

Lesson LXXXV.SEVENTH-CHORDS. S(7).

## Diatonic System

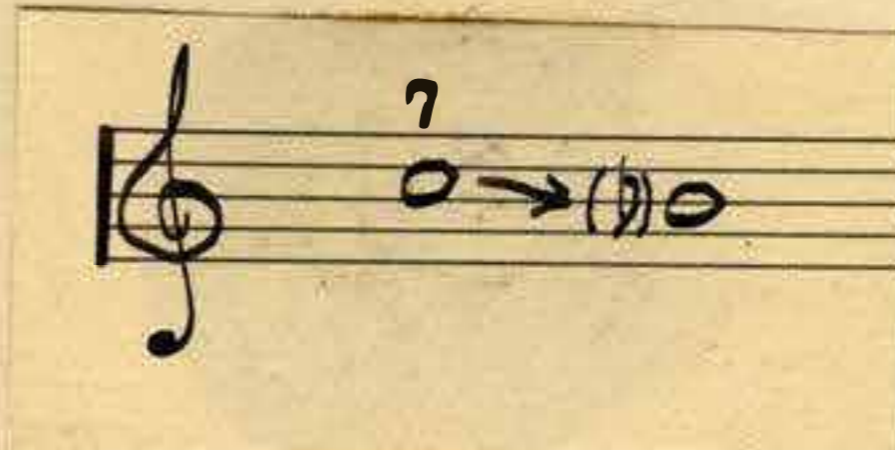
Fundamental Position	The First Inversion	The Second Inversion	The Third Inversion
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S(7) Seventh- Chord	S( <sup>6</sup> / <sub>5</sub> ) Fifth- Sixth Chord	S( <sup>4</sup> / <sub>3</sub> ) Third- Fourth Chord	S(2) Second Chord
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A seventh-chord including all inversions has 24 positions altogether.

The classical system of harmony is based on the postulate of resolving seventh: seventh moves one step down.

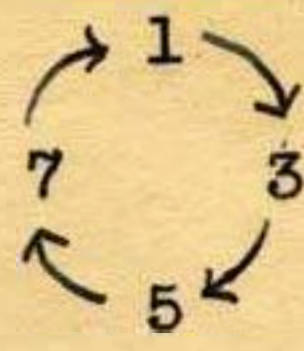


This postulate provides a medium for continuous progression of S(7) as well as establishes the entire system of diatonic continuity (cycles).

One movement is required to produce C<sub>3</sub>: the movement of the seventh alone. It results in a clockwise transformation.

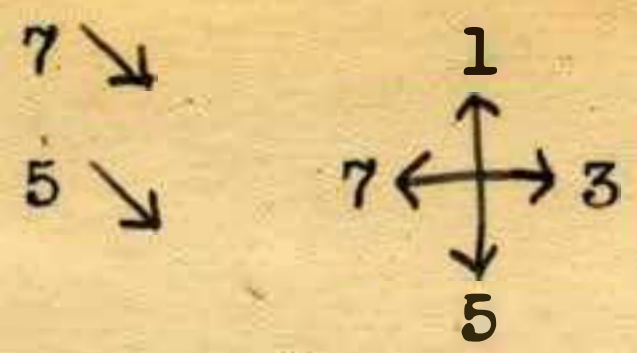


A musical staff in treble clef with a common time signature (C). The key signature has one flat (B-flat). The first chord is labeled C3 and has a '7' below it. An arrow points to the second chord, which has a '5' below it. The third chord has a '4' above and a '3' below. The fourth chord has a '2' below. The fifth chord has a '7' below. The staff ends with 'etc.' and an arrow pointing to the right.



Two movements are required to produce C<sub>5</sub>:  
 the movement of the seventh and of the fifth one step  
 down. It results in a crosswise transformation.

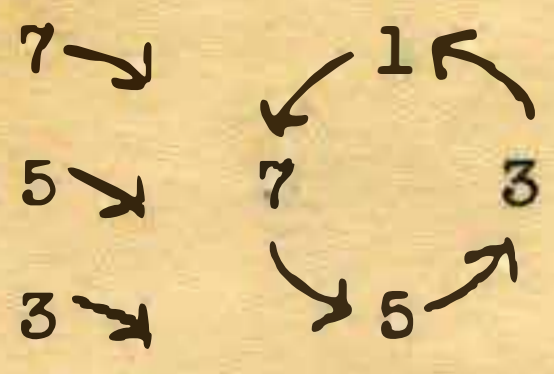
Two musical staves in treble clef with a common time signature (C). The key signature has one flat (B-flat). The top staff starts with a chord labeled C5 and a '7' below it. An arrow points to the second chord, which has a '4' above and a '3' below. The third chord has a '7' below. The staff ends with 'etc.'. The bottom staff starts with a chord with a '6' above and a '5' below. An arrow points to the second chord, which has a '2' below. The third chord has a '5' below. The staff ends with 'etc.'.



Three movements are required to produce C<sub>7</sub>:  
 the movement of the seventh, of the fifth and of the  
 third one step down. It results in a counter-clockwise  
 transformation.

Taking the chords over two from C<sub>3</sub> we obtain:

A musical staff in treble clef with a common time signature (C). The key signature has one flat (B-flat). The first chord is labeled C7 and has a '7' below it. The second chord has a '2' below. The third chord has a '4' above and a '3' below. The fourth chord has a '6' above and a '5' below. The staff ends with 'etc.'.



This type of music may be found among  
 contrapuntalists of XVII - XVIII Centuries. Palestrina,  
 Bach, Haendel obtained similar results by means of  
 suspensions.

Assigning a system of cycles we can produce



a continuity of  $S(7)$ . The starting chord may be taken in any position.

Example:  $C_5 + C_3 + C_5 + C_7 + C_3 + C_3 + C_5$

This continuity being entirely satisfactory harmonically may prove, in some cases, unsatisfactory melodically on account of continuous descending in all voices. This form, when desirable, may be eliminated by means of the two devices:

- (1) exchange of the common tones
- (2) octave inversion of the common tones

The same continuity of cycles assumes the following form:



Obviously  $C_1$  does not provide common tones, thus excluding the above devices.

As the continuity of the second type offers better melodic forms for all voices, it may be desirable to pre-set certain melodic forms in advance. For example, it is possible to obtain, by means of continuous  $C_5$ , the following form of descending through two parallel axes (b) or (d), as in the music of Frederick Chopin.



This may be harmonized as follows:

Two staves of music. The top staff is in treble clef and the bottom staff is in bass clef. Both staves show a descending sequence of notes with stems and beams, corresponding to the single staff above. The bottom staff uses a shorthand notation for notes. The sequence ends with 'etc.' in the top right corner.

Diatonic  $C_0$  becomes a necessity in order to avoid the excess of saturation typical of the continuity of  $S(7)$  with variable cycles.

The principle of moving continuously through  $C_0$  is based on the exchange and inversion of common tones.





The exchange and inversion of adjacent functions brings the utmost satisfaction. Nevertheless it is not desirable to use the two extreme functions for such purpose as they cause a certain amount of harshness.

1 ↔ 3      5 ↔ 7  
1 ↔ 3      3 ↔ 5      5 ↔ 7      1 ↔ 3  
5 ↔ 7

An example of continuity of the  $C_0$ :

The final form of continuity of  $S(7)$  consists of the mixtures of all cycles (including  $C_0$ ) based on a rhythmic composition of the coefficients of recurrence.



Example:  $2C_5 + C_0 + 2C_3 + C_0 + 2C_7 + C_0$

The image shows a piece of handwritten musical notation on a single staff. The staff is divided into two systems, each with a treble clef on the top line and a bass clef on the bottom line. The notation consists of various notes, including quarter notes, eighth notes, and sixteenth notes, some of which are beamed together. There are also rests and vertical lines indicating phrasing or breath marks. The handwriting is in black ink on aged, yellowish paper.



Lesson LXXXVI.Resolution of S(7).

Resolution of an S(7) into an S(5) in all positions and inversions may be defined as a transition from four functions to three functions.

S(5) in the four-part harmony and with a normal doubling (doubled root) consists of:

1, 1, 3, 5

S(7) consists of:

1, 3, 5, 7

When a transition occurs, obviously the root takes the place of the seventh. Therefore the resolution is provided through the motion of S(7) → S(7) and the substitution of one for the seven, i.e., the function which would become a seventh in the continuity of seventh-chords becomes a root-tone when a resolution is desired.

Example:

$\begin{array}{c} \leftarrow \\ \leftarrow \\ 7 \rightarrow 1 \\ 5 \rightarrow \textcircled{7} \rightarrow 1 \\ 3 \rightarrow 5 \\ 1 \rightarrow 3 \end{array}$

Note: Do not move S(7) → S(5) in the C<sub>0</sub>



Resolutions in the Diatonic Cycles.

OR, WHEN 4 IS UNDESIRABLE,  
BASS ASSUMES THE ROOT

This case provides an explanation why a tonic triad acquires a tripled root and loses its fifth.

Preparation of S(7)

There are three methods of preparation of S(7), i.e., of transition from S(5) to S(7):

- (1) suspending
- (2) descending
- (3) ascending





The first method is the only one producing the positive ( $C_3$ ,  $C_5$ ,  $C_7$ ) cycles.

The methods (2) and (3) are the outcome of the intrusion of melodic factors into harmony. They are obviously in conflict with the nature of harmony (like the groups with passing chords) as they produce the negative cycles, which in turn contradict the postulate of the resolving seventh universally observed in classical music.

The technique of preparation of the seventh consists of assigning a certain consonant function (1, 3, 5) to become a dissonant function (7) and to either sustain the assigned function of the S(5) over the bar line or to move it one step downward or upward.

The last two forms of a seventh must occur on a weak beat.

Exercise in different positions, inversions and cycles the S(5)  $\longrightarrow$  S(7) transition.

(1) Suspending:



$C_7$



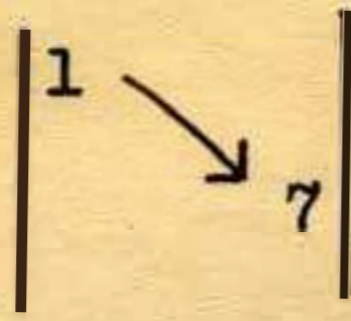
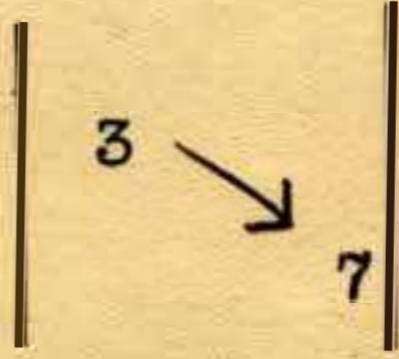
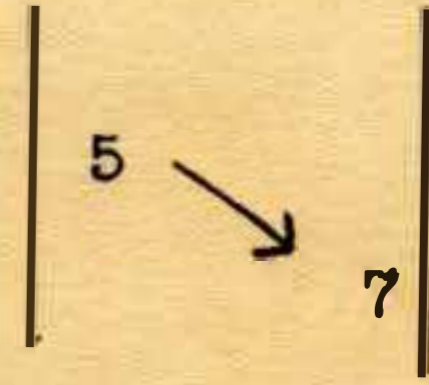
$C_5$



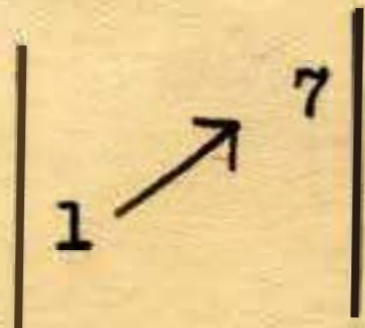
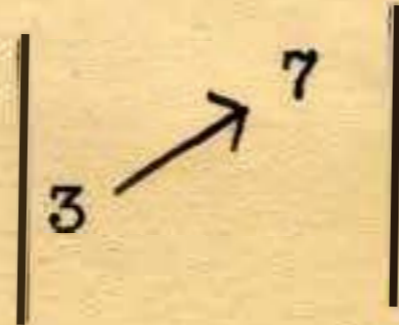
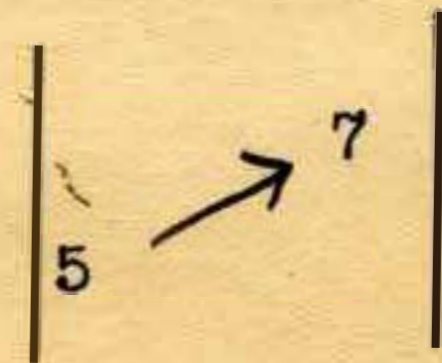
$C_3$



(2) Descending:

C<sub>0</sub>C<sub>-3</sub>C<sub>-5</sub>

(3) Ascending:

C<sub>-3</sub>C<sub>-5</sub>C<sub>-7</sub>

(Please see next page)



Preparation of S(7)

(1) Suspending

(2) Descending

(3) Ascending



The mixture of the zero, positive and negative cycles provides the final form of continuity based on S(5) and S(7).

For more efficient planning of such continuity use bar lines for the layout. The preparation of S(7) may be either positive or negative; the resolution - always positive.

Example:

The example consists of two parts: a sequence of cycle labels and a corresponding piano score.

**Sequence of Cycle Labels:**

Top line:  $S(5) + S(7) + S(5) + S(5) + S(7) + S(7) + S(5) + S(7) + S(7) + S(5) + S(5) + S(7) + S(5)$

Below the top line, bar lines separate the following cycle labels:  $C-3$ ,  $C7$ ,  $C5$ ,  $C0$ ,  $C3$ ,  $C-5$ .

Below these labels, a second line shows the resolution labels:  $C5$ ,  $C2$ ,  $C7$ ,  $C5$ ,  $C7$ ,  $C3$ .

**Piano Score:**

The piano score is written on two staves (treble and bass clef). It shows a sequence of chords and notes corresponding to the cycle labels above. The notes are marked with subscripts (e.g.,  $o_3$ ,  $o_7$ ,  $o_5$ ) and some are connected by lines, indicating a specific melodic or harmonic path.





Lesson LXXXVII.

The negative system of tonal cycles may be used as an independent system. The negative system is in reality a geometrical inversion of the positive system. Every principle, rule or regulation of the positive system becomes its own converse in the negative.

Chord structures become  $E_{(b)}$  of the original scale. Chord progressions are based on  $E_{(a)}$  which forms the  $C_{-3}$ . Clockwise transformations become counterclockwise and vice versa.

Chord Structures

Positive

Negative

Tonal Cycles:

Negative -  $\leftarrow$  + Positive  $\rightarrow$

Transformations:

$\rightarrow$  +  $\curvearrowright$   
 $\leftarrow$  -  $\curvearrowleft$



The postulate of resolving seventh for the negative system must be read: the negative seventh moves one step up. The C-<sub>5</sub> requires the negative seventh and negative fifth to move one step up. The C-<sub>7</sub> requires all the tones except the root to move up. This system may be of great advantage in building up climaxes.

Positive:

Negative:

C<sub>3</sub> C-<sub>3</sub>

The image shows two musical staves. The left staff, labeled C<sub>3</sub>, contains five chords in a sequence: C (root), A (seventh), F (fifth), D (third), and B (second). The right staff, labeled C-<sub>3</sub>, contains five chords in a sequence: C (root), E (seventh), G (fifth), B (third), and D (second).

The root-tone of the negative system is the seventh of the positive and vice-versa.

It is easy to see how the other cycles would operate.

C<sub>5</sub>C-<sub>5</sub>

C<sub>5</sub> C-<sub>5</sub>

The image shows two musical staves. The left staff, labeled C<sub>5</sub>, contains three chords in a sequence: C (root), G (seventh), and E (fifth). The right staff, labeled C-<sub>5</sub>, contains three chords in a sequence: C (root), E (seventh), and G (fifth).

C<sub>7</sub>C-<sub>7</sub>

C<sub>7</sub> C-<sub>7</sub>

The image shows two musical staves. The left staff, labeled C<sub>7</sub>, contains four chords in a sequence: C (root), B (seventh), G (fifth), and F (third). The right staff, labeled C-<sub>7</sub>, contains four chords in a sequence: C (root), F (seventh), G (fifth), and B (third).



If one wishes to read the negative system as if it were positive, the rules must be changed as follows:

The C-3 requires the ascending of 1  
 The C-5 " " " " 1 and 3  
 The C-7 " " " " 1, 3 and 5

Special Applications of S(7)

A. Groups

S(7) finds its application in  $G_4$ , either as the first or the last chord of the group.

The following forms are possible:

$S(5) + S(\frac{6}{4}) + S(\frac{6}{5})$	$S(7) + S(\frac{6}{4}) + S(6)$
$\longleftrightarrow$	$\longleftrightarrow$
$S(5) + S(\frac{6}{4}) + S(\frac{4}{3})$	$S(7) + S(\frac{6}{4}) + S(\frac{4}{3})$
$\longleftrightarrow$	$\longleftrightarrow$
$S(5) + S(\frac{6}{4}) + S(2)$	$S(7) + S(\frac{6}{4}) + S(2)$
$\longleftrightarrow$	$\longleftrightarrow$

The cycle between the extreme chords of  $G_4$  may be either  $C_0$ , or  $C_3$ , or  $C_5$ .

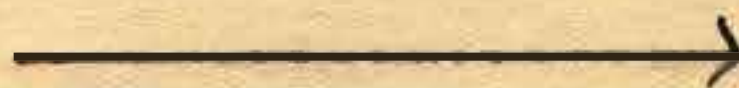


Besides  $G_3$  there is a special group where  $S(\frac{4}{3})$  is used as a passing chord. There are two forms of this group.

$$(A) \quad G_3^4(r) = S(5)^{\textcircled{3}} + S(\frac{4}{3}) + S(7) \quad \text{or } S(5)$$



$$(B) \quad G_3^4(l) = S(6)^{\textcircled{5}} + S(\frac{4}{3}) + S(7) \quad \text{or } S(5)$$



These two forms may be used in one direction only.

All positions are available.

Rule of voice-leading: bass and one of the voices of doubling move stepwise down. Common tones sustained.

The cycle between the extreme chords in the first form is  $C_3$ ; in the second form it is  $C_0$ .





B. Cadences.

The following applications of S(7) are commonly known:

- |     |        |             |       |
|-----|--------|-------------|-------|
| (1) | $IV_7$ | $I_4 - V_7$ | $I_5$ |
| (2) | $IV_5$ | " "         | "     |
| (3) | $II_5$ | " "         | "     |
| (4) | $II_3$ | " "         | "     |

In addition to this the following forms may be offered:

- |     |                           |               |       |
|-----|---------------------------|---------------|-------|
| (5) | Any of the previous forms | $I_4 - III_5$ | $I_5$ |
| (6) | "                         | $I_4 - VII_5$ | $I_5$ |

Besides these there are two ecclesiastic forms:

- |     |                            |       |
|-----|----------------------------|-------|
| (1) | $I_5 - IV^{(5)}_{(II_5)}$  | $I_5$ |
| (2) | $I_5 - IV^{(5)}_{(VII_3)}$ | $I_5$ |

(please see next page)



(1) (2) (3) (4)

etc. etc.

(5) (6) (1) (5) (2)

(1, 2, 3, 4) (1, 2, 3, 4)

etc. etc.



J O S E P H S C H I L L I N G E R

C O R R E S P O N D E N C E C O U R S E

With: Dr. Jerome Gross

Subject: Music

Lesson LXXXVIII.

Symmetric Zero Cycle (C<sub>0</sub>)

Symmetric C<sub>0</sub> offers an extraordinary versatility on S(7) as seven structures of the latter have been in use.

If evolving of the forms of S(7) would have been devised scientifically, they would be obtained in the following order.

Taking c - e - g - b<sup>b</sup> (4 + 3 + 3) as the most common form and producing variations thereof, we obtain two other forms:

c - e<sup>b</sup> - g - b<sup>b</sup> (3 + 4 + 3)  
and c - e<sup>b</sup> - g<sup>b</sup> - b<sup>b</sup> (3 + 3 + 4)

Taking another form, c - e - g - b (4 + 3 + 4), we obtain two other forms:

c - e - g<sup>#</sup> - b (4 + 4 + 3)  
and c - e<sup>b</sup> - g - b (3 + 4 + 4)

These two groups of three are distinctly different but as music has made the use of them for quite some time our ear does not find it objectionable any longer to mix all of them in one harmonic continuity.



Besides these six forms there is a  
 $c - e^b - g^b - b^{\bar{b}}$  (3 + 3 + 3 + 3) and might have been  
 $c - e - g^\# - b^\#$  (4 + 4 + 4 + 4) if there would not  
 be an objection to the fact that  $c - b^\#$  is an enhar-  
 monic octave.

A continuity on symmetric  $C_0$  of all seven  
 structures offers 5040 permutations. Thus a c - chord  
 alone can move (without changing its position and  
 without coefficients of recurrence being applied) for  
 $5040 \times 7 = 35,280$  chords.

A method of selecting the best of the  
 available progressions must be based on the following  
 principle: the best progressions on symmetric  $E_0$  are  
 due to identity of steps or to contrary motion.

Example

(1) Identity of Steps:



all semitones

(2) Contrary motion:







The principle of variation of the chord-structures and their positions remains the same as in S(5):

<u>Structure</u>		<u>Position</u>
Constant	—————	Variable
Variable	—————	Constant

S(7) in the following table has a dual system of indications: letter symbols and adjectives. The adjectives are chosen so that they do not adhere to the degrees of any scale but to structure alone. Thus, such a common adjective as "dominant" had to be sacrificed.

S(7) Table of Structures

The diagram shows a musical staff with seven measures, each containing a chord structure. The structures are labeled S1 through S7 and are associated with specific adjectives: MAJOR, MINOR, LARGE, SMALL, DIMINISHED, AUGMENTED I, and AUGMENTED II. The notes are written in a way that illustrates the unique structure of each chord.

An Example of Continuity in C<sub>0</sub>:

Structures:  $S_3 + S_7 + S_4 + S_1$

Coefficients ( $r_{5 \div 4}$ ):  $4S_3 + S_7 + 3S_4 + 2S_1 + 2S_2 +$   
 $+ 3S_7 + S_4 + 4S_1$



The image shows two systems of handwritten musical notation. Each system consists of a treble clef staff and a bass clef staff. The treble clef staves contain chords, often with a '3' below them, and various accidentals (sharps, flats, naturals). The bass clef staves contain single notes, some with accidentals. The notation is somewhat dense and appears to be a study or exercise in chord structures and their bass line accompaniment.

As in S(5), any combination of the forms of S(7) by 2, 3, 4, 5, 6 and 7 may be used.

Type III (Symmetric).

As in the previous cases when dealing with symmetrical tonics  $C_0$  may be applied either to any of the tonics or as a continuous change of chord structures occurring with each tonic.

When structures of S(5) and S(7) have to be specified in one continuity, they must have full indications:

$S_1(5)$ ;  $S_2(5)$ ;  $S_3(5)$ ;  $S_4(5)$  and

$S_1(7)$ ;  $S_2(7)$ ;  $S_3(7)$ ;  $S_4(7)$ ;  $S_5(7)$ ;  $S_6(7)$ ;

$S_7(7)$



Two Tonics ( $\sqrt{2}$ )

As the  $\sqrt{2}$  forms the center of the octave the progression  $1 \rightarrow \sqrt{2}$  ( $C \rightarrow F^\#$ ) is positive and  $\sqrt{2} \rightarrow 2$  ( $F^\# \rightarrow C$ ) is negative.

The system of Two Tonics which was continuous on  $S(5)$  becomes closed on  $S(7)$ . Transformations correspond to  $C_5$ .

*Example of continuity*

Three Tonics ( $\sqrt[3]{2}$ )

Continuous system: moves four times.

Transformations correspond to  $C_3$ . To obtain  $S(7)$



after an S(5) use the position which would correspond to continuous progression of S(7).

The image shows two systems of handwritten musical notation. The first system consists of two staves. The upper staff is in treble clef and contains a sequence of ten chords, each represented by a circle with notes and accidentals. The lower staff is in bass clef and contains ten single notes corresponding to the chords above. The second system also consists of two staves. The upper staff is in treble clef and contains ten chords, similar to the first system. Below each chord in this system is a label: S(7), S(5), S(7), S(5), S(7), S(5), S(7), S(5), S(7), S(5). The lower staff is in bass clef and contains ten single notes. A circled '7' is written below the first note of the bass staff in this system.

Example of Continuity:

The image shows a single system of handwritten musical notation with two staves. The upper staff is in treble clef and contains a sequence of ten chords, each represented by a circle with notes and accidentals. The lower staff is in bass clef and contains ten single notes corresponding to the chords above.





Four Tonics ( $\sqrt[4]{2}$ )

Closed system. Transformations correspond to  $C_3$ . S(7) after S(5) as in Three Tonics.

EXAMPLE OF CONTINUITY.

Six Tonics ( $\sqrt[6]{2}$ )

Continuous system: moves two times. Transformations correspond to  $C_7$ . S(7) after S(5) as in previous cases. Both positive and negative progressions are fully satisfactory. To obtain the negative progressions read the positive backwards.



Handwritten musical notation for the first system. The treble staff contains several chords with notes and accidentals, including a '5(7) -> +' annotation. A bar line is present. The bass staff contains notes with accidentals, including a '5' annotation below the first measure and an arrow pointing left above the final measure.

Handwritten musical notation for the second system. The treble staff contains several chords with notes and accidentals, with annotations '5(7)', '5(5)', '5(7)', '5(5)', '5(7) ->', '5(7)', '5(5)', '5(7)', '5(5)', and '5(7) ->' written below the notes. The bass staff contains notes with accidentals.

EXAMPLE OF CONTINUITY.

Handwritten musical notation for the third system, titled 'EXAMPLE OF CONTINUITY'. The treble staff contains several chords with notes and accidentals. The bass staff contains notes with accidentals.



Twelve Tonics ( $\sqrt[12]{2}$ )

Closed system. All specifications and applications as in Six Tonics.

Handwritten musical notation for the first system. It consists of two staves. The upper staff is in treble clef and contains twelve chords, each with a unique set of accidentals. The lower staff is in bass clef and contains twelve single notes, each corresponding to a chord above it. A label 'S(7) → +' is written on the left side of the upper staff. An arrow points to the right at the end of the system.

Handwritten musical notation for the second system. It consists of two staves. The upper staff contains twelve chords, with the first six having labels 'S(7)', 'S(5)', 'S(7)', 'S(5)', 'S(7)', and 'S(5)' below them. The lower staff contains twelve single notes. A double bar line is present after the sixth measure. An arrow points to the right at the end of the system.

EXAMPLE OF CONTINUITY

Handwritten musical notation for the 'EXAMPLE OF CONTINUITY' section. It consists of two staves. The upper staff contains a sequence of twelve chords with various accidentals. The lower staff contains a sequence of twelve single notes. The notes in the lower staff appear to be the root notes of the chords above them.



Lesson LXXXIX.Hybrid Five-Part Harmony

The technique of continuous S(7) makes it possible to evolve a hybrid five-part harmony, where bass is a constant root tone and the four upper functions assume variable forms of S(7) with respect to bass.

By placing an S(7) either on root, or third, or fifth, or seventh of the bass root we obtain all forms of S in five-part harmony. An S(5) has to be represented with the addition of 13<sup>th</sup> (the so-called "added sixth").

Forms of Chords in Hybrid Five-Part (4 + 1) Harmony

The 4 Upper Parts.	5	7	9	11	13
	3	5	7	9	11
	1	3	5	7	9
S	13	1	3	5	7

The Bass	1	1	1	1	1
-------------	---	---	---	---	---

The Forms of Tension	S(5)	S(7)	S(9)	S(11)	S(13)
----------------------------	------	------	------	-------	-------

It is possible to move continuously either form or any of the combinations of forms in any





rhythmic form of continuity. It is important to realize that the tonal cycles do not correspond in the upper four parts to the tonal cycles in the bass when the forms of tension are variable. For example, f - a - c - e may be 3 - 5 - 7 - 9 in a DS(9) as well as 7 - 9 - 11 - 13 in a GS(13). In such a case a progression  $C_5$  for the bass with  $S(9) \longrightarrow S(13)$  produces  $C_0$  for the upper four parts.

The principle of exchange and octave-inversion of the common tones holds true.

Three forms of harmonic continuity will be used in the following illustrations (these forms of continuity are applicable in the four-part harmony as well). When chord structures acquire greater tension and also when the compensation for the diatonic deficiency is required, it is often desirable to use preselected forms of chord-structures yet moving diatonically. Such system has a bass belonging to one definite diatonic scale, while the chord structures acquire various accidentals in order to produce a definite sonority. In the general classification of the harmonic progressions the latter type is known as diatonic-symmetric.

#### Three Types of Harmonic Progressions

- I. Diatonic
- II. Diatonic-Symmetric
- III. Symmetric



The following examples will be carried out in all three types of harmonic continuity. Constant and variable forms of tension will be offered.

In order to select a desirable form of structures for the forms of different tension it is advisable to select a scale first, as such a scale offers all forms of tension. For example, if the scale selected is  $\bar{c} - d - e - f^{\#} - g - a - b^b$ ,

$S(5) = c - e - g - a$ ;  $S(7) = c - e - g - b^b$ ;  
 $S(9) = c - e - g - b^b - d$ ;  $S(11) = c - g - b^b - d - f^{\#}$ ;  
 $S(13) = c - b^b - d - f^{\#} - a$ .

Though the same scale would be ideal for the progression, it is not impossible and not very undesirable to use any other scale for the chord-progressions.

(please see following pages)



Hybrid Five-Part Harmony

(Tables and Examples)

(1) Continuity of S(5) [monomials]

Scale: c - d - e - f<sup>#</sup> - g - a - b<sup>b</sup>

Type I.

Type II.

Type III.



(2) Continuity of S(7) [monomials]

Type I.

Type II.

Type III.





(3) Continuity of S(9) [monomials]

Type I.

Musical notation for Type I. The treble clef contains a sequence of ten chords, each with a sharp or flat sign. The bass clef contains single notes, some with sharp or flat signs, corresponding to the chords above.

Type II.

Musical notation for Type II. The treble clef contains a sequence of ten chords, each with a sharp or flat sign. The bass clef contains single notes, some with sharp or flat signs, corresponding to the chords above.

Type III.

Musical notation for Type III. The treble clef contains a sequence of chords, with mathematical symbols  $\sqrt{2}$ ,  $\sqrt[3]{2}$ , and  $\sqrt[4]{2}$  written below. The bass clef contains single notes, some with sharp or flat signs.

Musical notation for Type III. The treble clef contains a sequence of chords, with a  $\sqrt{2}$  symbol below. The bass clef contains single notes, some with sharp or flat signs. Arrows point to the right and left below the notes.

Musical notation for Type III. The treble clef contains a sequence of chords, with a  $\sqrt[12]{2}$  symbol below. The bass clef contains single notes, some with sharp or flat signs.



(4) Continuity of S(11) [monomials]

Type I.

Musical notation for Type I. The upper staff contains a sequence of 11 chords, each with a sharp sign and a flat sign. The lower staff contains a sequence of 11 notes, each with a sharp sign and a flat sign.

Type II.

Musical notation for Type II. The upper staff contains a sequence of 11 chords, each with a sharp sign and a flat sign. The lower staff contains a sequence of 11 notes, each with a sharp sign and a flat sign.

Type III.

Musical notation for Type III (first system). The upper staff contains a sequence of 11 chords, each with a sharp sign and a flat sign. The lower staff contains a sequence of 11 notes, each with a sharp sign and a flat sign. The first three chords are labeled with  $\sqrt{2}$ ,  $\sqrt[3]{2}$ , and  $\sqrt[4]{2}$  respectively. An arrow points from the second chord to the third.

Musical notation for Type III (second system). The upper staff contains a sequence of 11 chords, each with a sharp sign and a flat sign. The lower staff contains a sequence of 11 notes, each with a sharp sign and a flat sign. The first chord is labeled with  $\sqrt[6]{2}$ . A double-headed arrow is positioned below the first three chords.

Musical notation for Type III (third system). The upper staff contains a sequence of 11 chords, each with a sharp sign and a flat sign. The lower staff contains a sequence of 11 notes, each with a sharp sign and a flat sign. The first chord is labeled with  $\sqrt[12]{2}$ .



(5) Continuity of S(13) [monomials]

Type I.

Type II.

Type III.

Combinations by two (binomials), three (trinomials), four (quadrinomials) and five (quintinomials) may be devised in a similar way.



Table of Combinations

Arabic numbers in the following tables represent

Chord Structures (8)Combinations by 2

5 + 7	7 + 9	9 + 11	11 + 13
5 + 9	7 + 11	9 + 13	
5 + 11	7 + 13		
5 + 13			

10 combinations, 2 permutations each

Total:  $10 \times 2 = 20$

Combinations by 3

5 + 7 + 9	7 + 9 + 11	9 + 11 + 13
5 + 7 + 11	7 + 9 + 13	
5 + 7 + 13	7 + 11 + 13	
5 + 9 + 11		
5 + 9 + 13		
5 + 11 + 13		

10 combinations, 6 permutations each

Total:  $10 \times 6 = 60$





Combinations by 4

$$5 + 7 + 9 + 11$$

$$7 + 9 + 11 + 13$$

$$5 + 7 + 9 + 13$$

$$5 + 7 + 11 + 13$$

$$5 + 9 + 11 + 13$$

5 combinations, 24 permutations each

Total:  $5 \times 24 = 120$

Combinations by 5

$$5 + 7 + 9 + 11 + 13$$

1 combination, 120 permutations

Total:  $1 \times 120 = 120$

All other cases of trinomial, quadri-  
nomial, quintinomial and bigger combinations are  
treated as coefficients of recurrence.

Example:  $S^{\curvearrowright} = 2S(5) + S(7) + 2S(9) =$

$$= S(5) + S(5) + S(7) + S(9) + S(9),$$

i.e., a quintinomial with two identical pairs.



Coefficients of recurrence may be applied to the composition of continuity consisting of the forms of variable tension.

Examples

Type I.  $2S(5) + S(9) + S(13) + 2S(7)$

Type II.

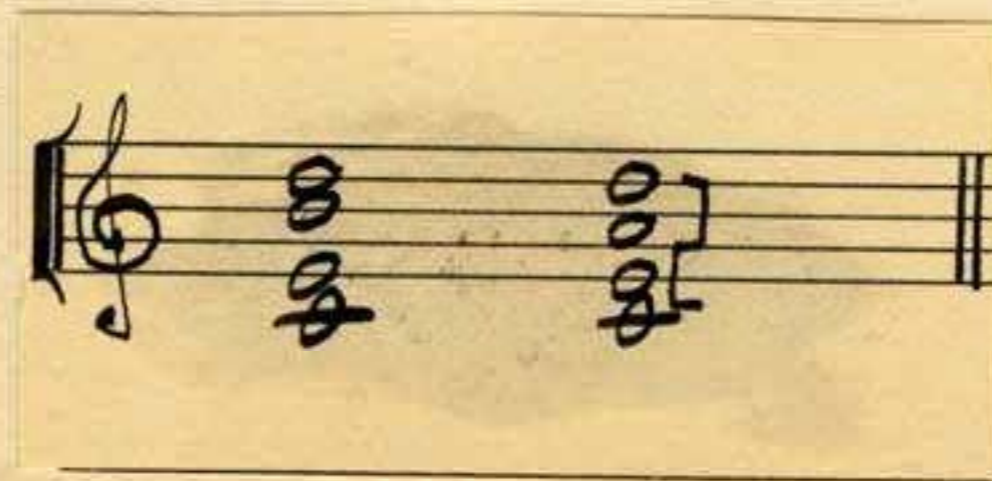
Type III.  $\sqrt[3]{2}$



Lesson XC.Ninth-Chords. S(9)Diatonic System

Ninth-chords in four-part harmony are used with the root-tone in the bass only, thus forming a hybrid four-part harmony [like S(5) with the doubled root]. The three upper parts consist of 3, 7 and 9. The seventh and the ninth are subject to resolution through the stepwise downward motion. If one function resolves at a time it is always the higher one (the ninth). A resolution of one function at a time produces  $C_0$ . Other cycles derive from the simultaneous resolutions of both functions (the ninth and the seventh). No consecutive S(9) are possible through this system [they alternate with S(7) and S(5)].

The reason for resolving the 9th and not the 7th first in  $C_0$  is the latter results in a chord-structure alien to the usual seven-unit diatonic scales (the intervals in the three upper voices are fourths).





Positions of S(9)

As bass remains constant, the three upper voices are subject to 6 permutations resulting in the corresponding distributions.

Table of Positions of S(9)

The image shows a grand staff with six measures. The bass clef line contains a constant bass note (C4). The treble clef line shows six different chordal permutations of the upper voices (E4, G4, B4) above the bass note. The permutations are: 1) E4, G4, B4; 2) E4, B4, G4; 3) G4, E4, B4; 4) G4, B4, E4; 5) B4, E4, G4; 6) B4, G4, E4.

Resolutions of S(9)

The image shows four resolutions of S(9) on a grand staff, labeled C<sub>0</sub>, C<sub>3</sub>, C<sub>5</sub>, and C<sub>7</sub>. Each resolution consists of a sequence of chords in the treble clef and a corresponding melodic line in the bass clef. C<sub>0</sub> shows a sequence of three chords with a bass line moving from C4 to E4. C<sub>3</sub> shows a sequence of three chords with a bass line moving from C4 to G4. C<sub>5</sub> shows a sequence of three chords with a bass line moving from C4 to B4. C<sub>7</sub> shows a sequence of two chords with a bass line moving from C4 to E4. The C<sub>3</sub> resolution is circled in red.

Resolutions (except C<sub>0</sub>) produce positive cycles only. C<sub>3</sub> is characteristic of Mozart, Clementi and others of the same period. C<sub>5</sub> (the second resolution) is the most commonly known, especially with b<sup>b</sup>





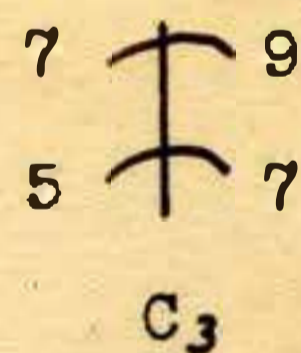
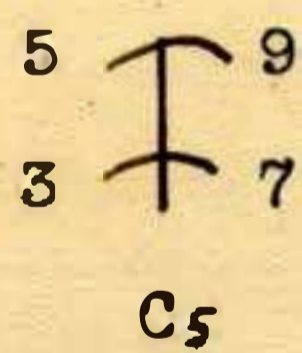
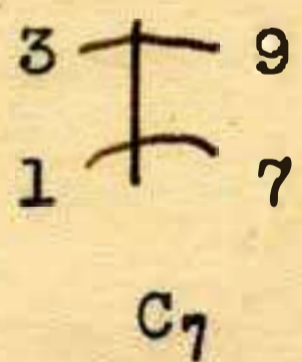
in the first chord (making a dominant chord of F-major of it).  $C_7$  is characteristic of Bach and contrapuntalists. They achieved such progression through the idea of two pairs of voices moving in thirds in contrary motion. Read the last bar with  $b^b$  and  $f^\sharp$  and add  $S(5)$  g-minor. All these cases of resolution were known to the classics through melodic manipulations (contrapuntal heritage) and not through the idea of independent structures we call  $S(9)$ .

Preparation of  $S(9)$  bears a great similarity with the preparation of  $S(7)$ . There is even an absolute correspondence in the cycles with respect to technical procedures.

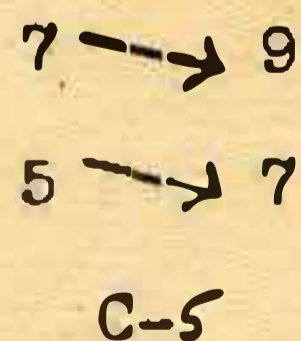
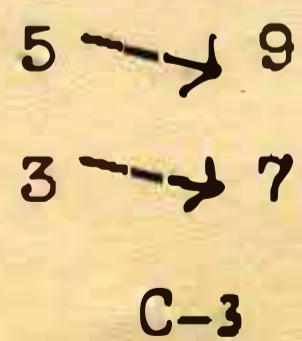
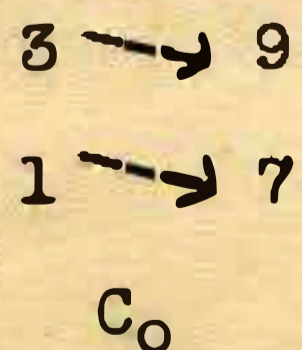
The same three methods constitute the technique of preparation (suspending, descending, ascending).

Table of Preparations.

(1) Suspending:



(2) Descending:





(3) Ascending:

3 → 9

5 → 9

7 → 9

1 → 7

3 → 7

5 → 7

C-3

C-5

C-7

Preparations of S(9)

(1)

C7 C5 C5 C3

(2)

C0 C0 C-3 C-5

(3)

C-3 C-5 C-7



It follows from the above chart that some of the preparations of S(9) require an S(5), some - S(7) and some allow both. It is practical to have S(5) or S(7) preparing S(9) with the root in the bass.

The first form of preparation was known to the classics as double suspension.

Example:

Similar cadence was used in major.

Another example of a characteristic classical cadence:



Example of Continuity Containing S(9):

The image shows two systems of handwritten musical notation for five-part harmony. Each system consists of a treble and bass staff. The first system has 7 measures with chords labeled C0, C5, E3, C7, C0, C5, C5, C5, E3, C-3, C0, C7, and G<sup>4</sup>/<sub>3</sub>. The second system has 5 measures with chords labeled C-5, E3, C7, C3, C7, C3, C0, C3, C5, and C5. Fingerings are indicated by numbers 1-5 below the notes.

Homework:

- (1) Make complete tables of preparations and resolutions from all positions.
- (2) Write diatonic continuity containing S(9).
- (3) Make some modal transpositions of the examples thus obtained.
- (4) Write continuity containing S(9) in the second type (diatonic-symmetric) of harmony. Select chord-structures from the examples of hybrid five-part harmony.





Lesson XCI.Ninth-Chords, S(9)Symmetric System.

The above described classical (preparation-resolution) technique commonly used in the diatonic system is applicable to the symmetric system as well. Symmetric roots correspond to the respective cycles:  $C_5$  - to  $\sqrt{2}$ ;  $C_3$  - to  $\sqrt[3]{2}$  and  $\sqrt[4]{2}$ ;  $C_7$  - to  $\sqrt[6]{2}$  and  $\sqrt[12]{2}$ . With this in view, continuity consisting of  $S(5)$ ,  $S(7)$  and  $S(9)$  and operated through the classical technique may be offered.

Symmetric  $C_0$  is quite fruitless when  $S(9)$  alone is used, as the upper three functions (3, 7, 9) produce an incomplete seventh-chord, the permutations of which ( $3 \leftrightarrow 7$ ,  $3 \leftrightarrow 9$ ) sound awkward with the exception of one:  $7 \leftrightarrow 9$ .

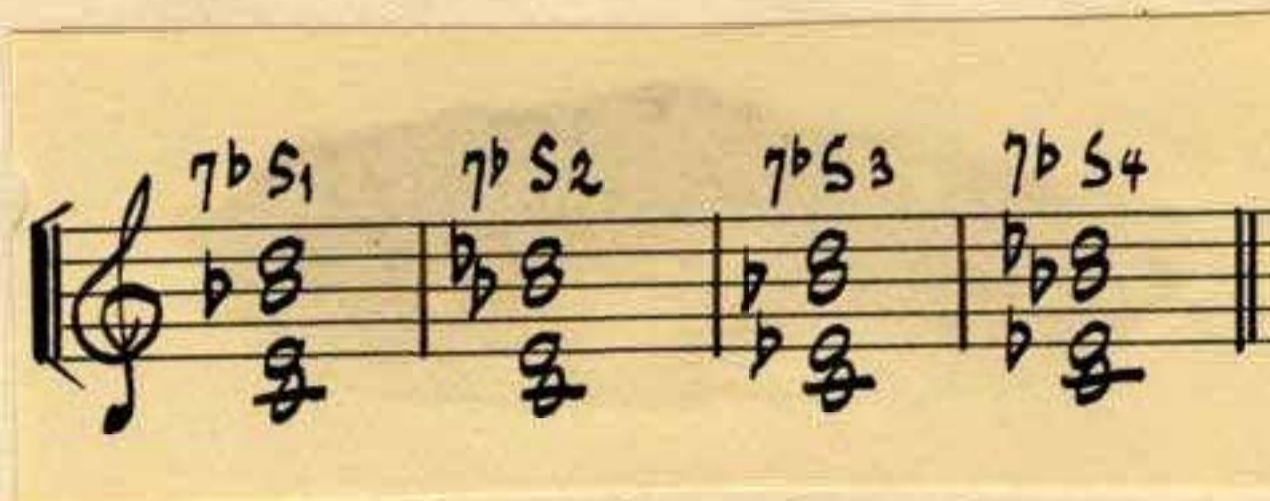
As  $S(9)$  in the hybrid four-part harmony is an incomplete structure (5 is omitted), the adjectives may be applied only with a certain allowance for the 5th.

There are two distinctly different families of  $S(9)$  not to be mixed except when in  $C_0$ :

- (1) The minor seventh family
- (2) The major seventh family



The minor 7th family includes the following structures:



You may attribute to them the following adjectives in their respective order:

$7^b S_1$  - large

$7^b S_2$  - diminished

$7^b S_3$  - minor

$7^b S_4$  - small

The major 7th family includes the following structures:



Their respective adjectives are:

$7^{\#} S_1$  - major

$7^{\#} S_2$  - augmented I

$7^{\#} S_3$  - augmented II



These are the only possible forms.

It seems that all combinations of the two families, except the ones producing consecutive seventh ( ${}^7\flat S_4 \leftrightarrow {}^7\sharp S_1$ ;  ${}^7\sharp S_2 \leftrightarrow {}^7\flat S_3$ ;  ${}^7\flat S_3 \leftrightarrow {}^7\sharp S_2$ ;  ${}^7\flat S_1 \leftrightarrow {}^7\flat S_4$ ), are satisfactory when in  $C_0$ . On the different roots the forms of  $S(9)$  must belong to one family.

Example of  $C_0$  Continuity:

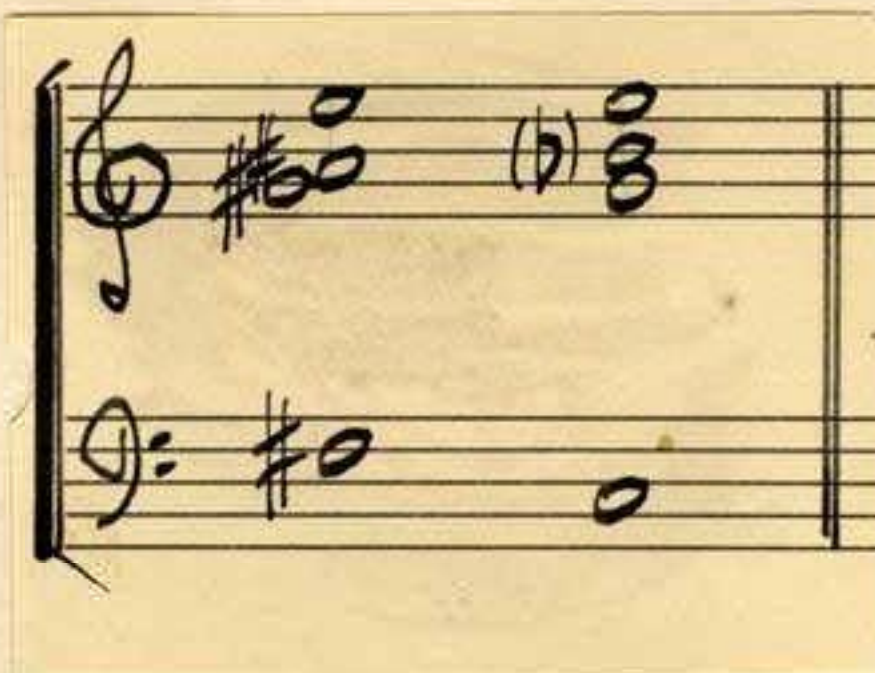
Full indication for  $S(9)$  when used in combinations with  $S(5)$  and  $S(7)$ :

- |                       |                       |                       |                    |
|-----------------------|-----------------------|-----------------------|--------------------|
| ${}^7\flat S_1(9)$ ;  | ${}^7\flat S_2(9)$ ;  | ${}^7\flat S_3(9)$ ;  | ${}^7\flat S_4(9)$ |
| ${}^7\sharp S_1(9)$ ; | ${}^7\sharp S_2(9)$ ; | ${}^7\sharp S_3(9)$ ; |                    |

Two Tonics ( $\sqrt{2}$ ). The technique corresponds to  $C_5$ .



To resolve the last chord of the preceding table use position (b) of the resolution technique.



Example of Continuity:

Three Tonics ( $\sqrt[3]{2}$ ). The technique corresponds to  $C_3$ .

RESOLUTION                      PREPARATION                      PROGRESSION

UNDESIRABLE:  
AWKWARD STEPS

etc.





In order to acquire a complete understanding of the voice-leading in the preceding table of progressions (9 - 6 - 9 - 6 etc.), reconstruct mentally an S(7) instead of an S(6). Then the first two chords will appear in the following positions:



It is clear now that d<sup>#</sup> and f<sup>x</sup> are the necessary 7 and 9 of the following chord.



Example of Continuity:





Four Tonics ( $\sqrt[4]{2}$ ). The technique corresponds to  $C_3$ .

Resolution

Preparation

Progression

Example of Continuity:

Six Tonics ( $\sqrt[6]{2}$ ). The technique corresponds to  $C_7$ .

Resolution

Preparation

Progression



The above consecutive sevenths are unavoidable with this technique.

The position of every S(9) is based on the assumption that the preceding chord was S(5) and not S(7).

Continuity: S(9) + S(7) + S(5)

Handwritten musical notation showing a sequence of chords on a grand staff. The treble clef staff contains seven chords with various accidentals (flats, sharps, naturals). The bass clef staff contains seven notes corresponding to the chords, with fingerings 9, 7, 5, 9, 7, 5-7, and 5 written below them.

The negative system which may be obtained by reading the above tables in position (b) is not as desirable with these media as the positive. The same concerns the following  $\sqrt[12]{2}$ . More plastic devices (general forms of transformations) will be offered later.

Twelve Tonics ( $\sqrt[12]{2}$ ). The technique corresponds to C<sub>7</sub>.

Handwritten musical notation illustrating the technique for Twelve Tonics. It is divided into three sections: RESOLUTION, PREPARATION, and PROGRESSION. The RESOLUTION section shows two chords. The PREPARATION section shows two chords. The PROGRESSION section shows a sequence of chords with various accidentals, ending with "etc."



Continuity:  $S(9) + S(7) + S(5)$

Handwritten musical notation on a staff with treble and bass clefs. The notation consists of two staves. The upper staff has a treble clef and the lower staff has a bass clef. The notes are written in a sequence of chords and single notes, with various accidentals (sharps, flats, naturals) and stems. Annotations include 'const abc' written below several notes, a 'K<sub>1</sub>' symbol, and 'etc.' at the end of the sequence.

Homework: Exercises in the different symmetric systems containing  $S(5)$ ,  $S(7)$  and  $S(9)$  with application of different structures and the  $C_0$  between the roots.





J O S E P H S C H I L L I N G E R

C O R R E S P O N D E N C E C O U R S E

With: Dr. Jerome Gross

Subject: Music

Lesson XCII.

Four-Part Harmony (Continuation)

Eleventh-Chords. S(11).

Diatonic System.

Eleventh-chords in four-part harmony are used with root-tone in the bass only, thus forming a hybrid four-part harmony [like S(5) with the doubled root]. The three upper parts consist of 7, 9, 11. An S(11) has an advantage over S(9) as the upper functions form a complete S(5). All three upper functions are subject to resolution through the stepwise downward motion. Resolutions of less than three upper functions produce  $C_0$ .

No consecutive S(11) are possible through this system. They alternate with the other structures.

For the reasons explained in the previous chapter the  $C_0$  resolutions must follow in the direction of the decreasing functions: first 11 must be resolved, then 9, then 7. When two functions resolve simultaneously they are 11 and 9. An S(11) allows a continuous chain of resolutions.

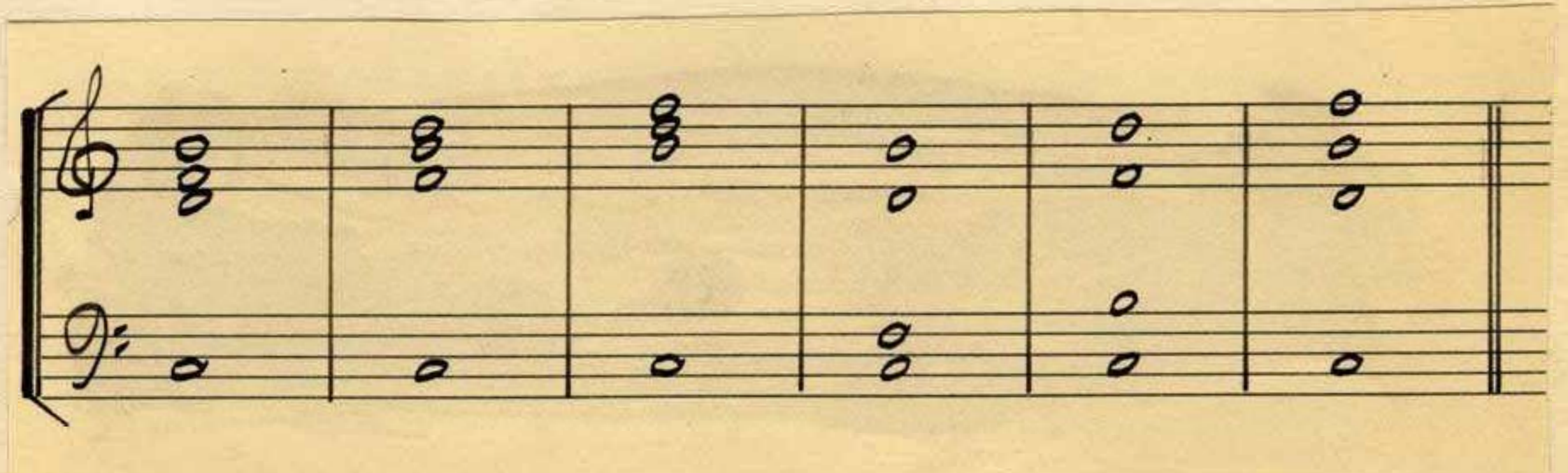


$$S(11) \ 11 \rightarrow S(9) \ 9 \rightarrow S(7) \ 7 \rightarrow S(6) \textcircled{3}$$

An eleventh-chord through resolution of the eleventh becomes a ninth-chord; a ninth-chord through resolution of the ninth becomes an incomplete seventh-chord (without a fifth), or a complete  $S\left(\frac{4}{3}\right)$  as in the corresponding resolutions of  $S(9)$ ; an incomplete seventh-chord through resolution of the seventh becomes a sixth-chord with the doubled third.

Positions of  $S(11)$ .

As bass remains constant, the three upper voices are subject to 6 permutations. Seventh, ninth and eleventh form a triad corresponding to a root, a third and a fifth while the bass is placed one degree higher. A c  $S(11)$  has an appearance of b  $S(5)$  with a bass raised one step.





Resolutions of S(11).

The musical notation consists of two staves. The upper staff is in treble clef and the lower in bass clef. The piece is divided into six measures, each with a context label above it: C<sub>0</sub>, C<sub>3</sub>, C<sub>3</sub>, C<sub>3</sub>, C<sub>3</sub>, and C<sub>5</sub>. The notes are as follows:

- Measure 1 (C<sub>0</sub>): Treble has two chords (S(11) and S(9)); Bass has two notes (S(11) and S(9)).
- Measure 2 (C<sub>3</sub>): Treble has two chords (S(11) and S(7)); Bass has two notes (S(11) and S(7)).
- Measure 3 (C<sub>3</sub>): Treble has two chords (S(11) and S(11/9)); Bass has two notes (S(11) and S(11/9)).
- Measure 4 (C<sub>3</sub>): Treble has four chords (S(11), S(9), S(7), S(6)); Bass has four notes (S(11), S(9), S(7), S(6)).
- Measure 5 (C<sub>3</sub>): Treble has three chords (S(11), S(6), S(2)); Bass has three notes (S(11), S(6), S(2)). A slur with 'n.' is over the last two notes of the bass staff.
- Measure 6 (C<sub>5</sub>): Treble has two chords (S(11) and S(7)); Bass has two notes (S(11) and S(7)).

As it follows from the above table, when S(11) resolves into S(9) in C<sub>0</sub>, S(9) has its proper structural constitution (i.e., 1, 3, 7, 9). For the same reason the C<sub>7</sub> resolution does not appear on this table, as the structural constitution of S(9), into which S(11) would resolve, is 1, 5, 7, 9 and this does not sound satisfactory according to our musical habits.

The musical notation shows two staves. The upper staff is in treble clef and the lower in bass clef. The piece is divided into two measures, each with a context label above it: C<sub>7</sub>. The notes are as follows:

- Measure 1 (C<sub>7</sub>): Treble has two chords (S(11) and S(9)); Bass has two notes (S(11) and S(9)).
- Measure 2 (C<sub>7</sub>): Treble has two chords (S(11) and S(9)); Bass has two notes (S(11) and S(9)).

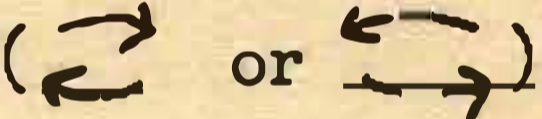
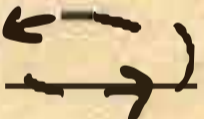
The above resolutions correspond to the classical resolutions of the triple suspensions.



Preparation of S(11) in the positive cycles has the cyclic correspondence with the preparation of S(7) and S(9) through suspensions. Nevertheless the manner of reasoning is somewhat different in this case.

As S(11) has an appearance of an S(5) with a bass placed one step higher, the most logical assumption is: take S(5), move its bass one step up and this will produce an S(11) of a proper structural constitution. In such a case the relation of the three stationary upper functions is  $C_0$ . Being common tones they may be inverted or exchanged.

The first case gives a clue to the preparation of other cycles (positive and negative as well).

The method of preparation implies merely the most gradual transformation ( or ) for the three upper functions.

To prepare S(11) after an S(5) in  $C_0$  move all upper functions down scalewise and leave the bass stationary (which is the converse of the first proposition).

(please see next page)





Preparations of S(11)

*C<sub>0</sub>*

S(5) S(11)    S(7) S(11)    S(4/3) S(11)    S(9) S(11)

*C<sub>7</sub>*

S(5) S(11)    S(5) S(11)    S(5) S(11)    S(7) S(11)

*C<sub>5</sub>*

S(5) S(11)    S(7) S(11)    S(7) S(11)

*C<sub>3</sub>*

S(5) S(11)    S(7) S(11)    S(4/3) S(11)    S(5) S(11)    S(7) S(11)    S(4/3) S(11)

*C<sub>-5</sub>*

S(5) S(11)    S(7) S(11)    S(4/3) S(11)    S(11)    S(5) S(11)    S(7) S(11)    S(4/3) S(11)



When all tones are in common in the three upper parts it is advisable to use the suspension (over the bar) method.

When some of the upper parts move and some remain stationary either the within the bar or the over the bar preparation may be used.

Characteristic progressions and cadences where all forms of tension [from S(5) to S(11)] are applied:

(please see next page)



Handwritten musical notation on a grand staff. The treble clef staff contains chords with fingerings (e.g., 5-3-7, 9-7-11, 9-7-3, 1-7, 3-5, 5-7, 9-7-11, 0, 0, 0). The bass clef staff contains notes with fingerings (7, 11, 9, 7, 6, 7, 11, 9, 7, 6). Handwritten annotations include "count abc" with arrows pointing to specific notes, and "etc." at the end.

Handwritten musical notation on a grand staff. The treble clef staff contains chords with fingerings (0-0-0, 1-7, 3-5, 0, 0, 0, 0, 0, 0, 0, 0). The bass clef staff contains notes with fingerings (7, 11, 9, 4-3, 7, 11, 5, 11, 7, 5, 11, 7, 5). The word "etc." is written in the middle of the staff.

Handwritten musical notation on a grand staff. The treble clef staff contains chords with various accidentals (sharps and flats) and fingerings. The bass clef staff contains notes with fingerings (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).

Example of Continuity Containing S(11).

Handwritten musical notation on a grand staff illustrating continuity. The treble clef staff contains chords with fingerings and accidentals (e.g., G<sup>4</sup>/<sub>3</sub>, c5, c5, c7, c0, c-5, c3, c0, c3). The bass clef staff contains notes with fingerings (5, 4-3, 7, 11, 7, 5, 11, 7, 11, 11, 2). The notation is organized into two systems of three staves each.



Lesson XCIII.Eleventh-Chords, S(11)Symmetric System.

The above described technique of diatonic progressions containing S(11) is applicable to the symmetric system as well. The cyclic correspondence previously used remains the same. Thus preparations of S(11) are possible in all systems of the symmetric roots, while resolutions can be performed only when the acting cycle is  $C_3$  ( $\sqrt[3]{2}$  and  $\sqrt[4]{2}$ ) and  $C_5$  ( $\sqrt{2}$ ). There is no difficulty with any preparation of S(11) after a resolution, as the latter always consists of 1, 3, 5 and therefore may be connected with the following chord through the usual transformations.

Contrary to S(9), S(11) produces a highly satisfactory  $C_0$ , due to the presence of all functions without gaps in the three upper parts.

As in the ninth-chords, there are two distinctly different families of S(11) not to be mixed except when in  $C_0$ . The distinction becomes even greater than before and the danger of mixing more dangerous.

The structural constitution of S(11) permits the classification of such structures as S(5) with regard to their three upper functions.





Forms of S(11).The Minor  
Seventh FamilyThe Major  
Seventh Family

Handwritten musical notation showing six chords in two systems. The first system is labeled "The Minor Seventh Family" and contains three chords:  $7b5_1$ ,  $7b5_2$ , and  $7b5_3$ . The second system is labeled "The Major Seventh Family" and contains three chords:  $7\#5_1$ ,  $7\#5_2$ , and  $7\#5_3$ . Each chord is represented by a treble clef staff with a chord symbol and a bass clef staff with a single note. Below the bass clef staves, the chords are labeled: "major", "Minor", "Augmented", "Major", "Minor", and "Diminished".

These are the only possible forms as the diminished in the first group equals (enharmonically) a diminished S(9) and the augmented in the second group equals (enharmonically) the second augmented S(9) with a fifth and without a third.

Handwritten musical notation showing four chords in two systems. The first system contains two chords:  $7b5_1$  and  $7b5_2$ . The second system contains two chords:  $7\#5_1$  and  $7\#5_2$ . Each chord is represented by a treble clef staff with a chord symbol and a bass clef staff with a single note. Below the bass clef staves, there are two equals signs (=) indicating enharmonic equivalence between the chords in the first and second systems.

The selection of better progressions in  $C_0$  for the continuity of S(11) must be analagous to the selection of forms for S(5). Consecutive seventh shall not be used.



Example of  $C_0$  Continuity.

Musical notation showing a sequence of chords in the treble clef and single notes in the bass clef. The treble clef starts with a D $\flat$  chord, followed by D $\flat$  and D $\flat$  with a sharp sign, then D $\flat$  and D $\flat$  with a sharp sign, and continues with various chromatic alterations. The bass clef contains single notes corresponding to the chord roots.

Full indications for  $S(11)$  when used in combinations with other structures:

$7^b S_1(11)$ ;  $7^b S_2(11)$ ;  $7^b S_3(11)$   
 $7^b S_4(11)$ ;  $7^b S_5(11)$ ;  $7^b S_6(11)$

Two Tonics ( $\sqrt{2}$ ). The technique corresponds to  $C_5$ . Clockwise or counterclockwise transformations for continuous  $S(11)$ .

Resolution

Preparation

Progression

Musical notation illustrating the concepts of Resolution, Preparation, and Progression. It shows chords in the treble clef and notes in the bass clef. The word "possible" is written under the first two chords. The notation is divided into three sections by double bar lines.

Continuous  $S(11)$

Musical notation for Continuous  $S(11)$ . It shows a sequence of chords in the treble clef and notes in the bass clef. A curved arrow indicates a clockwise transformation, and a straight arrow indicates a counterclockwise transformation.



You may consider the upper three parts either as 7, 9, 11 in  $\leftrightarrow$  and  $\curvearrowright$  transformations or as 1, 3, 5 with a displaced bass.

Example of Continuity.

Handwritten musical notation for 'Example of Continuity'. It consists of two staves. The upper staff is in treble clef and contains a sequence of chords and notes, including some chromatic alterations. The lower staff is in bass clef and contains a sequence of notes, some of which are marked with a sharp sign (#).

Three Tonics ( $\sqrt[3]{2}$ ). The technique corresponds to  $C_3$  or to the  $\leftrightarrow$  and  $\curvearrowright$  transformations.

Resolution

Preparation

Progression

Handwritten musical notation divided into three sections: 'Resolution', 'Preparation', and 'Progression'. Each section contains two staves of music. The 'Progression' section ends with the word 'etc.' written below the notes.

Progression

Handwritten musical notation for 'Progression', consisting of two staves of music.

Continuous S(11)

Handwritten musical notation for 'Continuous S(11)', consisting of two staves of music.



Example of Continuity.

Handwritten musical notation for 'Example of Continuity'. The top staff (treble clef) contains a sequence of chords:  $\text{C}_3$ ,  $\text{D}_3$ ,  $\text{E}_3$ ,  $\text{F}_3$ ,  $\text{G}_3$ ,  $\text{A}_3$ ,  $\text{B}_3$ ,  $\text{C}_4$ ,  $\text{B}_3$ ,  $\text{A}_3$ ,  $\text{G}_3$ ,  $\text{F}_3$ ,  $\text{E}_3$ ,  $\text{D}_3$ ,  $\text{C}_3$ . The bottom staff (bass clef) contains single notes corresponding to the roots of these chords: C, D, E, F, G, A, B, C, B, A, G, F, E, D, C.

Four Tonics ( $\sqrt[4]{2}$ ). The technique corresponds to  $\text{C}_3$ , or to the  $\curvearrowright$  and  $\curvearrowleft$  transformations.

Resolution

Preparation

Progression

Handwritten musical notation illustrating Resolution, Preparation, and Progression. The top staff (treble clef) shows chords:  $\text{C}_3$ ,  $\text{D}_3$ ,  $\text{E}_3$ ,  $\text{F}_3$ ,  $\text{G}_3$ ,  $\text{A}_3$ ,  $\text{B}_3$ ,  $\text{C}_4$ ,  $\text{B}_3$ ,  $\text{A}_3$ ,  $\text{G}_3$ ,  $\text{F}_3$ ,  $\text{E}_3$ ,  $\text{D}_3$ ,  $\text{C}_3$ . The bottom staff (bass clef) shows single notes: C, D, E, F, G, A, B, C, B, A, G, F, E, D, C. The word 'etc.' is written at the end of the progression.

Progression

Continuous S(11)

Handwritten musical notation illustrating Continuous S(11). The top staff (treble clef) shows chords:  $\text{C}_3$ ,  $\text{D}_3$ ,  $\text{E}_3$ ,  $\text{F}_3$ ,  $\text{G}_3$ ,  $\text{A}_3$ ,  $\text{B}_3$ ,  $\text{C}_4$ ,  $\text{B}_3$ ,  $\text{A}_3$ ,  $\text{G}_3$ ,  $\text{F}_3$ ,  $\text{E}_3$ ,  $\text{D}_3$ ,  $\text{C}_3$ . The bottom staff (bass clef) shows single notes: C, D, E, F, G, A, B, C, B, A, G, F, E, D, C. The word 'etc.' is written at the end of the progression. Arrows indicate the direction of the progression.

Example of Continuity.

Handwritten musical notation for a second 'Example of Continuity'. The top staff (treble clef) contains a sequence of chords:  $\text{C}_3$ ,  $\text{D}_3$ ,  $\text{E}_3$ ,  $\text{F}_3$ ,  $\text{G}_3$ ,  $\text{A}_3$ ,  $\text{B}_3$ ,  $\text{C}_4$ ,  $\text{B}_3$ ,  $\text{A}_3$ ,  $\text{G}_3$ ,  $\text{F}_3$ ,  $\text{E}_3$ ,  $\text{D}_3$ ,  $\text{C}_3$ . The bottom staff (bass clef) contains single notes: C, D, E, F, G, A, B, C, B, A, G, F, E, D, C.





With the complexity of the harmony above, the consecutive ninths (if they both are major and move on a whole tone) are perfectly admissible.

Six Tonics ( $\sqrt[6]{2}$ ).  $\rightleftarrows$  and  $\curvearrowright$  transformations only.

Continuous S(11)

A musical score for a six-tonic system. The top staff is a treble clef with a series of chords:  $b_2$ ,  $2$ ,  $\#_2$ ,  $\#_2$ ,  $b_3$ ,  $b_3$ ,  $\#_3$ . The bottom staff is a bass clef with notes:  $0$ ,  $0$ ,  $0$ ,  $\#0$ ,  $b0$ ,  $b0$ ,  $0$ . Arrows indicate transformations between the two systems.

Example of Continuity.

A musical score for a twelve-tonic system. The top staff shows a sequence of chords:  $\#_0$ ,  $b_0$ ,  $\#_0$ ,  $b_1$ ,  $\#_1$ ,  $b_1$ ,  $\#_1$ ,  $b_2$ ,  $\#_2$ ,  $b_2$ ,  $\#_2$ ,  $b_3$ ,  $\#_3$ ,  $b_3$ ,  $\#_3$ . The bottom staff shows notes:  $0$ ,  $0$ ,  $0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $0$ ,  $0$ ,  $\#0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $0$ ,  $0$ ,  $0$ ,  $\#0$ .

Twelve Tonics ( $\sqrt[12]{2}$ ).  $\rightleftarrows$  and  $\curvearrowright$  transformations only.

Continuous S(11)

A musical score for a six-tonic system. The top staff shows chords:  $b_2$ ,  $b_2$ ,  $\#_2$ ,  $b_3$ ,  $\#_3$ ,  $b_3$ ,  $\#_3$ ,  $b_4$ ,  $\#_4$ ,  $b_4$ ,  $\#_4$ ,  $b_5$ ,  $\#_5$ ,  $b_5$ ,  $\#_5$ , etc. The bottom staff shows notes:  $0$ ,  $b0$ ,  $b0$ ,  $\#0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $\#0$ ,  $0$ ,  $b0$ ,  $b0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $0$ ,  $b0$ ,  $b0$ ,  $b0$ ,  $b0$ .

Example of Continuity.

A musical score for a twelve-tonic system. The top staff shows chords:  $0$ ,  $b_1$ ,  $\#_1$ ,  $b_2$ ,  $\#_2$ ,  $b_3$ ,  $\#_3$ ,  $b_4$ ,  $\#_4$ ,  $b_5$ ,  $\#_5$ ,  $b_6$ ,  $\#_6$ ,  $b_7$ ,  $\#_7$ ,  $b_8$ ,  $\#_8$ . The bottom staff shows notes:  $0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $0$ ,  $\#0$ ,  $0$ ,  $b0$ ,  $b0$ ,  $b0$ ,  $b0$ ,  $0$ ,  $0$ ,  $b0$ ,  $b0$ ,  $b0$ ,  $b0$ .



Homework in the field of  $S(11)$  must correspond to that of  $S(9)$ , utilizing various structures, forms and progressions. The transformation technique is applicable to diatonic and diatonic-symmetric progressions as well.



Lesson XCIV.Hybrid Four-Part Harmony

The general technique of transformations for the groups with three functions may be adopted for the generalization of the forms of voice-leading in a hybrid four-part harmony. The three upper parts perform the transformations corresponding to the groups with three functions, and the bass remains constant.

The following technique is applicable to any type of harmonic progression (diatonic, diatonic-symmetric, symmetric). The specifications for the following forms of S are chosen with respect to their sonority. The ones marked with an asterisk in the following tables are less commonly used than the unmarked ones. The charts of transformations for the latter are worked out and you can easily supplement them for the ones marked with the asterisk.

(please see next page)



Forms of Hybrid Four-Part (3 + 1) Harmony

The Three upper parts.	5 3 1	5 3 13	7 5 3	7 3 1	9 7 3	9 7 1	11 9 7	13 9 7	13 11 7
The bass.	1	1	1	1	1	1	1	1	1
Forms of tension.	S(5)	S(5) <sup>*</sup>	S(7)	S(7) <sup>*</sup>	S(9)	S(9) <sup>*</sup>	S(11)	S(13)	S(13) <sup>*</sup>

When the numerals expressing the functions in a group are identical with the numerals of the following group, certain forms of transformation, such as constant abc, have to be eliminated on account of complete parallelism. When the numerals in the two allied groups are partly identical some of the forms (constant a, constant b, constant c) give either favorable or unfavorable partial parallelisms. The partial parallelisms are favorable when the parallel motion of functions forms desirable intervals with the bass. They are unfavorable when it causes consecutive motion of the seventh or ninth with the bass (consecutive seventh, consecutive ninth).

As the actual quality of voice-leading depends on the structures of the two allied chords, upon completion of all these charts in musical notation you will be able to make your preferential selection.





When the numerals in the two allied groups are either partly or totally different, often the constant abc transformation becomes the most favorable form of voice-leading. There is a natural compensation in this case: homogeneous structures are compensated by heterogeneous transformations and heterogeneous structures are compensated by homogeneous transformations. For example, if the allied groups both are S(5) the constant abc transformation would be impossible:  $1 \rightarrow 1, 3 \rightarrow 3, 5 \rightarrow 5$ , which gives consecutive octaves and fifths. On the contrary, when the functions have different numerals you acquire the smoothest voice-leading through this particular transformation. When two allied groups have different or partly different numerals for their functions, the first group becomes the original group and the following group becomes the prime group. When a transformation between such two groups is performed the prime group in turn becomes the original group for the next transformation.

The Original  
Group

a  
c      b

The Prime  
Group

a'  
c'      b'



For example, by connecting  $S(5) + S(9) + S(13)$  we obtain the following numerals in their corresponding order:

$S(5)$		$S(9)$		$S(13)$	
1		3		7	
5	3	9	7	13	9

When you connect the functions of  $S(5)$  with the functions of  $S(9)$  the first group is the original group, and the second -- the prime group. When you connect the functions of  $S(9)$  with  $S(13)$  the functions of  $S(9)$  form the original group, and the functions of  $S(13)$  -- the prime group.

Here is a complete table of transformations.

Forms of Transformations  
in the Homogeneous Groups

		Const. a	Const. b	Const. c	Const. abc
$a \rightarrow b$	$a \rightarrow c$	$a \rightarrow a$	$a \rightarrow c$	$a \rightarrow b$	$a \rightarrow a$
$b \rightarrow c$	$c \rightarrow b$	$b \rightarrow c$	$b \rightarrow b$	$b \rightarrow a$	$b \rightarrow b$
$c \rightarrow a$	$b \rightarrow a$	$c \rightarrow b$	$c \rightarrow a$	$c \rightarrow c$	$c \rightarrow c$




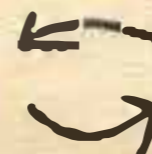
Forms of Transformations in  
the Heterogeneous Groups

The Original  
Group.

a  
c    b

The Prime  
Group.

a'  
c'    b'

		Const. a	Const. b	Const. c	Const. abc
a → b'	a → c'	a → a'	a → c'	a → b'	a → a'
b → c'	c → b'	b → c'	b → b'	b → a'	b → b'
c → a'	b → a'	c → b'	c → a'	c → c'	c → c'



Lesson XCV.

Here are all the combinations for the two allied groups taken, applied to all forms of tension.

Binomial Combinations of the Original  
and the Prime Groups.

$S(5) \leftrightarrow S(7)$	$S(7) \leftrightarrow S(9)$	$S(9) \leftrightarrow S(11)$	$S(11) \leftrightarrow S(13)$
$S(5) \leftrightarrow S(9)$	$S(7) \leftrightarrow S(11)$	$S(9) \leftrightarrow S(13)$	
$S(5) \leftrightarrow S(11)$	$S(7) \leftrightarrow S(13)$		
$S(5) \leftrightarrow S(13)$			

10 Combinations, 2 permutations each.

Total number of cases:  $10 \times 2 = 20$ .

Table of transformations for the twenty binomials consisting of one original and one prime group. Each S tension is represented in this table by one structure only. The sequence of the forms of transformations in this table remains the same for all cases:

- (1)  $\leftrightarrow$  ; (2)  $\leftarrow$  ; (3) Const. a; (4) Const. b;  
(5) Const. c; (6) Const. abc.





S(5) $\longrightarrow$ S(7)					
1 $\longrightarrow$ 5	1 $\longrightarrow$ 7	1 $\longrightarrow$ 3	1 $\longrightarrow$ 7	1 $\longrightarrow$ 5	1 $\longrightarrow$ 3
3 $\longrightarrow$ 7	3 $\longrightarrow$ 3	3 $\longrightarrow$ 7	3 $\longrightarrow$ 5	3 $\longrightarrow$ 3	3 $\longrightarrow$ 5
5 $\longrightarrow$ 3	5 $\longrightarrow$ 5	5 $\longrightarrow$ 5	5 $\longrightarrow$ 3	5 $\longrightarrow$ 7	5 $\longrightarrow$ 7

S(7) $\longrightarrow$ S(5)					
3 $\longrightarrow$ 3	3 $\longrightarrow$ 5	3 $\longrightarrow$ 1	3 $\longrightarrow$ 5	3 $\longrightarrow$ 3	3 $\longrightarrow$ 1
5 $\longrightarrow$ 5	5 $\longrightarrow$ 1	5 $\longrightarrow$ 5	5 $\longrightarrow$ 3	5 $\longrightarrow$ 1	5 $\longrightarrow$ 3
7 $\longrightarrow$ 1	7 $\longrightarrow$ 3	7 $\longrightarrow$ 3	7 $\longrightarrow$ 1	7 $\longrightarrow$ 5	7 $\longrightarrow$ 5

S(5) $\longrightarrow$ S(9)					
1 $\longrightarrow$ 7	1 $\longrightarrow$ 9	1 $\longrightarrow$ 3	1 $\longrightarrow$ 9	1 $\longrightarrow$ 7	1 $\longrightarrow$ 3
3 $\longrightarrow$ 9	3 $\longrightarrow$ 3	3 $\longrightarrow$ 9	3 $\longrightarrow$ 7	3 $\longrightarrow$ 3	3 $\longrightarrow$ 7
5 $\longrightarrow$ 3	5 $\longrightarrow$ 7	5 $\longrightarrow$ 7	5 $\longrightarrow$ 3	5 $\longrightarrow$ 9	5 $\longrightarrow$ 9

S(9) $\longrightarrow$ S(5)					
3 $\longrightarrow$ 3	3 $\longrightarrow$ 5	3 $\longrightarrow$ 1	3 $\longrightarrow$ 5	3 $\longrightarrow$ 3	3 $\longrightarrow$ 1
7 $\longrightarrow$ 5	7 $\longrightarrow$ 1	7 $\longrightarrow$ 5	7 $\longrightarrow$ 3	7 $\longrightarrow$ 1	7 $\longrightarrow$ 3
9 $\longrightarrow$ 1	9 $\longrightarrow$ 3	9 $\longrightarrow$ 3	9 $\longrightarrow$ 1	9 $\longrightarrow$ 5	9 $\longrightarrow$ 5



$S(5) \longrightarrow S(11)$					
$1 \longrightarrow 9$	$1 \longrightarrow 11$	$1 \longrightarrow 7$	$1 \longrightarrow 11$	$1 \longrightarrow 9$	$1 \longrightarrow 7$
$3 \longrightarrow 11$	$3 \longrightarrow 7$	$3 \longrightarrow 11$	$3 \longrightarrow 9$	$3 \longrightarrow 7$	$3 \longrightarrow 9$
$5 \longrightarrow 7$	$5 \longrightarrow 9$	$5 \longrightarrow 9$	$5 \longrightarrow 7$	$5 \longrightarrow 11$	$5 \longrightarrow 11$

$S(11) \longrightarrow S(5)$					
$7 \longrightarrow 3$	$7 \longrightarrow 5$	$7 \longrightarrow 1$	$7 \longrightarrow 5$	$7 \longrightarrow 3$	$7 \longrightarrow 1$
$9 \longrightarrow 5$	$9 \longrightarrow 1$	$9 \longrightarrow 5$	$9 \longrightarrow 3$	$9 \longrightarrow 1$	$9 \longrightarrow 3$
$11 \longrightarrow 1$	$11 \longrightarrow 3$	$11 \longrightarrow 3$	$11 \longrightarrow 1$	$11 \longrightarrow 5$	$11 \longrightarrow 5$

$S(5) \longrightarrow S(13)$					
$1 \longrightarrow 9$	$1 \longrightarrow 13$	$1 \longrightarrow 7$	$1 \longrightarrow 13$	$1 \longrightarrow 9$	$1 \longrightarrow 7$
$3 \longrightarrow 13$	$3 \longrightarrow 7$	$3 \longrightarrow 13$	$3 \longrightarrow 9$	$3 \longrightarrow 7$	$3 \longrightarrow 9$
$5 \longrightarrow 7$	$5 \longrightarrow 9$	$5 \longrightarrow 9$	$5 \longrightarrow 7$	$5 \longrightarrow 13$	$5 \longrightarrow 13$

$S(13) \longrightarrow S(5)$					
$7 \longrightarrow 3$	$7 \longrightarrow 5$	$7 \longrightarrow 1$	$7 \longrightarrow 5$	$7 \longrightarrow 3$	$7 \longrightarrow 1$
$9 \longrightarrow 5$	$9 \longrightarrow 1$	$9 \longrightarrow 5$	$9 \longrightarrow 3$	$9 \longrightarrow 1$	$9 \longrightarrow 3$
$13 \longrightarrow 1$	$13 \longrightarrow 3$	$13 \longrightarrow 3$	$13 \longrightarrow 1$	$13 \longrightarrow 5$	$13 \longrightarrow 5$



$$s(7) \longrightarrow s(9)$$

3 → 7	3 → 9	3 → 3	3 → 9	3 → 7	3 → 3
5 → 9	5 → 3	5 → 9	5 → 7	5 → 3	5 → 7
7 → 3	7 → 7	7 → 7	7 → 3	7 → 9	7 → 9

$$s(9) \longrightarrow s(7)$$

3 → 5	3 → 7	3 → 3	3 → 7	3 → 5	3 → 3
7 → 7	7 → 3	7 → 7	7 → 5	7 → 3	7 → 5
9 → 3	9 → 5	9 → 5	9 → 3	9 → 7	9 → 7

$$s(7) \longrightarrow s(11)$$

3 → 9	3 → 11	3 → 7	3 → 11	3 → 9	3 → 7
5 → 11	5 → 7	5 → 11	5 → 9	5 → 7	5 → 9
7 → 7	7 → 9	7 → 9	7 → 7	7 → 11	7 → 11

$$s(11) \longrightarrow s(7)$$

7 → 5	7 → 7	7 → 3	7 → 7	7 → 5	7 → 3
9 → 7	9 → 3	9 → 7	9 → 5	9 → 3	9 → 5
11 → 3	11 → 5	11 → 5	11 → 3	11 → 7	11 → 7



$S(7) \longrightarrow S(13)$					
$3 \rightarrow 9$	$3 \rightarrow 13$	$3 \rightarrow 7$	$3 \rightarrow 13$	$3 \rightarrow 9$	$3 \rightarrow 7$
$5 \rightarrow 13$	$5 \rightarrow 7$	$5 \rightarrow 13$	$5 \rightarrow 9$	$5 \rightarrow 7$	$5 \rightarrow 9$
$7 \rightarrow 7$	$7 \rightarrow 9$	$7 \rightarrow 9$	$7 \rightarrow 7$	$7 \rightarrow 13$	$7 \rightarrow 13$

$S(13) \longrightarrow S(7)$					
$7 \rightarrow 5$	$7 \rightarrow 7$	$7 \rightarrow 3$	$7 \rightarrow 7$	$7 \rightarrow 5$	$7 \rightarrow 3$
$9 \rightarrow 7$	$9 \rightarrow 3$	$9 \rightarrow 7$	$9 \rightarrow 5$	$9 \rightarrow 3$	$9 \rightarrow 5$
$13 \rightarrow 3$	$13 \rightarrow 5$	$13 \rightarrow 5$	$13 \rightarrow 3$	$13 \rightarrow 7$	$13 \rightarrow 7$

$S(9) \longrightarrow S(11)$					
$3 \rightarrow 9$	$3 \rightarrow 11$	$3 \rightarrow 7$	$3 \rightarrow 11$	$3 \rightarrow 9$	$3 \rightarrow 7$
$7 \rightarrow 11$	$7 \rightarrow 7$	$7 \rightarrow 11$	$7 \rightarrow 9$	$7 \rightarrow 7$	$7 \rightarrow 9$
$9 \rightarrow 7$	$9 \rightarrow 9$	$9 \rightarrow 9$	$9 \rightarrow 7$	$9 \rightarrow 11$	$9 \rightarrow 11$

$S(11) \longrightarrow S(9)$					
$7 \rightarrow 7$	$7 \rightarrow 9$	$7 \rightarrow 3$	$7 \rightarrow 9$	$7 \rightarrow 7$	$7 \rightarrow 3$
$9 \rightarrow 9$	$9 \rightarrow 3$	$9 \rightarrow 9$	$9 \rightarrow 7$	$9 \rightarrow 3$	$9 \rightarrow 7$
$11 \rightarrow 3$	$11 \rightarrow 7$	$11 \rightarrow 7$	$11 \rightarrow 3$	$11 \rightarrow 9$	$11 \rightarrow 9$





$$S(9) \longrightarrow S(13)$$

3 → 9	3 → 13	3 → 7	3 → 13	3 → 9	3 → 7
7 → 13	7 → 7	7 → 13	7 → 9	7 → 7	7 → 9
9 → 7	9 → 9	9 → 9	9 → 7	9 → 13	9 → 13

$$S(13) \longrightarrow S(9)$$

7 → 7	7 → 9	7 → 3	7 → 9	7 → 7	7 → 3
9 → 9	9 → 3	9 → 9	9 → 7	9 → 3	9 → 7
13 → 3	13 → 7	13 → 7	13 → 3	13 → 9	13 → 9

$$S(11) \longrightarrow S(13)$$

7 → 9	7 → 13	7 → 7	7 → 13	7 → 9	7 → 7
9 → 13	9 → 7	9 → 13	9 → 9	9 → 7	9 → 9
11 → 7	11 → 9	11 → 9	11 → 7	11 → 13	11 → 13

$$S(13) \longrightarrow S(11)$$

7 → 9	7 → 11	7 → 7	7 → 11	7 → 9	7 → 7
9 → 11	9 → 7	9 → 11	9 → 9	9 → 7	9 → 9
13 → 7	13 → 9	13 → 9	13 → 7	13 → 11	13 → 11



S(5) → S(7)

C<sub>3</sub>

C<sub>5</sub>

C<sub>7</sub>

S(7) → S(5)

C<sub>3</sub>

C<sub>5</sub>

C<sub>7</sub>



S(5) → S(9)

C<sub>3</sub>

Musical notation for C<sub>3</sub> chord progression. The system consists of two staves (treble and bass clef) with six measures. The notes are: Measure 1: C4, E4, G4; Measure 2: C4, E4, G4; Measure 3: C4, E4, G4; Measure 4: C4, E4, G4; Measure 5: C4, E4, G4; Measure 6: C4, E4, G4.

C<sub>5</sub>

Musical notation for C<sub>5</sub> chord progression. The system consists of two staves (treble and bass clef) with six measures. The notes are: Measure 1: C5, E5, G5; Measure 2: C5, E5, G5; Measure 3: C5, E5, G5; Measure 4: C5, E5, G5; Measure 5: C5, E5, G5; Measure 6: C5, E5, G5.

C<sub>7</sub>

Musical notation for C<sub>7</sub> chord progression. The system consists of two staves (treble and bass clef) with six measures. The notes are: Measure 1: C4, E4, G4, Bb4; Measure 2: C4, E4, G4, Bb4; Measure 3: C4, E4, G4, Bb4; Measure 4: C4, E4, G4, Bb4; Measure 5: C4, E4, G4, Bb4; Measure 6: C4, E4, G4, Bb4.

Please complete all tables.

Make additional tables for: S(5) → S(5); S(7) → S(7);  
 S(9) → S(9); S(11) → S(11);  
 S(13) → S(13).



It is easy to work out all cases in musical notation applying each case to all three tonal cycles.

As in the previous cases, continuity may be composed in all three types of harmony (diatonic, diatonic-symmetric and symmetric). Structures of different tension may be selected for the composition of continuity. Different individual styles depend upon the coefficients of recurrence applied to the structures of different tension.

The first of the following two examples of continuity is produced through the structures of constant form and tension [S(13)], and the second -- illustrates continuity of variable forms and tensions distributed through  $r_{3 \div 2}$ .

(please see next page)





Continuity of Groups with Identical Functions

$S(13) \rightarrow$  Type II. Scale:  $b^b$ -harm.,  $d_1$ .

Handwritten musical notation for Type II. The top staff shows a sequence of chords with accidentals (flats and naturals) and stems. The bottom staff shows a sequence of notes with arrows and labels "Const. 13" indicating relationships between the notes.

Continuity of Groups with Different Functions

$2S(9) + S(7) + S(13) + 2S(11);$  Type III.  $\sqrt[6]{2}$   
 $+ S(11)$

Handwritten musical notation for Type III. The top staff shows a sequence of chords with accidentals. The bottom staff shows a sequence of notes with arrows and labels "Const. abc" indicating relationships between the notes.



J O S E P H S C H I L L I N G E R

C O R R E S P O N D E N C E C O U R S E

With: Dr. Jerome Gross

Subject: Music

Lesson XCVI.

Generalization of Symmetric Progressions

The forms of symmetric progressions heretofore used in this course of Harmony were based on monomial symmetry of the uniform intervals of an octave.

In order to obtain various mixtures (binomials, trinomials and polynomials) of the original forms of symmetry within an octave, it is necessary to establish a general nomenclature for all intervals of an octave. As all intervals are special cases of the twelve-fold symmetry, any diatonic form may be considered a special case of symmetry as well.

The system of enumeration of intervals may follow the upward or downward direction from any established axis point. As both directions include all intervals (which means both positive and negative tonal cycles), the matter of preference must be determined by the quantitative predominance of the type of intervals generally used. It seems that the descending system is more practical, as smaller numbers express



the positive steps on three and four tonics, and the negative -- on six and twelve tonics.

In the following exposition the descending system will be used exclusively. This does not prevent you from using the ascending system.

Scales of Intervals within one Octave Range:

Descending System:

c → c = 0  
 c → b = 1  
 c → b<sup>b</sup> = 2  
 c → a = 3  
 c → a<sup>b</sup> = 4  
 c → g = 5  
 c → f<sup>#</sup> = 6  
 c → f = 7  
 c → e = 8  
 c → e<sup>b</sup> = 9  
 c → d = 10  
 c → d<sup>b</sup> = 11  
 c → c<sub>1</sub> = 12

Ascending System:

c → c = 0  
 c → d<sup>b</sup> = 1  
 c → d = 2  
 c → e<sup>b</sup> = 3  
 c → e = 4  
 c → f = 5  
 c → f<sup>#</sup> = 6  
 c → g = 7  
 c → a<sup>b</sup> = 8  
 c → a = 9  
 c → b<sup>b</sup> = 10  
 c → b = 11  
 c → c<sup>1</sup> = 12

Monomials

Two Tonics: 6 + 6

Three Tonics: 4 + 4 + 4 or 8 + 8 + 8

Four Tonics: 3 + 3 + 3 + 3 or 9 + 9 + 9 + 9

Six Tonics: 2 + 2 + 2 + 2 + 2 + 2 or 10 + 10 + 10 + 10 + 10 + 10

Twelve Tonics: 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1  
 or 11+11+11+11+11+11+11+11+11+11+11+11



Thus, each constant system<sup>of</sup> tonics becomes a form of monomial periodicity of a certain pitch-interval, expressible in the form of a constant number-value, which in turn expresses the quantity of semitones from the preceding pitch-unit.

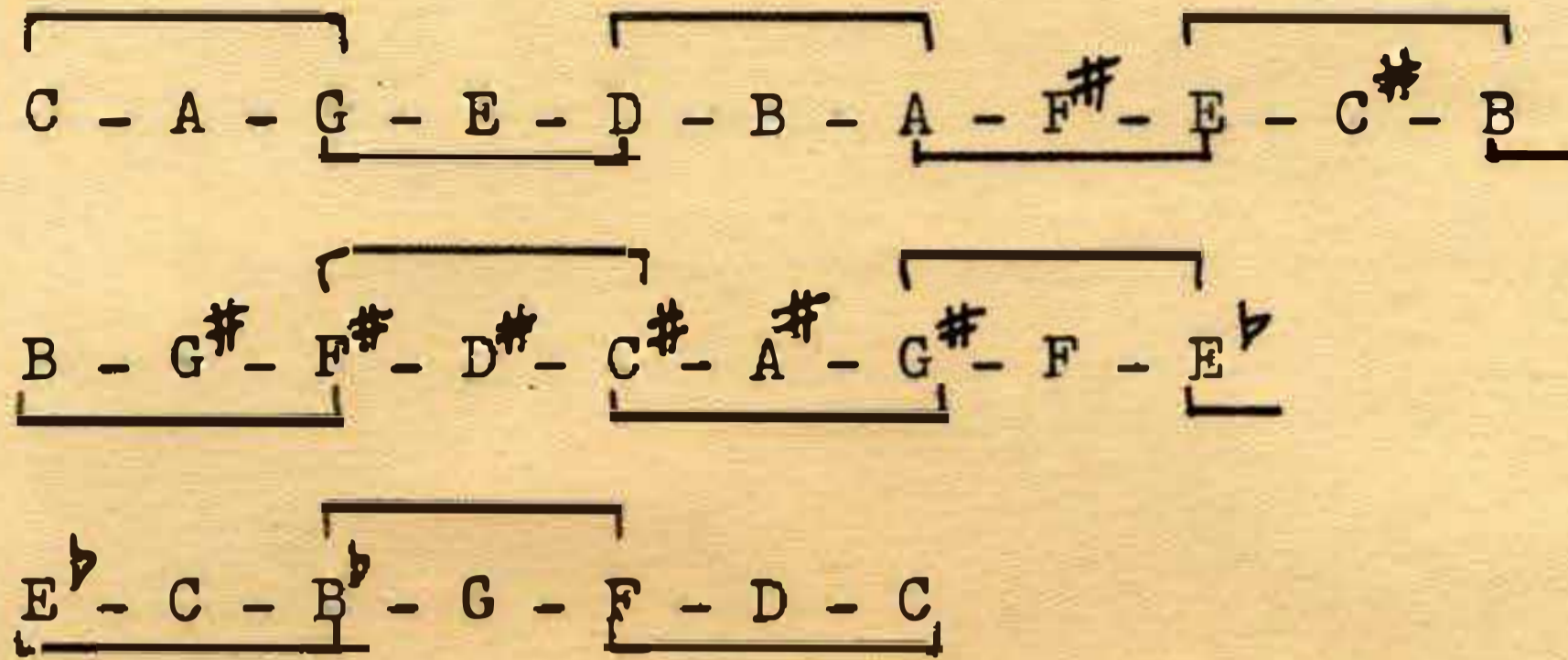
In the light of this system the problem of mixing the various tonics (or any interval-steps in general) becomes reduced to the process of composing binomials, trinomials or any more extended groups (such as rhythmic resultants, their modifications through permutations and powers, series of growth), i.e., to the rhythmic distribution of steps.

The vitality of such groups, i.e., the quantity of their recurrence until the completion of their cycle, depends upon the divisibility-properties of the sums of their interval-quantities. The total sum of all number-values expressing the intervals becomes a divisor to 12, or any multiple thereof. This signifies the motion of a certain group through an octave (or octaves).

For example, a binomial  $3 + 2$  has 12 recurrences until it completes its cycle, as  $3 + 2 = 5$ , and the smallest multiple of 12, divisible by 5 is 60. This is true of all prime numbers being used as divisors.







The above property makes the mixtures of three and four tonics very desirable when a long harmonic span is necessary without a need of the variety of steps.

The process of division serves as a testing tool of the vitality of compound symmetric groups.

Two tonics close after two cycles, as  $6 + 6 = 12$ , or  $\frac{12}{6} = 2$ ;

$r_{4+3}$  closes after one cycle, as  $3 + 1 + 2 + 2 + 1 + 3 = 12$ , and  $\frac{12}{12} = 1$ ;

$r_{5+4}$  closes after three cycles, as  $4 + 1 + 3 + 2 + 2 + 3 + 1 + 4 = 20$ , and  $\frac{60}{20} = 3$ .

A greater variety without deviating from a given style may be achieved by means of permutations of the members of a group. For example, a group with a short span may be revitalized through permutations:



$$(3+1+2) + (3+2+1) + (2+3+1) + (1+3+2) + (1+2+3) + (2+1+3)$$

$$\text{or: } \underbrace{C - A - G^\# - F^\#}_{\text{}} - \overbrace{E^\flat - D^\flat - C}^{\text{}} - \underbrace{B^\flat - G - F^\#}_{\text{}} - \overbrace{F^\flat - D - C}^{\text{}}$$

$$\underbrace{C - B^\flat - A - F^\#}_{\text{}} - \overbrace{E^\flat - E^\flat - C}^{\text{}}$$

The selection of number values is left to the composer's discretion. If he wants to obtain a tonic-dominant character of classical music, the only thing he needs is the excess of the value 5.

Anyone equipped with this method can dodge the extremities by a cautious selection of the coefficients of recurrence. For instance, in order to produce the style of progressions which lies somewhere between Wagner and Ravel it is necessary to have the 5, the 3, and the 10 in a certain proportion, like  $2_3 + 5 + 10$ , i.e.,

$$\underbrace{C - A - F^\# - C^\#}_{\text{}} - \overbrace{D^\# - C - A - E - F^\#}^{\text{}} \text{ etc.}$$

Naturally, the selection of the tensions and the forms of structures in definite proportions is as important as the selection of the forms of progressions when a certain definite style must be produced.



On the other hand, this method offers a wonderful pastime, as one can produce chord progressions from any number combinations. Thus, a telephone directory becomes a source of inspiration.

Example

Columbus 5 - 7573

5 + 7 + 5 + 7 + 3 is equivalent to  
C - G - C - G - C - A.

This progression closes after 4 cycles:

C - G - C - G - C - A - E - A - E - A - F<sup>#</sup> -  
F<sup>#</sup> - C<sup>#</sup> - F<sup>#</sup> - C<sup>#</sup> - F<sup>#</sup> - D<sup>#</sup> - A<sup>#</sup> - D<sup>#</sup> - A<sup>#</sup> - D<sup>#</sup> - C

When zeros occur in a number-combination they represent zero-steps, i.e., zero cycles (C<sub>0</sub>). Then the form of tension, the structure or the position of a chord has to be changed.

(please see next page)



Example of Continuity:

Progression:  $r_{5\div 3}$

The image shows a handwritten musical score on a single staff system. The top staff is in treble clef and contains a sequence of notes: C - A - G - F# - Eb - D - C - A. Below these notes are several chords, some with figured bass notation. The bottom staff is in bass clef and contains a sequence of notes: C - A - G - F# - Eb - D - C - A. The notes in the bass staff are mostly whole notes. There are several annotations in the score: "omit abc" written under the first two chords, "etc." at the end of the sequence, and some arrows pointing to specific notes. The chords are: C major (C-E-G), A major (A-C-E), G major (G-B-D), F# major (F#-A-C), Eb major (Eb-G-Bb), D major (D-F#-A), C major (C-E-G), and A major (A-C-E).





Lesson XCVII.Application of the Generalized Symmetric  
Progressions to Modulation

The rhythm of chord progressions expressed in number-values may serve the purpose of transition from one key to another. This procedure can be approached in two ways: (1) the connection concerns the tonic chords of the preceding and the following key; and (2) any chord of the preceding key, in its relation to any chord of the following key. The last case requires movement through diatonic cycles in both the preceding and the following key.

The technique of performing modulations, based on the rhythm of symmetric progressions, consists of two steps: (1) the detection of the number-value expressing the interval between the two chords, where such connection must be established; (2) composition of a rhythmic group from the numeral expressing the interval between the abovementioned chords. For example, if one wants to perform a modulation by means of symmetric progressions from the chord C (which may or may not be in the key of C) to the chord E<sup>b</sup> (which may or may not be in the key of E<sup>b</sup>), the first procedure to perform is to compose rhythm from the interval 9. The knowledge of the Theory of



Rhythm offers many ways of composing such groups: composition of binomials, trinomials or larger groups from the original number, or any permutations thereof.

The quantity of the terms in a group will define the number of chords for the modulatory transition. Breaking up number 9 into binomials, we obtain:  $8 + 1$ ,  $7 + 2$ ,  $6 + 3$ ,  $5 + 4$ , and their reciprocals. When a binomial is used in this sense, the two chords are connected through one intermediate chord. For example, taking  $5 + 4$  we acquire:  $C - G - E^b$ . If more chords are desired any other rhythmic group may be devised from number 9. For example,  $4 + 1 + 4$ , which will give  $C - A^b - G - E^b$ , i.e., two intermediate chords. When a number-value expressing the interval between the two chords to be connected through modulation is a small number, it is necessary to add the invariant 12. This places the same pitch-unit (or the root of the chord) into a different octave, without changing its intonation. For example, if a modulation from a chord of  $C$  to the chord of  $B^b$  is required, such addition becomes very desirable.

$$C \longrightarrow B^b = 2$$

$$B^b \longrightarrow B_1^b = 12$$

$$12 + 2 = 14$$



Some possible rhythms derived from the value 14:

$$7 + 7 = C - F - B^b$$

$$5 + 2 + 2 + 5 = C - G - F - E^b - B^b$$

In cases like this rhythmic resultants may be used as well, providing the necessary changes are made.

$$r_{4 \div 3} = 3 + 1 + 2 + 2 + 1 + 3$$

Readjustment:

$$3 + 1 + 2 + 2 + 1 + 3 + 2 = C - A - A^b - F^\# - F^b - E^b - C - B^b$$

Or:

$$r_{5 \div 3} = 3 + 2 + 1 + 3 + 1 + 2 + 3$$

Readjustment:

$$3 + 2 + 1 + 2 + 1 + 2 + 3 = C - A - G - F^\# - E - E^b - D^b - B^b$$

Thus, all these procedures guarantee the appearance of the desirable  $B^b$  point.

When a modulation of still greater extension is required, the invariant of addition becomes 24, 36, or even a higher multiple of 12, from which rhythmic groups may be composed.

Many persons engaged in the work of arranging find this type of transition more effective than the modulations proper. Naturally, the selection of the structures of different tension and form may be made according to the requirements of the general style of harmony used in a particular arrangement.



The best modulations will result from the symmetry that may be detected in a given piece of music. Even when tonic-dominant progression is characteristic of harmonic continuity, this method may be used with success, as it simply requires the composition of a rhythmic group, where the original value is 5. In this seemingly limited case there is still a choice of steps: 4 + 1; 3 + 2; 2 + 3; 1 + 4.

Examples of Modulations  
Through Symmetric Groups

(1) Key of C to Key of  $E^b$ ;  $i = 9$

Symmetric Group:  $1 + 3 + 1 + 3 + 1$  ( $r_3$  of  $\frac{9}{3}$  series)





(2) Key of C to Key of E<sup>b</sup>

Chords to be Connected: D — B<sup>b</sup>; i = 4;

$$4 + 12 = 16$$

Symmetric Group:  $r_{4 \div 3} = 3 + 1 + 2 + 1 + 1 + 1 +$   
 $+ 1 + 2 + 1 + 3$

The image shows a handwritten musical score on a yellowed page. It features two staves. The upper staff is in treble clef and contains a series of chords, each represented by a group of notes with various accidentals (sharps, flats, naturals). Above the first two chords, the labels 'c3' and 'c5' are written. The lower staff is in bass clef and contains a sequence of single notes with accidentals. Some notes in the bass staff are connected to the chords in the upper staff by vertical lines, indicating their position within the chords. The notes in the bass staff include flats, naturals, and sharps.



Lesson XCVIII.Chromatic System of Harmony.

The basis of this system is transformation of diatonic chordal functions into chromatic chordal functions and back into diatonic. Chromatic continuity evolved from this basis emphasizes various phenomena of harmony which do not confine to diatonic or symmetric systems. The usually known modulations are but a special case of the chromatic system. Chord progressions usually known as "alien" chord progressions find their exhaustive explanation in this system.

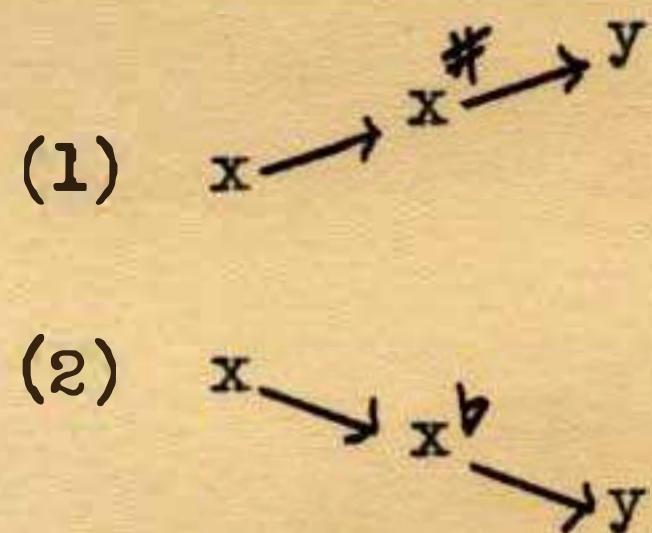
Wagner was the first composer to manipulate intuitively with this type of harmonic continuity. Not having any theoretical basic principle of handling such progressions, Wagner often wrote them in an enharmonically confusing way. (J.S. Bach made an unsuccessful attempt to move in chromatic systems. See "Well Tempered Clavichord" - Vol. I, Fugue 6 - bar 16). It is necessary, for analytical purposes, to rewrite such music in proper notation, i.e., chromatically and not enharmonically. A more consistent notation of chromatic continuity may be found among the followers of Wagnerian harmony, such as Borodin and Rimsky-Korsakov.

The chromatic system of harmonic continuity



is based on progressions of chromatic groups. Every chromatic group consists of three chords, which express the following mechanical process: balance - tension - release. These three moments correspond to the diatonic - chromatic - diatonic transformation. A chromatic group may consist of one or more simultaneous operations. Such operations are alterations of diatonic tones into chromatic tones, by raising or lowering them. The initial diatonic tone of a chromatic group retains its name, while being altered, and changes it during the moment of release.

The two forms of chromatic operations are:



In application to musical names it may become, for instance,  $g - g^{\#} - a$  or  $g - g^{\flat} - f$ . Such steps are always semitones. At such moment of release, in a chromatic group, a new chordal function (and in some cases the same) becomes the starting point of the next chromatic group, thus evolving into an infinite chromatic continuity.

Such continuity acquires the following appearance:



d - ch - d

d - ch - d

etc.

d - ch - d

Chromatic continuity in such form offers a very practical bar distribution by placing two chords in a bar. Such distribution places the release on the downbeat and sounds satisfactory to our ear, probably due to the habit of hearing them in such distribution.

As in the diatonic progressions, the commonness of tones, or the resolution of chordal functions, or as in the symmetric progressions the symmetric roots become the stimuli of motion, likewise in the chromatic progressions such stimuli are the chromatic alterations of the diatonic tones.

Besides the form of continuity of chromatic groups offered in the preceding diagram, two other forms are possible. Thus, the latter do not necessarily require the technique of the chromatic system. The first of these forms of continuity produces an overlapping, over one term:

(1) d - ch - d

d - ch - d

d - ch - d i.e.,

the second part produces the first term of a chromatic group, while the first one produces the second term.





(2)  $\overline{d - ch - d}$

$\underline{d - ch - d}$

i.e., two or more parts of harmony coincide in their transformation in time, though the form of transformation may be different in each part.

Any chord acquiring a chromatic alteration becomes more intense than the corresponding form of tension, without it. If the middle term of a chromatic group has to be intensified, the following forms of tension may constitute a chromatic group:

S(5)	S(7)	S(5)
S(5)	S(7)	S(7)
S(7)	S(7)	S(5)
S(7)	S(7)	S(7)

The only combination which is undesirable, as it produces an effect of weakness, is when the middle term is S(5).

Operations in a given chromatic group correspond to a group of chordal functions which may be assigned to any form of alterations. As for technical reasons the 4-part harmony is limited to S(5) and S(7) forms, with their inversions, all transformations of functions in the chromatic group deal with the four lower functions (9, 11 and 13 are excluded).



Numerical Table of Transformations  
for the Chromatic Groups.

1-1-1	3-3-3	5-5-5	7-7-7
<hr/>			
1-1-3	3-3-1	5-5-1	7-7-1
1-3-1	3-1-3	5-1-5	7-1-7
3-1-1	1-3-3	1-5-5	1-7-7
<hr/>			
1-1-5	3-3-5	5-5-3	7-7-3
1-5-1	3-5-3	5-3-5	7-3-7
5-1-1	5-3-3	3-5-5	3-7-7
<hr/>			
1-1-7	3-3-7	5-5-7	7-7-5
1-7-1	3-7-3	5-7-5	7-5-7
7-1-1	7-3-3	7-5-5	5-7-7
<hr/>			
1-3-5	1-3-7	1-5-7	3-5-7
1-5-3	1-7-3	1-7-5	3-7-5
5-1-3	7-1-3	7-1-5	7-3-5
3-1-5	3-1-7	5-1-7	5-3-7
3-5-1	3-7-1	5-7-1	5-7-3
5-3-1	7-3-1	7-5-1	7-5-3
<hr/>			

Some of these combinations must be excluded because of the adherence of the Seventh to the classical system of voice-leading (descending resolution).



The preceding table offers 16 different versions for each starting function (1, 3, 5, 7). In addition to this, any middle chord of a chromatic group may assume one of the seven forms of S(7), and any of the last chords of a chromatic group -- either four forms of S(5) or seven forms of S(7). Thus, each starting point offers either 28 or 49 forms. The total number of starting points for one function equals 16. These quantities must be multiplied by 16 in order to show the total number of cases.

$$28 \times 16 = 448$$

$$49 \times 16 = 784$$

This applies to one initial function only, and as any group may start with either of the four functions, the total quantity is  $4(784 + 448) = 4,928$ . A number of these cases eventually excludes themselves on account of the abovementioned limitations caused by the traditional voice-leading.

The actual realization of chromatic groups must be performed from the two fundamental bases: the major and the minor. The concept of a harmonic basis expresses any three adjacent chordal functions, such as:

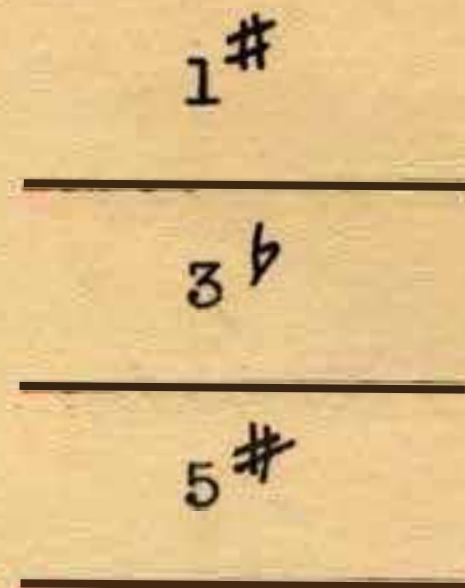
5	7	9	11	13
3	5	7	9	11
1	3	5	7	9



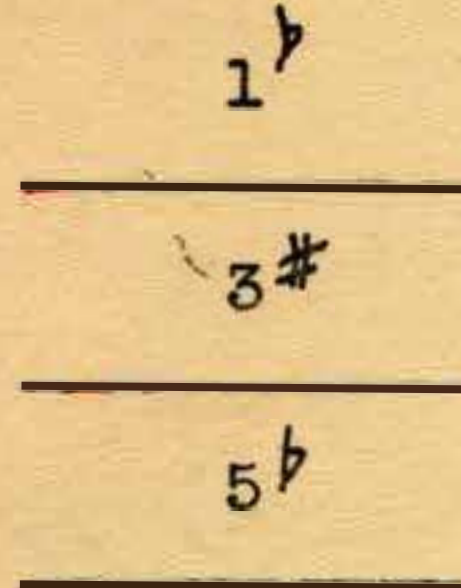
Due to practical limitations this course of Harmony will deal with the first  $\begin{matrix} 5 \\ (3) \\ 1 \end{matrix}$  basis only.

The terms major and minor correspond to the structural constitution in the usual sense: major = 4 + 3, and minor = 3 + 4. All fundamental chromatic operations are derived from these two bases.

Major Basis



Minor Basis



These six forms of chromatic operations (3 from each basis) are used independently. Chromatic operations available from the major basis are: raising of the root-tone, lowering of the third, raising of the fifth. They are the opposite from the minor basis.

(please see following pages)





# Examples of Chromatic Traps: One Operation

## Table of Transformations

1-1-1                      1-1-3                      1-3-1

1-1-1: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

1-1-3: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

1-3-1: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

3-1-1

3-1-1: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

1-1-5                      1-5-1

1-1-5: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

1-5-1: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

5-1-1                      1-1-7

5-1-1: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

1-1-7: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

1-3-5                      3-5-1

1-3-5: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.

3-5-1: Treble clef notes: G4, A4, B4, C5. Bass clef notes: G3, A3, B3, C4.



The image shows three systems of handwritten musical notation on aged paper. Each system consists of two staves: a treble clef staff on top and a bass clef staff on the bottom. The notation is organized into measures by vertical bar lines.

- System 1:** Labeled with "5-1-3" above the first measure and "1-5-3" above the second measure. It contains four measures of chords. The first measure has a circled note in the treble staff. The second measure has a circled note in the bass staff.
- System 2:** Labeled with "5-3-1" above the first measure and "3-1-5" above the second measure. It contains four measures of chords. The first measure has a circled note in the treble staff. The second measure has a circled note in the bass staff.
- System 3:** Labeled with "3-3-3" above the first measure. It contains three measures of chords. The first measure has a circled note in the treble staff.

Try to find the remaining cases through the table of transformations of the chordal functions. Please remember that the classical system of voice-leading must be carried out through chromatic continuity. A Seventh either descends or remains (as in traditional cadences); it may even go up one semitone, due to the chord structure, yet it positively must retain its original name, like  $d - d^\sharp$ .



Through the selection of different chromatic groups (which may be used with coefficients of recurrence) a chromatic continuity may be composed.

With the amount of explanation offered so far, every last chord of the preceding group (and therefore the first chord of the following group) must be major or minor, as the operations from other bases will be explained in the following lesson.

Example of Chromatic Continuity

The image shows a handwritten musical score on two staves. The top staff is in treble clef and the bottom staff is in bass clef. The music consists of a sequence of chords, with some notes marked with accidentals (sharps and flats). Above the top staff, there are two groups of notes with brackets and numbers above them: the first group has notes with '5', '5', and '1' above them; the second group has notes with '5', '3', and '7' above them. Below the bottom staff, there are two groups of notes with brackets and numbers below them: the first group has notes with '1', '1', and '3' below them; the second group has notes with '3', '3', and '3' below them. The notation is somewhat messy and appears to be a student's or composer's draft.



Lesson XCIX.Operations from  $S_3(5)$  and  $S_4(5)$  bases

As 3 of  $S_3(5)$  is identical with 3 of  $S_4(5)$ , the fundamental operations correspond to  $S_3(5)$ . They are:

- (1) raising of 1
- (2) lowering of 3

Function 5 does not participate in the fundamental operations, as it is already altered. As the form of the middle chord is pre-selected, the fifth requires rectification in many cases though it retains its name. All forms of doublings are acceptable.

As 3 of  $S_4(5)$  is identical with 3 of  $S_3(5)$ , the fundamental operations correspond to  $S_3(5)$ . They are:

- (1) lowering of 1
- (2) raising of 3

Fifth does not participate in the fundamental operations, but may be rectified.

Figure I.Operations from an augmented basis.

(please see next page)





FIG I

First system of Figure I. Treble clef staff: Measure 1 (F#4, C5, G4), Measure 2 (F#4, C5, G4), Measure 3 (F#4, C5, G4). Bass clef staff: Measure 1 (C3), Measure 2 (C3), Measure 3 (C3). Fingerings: 1, 1, 3; 1, 1, 3; 3, 1, 1.

Second system of Figure I. Treble clef staff: Measure 1 (F#4, C5, G4), Measure 2 (F#4, C5, G4), Measure 3 (F#4, C5, G4). Bass clef staff: Measure 1 (C3), Measure 2 (C3), Measure 3 (C3). Fingerings: 3, 7, 1; 3, 7, 1; 3, 7, 1.

Figure II.

Operations from a diminished basis.

FIG II

First system of Figure II. Treble clef staff: Measure 1 (F#4, C5, G4), Measure 2 (F#4, C5, G4), Measure 3 (F#4, C5, G4). Bass clef staff: Measure 1 (C3), Measure 2 (C3), Measure 3 (C3). Fingerings: 1, 1, 3; 1, 1, 3; 1, 1, 3.

Second system of Figure II. Treble clef staff: Measure 1 (F#4, C5, G4), Measure 2 (F#4, C5, G4), Measure 3 (F#4, C5, G4). Bass clef staff: Measure 1 (C3), Measure 2 (C3), Measure 3 (C3). Fingerings: 1, 1, 3; 1, 1, 3; 1, 1, 3.



Figure III.Chromatic continuity including all bases.

FIG III

The image shows two systems of handwritten musical notation. The first system consists of a treble clef staff with several chords and a bass clef staff with a chromatic line of notes. The second system also consists of a treble clef staff with chords and a bass clef staff with a chromatic line. The notation is handwritten and includes various accidentals and ties.

Chromatic Alteration of the Seventh.

Due to the classical tendency of a downward resolution of the seventh, chromatic alterations follow the same direction. Lowering of the seventh (both major and minor) can be carried out from all forms of S(7). If the seventh is minor, it is more practical to have it as sharp or natural, as lowering of the flat produces a double-flat. Do not operate from a diminished seventh.



Figure IV.

Examples of operations from the Seventh.

FIG. IV

The image shows two systems of musical notation, each consisting of a treble and bass staff. The notation is handwritten and includes various musical symbols such as clefs, notes, rests, and accidentals. The first system has four measures, and the second system also has four measures. Some notes are marked with numbers 1 through 5, likely indicating specific operations or intervals. The title 'FIG. IV' is written above the first system.

We can incorporate now all the single operations into the final form of chromatic continuity.

Figure V.

Operations from 1, 3, 5 and 7.

All bases.

FIG V

The image shows two staves of musical notation. The notation is handwritten and includes various musical symbols such as clefs, notes, rests, and accidentals. The title 'FIG V' is written above the first staff. The notation includes various musical symbols such as clefs, notes, rests, and accidentals, with some notes marked with numbers 1 through 7, likely indicating specific operations or intervals. The title 'FIG V' is written above the first staff.



J O S E P H S C H I L L I N G E R

C O R R E S P O N D E N C E C O U R S E

With: Dr. Jerome Gross

Subject: Music

Lesson C.

Parallel Double Chromatics

(Double chromatic operations)

Parallel double chromatics occur when fundamental operations are performed from an opposite base. In such a case the rectification of the third is required.

If, for example, we decide to lower 1 of  $S_1(5)$  basis, it becomes necessary to alter 3 to its proper basis, i.e. to lower in this case.

We shall consider the alterations of 1 and 5 as fundamental and the correction of 3 as complementary chromatics.

The following table represents all operations.

Figure VI.

Parallel Double Chromatics.

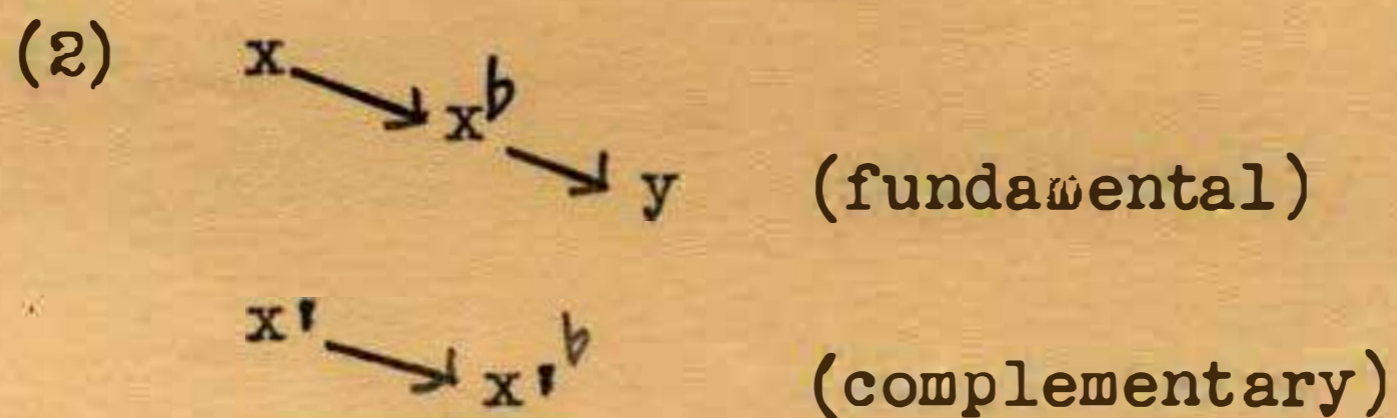
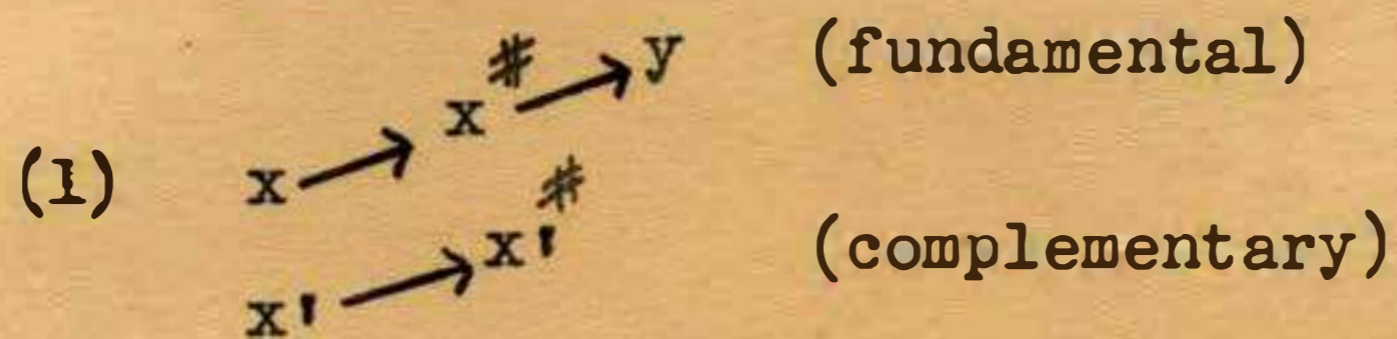
$S_1(5)$ basis		$S_2(5)$ basis	
Fundamental	1 <sup>b</sup>	Fundamental	1 <sup>#</sup>
Complementary	3 <sup>b</sup>	Complementary	3 <sup>#</sup>
<hr/>		<hr/>	
Fundamental	5 <sup>b</sup>	Fundamental	5 <sup>#</sup>
Complementary	3 <sup>b</sup>	Complementary	3 <sup>#</sup>





Fundamental chromatics represent the middle term of a complete chromatic group, whereas the complementary chromatics do not necessarily perform the conclusive movement designated by their alterations.

Thus, the scheme of chromatic groups for the parallel double chromatics appears as follows:



For example, if  $c - c^b - b^b$  is a fundamental operation, the complementary chromatic is:  $e - e^b$ . The complementary chromatic  $e^b$  does not necessarily move into  $d$ . It may remain or even move upward, depending on the chordal function assigned to it.

The same is true of the ascending chromatics. If  $c - c^{\#} - d$  is the fundamental operation, the complementary chromatic is  $e^b - e$ . The complementary chromatic  $e$  does not necessarily move into  $f$ . It may remain or even move downward, depending on the chordal function assigned to it.



The assignment of chordal functions must be performed for the two simultaneous operations: fundamental and complementary. It is practical to designate the ascending alterations as:  $\begin{matrix} 3 \\ 1 \end{matrix}$  or  $\begin{matrix} 5 \\ 3 \end{matrix}$ , and the descending -- as:  $\begin{matrix} 7 \\ 5 \end{matrix}$  or  $\begin{matrix} 5 \\ 3 \end{matrix}$ .

This protects harmonic continuity from a wrong direction and sometimes from an excess of accidentals. This remark refers to the middle term of a chromatic group.

Figure VII.

Examples of Double Parallel Chromatics.

(please see next page)



# FIG VII

S1 (5) BASIS =  $\begin{matrix} 3 \rightarrow \\ 1 \rightarrow \end{matrix}$

Handwritten musical notation for S1 (5) BASIS = 3/1. It consists of two systems of two staves each. The first system shows a treble clef staff with notes and accidentals, and a bass clef staff with notes. The second system continues the notation. The notes in the treble clef are grouped with vertical lines and numbers 1, 3, 5, and 7, indicating specific intervals or positions.

S1 (5) BASIS =  $\begin{matrix} 5 \rightarrow \\ 3 \rightarrow \end{matrix}$

Handwritten musical notation for S1 (5) BASIS = 5/3. It consists of two systems of two staves each. The first system shows a treble clef staff with notes and accidentals, and a bass clef staff with notes. The notes in the treble clef are grouped with vertical lines and numbers 1, 3, 5, and 7, indicating specific intervals or positions.

S2 (5)  $\begin{matrix} 3 \rightarrow \\ 1 \rightarrow \end{matrix}$

Handwritten musical notation for S2 (5) 3/1. It consists of two systems of two staves each. The first system shows a treble clef staff with notes and accidentals, and a bass clef staff with notes. The notes in the bass clef are grouped with vertical lines and numbers 1, 3, 5, and 7, indicating specific intervals or positions.

S2 (5)  $\begin{matrix} 5 \rightarrow \\ 3 \rightarrow \end{matrix}$

Handwritten musical notation for S2 (5) 5/3. It consists of two systems of two staves each. The first system shows a treble clef staff with notes and accidentals, and a bass clef staff with notes. The notes in the treble clef are grouped with vertical lines and numbers 1, 3, 5, and 7, indicating specific intervals or positions.



By assigning the opposite bases, we can obtain double parallel chromatics at any desirable place of chromatic continuity.

Figure VIII.

Continuity of Double Parallel Chromatics.

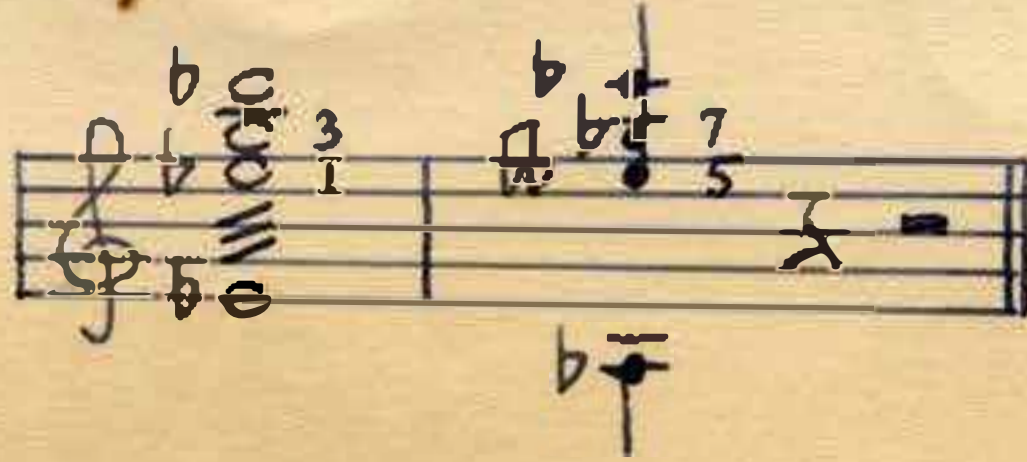
FIG VIII

Double parallel chromatics are the quintessence of chromatic style in harmony. It created the unmistakable character of Wagner and the post-Wagnerian music. While the analysis of Borodin, Rimsky-Korsakov, Frank and Delius does not present any difficulties for the analyst familiar with this theory, the music of Wagner often requires transcribing into chromatic notation. One of the progressions typical of the later Wagner's period (we find much of it in "Parsifal") is:





Being transcribed into chromatic notation it acquires the following appearance:



This corresponds to  $S_1(5)$  basis:

3 →  
1 →

There are many instances when double parallel chromatics are evolved on a basis of passing chromatic tones. They are abundant in the music of Rimsky-Korsakov, Borodin and, lately, became very common in the American popular and show songs ("Cuban Long Song", "The Man I Love"). The source of passing chromatic tones, the technique of which we shall discuss later, is more Chopin than Wagner or the post-Wagnerians.



Lesson CI.

Triple and Quadruple

Parallel Chromatics

Triple parallel chromatics occur when 1 is raised in  $S_4(5)$  basis. This, being the fundamental operation, requires the correction of the third ( $3^\sharp$ ) and of the fifth ( $5^\sharp$ ). The triple alterations become

5		7
3	or	5
1		3

Figure IX.

Triple parallel chromatics.

FIG IX

The figure shows two systems of musical notation, each consisting of a treble and bass staff. The first system is divided into two measures by a double bar line. The second system is also divided into two measures. The notation includes various symbols such as notes, rests, and accidentals, with some numbers (1, 3, 5, 7) written below the notes to indicate specific intervals or fingerings. In the first measure of the first system, there are three upward-pointing arrows above the treble staff. The second system shows similar notation with different interval markings.



Quadruple parallel chromatics occur when 1 is raised in  $S_5(7)$  basis [diminished seventh-chord]. This requires the alteration of all remaining functions, i.e.  $3^\sharp$ ,  $5^\sharp$  and  $7^\sharp$ . This is the only interpretation satisfying the cases of chromatic parallel motion of the diminished seventh-chords. See Beethoven's Piano Sonata No. 7 Largo (bar 20 from the end and the following 5 bars in relation to the adjacent harmonic context). Such a continuous chain of quadruple parallelisms takes place when the same operation is performed several times in succession.

As chromatic system is limited to four functions (1, 3, 5, 7), quadruple parallel chromatics remain with their original assignments (while being altered).

Figure X.

Quadruple Parallel Chromatics

FIG X



By combining all forms of chromatic operations, i.e. single, double, triple and quadruple, we obtain the final form of mixed chromatic continuity.

Figure XI.

Continuity of Mixed Chromatic Operations.

**FIG XI**

The figure displays four staves of handwritten musical notation. The first two staves are grouped together, and the last two are grouped together. Each staff contains a series of notes with various accidentals (sharps, flats, naturals) and chromatic lines. The notation is complex, illustrating the continuity of mixed chromatic operations and enharmonic treatments. The first two staves use a treble clef and a bass clef, while the last two staves use a treble clef and a bass clef. The notes are connected by lines, indicating chromatic movement. Some notes have 'x' marks above them, possibly indicating specific operations or enharmonic equivalents.

Enharmonic Treatment of the Chromatic System.

By reversing the original directions of chromatic operations we more than double the original resources of the chromatic system.

Enharmonic treatment of chromatic groups





consists of substituting raising for lowering and vice-versa. This changes the original direction of a group and brings to new points of release in its third term.

The following formula expresses all conditions necessary for the enharmonic treatment.

$$(1) \quad x \rightarrow x^{\#} = y^{\flat} \rightarrow z \quad (1, 3, 5, 7)$$

$$(2) \quad x \rightarrow x^{\flat} = y^{\#} \rightarrow z \quad (1, 3, 5, 7)$$

Progressions of this kind are characteristic of post-Wagnerian composers (Borodin's "Prince Igor", Rimsky-Korsakov's "Coq D'Or" and "Khovanschina").\*)

Figure XII.

Examples of enharmonic treatment  
of the chromatic system.

(please see following pages)

---

\*) by Moussorgsky.



FIG XIII

S<sub>1</sub>(5) 1 → ↘

The first system consists of two staves. The upper staff contains two measures of chords: the first measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4); the second measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4). The lower staff contains two measures of notes: the first measure has notes C4, E4, G4, and the second measure has notes C4, E4, G4.

3 → →

The second system consists of two staves. The upper staff contains two measures of chords: the first measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4); the second measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4). The lower staff contains two measures of notes: the first measure has notes C4, E4, G4, and the second measure has notes C4, E4, G4.

The third system consists of two staves. The upper staff contains two measures of chords: the first measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4); the second measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4). The lower staff contains two measures of notes: the first measure has notes C4, E4, G4, and the second measure has notes C4, E4, G4.

5 → ↘

The fourth system consists of two staves. The upper staff contains two measures of chords: the first measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4); the second measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4). The lower staff contains two measures of notes: the first measure has notes C4, E4, G4, and the second measure has notes C4, E4, G4.

The fifth system consists of two staves. The upper staff contains two measures of chords: the first measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4); the second measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4). The lower staff contains two measures of notes: the first measure has notes C4, E4, G4, and the second measure has notes C4, E4, G4.

S<sub>2</sub>(5) 1 → →

The sixth system consists of two staves. The upper staff contains two measures of chords: the first measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4); the second measure has a C major triad (C4, E4, G4) and a C minor triad (C4, E♭4, G4). The lower staff contains two measures of notes: the first measure has notes C4, E4, G4, and the second measure has notes C4, E4, G4.



(Figure XII, cont.)

Handwritten musical notation for the first system. The treble clef staff contains two measures. The first measure shows a chord with a 5 above it and a slur over it, with fingerings 1, 3, 5, 7. The second measure shows a chord with a 3 above it and a slur over it, with fingerings 1, 3, 5. The bass clef staff contains two measures with single notes.

Handwritten musical notation for the second system. The treble clef staff contains two measures. The first measure shows a chord with a slur over it and fingerings 1, 3, 5, 7. The second measure shows a chord with a slur over it and fingerings 1, 3, 5, 7. The bass clef staff contains two measures with single notes.

Handwritten musical notation for the third system. The treble clef staff contains two measures. The first measure shows a chord with a slur over it and fingerings 1, 3, 5. The second measure shows a chord with a 5 above it and a slur over it, with fingerings 1, 3, 5, 7. The bass clef staff contains two measures with single notes.

Handwritten musical notation for the fourth system. The treble clef staff contains two measures. The first measure shows a chord with a slur over it and fingerings 1, 3, 5, 7. The second measure shows a chord with a slur over it and fingerings 1, 3, 5, 7. The bass clef staff contains two measures with single notes.



In cases of double and triple chromatics, all or some of the altered functions can be enharmonized.

Figure XIII.

Enharmonic treatment of double and triple chromatics.

FIG XIII.

The image shows a piece of aged paper with handwritten musical notation. At the top right, the page number '13.' is printed. Below it, a paragraph explains that in cases of double and triple chromatics, some altered functions can be enharmonized. This is followed by the title 'Figure XIII.' and a subtitle 'Enharmonic treatment of double and triple chromatics.', both underlined. The main content is a photograph of a handwritten musical score on a separate piece of paper. The score is titled 'FIG XIII.' and consists of two staves. The upper staff uses a soprano clef and contains several chords and notes, some with accidentals and fingerings. The lower staff uses an alto clef and contains notes corresponding to the upper staff. The notation is handwritten and somewhat stylized.





Lesson CII.Overlapping Chromatic Groups.

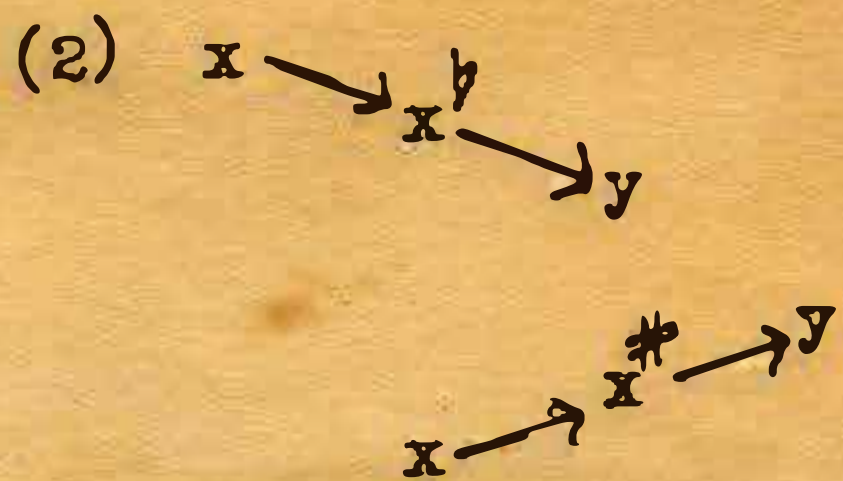
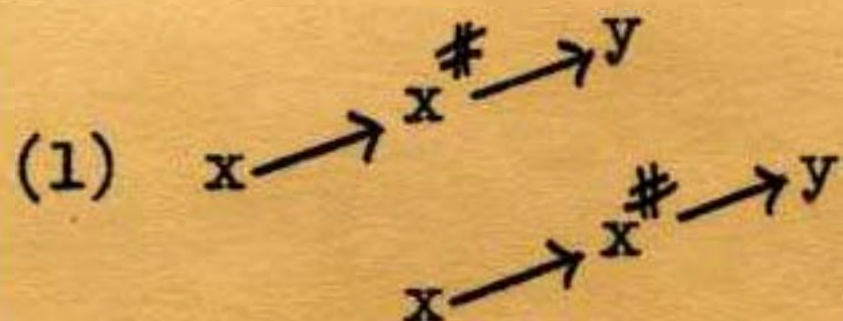
Overlapping groups produce a highly saturated form of chromatic continuity. The alterations in the two overlapping groups may be either both ascending, or both descending, or one of the groups can be ascending, while the other descending. The choice of ascending and descending groups depends on the possibilities presented by the preceding groups during the moment of alteration.

The general form of overlapping chromatic groups is:

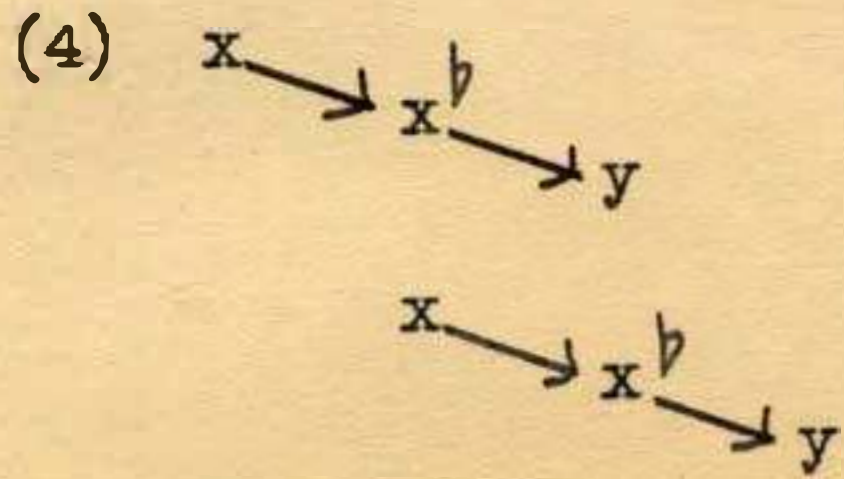
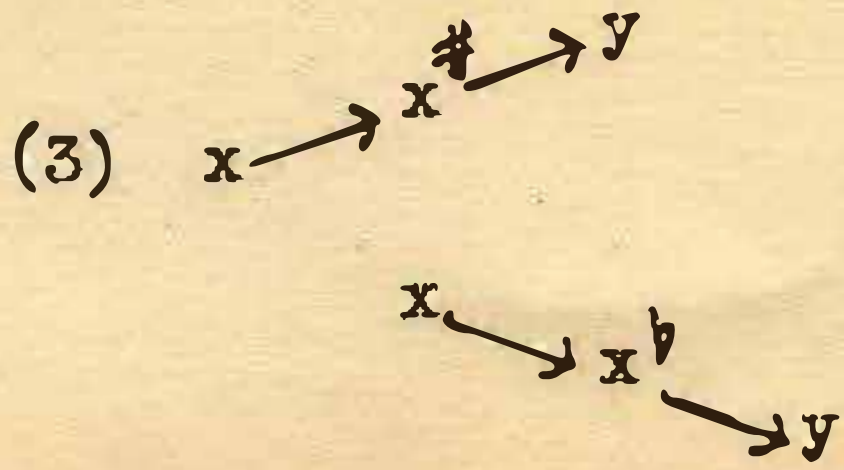
$$d - ch - d$$

$$d - ch - d$$

This scheme, being applied to ascending and descending alterations, offers 4 variants.







Thus, parallel as well as contrary forms are possible.

Each of the mutually overlapping groups has a single chromatic operation.

Figure XIV.

Examples of overlapping chromatic groups.

**FIG XIV**

The figure displays two systems of overlapping chromatic groups. Each system is written on two staves, one with a treble clef and one with a bass clef. The notes are connected by lines, and accidentals (sharps and flats) are used to indicate chromatic movement. The first system shows a chromatic scale in the bass clef and a corresponding scale in the treble clef. The second system shows a chromatic scale in the treble clef and a corresponding scale in the bass clef.



The sequence in which such groups can be constructed is as follows:

In the first example of Figure XIV (and similar procedure refers to all cases) we write the first chord first:



The next step is to make operations in one voice. In this example  $1^\sharp$  was chosen in the bass:



The next step is to construct the middle chord of this group: ( $1^\sharp$  was assumed to remain  $1$ , which gave the  $C^\sharp$  seventh-chord):





The next step is to estimate the possibilities of other voices with regard to chromatic alterations.

The  $b \rightarrow b^b$  step permits to construct a chord which necessitates the inclusion of  $d$  and  $b^b$ . Another possibility might have been to produce  $g \rightarrow g^\sharp$ , which would also permit the use of  $d$  in the bass. See the second example of Figure XIV. The third possibility might have been the step  $e \rightarrow e^\sharp$ , in the alto voice, which also permits the use of  $d$ . Even steps like  $e \rightarrow e^b$  or  $g \rightarrow g^b$  would be possible, though the latter require an augmented  $S(7)$ , i.e. (reading upward)  $d - g^b - e^b - b^b$ .

Figure XV.

Continuity of Overlapping Chromatic Groups.

**FIG XV**

The figure shows two staves of handwritten musical notation. The top staff is in treble clef and the bottom staff is in bass clef. Both staves contain a sequence of notes with various chromatic alterations (sharps, flats, and double flats) and slurs. The notation is somewhat messy and appears to be a working draft or a specific example of chromatic movement in a chord progression.





Lesson CIII.Coinciding Chromatic Groups.

The technique of evolving coinciding chromatic groups is quite different from all the chromatic techniques previously described. It is more similar to the technique of passing chromatic tones, at which we shall arrive later.

Coinciding chromatic groups are evolved as a form of contrary motion in two voices being a doubling of the chord, with which the group begins.

The general form of a coinciding chromatic group is:

d - ch - d

d - ch - d

Contrary directions of the chromatic operations can be either outward or inward:

(1)  $x \rightarrow x^{\#} \rightarrow y$

$x \rightarrow x^b \rightarrow y$

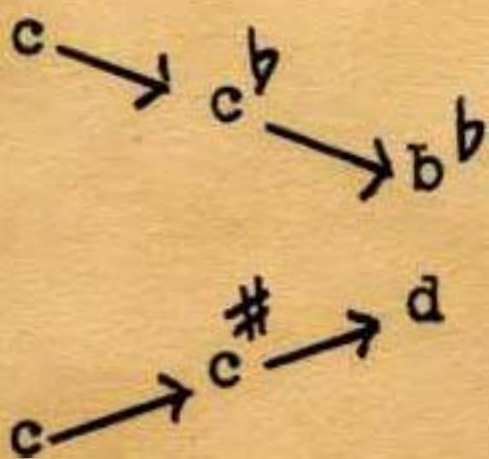
(2)  $x \rightarrow x^b \rightarrow y$

$x \rightarrow x^{\#} \rightarrow y$



The assignment of the two remaining functions in the middle chord of a coinciding group can be performed by sonority, i.e. enharmonically.

For instance, in a group



the  $\begin{matrix} c^\flat \\ c^\sharp \end{matrix}$  interval can be read enharmonically, i.e. as

$\begin{matrix} b \\ c^\sharp \\ c \end{matrix}$  in which case it becomes  $\begin{matrix} 7 & 9 \\ 1 & 3 \end{matrix}$  etc. It is easy

then to find the two remaining functions, like 3 and 5. Thus, we can construct a chord  $c^\# - e - g - b$ .

As coinciding chromatics result from doublings, it is very important to have full control of the variable doublings technique. Thus the doubling of 1, 3, 5 and also 7 (major or minor) must be used intentionally in all forms and inversions of S(5) and S(7). The latter, naturally, for obtaining the doubled 7.

#### Figure XVI.

#### Examples of Coinciding Chromatic Groups.

(Notation of chromatic operations as in all other forms of chromatic groups).

(please see next page)



# FIG XVI

①

Handwritten musical notation for system 1, measures 1-4. The system consists of two staves: a treble clef staff on top and a bass clef staff on the bottom. Each measure contains two chords. The notes in the treble staff are generally higher than those in the bass staff. The notation includes various accidentals such as flats (b) and sharps (#).

③

Handwritten musical notation for system 2, measures 1-4. The system consists of two staves: a treble clef staff on top and a bass clef staff on the bottom. Each measure contains two chords. The notation includes various accidentals such as flats (b) and sharps (#).

⑤

Handwritten musical notation for system 3, measures 1-5. The system consists of two staves: a treble clef staff on top and a bass clef staff on the bottom. Each measure contains two chords. The notation includes various accidentals such as flats (b) and sharps (#).

⑦

Handwritten musical notation for system 4, measures 1-5. The system consists of two staves: a treble clef staff on top and a bass clef staff on the bottom. Each measure contains two chords. The notation includes various accidentals such as flats (b) and sharps (#).



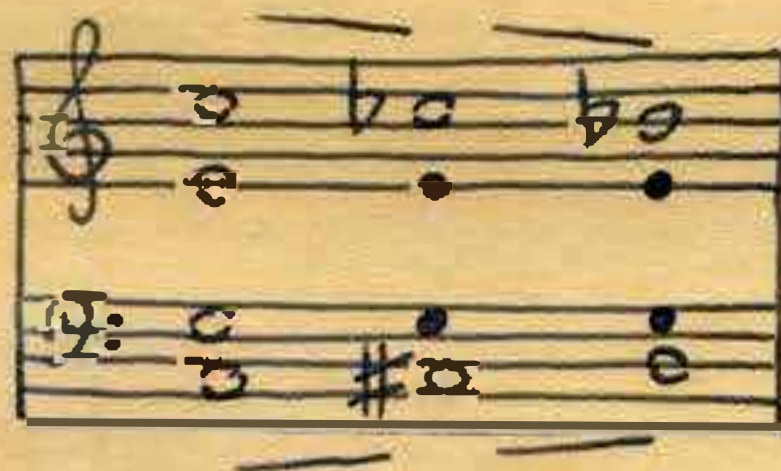
It is important to take into consideration, while executing the coinciding chromatic groups, that the first procedure is to establish the chromatic operations



and the second procedure is to add the two missing functions.



After performing this, the final step is to assign the functions in the last chord of the group.



All coinciding groups are reversible.

When moving from an octave inward by semitones, the

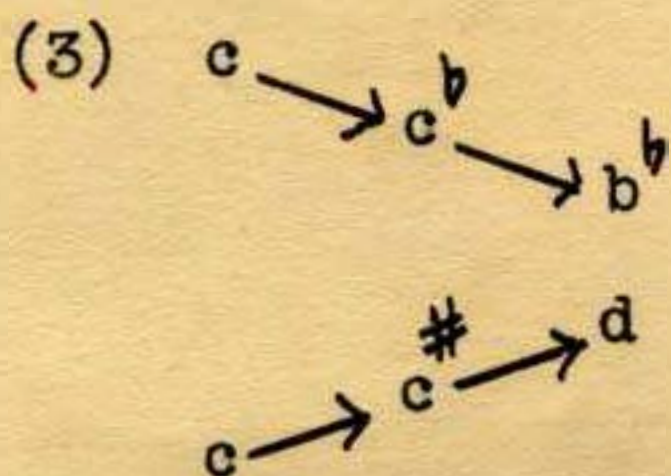
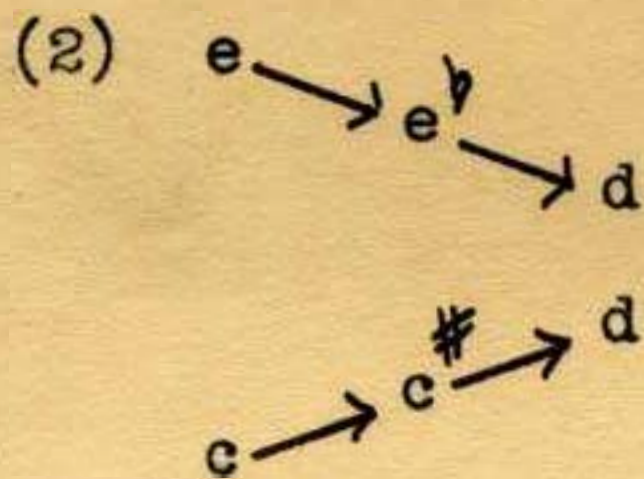
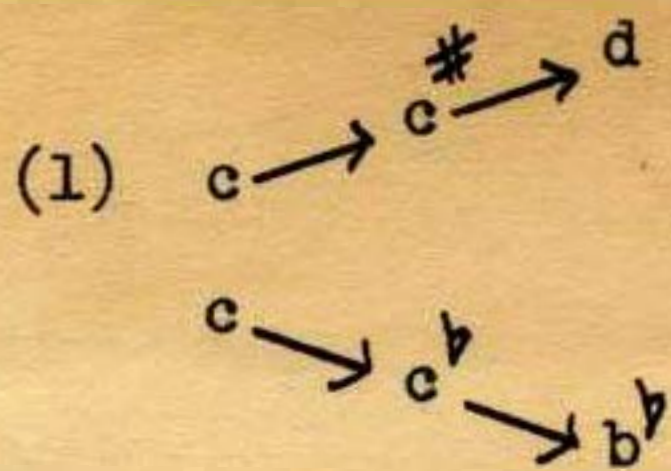




last term of the group produces a minor sixth. When moving outward from unison or octave, the last term of the group produces a major third.

It is important to take these considerations into account while evolving a continuity of coinciding chromatic groups. Any such group can start from any two voices producing (vertically) a unison, an octave, a major third or a minor sixth.

The following are all movements and directions with respect to c.





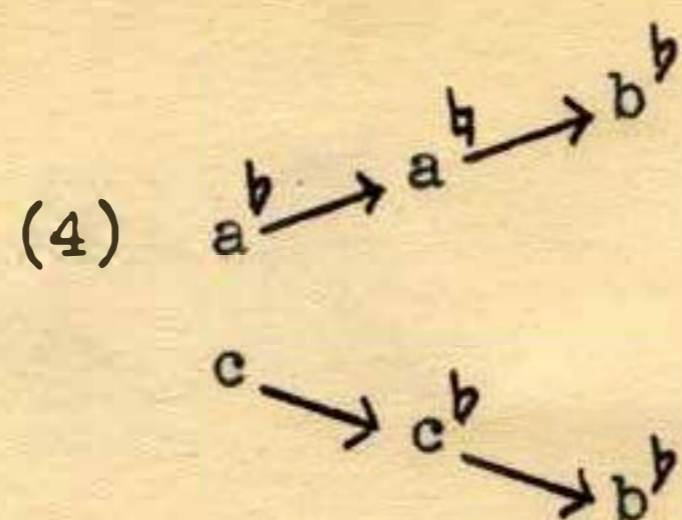


Figure XVII.

Continuity of Coinciding Chromatic Groups.

**FIG. XVII**

The musical notation consists of two staves. The top staff is in treble clef and the bottom in bass clef. The notation shows a series of chords and single notes connected by lines, illustrating the continuity of coinciding chromatic groups. A checkmark is placed under the first few notes of the top staff.

All techniques of chromatic harmony can be utilized in the mixed forms of chromatic continuity.

