twelve Tonics ($\sqrt[12]{2}$). and transformations only.

Continuous S(11)

Example of Continuity.



Homework in the field of S(11) must correspond to that of S(9), utilizing various structures, forms and progressions. The transformation technique is applicable to diatonic and diatonic-symmetric progressions as well.

Lesson XCIV.

Hybrid Four-Part Harmony

The general technique of transformations for the groups with three functions may be adopted for the generalization of the forms of voice-leading in a hybrid four-part harmony. The three upper parts perform the transformations corresponding to the groups with three functions, and the bass remains constant.

The following technique is applicable to any type of harmonic progression (diatonic, diatonic-symmetric, symmetric). The specifications for the following forms of S are chosen with respect to their sonority. The ones marked with an asterisk in the following tables are less commonly used than the unmarked ones. The charts of transformations for the latter are worked out and you can easily supplement them for the ones marked with the asterisk.

(please see next page)

Forms of Hybrid Four-Part (3 + 1) Harmony

The Three upper parts.	5 3 1	5 3 13	7 5 3	7 3 1	9 7 3	9 7 1	11 9 7	13 9 7	13 11 7
The bass.	1	1	1	1	1	1	1	1	1
Forms of tension.	S(5)	* S(5)	S(7)	* S(7)	S(9)	* S(9)	S(11)	S(13)	* S(13)

When the numerals expressing the functions in a group are identical with the numerals of the following group, certain forms of transformation, such as constant abc, have to be eliminated on account of complete parallelism. When the numerals in the two allied groups are partly identical some of the forms (constant a, constant b, constant c) give either favorable or unfavorable partial parallelisms. The partial parallelisms are favorable when the parallel motion of functions forms desirable intervals with the bass. They are unfavorable when it causes consecutive motion of the seventh or ninth with the bass (consecutive seventh, consecutive ninth).

As the actual quality of voice-leading depends on the structures of the two allied chords, upon completion of all these charts in musical notation you will be able to make your preferential selection.

When the numerals in the two allied groups are either partly or totally different, often the constant abc transformation becomes the most favorable form of voice-leading. There is a natural compensation in this case: homogeneous structures are compensated by heterogeneous transformations and heterogeneous structures are compensated by homogeneous transformations. For example, if the allied groups both are S(5) the constant abc transformation would be impossible: $1 \rightarrow 1$, $3 \rightarrow 3$, $5 \rightarrow 5$, which gives consecutive octaves and fifths. On the contrary, when the functions have different numerals you acquire the smoothest voice-leading through this particular transformation. When two allied groups have different or partly different numerals for their functions, the first group becomes the <u>original</u> group and the following group becomes the <u>prime</u> group. When a transformation between such two groups is performed the prime group in turn becomes the original group for the next transformation.

The Original Group	The Prime Group
a	a'
c b	c' b'

For example, by connecting S(5) + S(9) + + S(13) we obtain the following numerals in their corresponding order:

S(5)		S(9)		S(13)	
1		3		7	
5	3	9	7	13	9

When you connect the functions of S(5) with the functions of S(9) the first group is the original group, and the second -- the prime group. When you connect the functions of S(9) with S(13) the functions of S(9) form the original group, and the functions of S(13) -- the prime group.

Here is a complete table of transformations.

<u>Forms of Transformations</u>

<u>in the Homogeneous Groups</u>

		Const. a	Const. b	Const. c	Const. abc
a c b	a c b	a c b	a c b	a c b	a c b
a → b	a → c	a → a	a → c	a → b	a → a
b → c	c → b	b → c	b → b	b → a	b → b
c → a	b → a	c → b	c → a	c → c	c → c

Forms of Transformations in the Heterogeneous Groups

The Original Group.

a

c b

The Prime Group.

a'

c' b'

		Const. a	Const. b	Const. c	Const. abc
a → b'	a → c'	a → a'	a → c'	a → b'	a → a'
b → c'	c → b'	b → c'	b → b'	b → a'	b → b'
c → a'	b → a'	c → b'	c → a'	c → c'	c → c'

Lesson XCV.

Here are all the combinations for the two allied groups taken, applied to all forms of tension.

Binomial Combinations of the Original and the Prime Groups.

$S(5) \longleftrightarrow S(7)$	$S(7) \longleftrightarrow S(9)$	$S(9) \longleftrightarrow S(11)$	$S(11) \longleftrightarrow S(13)$
$S(5) \longleftrightarrow S(9)$	$S(7) \longleftrightarrow S(11)$	$S(9) \longleftrightarrow S(13)$	
$S(5) \longleftrightarrow S(11)$	$S(7) \longleftrightarrow S(13)$		
$S(5) \longleftrightarrow S(13)$			

10 Combinations, 2 permutations each.

Total number of cases: 10 x 2 = 20.

Table of transformations for the twenty binomials consisting of one original and one prime group. Each S tension is represented in this table by one structure only. The sequence of the forms of transformations in this table remains the same for all cases:
(1) $\rightleftarrows$; (2) $\leftrightarrows$; (3) Const. a; (4) Const. b;
(5) Const. c; (6) Const. abc.

		S(5) ⟶ S(7)			
1 → 5 3 → 7 5 → 3	1 → 7 3 → 3 5 → 5	1 → 3 3 → 7 5 → 5	1 → 7 3 → 5 5 → 3	1 → 5 3 → 3 5 → 7	1 → 3 3 → 5 5 → 7

		S(7) ⟶ S(5)			
3 → 3 5 → 5 7 → 1	3 → 5 5 → 1 7 → 3	3 → 1 5 → 5 7 → 3	3 → 5 5 → 3 7 → 1	3 → 3 5 → 1 7 → 5	3 → 1 5 → 3 7 → 5

		S(5) ⟶ S(9)			
1 → 7 3 → 9 5 → 3	1 → 9 3 → 3 5 → 7	1 → 3 3 → 9 5 → 7	1 → 9 3 → 7 5 → 3	1 → 7 3 → 3 5 → 9	1 → 3 3 → 7 5 → 9

		S(9) ⟶ S(5)			
3 → 3 7 → 5 9 → 1	3 → 5 7 → 1 9 → 3	3 → 1 7 → 5 9 → 3	3 → 5 7 → 3 9 → 1	3 → 3 7 → 1 9 → 5	3 → 1 7 → 3 9 → 5

		S(5)⟶S(11)			
1⟶9 3⟶11 5⟶7	1⟶11 3⟶7 5⟶9	1⟶7 3⟶11 5⟶9	1⟶11 3⟶9 5⟶7	1⟶9 3⟶7 5⟶11	1⟶7 3⟶9 5⟶11

		S(11)⟶S(5)			
7⟶3 9⟶5 11⟶1	7⟶5 9⟶1 11⟶3	7⟶1 9⟶5 11⟶3	7⟶5 9⟶3 11⟶1	7⟶3 9⟶1 11⟶5	7⟶1 9⟶3 11⟶5

		S(5)⟶S(13)			
1⟶9 3⟶13 5⟶7	1⟶13 3⟶7 5⟶9	1⟶7 3⟶13 5⟶9	1⟶13 3⟶9 5⟶7	1⟶9 3⟶7 5⟶13	1⟶7 3⟶9 5⟶13

		S(13)⟶S(5)			
7⟶3 9⟶5 13⟶1	7⟶5 9⟶1 13⟶3	7⟶1 9⟶5 13⟶3	7⟶5 9⟶3 13⟶1	7⟶3 9⟶1 13⟶5	7⟶1 9⟶3 13⟶5

$S(7) \longrightarrow S(9)$

$3 \to 7$	$3 \to 9$	$3 \to 3$	$3 \to 9$	$3 \to 7$	$3 \to 3$
$5 \to 9$	$5 \to 3$	$5 \to 9$	$5 \to 7$	$5 \to 3$	$5 \to 7$
$7 \to 3$	$7 \to 7$	$7 \to 7$	$7 \to 3$	$7 \to 9$	$7 \to 9$

$S(9) \longrightarrow S(7)$

$3 \to 5$	$3 \to 7$	$3 \to 3$	$3 \to 7$	$3 \to 5$	$3 \to 3$
$7 \to 7$	$7 \to 3$	$7 \to 7$	$7 \to 5$	$7 \to 3$	$7 \to 5$
$9 \to 3$	$9 \to 5$	$9 \to 5$	$9 \to 3$	$9 \to 7$	$9 \to 7$

$S(7) \longrightarrow S(11)$

$3 \to 9$	$3 \to 11$	$3 \to 7$	$3 \to 11$	$3 \to 9$	$3 \to 7$
$5 \to 11$	$5 \to 7$	$5 \to 11$	$5 \to 9$	$5 \to 7$	$5 \to 9$
$7 \to 7$	$7 \to 9$	$7 \to 9$	$7 \to 7$	$7 \to 11$	$7 \to 11$

$S(11) \longrightarrow S(7)$

$7 \to 5$	$7 \to 7$	$7 \to 3$	$7 \to 7$	$7 \to 5$	$7 \to 3$
$9 \to 7$	$9 \to 3$	$9 \to 7$	$9 \to 5$	$9 \to 3$	$9 \to 5$
$11 \to 3$	$11 \to 5$	$11 \to 5$	$11 \to 3$	$11 \to 7$	$11 \to 7$

$S(7) \longrightarrow S(13)$

$3 \to 9$ $5 \to 13$ $7 \to 7$	$3 \to 13$ $5 \to 7$ $7 \to 9$	$3 \to 7$ $5 \to 13$ $7 \to 9$	$3 \to 13$ $5 \to 9$ $7 \to 7$	$3 \to 9$ $5 \to 7$ $7 \to 13$	$3 \to 7$ $5 \to 9$ $7 \to 13$

$S(13) \longrightarrow S(7)$

$7 \to 5$ $9 \to 7$ $13 \to 3$	$7 \to 7$ $9 \to 3$ $13 \to 5$	$7 \to 3$ $9 \to 7$ $13 \to 5$	$7 \to 7$ $9 \to 5$ $13 \to 3$	$7 \to 5$ $9 \to 3$ $13 \to 7$	$7 \to 3$ $9 \to 5$ $13 \to 7$

$S(9) \longrightarrow S(11)$

$3 \to 9$ $7 \to 11$ $9 \to 7$	$3 \to 11$ $7 \to 7$ $9 \to 9$	$3 \to 7$ $7 \to 11$ $9 \to 9$	$3 \to 11$ $7 \to 9$ $9 \to 7$	$3 \to 9$ $7 \to 7$ $9 \to 11$	$3 \to 7$ $7 \to 9$ $9 \to 11$

$S(11) \longrightarrow S(9)$

$7 \to 7$ $9 \to 9$ $11 \to 3$	$7 \to 9$ $9 \to 3$ $11 \to 7$	$7 \to 3$ $9 \to 9$ $11 \to 7$	$7 \to 9$ $9 \to 7$ $11 \to 3$	$7 \to 7$ $9 \to 3$ $11 \to 9$	$7 \to 3$ $9 \to 7$ $11 \to 9$

S(9) ⟶ S(13)

3 ⟶ 9	3 ⟶ 13	3 ⟶ 7	3 ⟶ 13	3 ⟶ 9	3 ⟶ 7
7 ⟶ 13	7 ⟶ 7	7 ⟶ 13	7 ⟶ 9	7 ⟶ 7	7 ⟶ 9
9 ⟶ 7	9 ⟶ 9	9 ⟶ 9	9 ⟶ 7	9 ⟶ 13	9 ⟶ 13

S(13) ⟶ S(9)

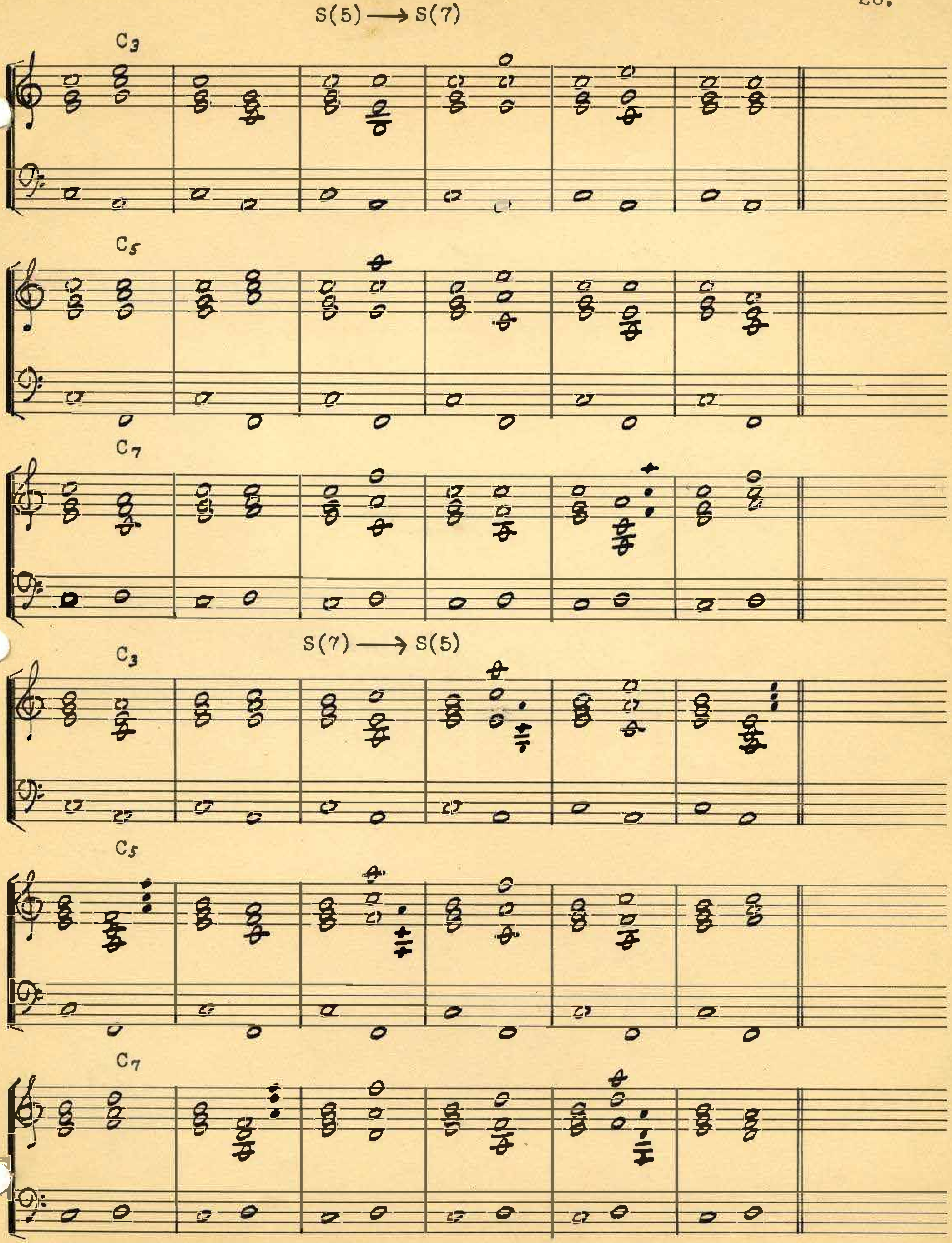
7 ⟶ 7	7 ⟶ 9	7 ⟶ 3	7 ⟶ 9	7 ⟶ 7	7 ⟶ 3
9 ⟶ 9	9 ⟶ 3	9 ⟶ 9	9 ⟶ 7	9 ⟶ 3	9 ⟶ 7
13 ⟶ 3	13 ⟶ 7	13 ⟶ 7	13 ⟶ 3	13 ⟶ 9	13 ⟶ 9

S(11) ⟶ S(13)

7 ⟶ 9	7 ⟶ 13	7 ⟶ 7	7 ⟶ 13	7 ⟶ 9	7 ⟶ 7
9 ⟶ 13	9 ⟶ 7	9 ⟶ 13	9 ⟶ 9	9 ⟶ 7	9 ⟶ 9
11 ⟶ 7	11 ⟶ 9	11 ⟶ 9	11 ⟶ 7	11 ⟶ 13	11 ⟶ 13

S(13) ⟶ S(11)

7 ⟶ 9	7 ⟶ 11	7 ⟶ 7	7 ⟶ 11	7 ⟶ 9	7 ⟶ 7
9 ⟶ 11	9 ⟶ 7	9 ⟶ 11	9 ⟶ 9	9 ⟶ 7	9 ⟶ 9
13 ⟶ 7	13 ⟶ 9	13 ⟶ 9	13 ⟶ 7	13 ⟶ 11	13 ⟶ 11


S(5) ⟶ S(7)
C3
C5
C7
S(7) ⟶ S(5)
C3
C5
C7

$S(5) \longrightarrow S(9)$

Please complete all tables.

Make additional tables for: $S(5) \longrightarrow S(5)$; $S(7) \longrightarrow S(7)$;
$S(9) \longrightarrow S(9)$; $S(11) \longrightarrow S(11)$;
$S(13) \longrightarrow S(13)$.

It is easy to work out all cases in musical notation applying each case to all three tonal cycles.

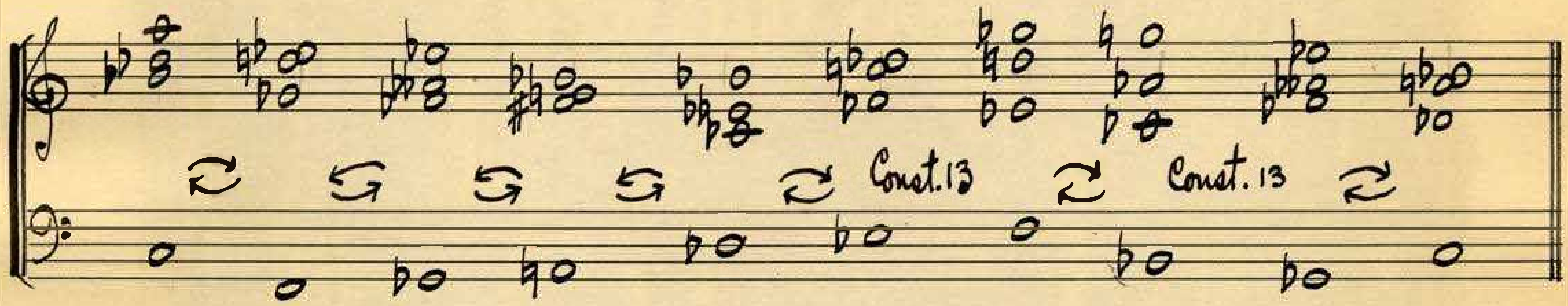
As in the previous cases, continuity may be composed in all three types of harmony (diatonic, diatonic-symmetric and symmetric). Structures of different tension may be selected for the composition of continuity. Different individual styles depend upon the coefficients of recurrence applied to the structures of different tension.

The first of the following two examples of continuity is produced through the structures of constant form and tension [S(13)], and the second -- illustrates continuity of variable forms and tensions distributed through $r_{3 \div 2}$.

(please see next page)

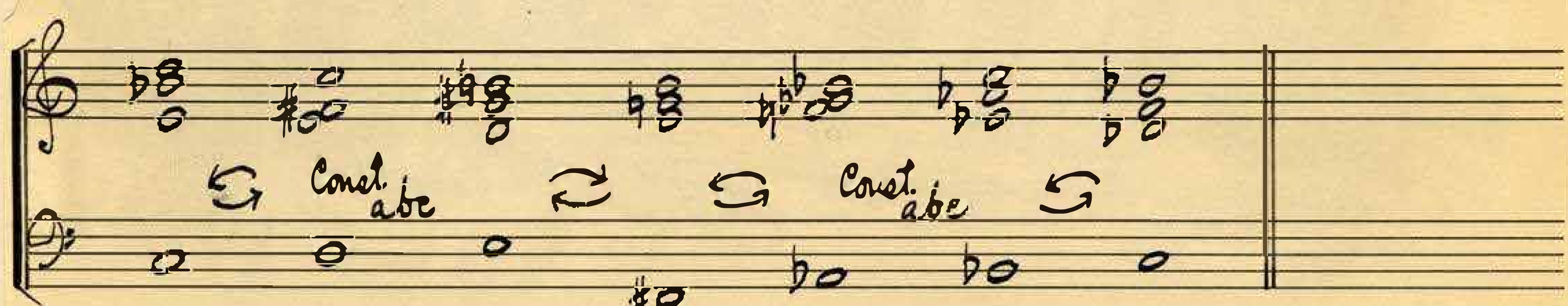
Continuity of Groups with Identical Functions

S(13) ⟶ Type II. Scale: $b^{\flat}$-harm., d_7.



Continuity of Groups with Different Functions

2S(9) + S(7) + S(13) + 2S(11); Type III. $\sqrt[6]{2}$
+ S(11)



<u>J O S E P H   S C H I L L I N G E R</u>

C O R R E S P O N D E N C E   C O U R S E

<u>With: Dr. Jerome Gross</u>   <u>Subject: Music</u>

<u>Lesson XCVI.</u>

<u>Generalization of Symmetric Progressions</u>

The forms of symmetric progressions heretofore used in this course of Harmony were based on monomial symmetry of the uniform intervals of an octave.

In order to obtain various mixtures (binomials, trinomials and polynomials) of the original forms of symmetry within an octave, it is necessary to establish a general nomenclature for all intervals of an octave. As all intervals are special cases of the twelve-fold symmetry, any diatonic form may be considered a special case of symmetry as well.

The system of enumeration of intervals may follow the upward or downward direction from any established axis point. As both directions include <u>all</u> intervals (which means both positive and negative tonal cycles), the matter of preference must be determined by the quantitative predominance of the type of intervals generally used. It seems that the descending system is more practical, as smaller numbers express

the positive steps on three and four tonics, and the negative -- on six and twelve tonics.

In the following exposition the descending system will be used exclusively. This does not prevent you from using the ascending system.

Scales of Intervals within one Octave Range:

Descending System:	Ascending System:
$c \longrightarrow c = 0$	$c \longrightarrow c = 0$
$c \longrightarrow b = 1$	$c \longrightarrow d^{\flat} = 1$
$c \longrightarrow b^{\flat} = 2$	$c \longrightarrow d = 2$
$c \longrightarrow a = 3$	$c \longrightarrow e^{\flat} = 3$
$c \longrightarrow a^{\flat} = 4$	$c \longrightarrow e = 4$
$c \longrightarrow g = 5$	$c \longrightarrow f = 5$
$c \longrightarrow f^{\sharp} = 6$	$c \longrightarrow f^{\sharp} = 6$
$c \longrightarrow f = 7$	$c \longrightarrow g = 7$
$c \longrightarrow e = 8$	$c \longrightarrow a^{\flat} = 8$
$c \longrightarrow e^{\flat} = 9$	$c \longrightarrow a = 9$
$c \longrightarrow d = 10$	$c \longrightarrow b^{\flat} = 10$
$c \longrightarrow d^{\flat} = 11$	$c \longrightarrow b = 11$
$c \longrightarrow c_{1} = 12$	$c \longrightarrow c^{1} = 12$

Monomials

Two Tonics: 6 + 6

Three Tonics: 4 + 4 + 4 or 8 + 8 + 8

Four Tonics: 3 + 3 + 3 + 3 or 9 + 9 + 9 + 9

Six Tonics: 2 + 2 + 2 + 2 + 2 + 2 or 10 + 10 + 10 + 10 + 10 + 10

Twelve Tonics: 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 or 11+11+11+11+11+11+11+11+11+11+11+11

Thus, each constant system of tonics becomes a form of monomial periodicity of a certain pitch-interval, expressible in the form of a constant number-value, which in turn expresses the quantity of semitones from the preceding pitch-unit.

In the light of this system the problem of mixing the various tonics (or any interval-steps in general) becomes reduced to the process of composing binomials, trinomials or any more extended groups (such as rhythmic resultants, their modifications through permutations and powers, series of growth), i.e., to the rhythmic distribution of steps.

The vitality of such groups, i.e., the quantity of their recurrence until the completion of their cycle, depends upon the divisibility-properties of the sums of their interval-quantities. The total sum of all number-values expressing the intervals becomes a divisor to 12, or any multiple thereof. This signifies the motion of a certain group through an octave (or octaves).

For example, a binomial 3 + 2 has 12 recurrences until it completes its cycle, as 3 + 2 = 5, and the smallest multiple of 12, divisible by 5 is 60. This is true of all prime numbers being used as divisors.

C – A – G – E – D – B – A – F♯ – E – C♯ – B

B – G♯ – F♯ – D♯ – C♯ – A♯ – G♯ – F – E♭

E♭ – C – B♭ – G – F – D – C

The above property makes the mixtures of three and four tonics very desirable when a long harmonic span is necessary without a need of the variety of steps.

The process of division serves as a testing tool of the vitality of compound symmetric groups.

Two tonics close after two cycles, as $6 + 6 = 12$, or $\frac{12}{6} = 2$;

$r_{4\div3}$ closes after one cycle, as $3 + 1 + 2 + 2 + 1 + 3 = 12$, and $\frac{12}{12} = 1$;

$r_{5\div4}$ closes after three cycles, as $4 + 1 + 3 + 2 + 2 + 3 + 1 + 4 = 20$, and $\frac{60}{20} = 3$.

A greater variety without deviating from a given style may be achieved by means of permutations of the members of a group. For example, a group with a short span may be revitalized through permutations:

(3+1+2) + (3+2+1) + (2+3+1) + (1+3+2) + (1+2+3) + (2+1+3)

or: C - A - $G^{\sharp}$ - $F^{\sharp}$ - $E^{\flat}$ - $D^{\flat}$ - C - $B^{\flat}$ - G - $F^{\sharp}$ - $F^{\natural}$ - D - C

C - $B^{\natural}$ - A - $F^{\sharp}$ - $E^{\natural}$ - $E^{\flat}$ - C

The selection of number values is left to the composer's discretion. If he wants to obtain a tonic-dominant character of classical music, the only thing he needs is the excess of the value 5.

Anyone equipped with this method can dodge the extremities by a cautious selection of the coefficients of recurrence. For instance, in order to produce the style of progressions which lies somewhere between Wagner and Ravel it is necessary to have the 5, the 3, and the 10 in a certain proportion, like

$2_3 + 5 + 10$, i.e.,

C - A - $F^{\sharp}$ - $C^{\sharp}$ - $D^{\sharp}$ - C - A - E - $F^{\sharp}$ etc.

Naturally, the selection of the tensions and the forms of structures in definite proportions is as important as the selection of the forms of progressions when a certain definite style must be produced.

On the other hand, this method offers a wonderful pastime, as one can produce chord progressions from any number combinations. Thus, a telephone directory becomes a source of inspiration.

Example

Columbus 5 - 7573

5 + 7 + 5 + 7 + 3 is equivalent to C - G - C - G - C - A.

This progression closes after 4 cycles:

C - G - C - G - C - A - E - A - E - A - F# -

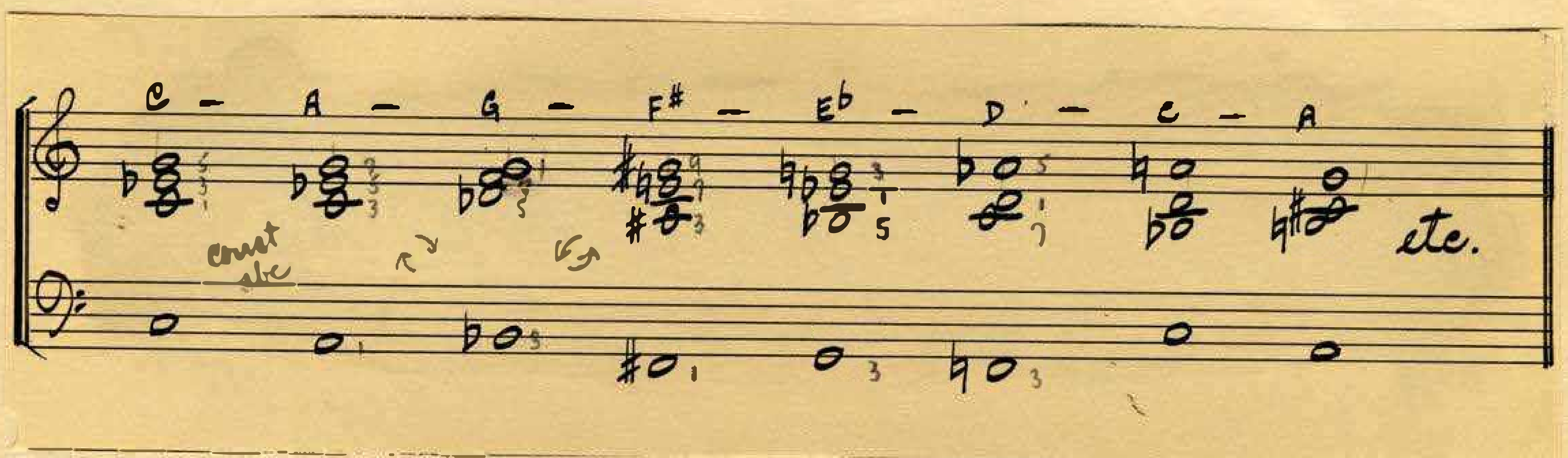
F# - C# - F# - C# - F# - D# - A# - D# - A# - D# - C

When zeros occur in a number-combination they represent zero-steps, i.e., zero cycles (C_0). Then the form of tension, the structure or the position of a chord has to be changed.

(please see next page)

Example of Continuity:

Progression: $r_{5 \div 3}$



Lesson XCVII.

Application of the Generalized Symmetric Progressions to Modulation

The rhythm of chord progressions expressed in number-values may serve the purpose of transition from one key to another. This procedure can be approached in two ways: (1) the connection concerns the tonic chords of the preceding and the following key; and (2) any chord of the preceding key, in its relation to any chord of the following key. The last case requires movement through diatonic cycles in both the preceding and the following key.

The technique of performing modulations, based on the rhythm of symmetric progressions, consists of two steps: (1) the detection of the number-value expressing the interval between the two chords, where such connection must be established; (2) composition of a rhythmic group from the numeral expressing the interval between the abovementioned chords. For example, if one wants to perform a modulation by means of symmetric progressions from the chord C (which may or may not be in the key of C) to the chord E♭ (which may or may not be in the key of E♭), the first procedure to perform is to compose rhythm from the interval 9. The knowledge of the Theory of

Rhythm offers many ways of composing such groups: composition of binomials, trinomials or larger groups from the original number, or any permutations thereof.

The quantity of the terms in a group will define the number of chords for the modulatory transition. Breaking up number 9 into binomials, we obtain: 8 + 1, 7 + 2, 6 + 3, 5 + 4, and their reciprocals. When a binomial is used in this sense, the two chords are connected through one intermediate chord. For example, taking 5 + 4 we acquire: C - G - E$^{\flat}$. If more chords are desired any other rhythmic group may be devised from number 9. For example, 4 + 1 + 4, which will give C - A$^{\flat}$ - G - E$^{\flat}$, i.e., two intermediate chords. When a number-value expressing the interval between the two chords to be connected through modulation is a small number, it is necessary to add the invariant 12. This places the same pitch-unit (or the root of the chord) into a different octave, without changing its intonation. For example, if a modulation from a chord of C to the chord of B$^{\flat}$ is required, such addition becomes very desirable.

$$C \longrightarrow B^{\flat} = 2$$

$$B^{\flat} \longrightarrow B_1^{\flat} = 12$$

$$12 + 2 = 14$$

Some possible rhythms derived from the value 14:

$7 + 7 = C - F - B^{\flat}$

$5 + 2 + 2 + 5 = C - G - F - E^{\flat} - B^{\flat}$

In cases like this rhythmic resultants may be used as well, providing the necessary changes are made.

$r_{4 \div 3} = 3 + 1 + 2 + 2 + 1 + 3$

Readjustment:

$3 + 1 + 2 + 2 + 1 + 3 + 2 = C - A - A^{\flat} - F^{\sharp} - F^{\flat} - E^{\flat} - C - B^{\flat}$

Or:

$r_{5 \div 3} = 3 + 2 + 1 + 3 + 1 + 2 + 3$

Readjustment:

$3 + 2 + 1 + 2 + 1 + 2 + 3 = C - A - G - F^{\sharp} - E - E^{\flat} - D^{\flat} - B^{\flat}$

Thus, all these procedures guarantee the appearance of the desirable $B^{\flat}$ point.

When a modulation of still greater extension is required, the invariant of addition becomes 24, 36, or even a higher multiple of 12, from which rhythmic groups may be composed.

Many persons engaged in the work of arranging find this type of transition more effective than the modulations proper. Naturally, the selection of the structures of different tension and form may be made according to the requirements of the general style of harmony used in a particular arrangement.

The best modulations will result from the symmetry that may be detected in a given piece of music. Even when tonic-dominant progression is characteristic of harmonic continuity, this method may be used with success, as it simply requires the composition of a rhythmic group, where the original value is 5. In this seemingly limited case there is still a choice of steps: 4 + 1; 3 + 2; 2 + 3; 1 + 4.

Examples of Modulations
Through Symmetric Groups

(1) Key of C to Key of $E^{\flat}$; $i = 9$

Symmetric Group: 1 + 3 + 1 + 3 + 1 (r_3 of $\frac{9}{9}$ series)

(2) Key of C to Key of E♭

Chords to be Connected: D —— B♭; i = 4;

4 + 12 = 16

Symmetric Group: $r_{4 \div 3}$ = 3 + 1 + 2 + 1 + 1 + 1 +

+ 1 + 2 + 1 + 3

Lesson XCVIII.

Chromatic System of Harmony

The basis of this system is transformation of diatonic chordal functions into chromatic chordal functions and back into diatonic. Chromatic continuity evolved from this basis emphasizes various phenomena of harmony which do not confine to diatonic or symmetric systems. The usually known modulations are but a special case of the chromatic system. Chord progressions usually known as "alien" chord progressions find their exhaustive explanation in this system.

Wagner was the first composer to manipulate intuitively with this type of harmonic continuity. Not having any theoretical basic principle of handling such progressions, Wagner often wrote them in an enharmonically confusing way. (J.S. Bach made an unsuccessful attempt to move in chromatic systems. See "Well Tempered Clavichord" - Vol. I, Fugue 6 - bar 16). It is necessary, for analytical purposes, to rewrite such music in proper notation, i.e., chromatically and not enharmonically. A more consistent notation of chromatic continuity may be found among the followers of Wagnerian harmony, such as Borodin and Rimsky-Korsakov.

The chromatic system of harmonic continuity

is based on progressions of chromatic groups. Every chromatic group consists of three chords, which express the following mechanical process: balance - tension - release. These three moments correspond to the diatonic - chromatic - diatonic transformation. A chromatic group may consist of one or more simultaneous operations. Such operations are alterations of diatonic tones into chromatic tones, by raising or lowering them. The initial diatonic tone of a chromatic group retains its name, while being altered, and changes it during the moment of release.

The two forms of chromatic operations are:

(1) $x \nearrow x^{\sharp} \nearrow y$

(2) $x \searrow x^{\flat} \searrow y$

In application to musical names it may become, for instance, g - $g^{\sharp}$ - a or g - $g^{\flat}$ - f. Such steps are always semitones. At such moment of release, in a chromatic group, a new chordal function (and in some cases the same) becomes the starting point of the next chromatic group, thus evolving into an infinite chromatic continuity.

Such continuity acquires the following appearance:

d - ch - d

d - ch - d etc.

d - ch - d

Chromatic continuity in such form offers a very practical bar distribution by placing two chords in a bar. Such distribution places the release on the downbeat and sounds satisfactory to our ear, probably due to the habit of hearing them in such distribution.

As in the diatonic progressions, the commonness of tones, or the resolution of chordal functions, or as in the symmetric progressions the symmetric roots become the stimuli of motion, likewise in the chromatic progressions such stimuli are the chromatic alterations of the diatonic tones.

Besides the form of continuity of chromatic groups offered in the preceding diagram, two other forms are possible. Thus, the latter do not necessarily require the technique of the chromatic system. The first of these forms of continuity produces an overlapping, over one term:

(1) d - ch - d

d - ch - d

d - ch - d i.e.,

the second part produces the first term of a chromatic group, while the first one produces the second term.

(2) d - ch - d

d - ch - d

i.e., two or more parts of harmony coincide in their transformation in time, though the form of transformation may be different in each part.

Any chord acquiring a chromatic alteration becomes more intense than the corresponding form of tension, without it. If the middle term of a chromatic group has to be intensified, the following forms of tension may constitute a chromatic group:

S(5)	S(7)	S(5)
S(5)	S(7)	S(7)
S(7)	S(7)	S(5)
S(7)	S(7)	S(7)

The only combination which is undesirable, as it produces an effect of weakness, is when the middle term is S(5).

Operations in a given chromatic group correspond to a group of chordal functions which may be assigned to any form of alterations. As for technical reasons the 4-part harmony is limited to S(5) and S(7) forms, with their inversions, all transformations of functions in the chromatic group deal with the four lower functions (9, 11 and 13 are excluded).

Numerical Table of Transformations for the Chromatic Groups.

1-1-1	3-3-3	5-5-5	7-7-7
1-1-3	3-3-1	5-5-1	7-7-1
1-3-1	3-1-3	5-1-5	7-1-7
3-1-1	1-3-3	1-5-5	1-7-7
1-1-5	3-3-5	5-5-3	7-7-3
1-5-1	3-5-3	5-3-5	7-3-7
5-1-1	5-3-3	3-5-5	3-7-7
1-1-7	3-3-7	5-5-7	7-7-5
1-7-1	3-7-3	5-7-5	7-5-7
7-1-1	7-3-3	7-5-5	5-7-7
1-3-5	1-3-7	1-5-7	3-5-7
1-5-3	1-7-3	1-7-5	3-7-5
5-1-3	7-1-3	7-1-5	7-3-5
3-1-5	3-1-7	5-1-7	5-3-7
3-5-1	3-7-1	5-7-1	5-7-3
5-3-1	7-3-1	7-5-1	7-5-3

Some of these combinations must be excluded because of the adherence of the Seventh to the classical system of voice-leading (descending resolution).

The preceding table offers 16 different versions for each starting function (1, 3, 5, 7). In addition to this, any middle chord of a chromatic group may assume one of the seven forms of S(7), and any of the last chords of a chromatic group -- either four forms of S(5) or seven forms of S(7). Thus, each starting point offers either 28 or 49 forms. The total number of starting points for one function equals 16. These quantities must be multiplied by 16 in order to show the total number of cases.

$$28 \times 16 = 448$$

$$49 \times 16 = 784$$

This applies to one initial function only, and as any group may start with either of the four functions, the total quantity is 4(784 + 448) = 4, 928. A number of these cases eventually excludes themselves on account of the abovementioned limitations caused by the traditional voice-leading.

The actual realization of chromatic groups must be performed from the two fundamental bases: the major and the minor. The concept of a <u>harmonic basis</u> expresses any three adjacent chordal functions, such as:

5	7	9	11	13
3	5	7	9	11
1	3	5	7	9

Due to practical limitations this course of Harmony will deal with the first $\overset{5}{\underset{1}{(3)}}$ basis only. The terms major and minor correspond to the structural constitution in the usual sense: major = 4 + 3, and minor = 3 + 4. All fundamental chromatic operations are derived from these two bases.

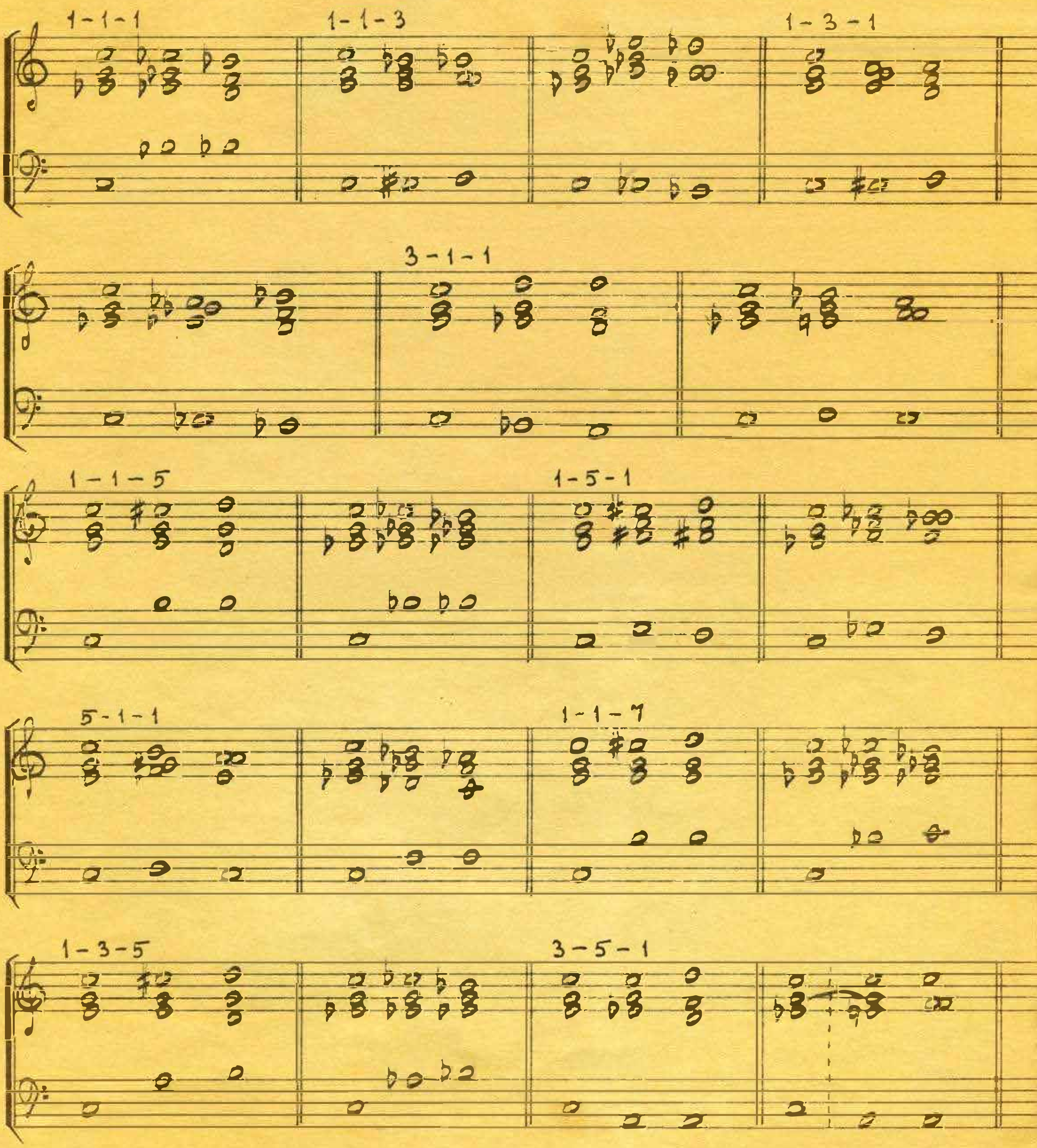
Major Basis	Minor Basis
$1^{\sharp}$	$1^{\flat}$
$3^{\flat}$	$3^{\sharp}$
$5^{\sharp}$	$5^{\flat}$

These six forms of chromatic operations (3 from each basis) are used independently. Chromatic operations available from the major basis are: raising of the root-tone, lowering of the third, raising of the fifth. They are the opposite from the minor basis.

(please see following pages)

Examples Of Chromatic Groups: One Operation

Table Of Transformations



Try to find the remaining cases through the table of transformations of the chordal functions. Please remember that the classical system of voice-leading must be carried out through chromatic continuity. A Seventh either descends or remains (as in traditional cadences); it may even go up one semitone, due to the chord structure, yet it positively must retain its original name, like d - d#.

Through the selection of different chromatic groups (which may be used with coefficients of recurrence) a chromatic continuity may be composed.

With the amount of explanation offered so far, every last chord of the preceding group (and therefore the first chord of the following group) must be major or minor, as the operations from other bases will be explained in the following lesson.

Example of Chromatic Continuity

Lesson XCIX.

Operations from $S_3(5)$ and $S_4(5)$ bases

As 3 of $S_3(5)$ is identical with 3 of $S_1(5)$, the fundamental operations correspond to $S_1(5)$. They are:

(1) raising of 1

(2) lowering of 3

Function 5 does not participate in the fundamental operations, as it is already altered. As the form of the middle chord is pre-selected, the fifth requires rectification in many cases though it retains its name. All forms of doublings are acceptable.

As 3 of $S_4(5)$ is identical with 3 of $S_2(5)$, the fundamental operations correspond to $S_2(5)$. They are:

(1) lowering of 1

(2) raising of 3

Fifth does not participate in the fundamental operations, but may be rectified.

Figure I.

Operations from an augmented basis.

(please see next page)

FIG I

Figure II.

Operations from a diminished basis.

FIG II

Figure III.

Chromatic continuity including all bases.

Chromatic Alteration of the Seventh.

Due to the classical tendency of a downward resolution of the seventh, chromatic alterations follow the same direction. Lowering of the seventh (both major and minor) can be carried out from all forms of S(7). If the seventh is minor, it is more practical to have it as sharp or natural, as lowering of the flat produces a double-flat. Do not operate from a diminished seventh.

Figure IV.

Examples of operations from the Seventh.

We can incorporate now all the single operations into the final form of chromatic continuity.

Figure V.

Operations from 1, 3, 5 and 7.

All bases.

JOSEPH SCHILLINGER

CORRESPONDENCE COURSE

With: Dr. Jerome Gross Subject: Music

Lesson C.

Parallel Double Chromatics

(Double chromatic operations)

Parallel double chromatics occur when fundamental operations are performed from an opposite base. In such a case the rectification of the third is required.

If, for example, we decide to lower 1 of $S_1(5)$ basis, it becomes necessary to alter 3 to its proper basis, i.e. to lower in this case.

We shall consider the alterations of 1 and 5 as fundamental and the correction of 3 as complementary chromatics.

The following table represents all operations.

Figure VI.

Parallel Double Chromatics.

$S_1(5)$ basis		$S_2(5)$ basis	
Fundamental	1♭	Fundamental	1♯
Complementary	3♭	Complementary	3♯
Fundamental	5♭	Fundamental	5♯
Complementary	3♭	Complementary	3♯

Fundamental chromatics represent the middle term of a complete chromatic group, whereas the complementary chromatics do not necessarily perform the conclusive movement designated by their alterations.

Thus, the scheme of chromatic groups for the parallel double chromatics appears as follows:

(1) $x \nearrow x^{\sharp} \nearrow y$ (fundamental)

$x' \nearrow x'^{\sharp}$ (complementary)

(2) $x \searrow x^{\flat} \searrow y$ (fundamental)

$x' \searrow x'^{\flat}$ (complementary)

For example, if $c - c^{\flat} - b^{\flat}$ is a fundamental operation, the complementary chromatic is: $e - e^{\flat}$. The complementary chromatic $e^{\flat}$ does not necessarily move into d. It may remain or even move upward, depending on the chordal function assigned to it.

The same is true of the ascending chromatics. If $c - c^{\sharp} - d$ is the fundamental operation, the complementary chromatic is $e^{\flat} - e$. The complementary chromatic e does not necessarily move into f. It may remain or even move downward, depending on the chordal function assigned to it.

The assignment of chordal functions must be performed for the two simultaneous operations: fundamental and complementary. It is practical to designate the ascending alterations as: $\genfrac{}{}{0pt}{}{3}{1}$ or $\genfrac{}{}{0pt}{}{5}{3}$, and the descending -- as: $\genfrac{}{}{0pt}{}{7}{5}$ or $\genfrac{}{}{0pt}{}{5}{3}$.

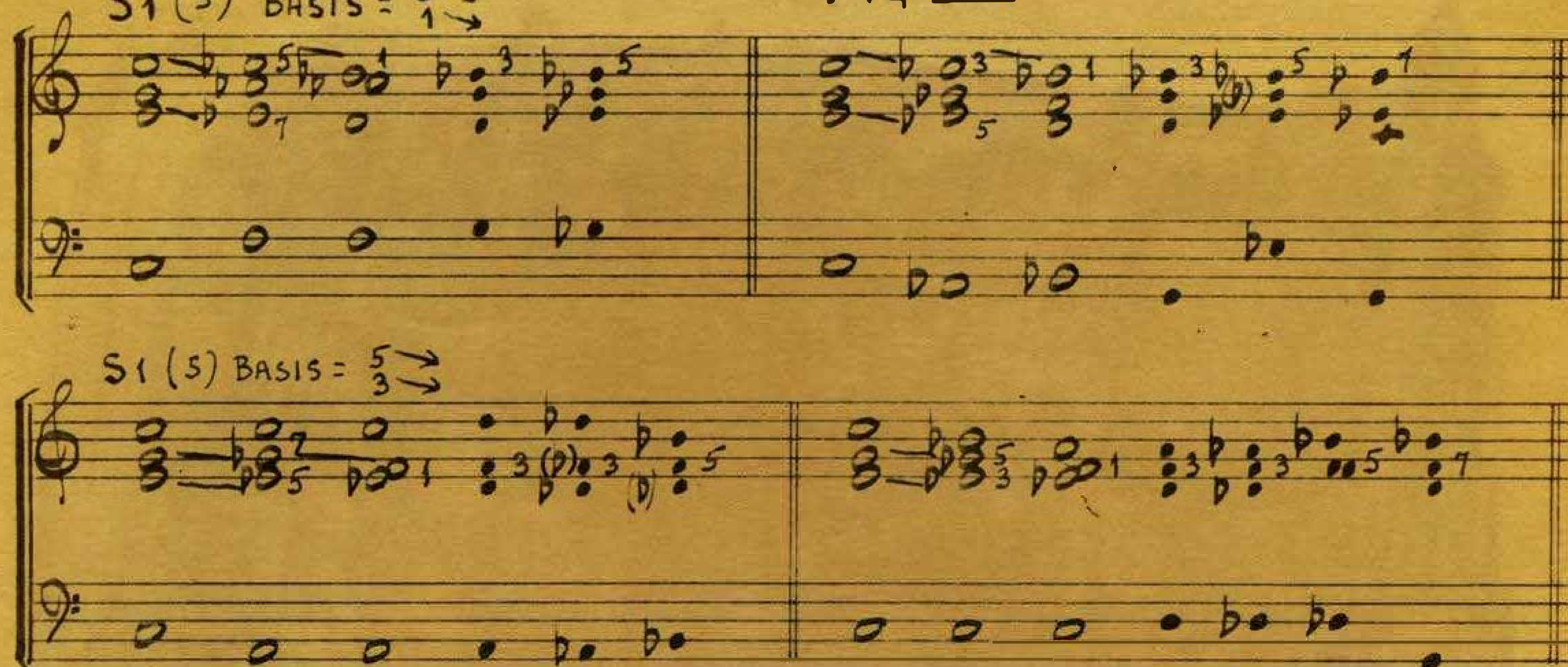
This protects harmonic continuity from a wrong direction and sometimes from an excess of accidentals. This remark refers to the middle term of a chromatic group.

<u>Figure VII.</u>

<u>Examples of Double Parallel Chromatics.</u>

(please see next page)

FIG VII



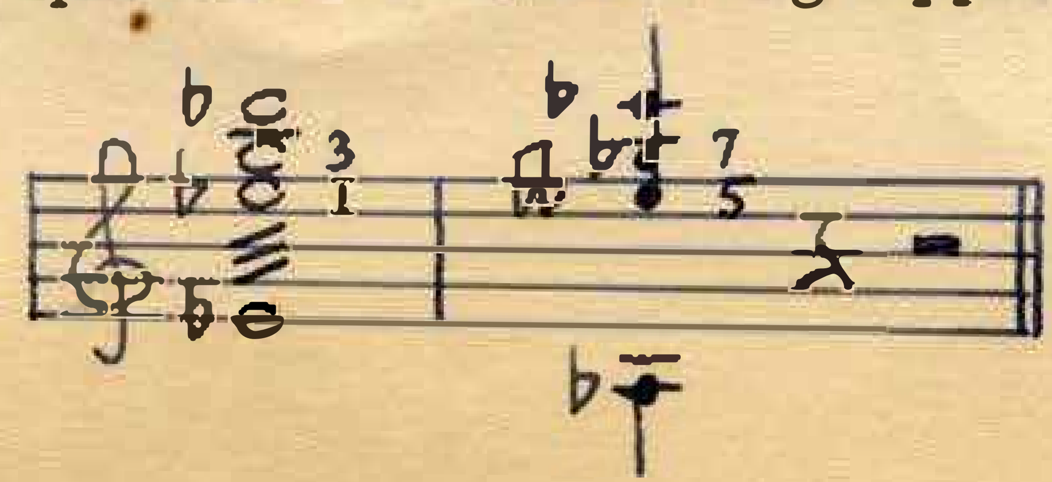
By assigning the opposite bases, we can obtain double parallel chromatics at any desirable place of chromatic continuity.

Figure VIII.

Continuity of Double Parallel Chromatics.

Double parallel chromatics are the quintessence of chromatic style in harmony. It created the unmistakable character of Wagner and the post-Wagnerian music. While the analysis of Borodin, Rimsky-Korsakov, Frank and Delius does not present any difficulties for the analyst familiar with this theory, the music of Wagner often requires transcribing into chromatic notation. One of the progressions typical of the later Wagner's period (we find much of it in "Parsifal") is:

Being transcribed into chromatic notation it acquires the following appearance:



This corresponds to $S_1(5)$ basis: $3 \searrow$ $1 \searrow$

There are many instances when double parallel chromatics are evolved on a basis of passing chromatic tones. They are abundant in the music of Rimsky-Korsakov, Borodin and, lately, became very common in the American popular and show songs ("Cuban Long Song", "The Man I Love"). The source of passing chromatic tones, the technique of which we shall discuss later, is more Chopin than Wagner or the post-Wagnerians.

Lesson CI.

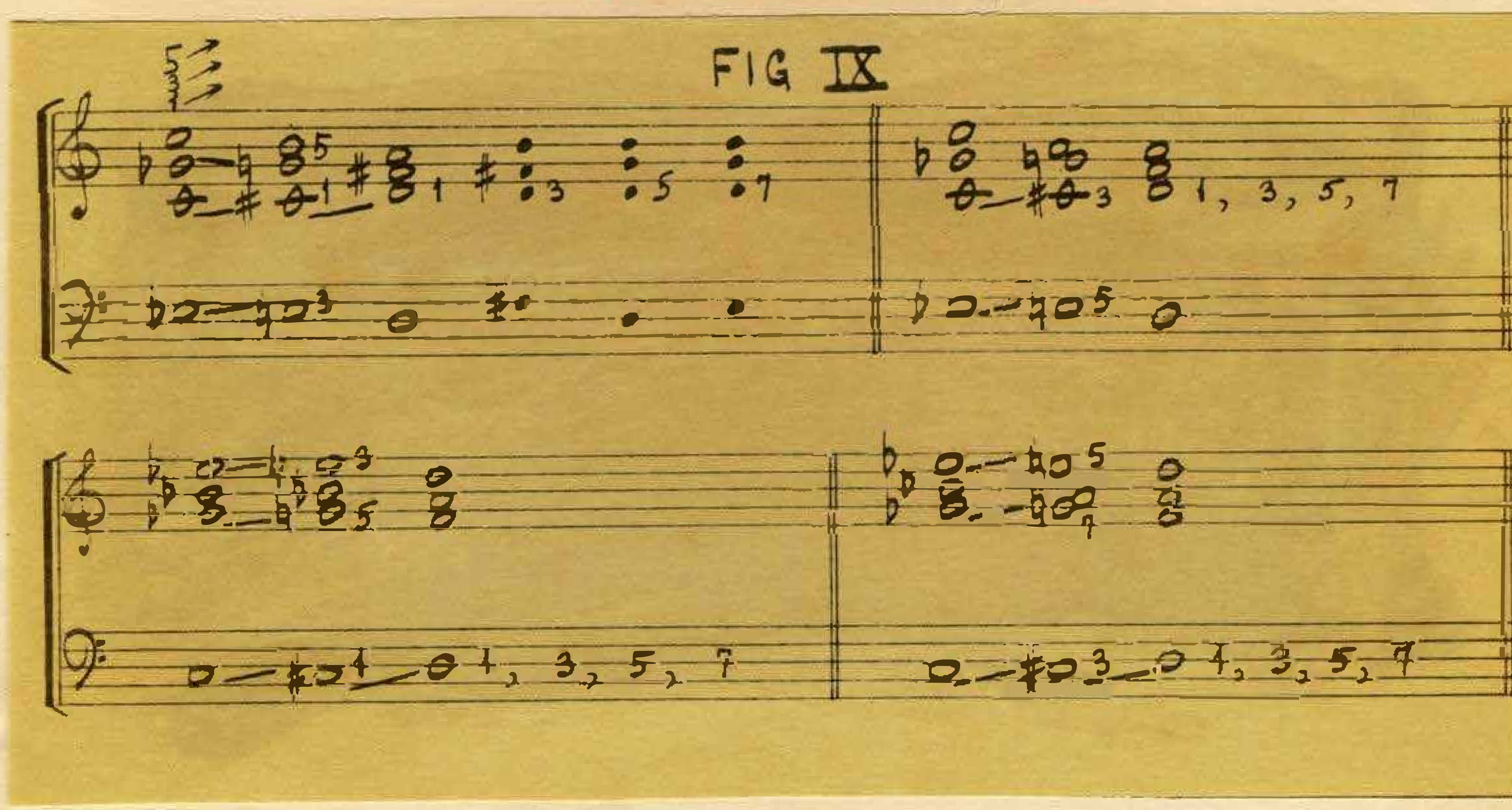
Triple and Quadruple Parallel Chromatics

Triple parallel chromatics occur when 1 is raised in $S_4(5)$ basis. This, being the fundamental operation, requires the correction of the third ($3^{\#}$) and of the fifth ($5^{\#}$). The triple alterations become

5		7
3	or	5
1		3

Figure IX.

Triple parallel chromatics



Quadruple parallel chromatics occur when 1 is raised in $S_5(7)$ basis [diminished seventh-chord]. This requires the alteration of all remaining functions, i.e. $3^{\#}$, $5^{\#}$ and $7^{\#}$. This is the only interpretation satisfying the cases of chromatic parallel motion of the diminished seventh-chords. See Beethoven's Piano Sonata No. 7 Largo (bar 20 from the end and the following 5 bars in relation to the adjacent harmonic context). Such a continuous chain of quadruple parallelisms takes place when the same operation is performed several times in succession.

As chromatic system is limited to four functions (1, 3, 5, 7), quadruple parallel chromatics remain with their original assignments (while being altered).

Figure X.

Quadruple Parallel Chromatics

By combining all forms of chromatic operations, i.e. single, double, triple and quadruple, we obtain the final form of mixed chromatic continuity.

Figure XI.

Continuity of Mixed Chromatic Operations

Enharmonic Treatment of the Chromatic System

By reversing the original directions of chromatic operations we more than double the original resources of the chromatic system.

Enharmonic treatment of chromatic groups

consists of substituting raising for lowering and vice-versa. This changes the original direction of a group and brings to new points of release in its third term.

The following formula expresses all conditions necessary for the enharmonic treatment.

$$(1)\quad x \nearrow x^{\sharp} = y^{\flat} \searrow z \quad (1, 3, 5, 7)$$

$$(2)\quad x \searrow x^{\flat} = y^{\sharp} \nearrow z \quad (1, 3, 5, 7)$$

Progressions of this kind are characteristic of post-Wagnerian composers (Borodin's "Prince Igor", Rimsky-Korsakov's "Coq D'Or" and "Khovanschina").*)

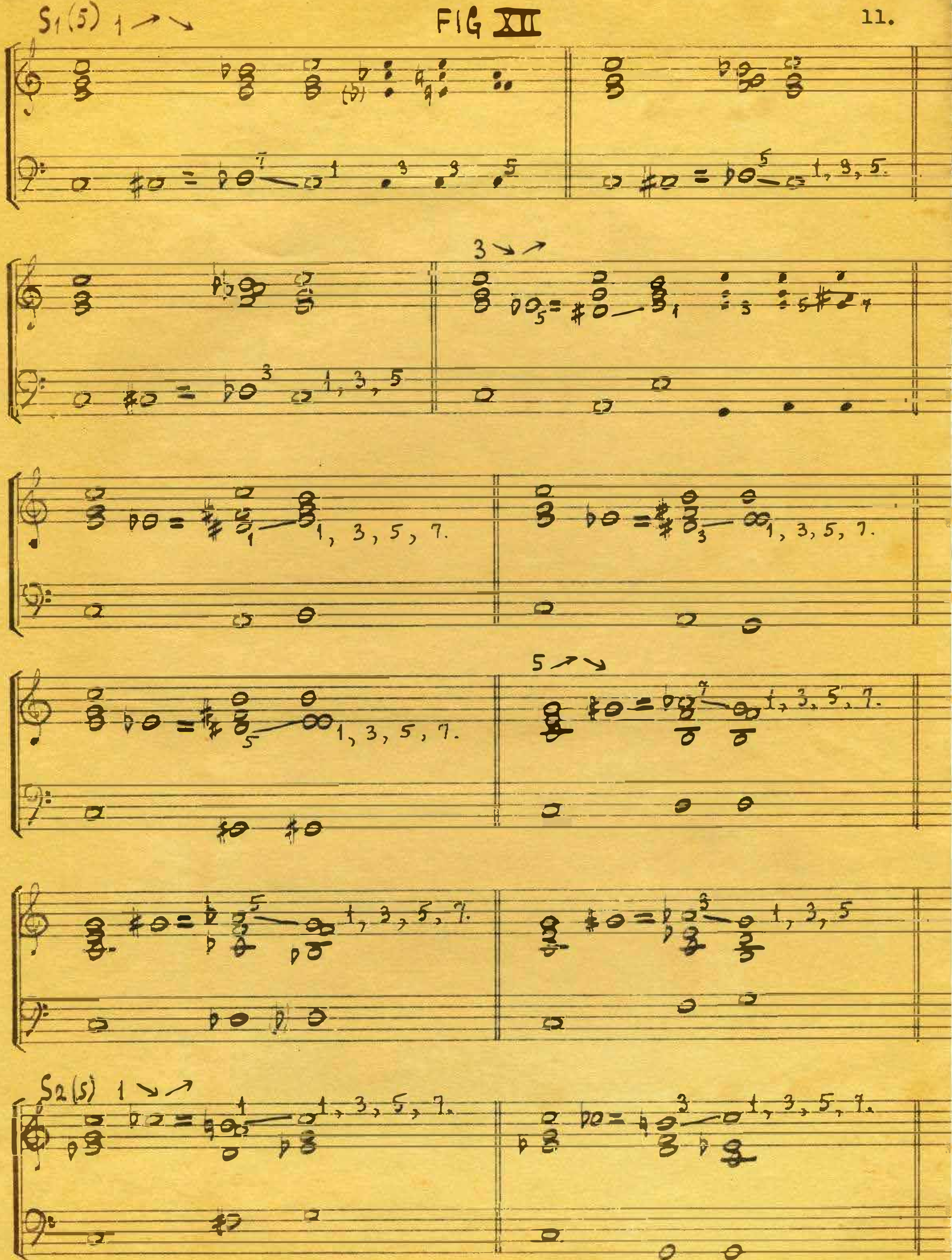
Figure XII.

Examples of enharmonic treatment of the chromatic system.

(please see following pages)

*) by Moussorgsky.

FIG XII



(Figure XII, cont.)

In cases of double and triple chromatics, all or some of the altered functions can be enharmonized.

Figure XIII.

Enharmonic treatment of double and triple chromatics.

Lesson CII.

Overlapping Chromatic Groups.

Overlapping groups produce a highly saturated form of chromatic continuity. The alterations in the two overlapping groups may be either both ascending, or both descending, or one of the groups can be ascending, while the other descending. The choice of ascending and descending groups depends on the possibilities presented by the preceding groups during the moment of alteration.

The general form of overlapping chromatic groups is:

$$\begin{array}{ll} d - ch - d & \\ & d - ch - d \end{array}$$

This scheme, being applied to ascending and descending alterations, offers 4 variants.

(1) $$\begin{array}{ll} x \nearrow x^{\sharp} \nearrow y & \\ & x \nearrow x^{\sharp} \nearrow y \end{array}$$

(2) $$\begin{array}{ll} x \searrow x^{\flat} \searrow y & \\ & x \nearrow x^{\sharp} \nearrow y \end{array}$$

(3) $x \rightarrow x^{\sharp} \rightarrow y$

$x \rightarrow x^{\flat} \rightarrow y$

(4) $x \rightarrow x^{\flat} \rightarrow y$

$x \rightarrow x^{\flat} \rightarrow y$

Thus, parallel as well as contrary forms are possible.

Each of the mutually overlapping groups has a <u>single</u> chromatic operation.

<u>Figure XIV.</u>

<u>Examples of overlapping chromatic groups.</u>

The sequence in which such groups can be constructed is as follows:

In the first example of Figure XIV (and similar procedure refers to all cases) we write the first chord first:

The next step is to make operations in one voice. In this example $1^{\#}$ was chosen in the bass:

The next step is to construct the middle chord of this group: ($1^{\#}$ was assumed to remain 1, which gave the $C^{\#}$ seventh-chord):

The next step is <u>to estimate the possibilities of other voices with regard to chromatic alterations.</u>

The b ⟶ $b^{\flat}$ step permits to construct a chord which necessitates the inclusion of d and $b^{\flat}$. Another possibility might have been to produce g ⟶ $g^{\sharp}$, which would also permit the use of d in the bass. See the second example of Figure XIV. The third possibility might have been the step e ⟶ $e^{\sharp}$, in the alto voice, which also permits the use of d. Even steps like e ⟶ $e^{\flat}$ or g ⟶ $g^{\flat}$ would be possible, though the latter require an augmented S(7), i.e. (reading upward) d - $g^{\flat}$ - $e^{\flat}$ - $b^{\flat}$.

Figure XV.

Continuity of Overlapping Chromatic Groups.

Lesson CIII.

Coinciding Chromatic Groups.

The technique of evolving coinciding chromatic groups is quite different from all the chromatic techniques previously described. It is more similar to the technique of passing chromatic tones, at which we shall arrive later.

Coinciding chromatic groups are evolved as a form of contrary motion in two voices being a doubling of the chord, with which the group begins.

The general form of a coinciding chromatic group is:

d - ch - d

d - ch - d

Contrary directions of the chromatic operations can be either outward or inward:

(1) $x \to x^{\sharp} \to y$

$x \to x^{\flat} \to y$

(2) $x \to x^{\flat} \to y$

$x \to x^{\sharp} \to y$

The assignment of the two remaining functions in the middle chord of a coinciding group can be performed by sonority, i.e. enharmonically.

For instance, in a group

$$\begin{array}{l} c \rightarrow c^{\flat} \rightarrow b^{\flat} \\ c \rightarrow c^{\sharp} \rightarrow d \end{array}$$

the $\begin{matrix} c^{\flat} \\ c^{\sharp} \end{matrix}$ interval can be read enharmonically, i.e. as $\begin{matrix} b \\ c^{\sharp} \end{matrix}$ in which case it becomes $\begin{matrix} 7 \\ 1 \end{matrix}$ or $\begin{matrix} 9 \\ 3 \end{matrix}$ etc. It is easy then to find the two remaining functions, like 3 and 5. Thus, we can construct a chord $c^{\sharp}$ - e - g - b.

As coinciding chromatics result from doublings, it is very important to have full control of the variable doublings technique. Thus the doubling of 1, 3, 5 and also 7 (major or minor) must be used intentionally in all forms and inversions of S(5) and S(7). The latter, naturally, for obtaining the doubled 7.

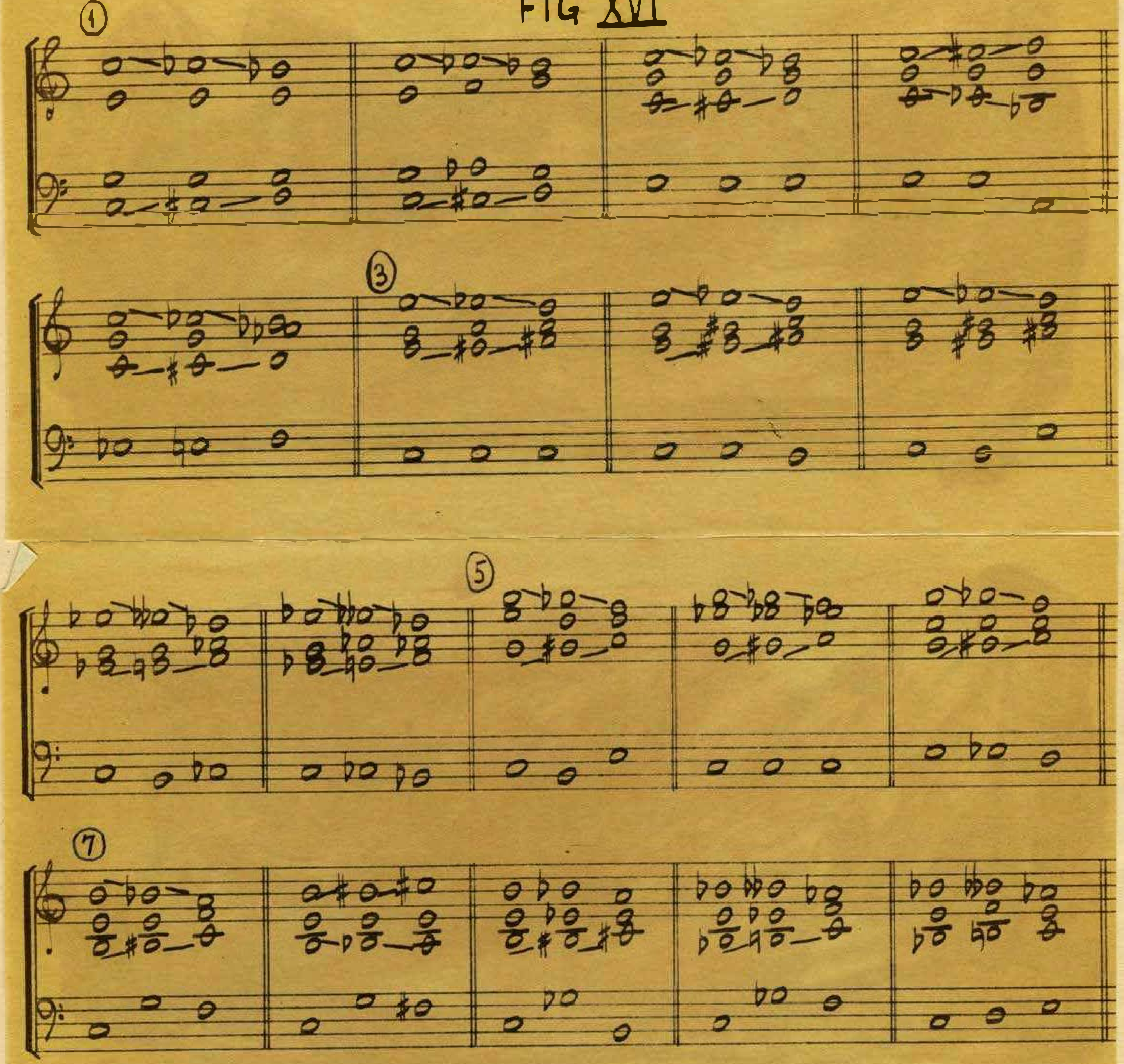
<u>Figure XVI.</u>

<u>Examples of Coinciding Chromatic Groups.</u>

(Notation of chromatic operations as in all other forms of chromatic groups).

(please see next page)

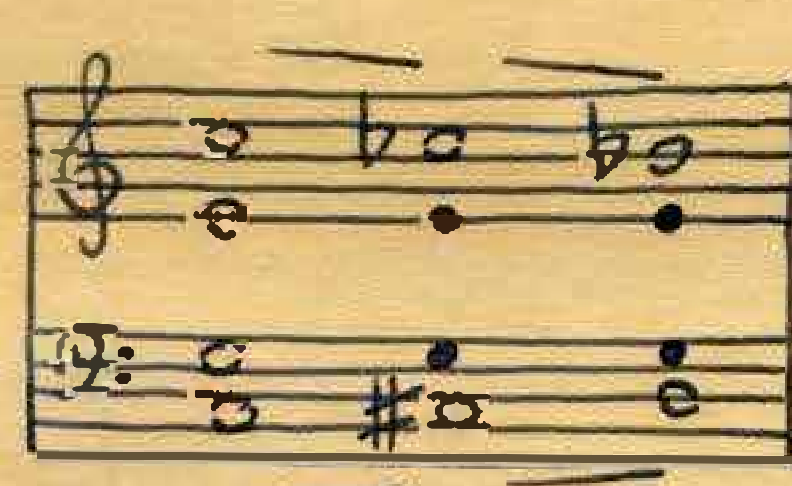
FIG XVI



It is important to take into consideration, while executing the coinciding chromatic groups, that the first procedure is to establish the chromatic operations

and the second procedure is to add the two missing functions.

After performing this, the final step is to assign the functions in the last chord of the group.



<u>All coinciding groups are reversible.</u> When moving from an octave inward by semitones, the

last term of the group produces a minor sixth. When moving outward from unison or octave, the last term of the group produces a major third.

It is important to take these considerations into account while evolving a continuity of coinciding chromatic groups. Any such group can start from any two voices producing (vertically) a unison, an octave, a major third or a minor sixth.

The following are all movements and directions with respect to c.

(1) $c \nearrow c^{\sharp} \nearrow d$

$c \searrow c^{\flat} \searrow b^{\flat}$

(2) $e \searrow e^{\flat} \searrow d$

$c \nearrow c^{\sharp} \nearrow d$

(3) $c \searrow c^{\flat} \searrow b^{\flat}$

$c \nearrow c^{\sharp} \nearrow d$

(4) $a^{\flat} \rightarrow a^{\natural} \rightarrow b^{\flat}$

$c \rightarrow c^{\flat} \rightarrow b^{\flat}$

Figure XVII.

Continuity of Coinciding Chromatic Groups.

All techniques of chromatic harmony can be utilized in the mixed forms of chromatic continuity.