Lesson CXLVIII.

THEORY OF HARMONIC INTERVALS

(Introduction to Counterpoint)

A sequence of two pitch-units produces a melodic interval. A simultaneous combination of two pitch-units produces a harmonic interval. The technique of correlation of simultaneous melodies depends entirely upon the composition of harmonic intervals. Any number of simultaneous parts (voices) in counterpoint are formed by the pairs. These pairs may be conceived as voices immediately adjacent in pitch, as well as in any other form of vertical arrangement (i.e. over 1, over 2, etc.).

The success of harmonic versatility of counterpoint depends upon the manifold of harmonic intervals used in a certain style. Limited quantity of harmonic intervals results in limited forms of the harmonic versatility of counterpoint. Thus, the study of harmonic intervals becomes one of the important prerequisites of counterpoint.

Harmonic intervals have dual origin:

1. Physical
2. Musical

The physical origin of harmonic intervals leads back to the simplest ratios. The musical origin

of intervals is based on selective and combinatory processes. All semitones, i.e. units of the equal temperament of twelve, are the structural units of all other harmonic intervals available in such equal temperament. As they appear in our hearing, they amount to the following forms:

i = 1, i = 2, i = 3, i = 4,
i = 5, i = 6, i = 7, i = 8,
i = 9, i = 10, i = 11, l = 12

This completes the entire selection within one octave range. An addition of intervals to an octave produces musically identical intervals over one octave, as the similarity of different pitch-units within the ratio of 2 to 1 is so great that they even have identical musical names. The system of musical notation introduces, among other forms of confusion, the dual system of the interval nomenclature. Thus, an interval containing three semitones may be called either a minor third or an augmented second.

Simple ratios of acoustical intervals are merely approximate equivalents of the harmonic intervals of equal temperament. It is not scientifically correct to think the way the majority of acousticians do, that a 5 to 4 ratio is an equivalent of a major third or a 6 to 5, of a minor third, or a 7 to 4, of

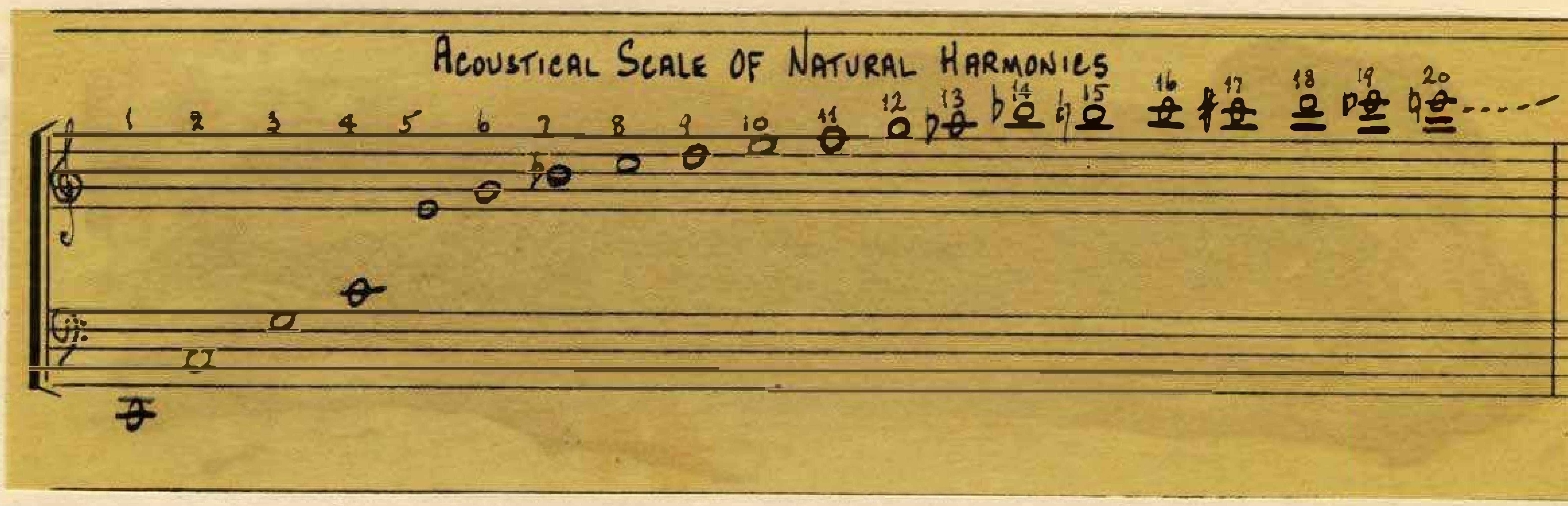
a minor seventh, etc., as these intervals deviate considerably from their equivalents in equal temperament.

It is utterly impossible to follow the methods established by some acousticians in studying the type and quality of intervals in the equal temperament of twelve as compared to their equivalents in the simple acoustical ratios. The so-called consonance is a totally different type of intervals musically or acoustically. If music had to use acoustical consonances only, yet being confined to an equal temperament of twelve, the only real consonance would be an octave, i.e. no two pitch-units bearing different names would ever be used, and we would never have either any harmony or counterpoint. The reason for this is that no other intervals than an octave or a perfect fifth, with a certain allowance, are consonances within the equal temperament. All other intervals are quite complicated ratios. Thus, the art of music has its own possibilities based on the limitations within a given manifold of our tuning system.

Acoustical consonances produce a so-called <u>natural harmonic scale,</u> which consists of a fundamental with all its partials appearing in the sequence of a natural harmonic series (i.e. 1, 2, 3,

4, 5, 6, 7, 8, 9, etc.). The ratios of acoustical consonances are equivalent to the ratios of vibrations producing pitches. For example, a $\frac{3}{2}$ ratio means that if the actual quantities representing both the numerator and the denominator were multiplied by a considerable number value, they would actually sound as pitches. While $\frac{3}{2}$, as such, sounds to our ear as the resultant of interference of 3 to 2, $\frac{300}{200}$ cycles per second sounds to our ear as a perfect fifth.

Figure I.



Our ear accepts pitch-units and their ratios as they reach said ear and the auditory consciousness and not as they are induced upon us in the traditional musical schooling. For example,

a melody played simultaneously in the key of c and in the key of b next to it, or a seventh above, sounds decidedly disturbing to musicians of our time. Yet an interval that is musically identical is acoustically so different that being placed three octaves apart it produces a musically consonant impression. The reason for this is that in such absolute intervals as seventh three octaves apart approximates the the 15 to 1 ratio, i.e. the sound of a 15th harmonic in relation to its fundamental. And when the pitches are so far apart the deviation from equal temperament becomes less obvious for our pitch discrimination. The following tables offer a group of examples illustrating musically consonant intervals which are usually classified as dissonances, and with their correspondence to the proper location of harmonics. In all these cases no octave substitution can be made without affecting the actual state of consonance.

Figure II.

(please see next page)

(Fig. II)

Likewise, musical consonances being placed into a wrong pitch register, such as low register, produce upon our ear an effect of musical dissonances. The reason for this is that being an approximation of simple ratios they require the placement of their fundamentals at such low frequencies that they are below the range of audibility. For example, a major third being associated with $\frac{5}{4}$ ratio would require that the fundamental be located two octaves below the fourth harmonic. Music being played in major thirds in the contra-octave simply would not permit the physical existence of such fundamental.

The following tables offer three <u>examples</u> of the low setting of intervals.

<u>Figure III.</u>

(please see next page)

(Fig. III)

With this understanding in mind we can see that no serious theory of resolution of dissonant intervals may be devised without specifica-

tions to exact octave location of the interval. Thus, when we come to the theory of resolution of intervals it will merely be offered for the purpose of the versatile treatment of the progressions of harmonic intervals, and not for the purpose of extermination of dissonances. Esthetically as well as physiologically we desire sequences of tension and release, and as different harmonic intervals produce different degrees of tension the versatility of the sequence of intervals will satisfy such requirements.

It has often been the case that music written according to the rules and regulations of the dogmatic counterpoint does not sound esthetically as convincing as its counterpart in the XVI or XVII Century. This inferior quality is due to the limited quantity of harmonic intervals and the forms of treatment of the latter.

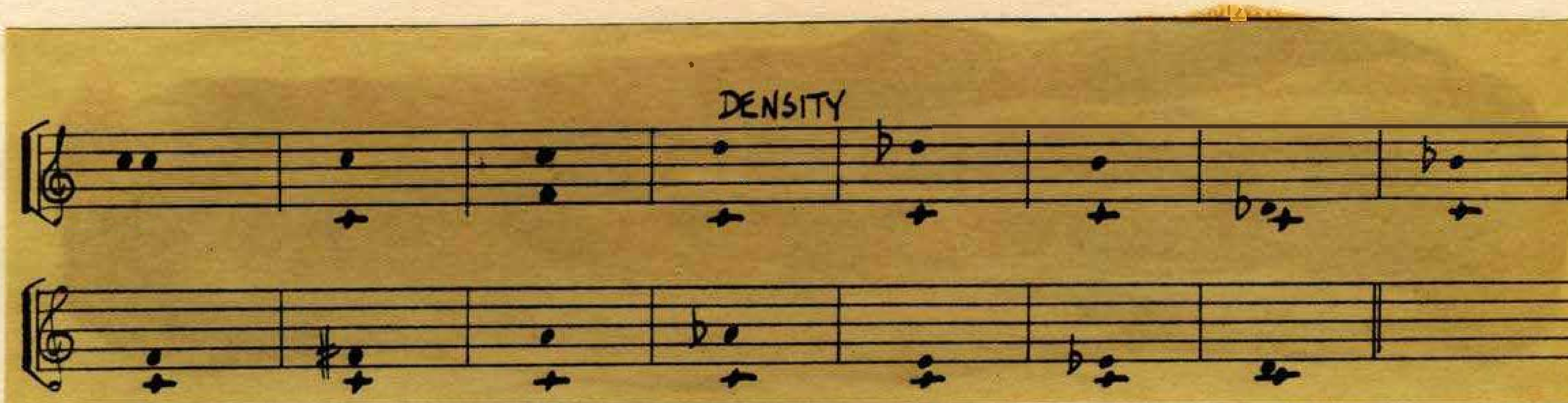
A. Classification of Harmonic Intervals within the Equal Temperament of Twelve

All harmonic intervals may be classified into two groups:

1. With regard to their <u>density</u>, i.e. the fullness of sonority, and
2. With regard to their <u>tension</u>, i.e. their dissonant quality.

Classification of density evolves from the intervals producing the emptiest effect upon our ear up to the intervals producing the fullest effect. The following table is only an approximate one; nevertheless, it serves the purpose with a certain degree of approximation, i.e. the first few intervals sound decidedly empty and the last few sound decidedly full, while in the center there are a few intermediate ones.

Figure IV.



Classification of tension is based upon the separation of consonances from the dissonances and the separation of the consonances and dissonances by name from the consonances and dissonances by sonority. All cases when consonances and dissonances correspond respectively by name and sonority imply the diatonic intervals. And all cases when

consonances and dissonances do not correspond to their original names produce chromatic intervals. The group of diatonic consonances includes perfect unisons, perfect octaves, perfect fifths, perfect fourths, major thirds, minor thirds, major sixths, minor sixths. The group of diatonic dissonances includes major and minor seconds, major and minor sevenths, major and minor ninths. All the chromatic intervals are classified into augmented and diminished.

The Augmented Intervals:

Unison, 2nd 3rd, 4th, 5th, 6th.

The Diminished Intervals:

Octave, 7th, 6th, 5th, 4th, 3rd.

Figure V.

The augmented	unison	is	equivalent	to	minor 2nd	by sonority.
"	"	2nd	"	"	" major 3rd	" "
"	"	3rd	"	"	" perfect 4th	" "
"	"	4th	"	"	" no diatonic interval.	
"	"	5th	"	"	" minor 6th	by sonority.
"	"	6th	"	"	" minor 7th	" "
The diminished	octave		"	"	" major 7th	" "
"	"	7th	"	"	" major 6th	" "
"	"	6th	"	"	" perfect 5th	" "
"	"	5th	"	"	" no diatonic interval.	
"	"	4th	"	"	" major 3rd	by sonority.
"	"	3rd	"	"	" major 2nd	" "

Thus, the following intervals are consonances by sonority. The augmented 2nd, 3rd, 5th; the diminished 7th, 6th, 4th. All other chromatic intervals will be treated as dissonances with the resolutions corresponding either to diatonic or to chromatic dissonances.

Lesson CXLIX.

B. Resolution of Harmonic Intervals

The necessity of varying tension implies the procedure known as resolution of intervals. It is important to realize that the variation of tension may be gradual as well as sudden, i.e. the transition from a more dissonant harmonic interval to a less dissonant one and finally into a fully consonant one is as desirable as a direct transition from extreme tension to full consonance.

In the following tables intervals such as perfect 4th and 5th are included as well, not for the purpose of relieving them from tension, but for the purpose of devising different useful manipulations forming contrapuntal sequences. The quantity of resolutions known to a composer has a definite effect upon the harmonic versatility of his counterpoint. For example, if one knows only four resolutions of a major 2nd (which is the usual case) as compared to the twelve possible resolutions, the amount of musical possibilities is considerably less. Thinking in terms of variations one can see that the number of permutations available from four or from twelve elements is so different in quantity that they cannot even be compared (the first giving twenty-four variations

and the second giving 479,001,600 variations). It is easy to see that having such losses on the quantity of resolutions of each harmonic interval, the loss on the total of versatility of counterpoint is incalculable. There is no need in memorizing all the details of the resolution of intervals, as there are general underlying principles evolved through the tradition of centuries.

1. All diatonic intervals resolve through either outward or inward or oblique motion of each voice on a semitone or a whole tone. *

2. When a resolution is obtained through oblique motion the sustained voice may produce a leap on a melodic interval of a perfect 4th, either up or down.

3. All intervals known as 2nds have a tendency to expand. All intervals known as 7ths have a tendency to contract. All 7ths are the exact equivalents of 2nds in the octave inversion (i.e. all pitch-units are identical with those of the 2nds). All the 9ths have a tendency to contract. All the 4ths and 5ths are neutral, i.e. they either expand or contract.

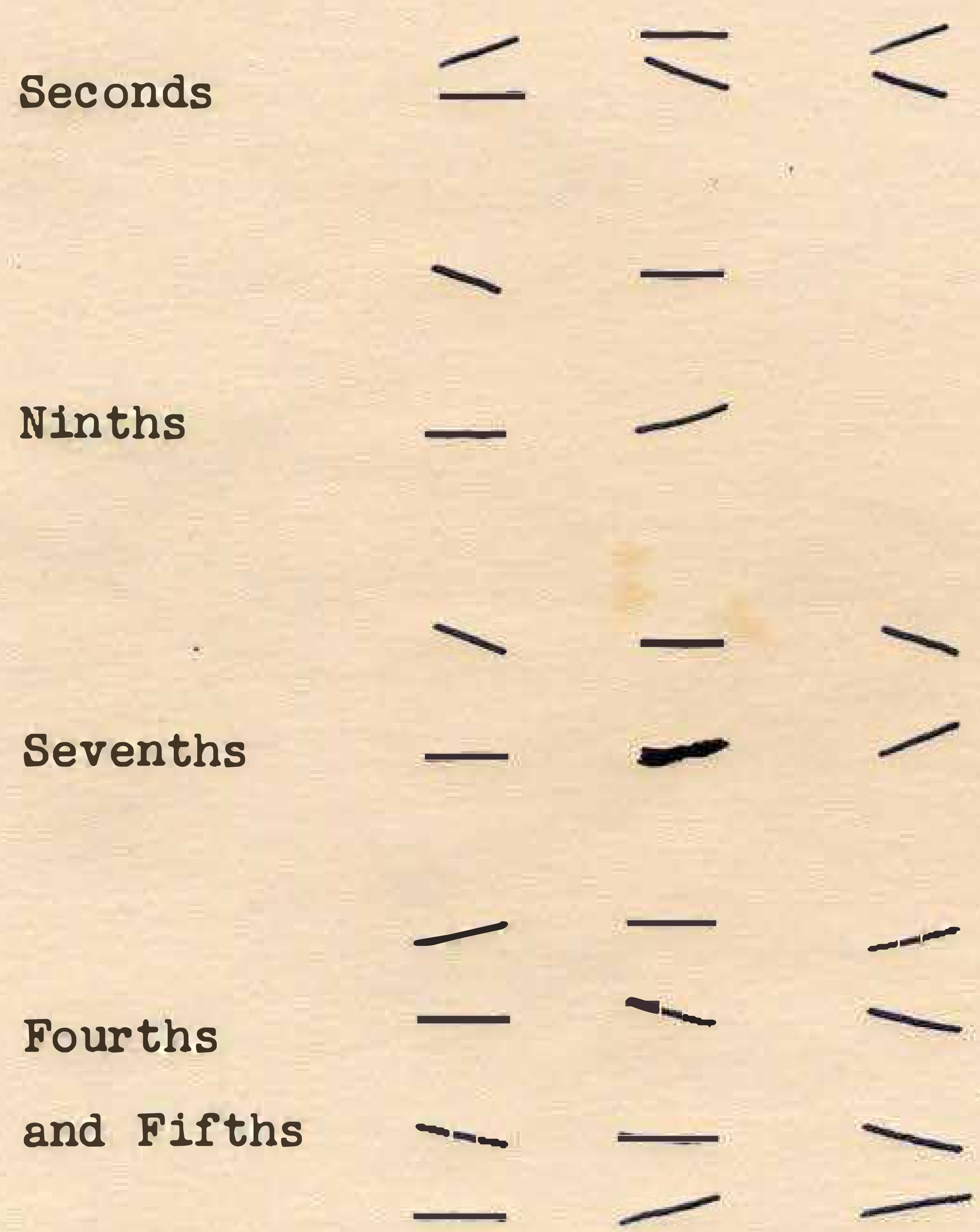
Thus, the entire range of permutations of semitones and whole tones, with their respective

* An i = 3 is also correct when such an interval represents two adjacent musical names (c - d#, for example).

directions, constitutes the entire manifold of resolutions.

Refer to Resolution of Diatonic Intervals chart below.

Resolution of Diatonic Intervals



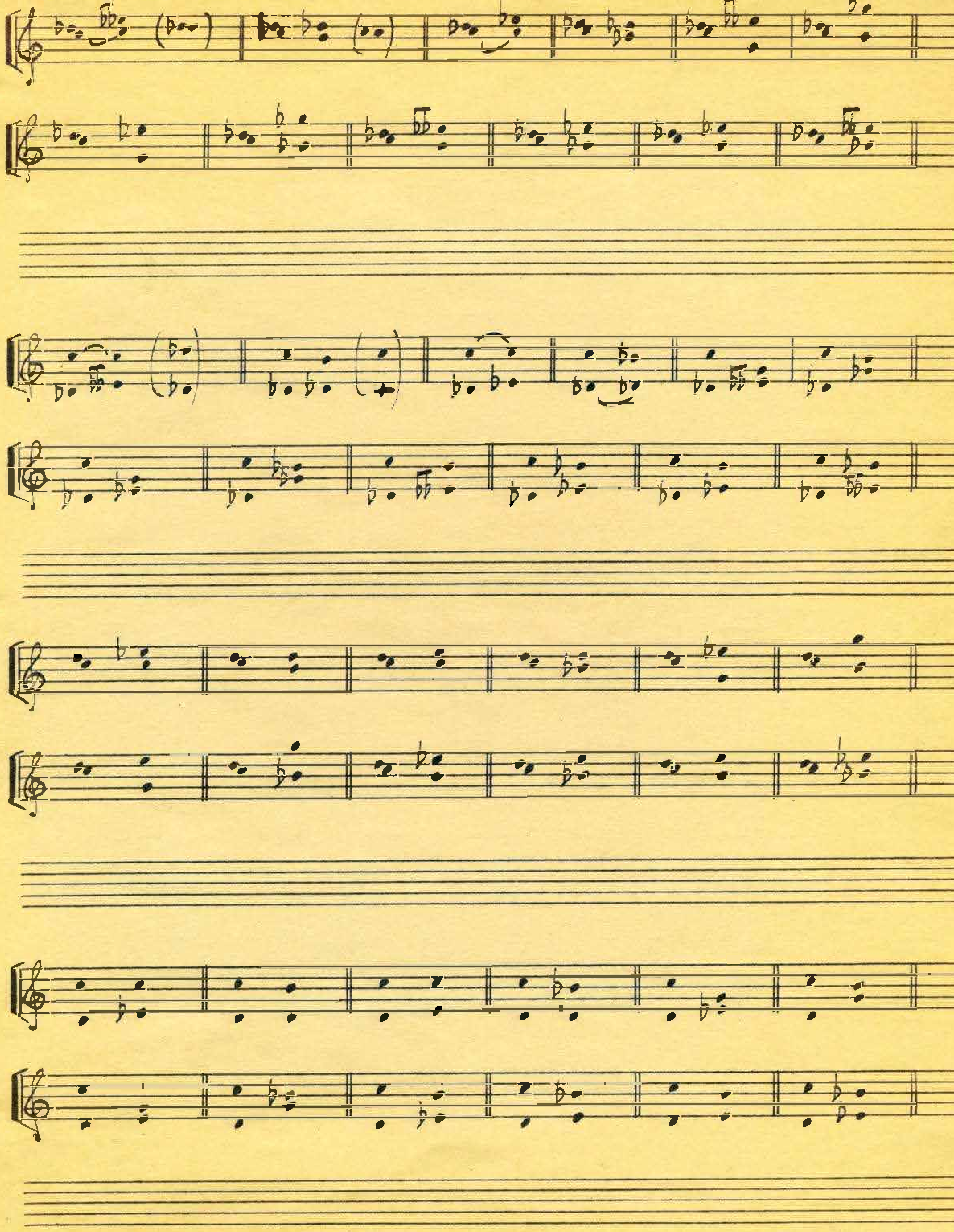
(enharmonic)

The following is a complete table of resolutions of diatonic intervals. The intervals in parentheses are the secondary resolutions. They are used in all cases when the first resolution produces a dissonance.

Figure VI.

SECONDS AND SEVENTHS

6.



FOURTHS AND FIFTHS

NINTHS

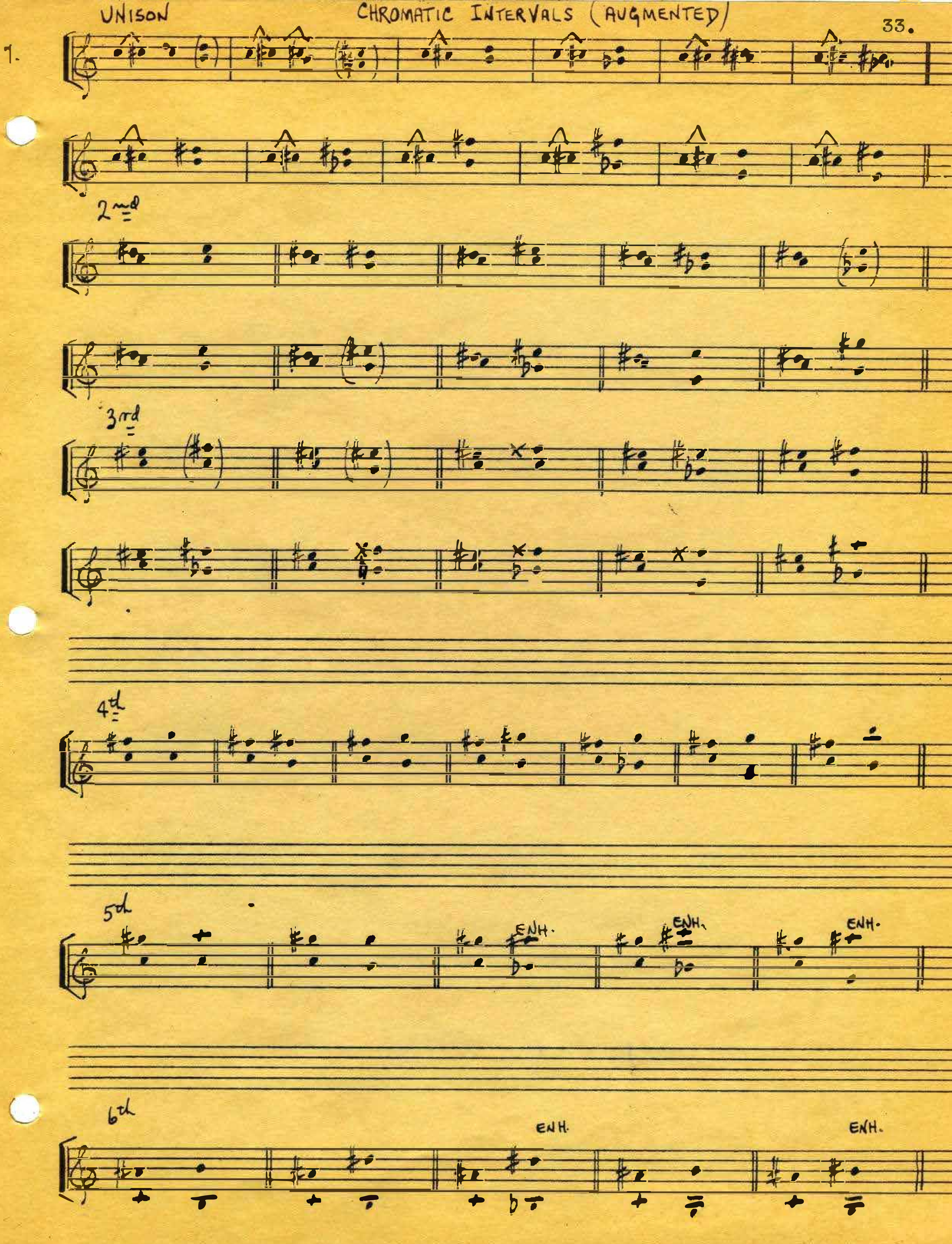
All chromatic intervals which are augmented have a tendency of expansion. And all chromatic intervals which are diminished have a tendency of contraction. The method of reasoning in resolving augmented or diminished intervals is as follows: $\genfrac{}{}{0pt}{}{d^{\sharp}}{c}$ is a 2nd derived through augmentation of a major second, either through altering of d to $d^{\sharp}$ or of $c^{\sharp}$ to $c^{\natural}$. Thus, originally it might have been a 2nd $\genfrac{}{}{0pt}{}{d}{c}$ or $\genfrac{}{}{0pt}{}{d^{\sharp}}{c^{\sharp}}$. Considering the dual origin of such interval we find the respective resolutions: if $d^{\sharp}$ is the alteration of d, it has the inertia of moving further in the same direction, i.e. to e; or if $c^{\natural}$ is the alteration of $c^{\sharp}$, it has the inertia of moving to $b^{\natural}$. Such two steps taken individually or simultaneously constitute the fundamental resolutions. An analogous procedure must be applied to the diminished intervals where the diminutions are produced through inward alteration.

The following is a complete table of resolutions of chromatic intervals. When a chromatic interval resolves into a consonance by sonority, the sign "enh." is placed above it (enharmonic). When the interval of resolution is surrounded by parenthesis, the interval of resolution is a dissonance.

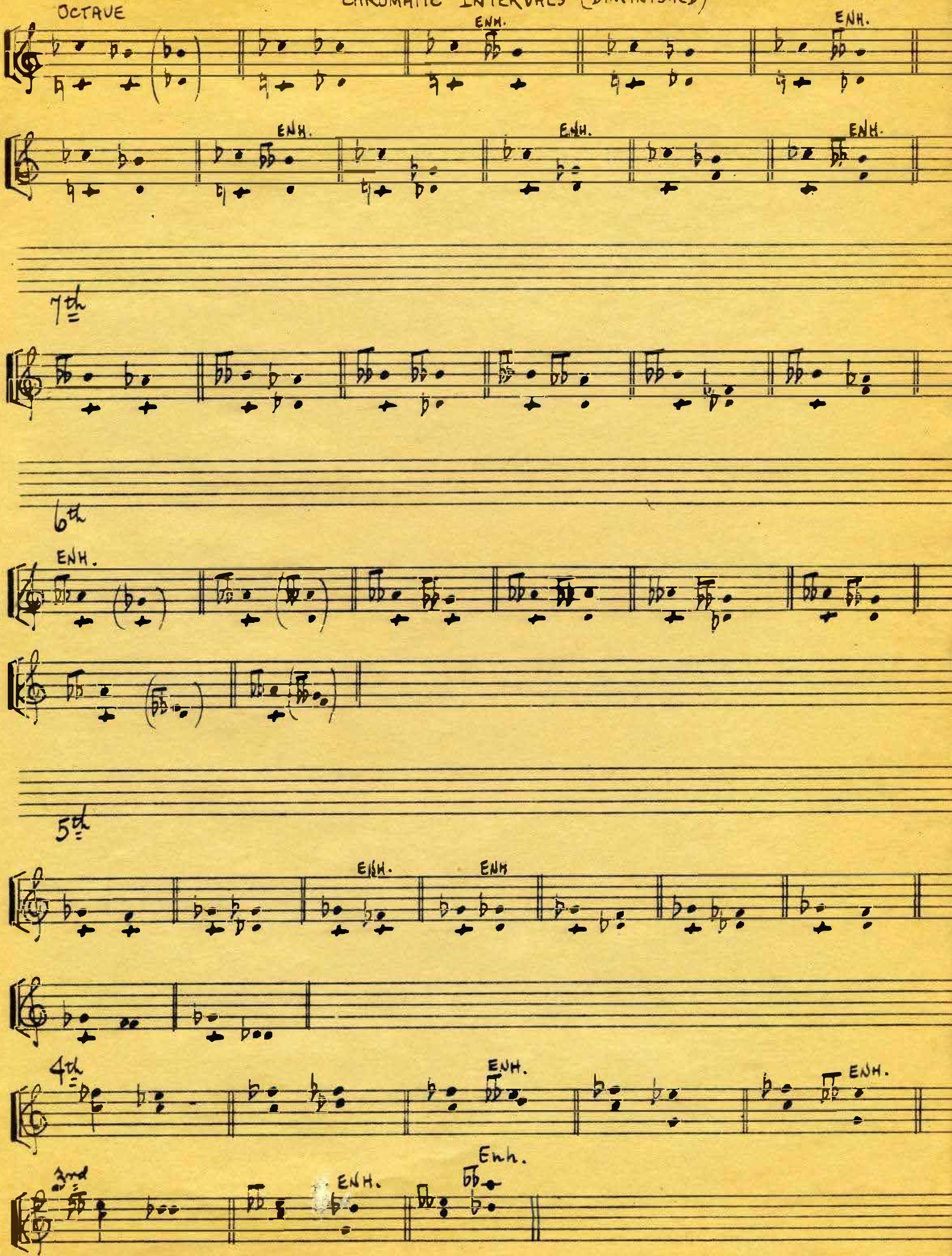
<u>Figure VII.</u>

(please see next page)

CHROMATIC INTERVALS (AUGMENTED)



CHROMATIC INTERVALS (DIMINISHED)



In the old counterpoint we often find a different type of resolutions from the ones described above. They were known as <u>kambiata</u> resolutions, which are conceived as a melodic step of a 3rd instead of a 2nd. No good reason has ever been given why such resolutions would be used. I offer an hypothesis for the explanation of these resolutions which I believe is the only one to be correct.

As the tradition of old counterpoint was developed, while the pentatonic (5 units) scales were in use, some of the pitch-units of full diatonic (heptatonic = 7 units) scales were absent. Thus, if we find that in an interval $\begin{matrix} d \\ c \end{matrix}$, d moves to e, while c moves to a (instead of b), a kambiata takes place merely because such scale may be a pentatonic scale and the unit b does not exist.

This approach offers us a definite principle of resolution of intervals in the scales which have not been in use in the classical traditional music confining all the resolutions merely to the next step with the following musical name. For example, in harmonic a minor, the interval $\begin{matrix} a \\ g\sharp \end{matrix}$ may be resolved through movement of the lower voice only to $f\natural$, as no other pitch-unit with the name f exists in such scale.

This concludes the Theory of Harmonic Intervals.

J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross   Subject: Music

Lesson CL.

Theory of Correlated Melodies.

(Counterpoint)

As counterpoint represents a system of correlation of melodies in simultaneity and continuity, it is absolutely essential to be thoroughly familiar with the constitution of melody. Only by being familiar with the material of the Theory of Melody is the successful accomplishment of such task possible. Correlation of melodies is usually considered to be one of the most difficult procedures. As the structural constitution of one melody is unknown theoretically, the combination of two unknown quantities is an entirely fantastic task to undertake.

It is not only a problem of putting two voices together, but a problem of either combining two melodies already made, or a composition of two melodies with distinct individual characteristics. As <u>each</u> melody consists of several components, such as the rhythm of durations, attacks, melodic forms, the forms of trajectorial motion, etc., the correlation of <u>two</u> melodies in addition to the above described components

adds one more: harmonic correlation. Thus, counterpoint can briefly be defined as a system of correlation of rhythmic, melodic and harmonic forms in two or more conjugated melodies.

As the forms concerning one individual melody are known through the previous material, we will first cover the field of harmonic correlation which is based on the Theory of Harmonic Intervals. After covering this particular branch we shall return to the other forms of correlation for the purpose of achieving the final results offered by the contrapuntal technique.

A. Two-Part Counterpoint

The fundamental technique in writing two-part counterpoint is based on writing one new melody to a given melody. A given melody is usually abstracted from its rhythm of durations, thus producing a purely melodic form which may be taken from a choral as well as from a popular song. The usual way of presenting such an abstracted melodic form is in whole notes. Such a melodic form is usually known as Cantus Firmus (firm chant = canonic or established chant). Our abbreviations for Cantus Firmus will be C.F. and for the melody written to it, counterpoint or C.P. The first forms of counterpoint will be classified through the quantity of attacks in C.P. as against one attack in C.F. Thus,

all the fundamental forms of counterpoint will be as follows:

$$\frac{CP}{CF} = 1, 2, 3 \ldots\ldots n$$

$$\frac{CP}{CF} = a$$

This form of counterpoint, through international agreement for a number of centuries, implies the usage of consonances only. As we shall have four fundamental forms of harmonic correlation and some of these forms will be polytonal (i.e., there will be two different keys used simultaneously), we will have to use consonances by name and by sonority. The positive requirements for harmonic correlation in 2-part C.P. are:

a. The variety of types of intervals (i.e., intervals expressed by different numbers).

b. The variety of density.

c. Well defined cadences expressed through the leading tones moving into their axes.

d. Crossing of C.P. and C.F. is permissible <u>when necessary.</u>

The negative requirements are:

a. The elimination of consecutive intervals which are perfect unisons, octaves, 4ths and 5ths.

No consecutive dissonances. Thus, the only intervals to be used in parallel motion are thirds and sixths.

b. Motion toward such intervals only through contrary (outward or inward) directions.

c. No repetition of the same pitch-unit in CP unless it is in a different octave.

The forms of harmonic relations previously used in time continuity (see Theory of Pitch Scales) will be used in counterpoint as the forms of simultaneous harmonic correlation.

Forms of Harmonic Correlation

1. U. - U. Unitonal - Unimodal (identical scale structure and key signature).

2. U. - P. Unitonal - Polymodal (a family scale with identical key signature).

3. P. - U. Polytonal - Unimodal (identical scale structure, different key signature).

4. P. - P. Polytonal - Polymodal (different scale structure, different key signature).

In the XIV Century, in the case of Guillaume de Machault* we find a fully developed type 2, and in some cases an undeveloped type 3. Only the

* The phonograph records of a Mass written by this composer for the coronation of Charles V are available. (Les Paraphonistes de St. Jean des Matines and Brass Ensemble conducted by Van). The reconstruction of Machault's 2 and 3-Part Madrigals in our musical notation is published by the Historic Musicological Society of Leipzig in 1926. Not available in U.S.A.

ignorance and vanity of the contemporary composers make them believe that they are the discoverers of polytonal counterpoint. The greatest joke is on the modern French composers who make the claim of priority, not being aware that their direct musical ancestors were the originators of this style centuries ago. It is also unfortunate that the idea of polytonality goes hand in hand with the so-called "dissonant counterpoint", i.e., the counterpoint of continuous tension without release. Music based on polytonality with resolutions is a very fruitful, highly promising and almost undiscovered field.

The usual length of C. F. is about 5, 7, 9 or more bars, preferably in odd numbers (this requirement is traditional). The selection of different key signatures for the types 3 and 4 is entirely optional. Any two scales, the root tones of which produce a consonance, may be used for this type of counterpoint. The best way of constructing exercises is the placement of C.F. on a central staff surrounded by two staves below, and two staves above, assigning each staff for a different type of counterpoint.

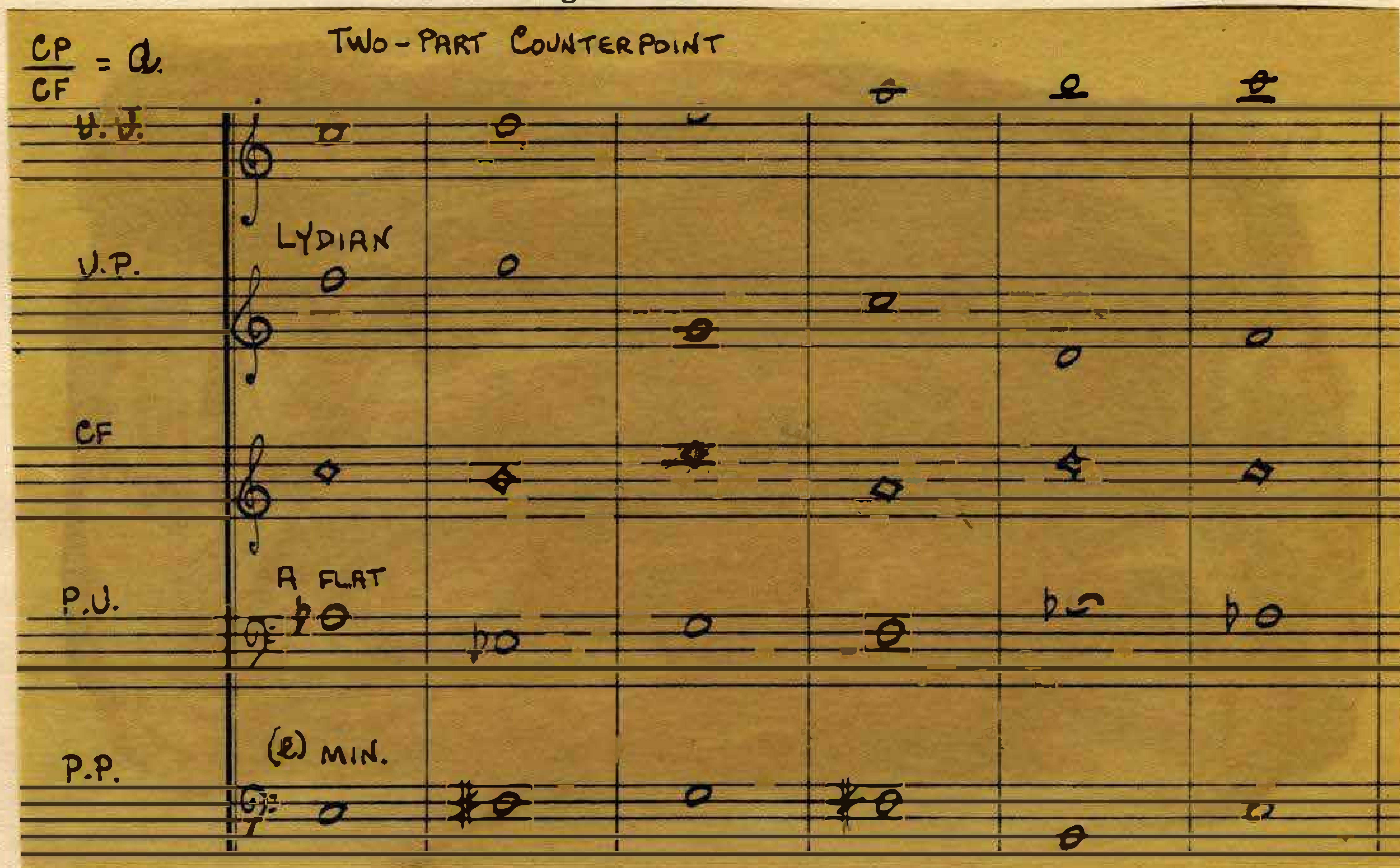
In the following group of exercises each part must be played individually with C.F. Thus, each example produces four types of counterpoint with a

historical emphasis of eight centuries, as the first and second types were considerably developed during the middle ages, and the third and the fourth types are mostly used in the music of today.

It is important to realize that all forms of traditional contrapuntal writing were based on the conception of each melody being in a different mode, and we can even trace the polytonal forms (though in their embryonic form) as far back as the XIII Century.

Lesson CLI.

Figure I.



As a temporary device for harmonic accompaniment, double pedal point may be used in addition to 2-part counterpoint. The root tones of both contrapuntal parts become the axes which must be assigned as chordal functions of a double pedal point. For example, counterpoint type 1 (giving the same pitch-units for both voices) may be considered as a root tone or a 3rd or a 5th, etc., of a simple chord structure. Then, having c as a axis for both contrapuntal parts, the pedal point will become

$\genfrac{}{}{0pt}{}{g}{c}$ or $\genfrac{}{}{0pt}{}{e}{a}$, $\genfrac{}{}{0pt}{}{c}{f}$, etc. This device is applicable to all four types of counterpoint. For example, in type 2, if one contrapuntal part is ionian c and the other aeolian a, they may represent a root and a 3rd, or a 3rd and a 5th, etc., respectively. The pedal point in such case will be $\genfrac{}{}{0pt}{}{e}{a}$ or $\genfrac{}{}{0pt}{}{c}{f}$, etc. In the types 3 and 4 with such two axes as c and a♭, we may use $\genfrac{}{}{0pt}{}{e\flat}{a\flat}$ or $\genfrac{}{}{0pt}{}{c}{f}$, etc. as pedal points. Each double pedal point must last through the entire contrapuntal continuity.

More flexible forms of harmonization of the 2-part counterpoint will be offered later.

$$\frac{CP}{CF} = 2\,a$$

In devising two attacks of a counterpoint against one attack of the C.F., the following combinations of harmonic intervals are possible:

(c - consonance; d - dissonance)

c - c

c - d

d - c

d - d*

In the old counterpoint all these cases were used in both <u>strict</u> and <u>free</u> style, with the exception of a

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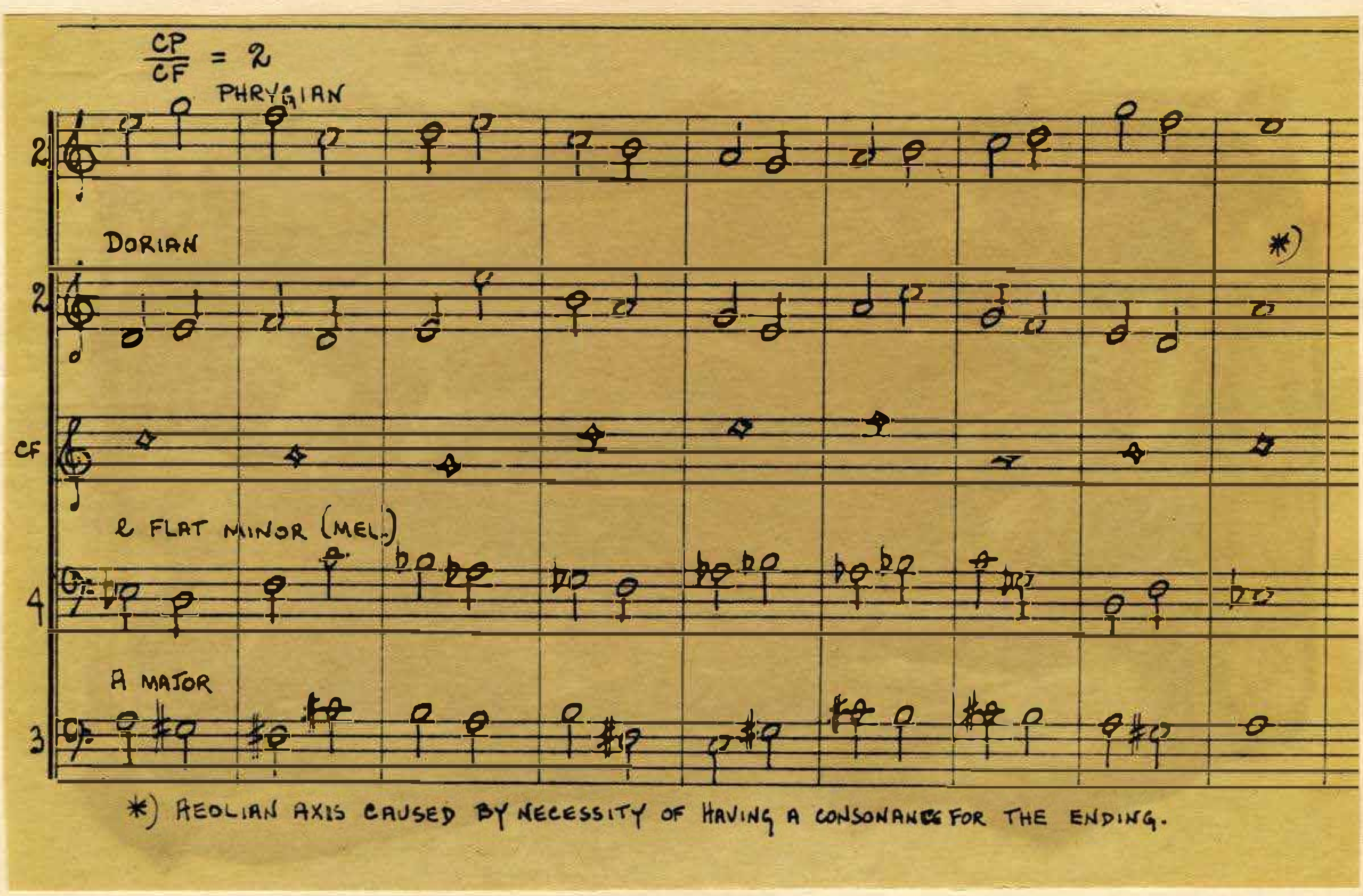
*In scalewise contrary motion only.

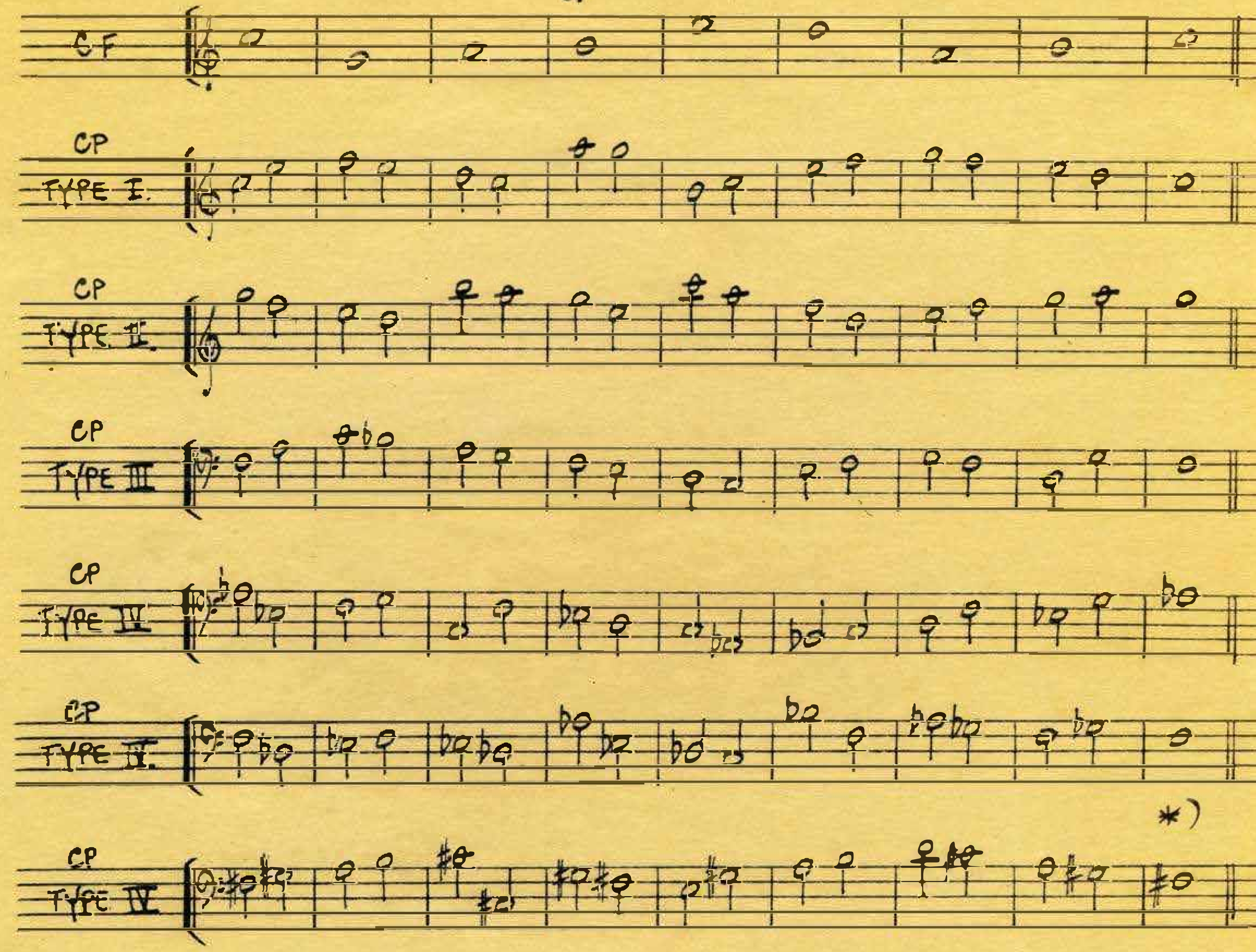
dissonance being on the first beat.

Thus, each bar may start with either a consonance or a dissonance. And, in the case of $\frac{CP}{CF} = 2$, all dissonances require immediate resolutions. Here are a few examples of such contrapuntal exercises.

Figure II.

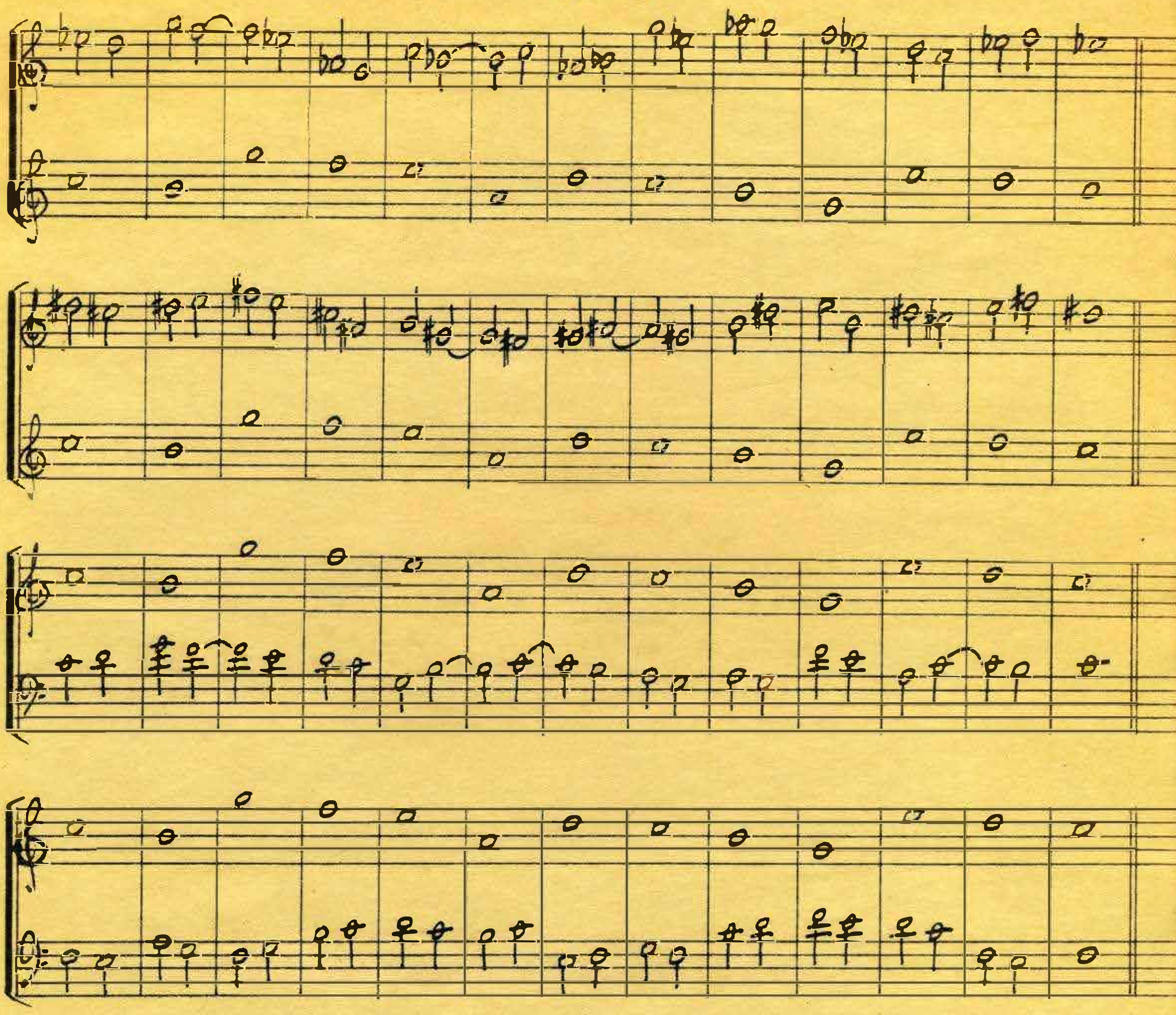


$\frac{CP}{CF} = 2$



*) THE ALLOWANCE IS MADE FOR A WEAK DISSONANCE.

COUNTERPOINT.



Lesson CLII.

$$\frac{CP}{CF} = 3\ a$$

Three attacks of CP against one attack of CF offer the following combinations of harmonic intervals:

c - c - c

c - d - c

d - c - c

c - c - d resolution

d - c - d resolution

d - d*- c

c - d - d*

The d - c - c combination offers a new device which becomes possible with three and more attacks. We shall call it a <u>delayed</u> (or indirect) <u>resolution.</u> Instead of resolving a tense interval we move it to another consonance, after which we resolve the dissonance.

This device accomplishes two things:

(1) it produces a psychological suspense, thus making music more intriguing;

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* In scalewise contrary motion only

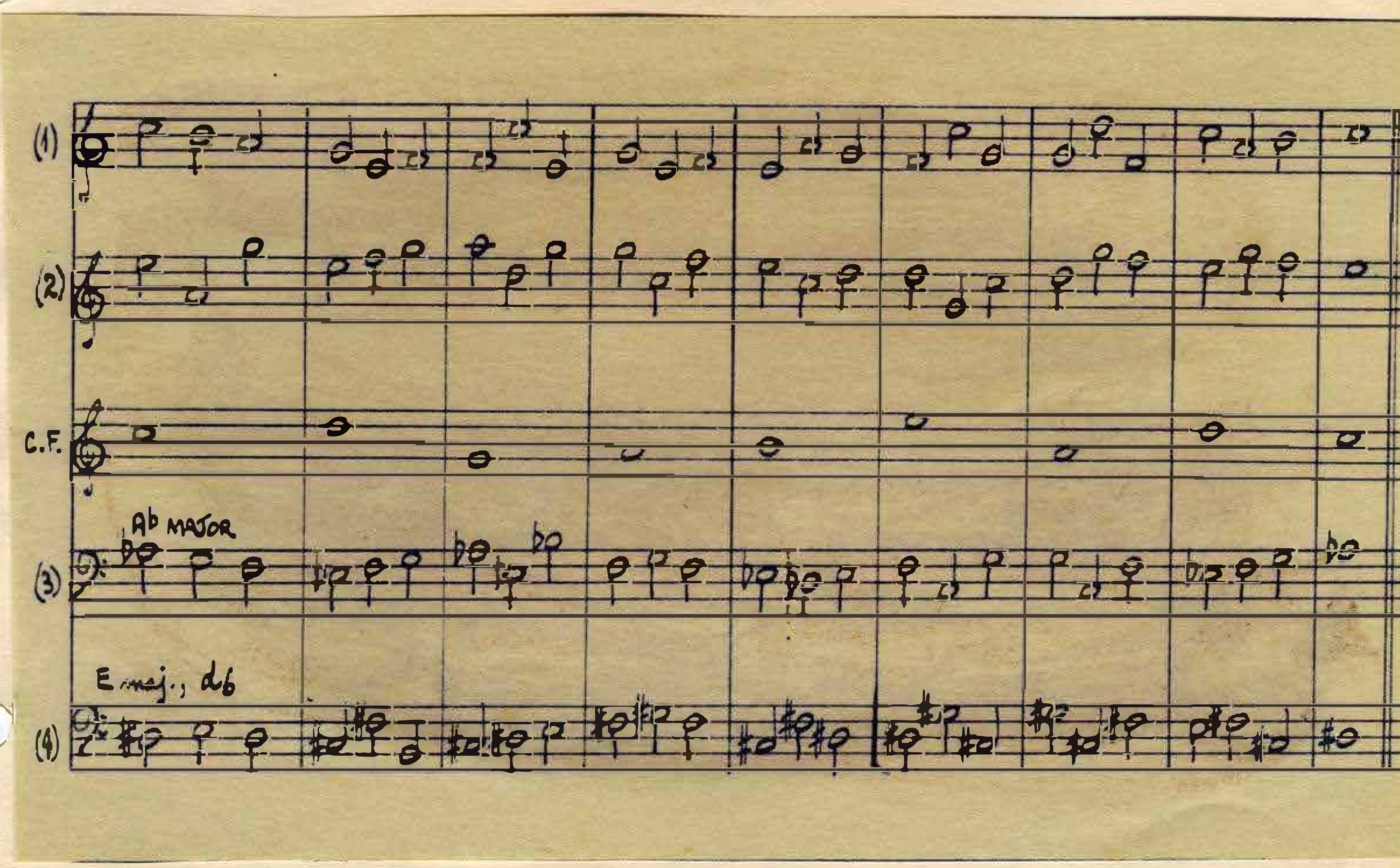
(2) it produces ipso facto a more expressive melodic form.

Examples of Delayed Resolutions

Figure III.

Examples of $\frac{CP}{CF} = 3\,a$

Figure IV.



$$\frac{CP}{CF} = 4a$$

Four attacks of C.P. against one attack of C.F. offer the following combinations of harmonic intervals:

c - c - c - c
c - c - c - d resolution
c - c - d - c
c - d - c - c
d - c - c - c
c - c - d - d*
c - d - d*- c
d - d** c - c
d - c - c - d resolution
c - d - c - d resolution
d - c - d - c

There are wider possibilities in the field of delayed resolution for $\frac{CP}{CF} = 4$.

Parallel axes, centrifugal and centripetal forms become more prominent.

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*In scalewise contrary motion only.

**Either as * or two independent dissonances, both of which are resolved by the following c - c in any order.

Examples of Delayed Resolutions

Figure V.

It is also useful to know all the advantageous starting points for the scalewise passages ending with a consonance.

Examples of Passages Ending with a Consonance.

Figure VI.

Examples of $\frac{CP}{CF} = 4$ a

Figure VII.

Lesson CLIII.

$$\frac{CP}{CF} = 5\ a$$

It is no longer necessary to tabulate all the possible combinations of c and d.

Best melodic quality of CP results from an extensive use of delayed resolutions. The latter being combined with the variety of intervals and with the scalewise passages produce most versatile forms of melody.

The devices for delayed resolution, impossible for less attacks than five, are as follows:

$d_1\ d_2\ c\ \ d_1\ c$, i.e.: the first dissonance is followed by the second dissonance with its resolution, then by the repetition of the first dissonance with its resolution;

$d_1\ d_2\ c\ \ d_2\ c$, i.e.: the first dissonance is followed by the second dissonance without resolution, followed by the resolution of the first dissonance, then by the repetition of the second dissonance followed by its resolution.

Examples of Delayed Resolutions.

Figure VIII.

(please see next page)

Scalewise Passages Ending with a Consonance

Figure IX.

Examples of $\frac{CP}{CF} = 5\ a$

Figure X.

$$\frac{CF}{CP} = 6a$$

The new devices for delayed resolutions possible with six attacks:

$d_1\ d_2\ d_1\ c\quad d_2\ c$, i.e.: the first dissonance, the second dissonance, the repetition of the first dissonance with its resolution, the repetition of the second dissonance with its resolution;

$d_1\ d_2\ c\quad d_2\ c\quad c$, i.e.: the first dissonance, the second dissonance, the resolution of the first dissonance, the repetition of the second dissonance, the delay, the resolution of the second dissonance;

$d_1\ d_2\ c\quad d_1\ c\quad c$, i.e.: the first dissonance, the second dissonance with its resolution, the repetition of the first dissonance, a delay, resolution of the first dissonance;

$d_1\ c\quad c\quad d_2\ c\quad c$, i.e. a combination of two groups by three, each consisting of a dissonance, a delay and a resolution.

Other combinations can be devised in a similar way. For example: $d_1\ c\quad d_2\ c\quad d_2\ c$, which is a combination of 2 + 4.

While using six attacks against CF, it is easy to devise a great variety of melodic forms and interference pattern (see: Melodization of Harmony).

Examples of Delayed Resolutions.

Figure XI.

Examples of Scalewise Passages Ending with

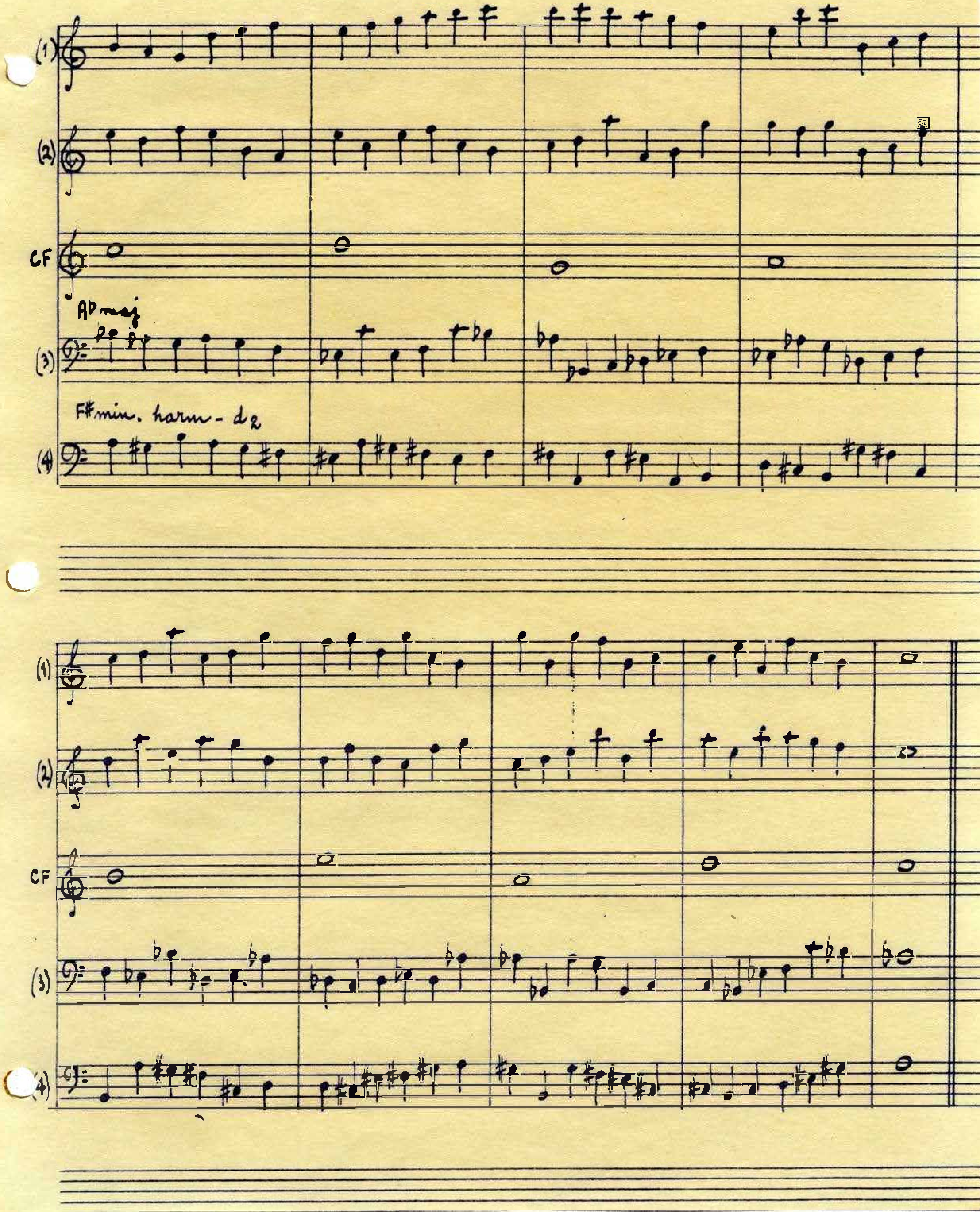
a Consonance.

Figure XII.

Examples of $\frac{CP}{CF}$ = 6 a

Figure XIII.

(please see next page)


(1)
(2)
CF
A♭ maj
(3)
F# min. harm - d2
(4)
(1)
(2)
CF
(3)
(4)

$$\frac{CP}{CF} = 7\ a$$

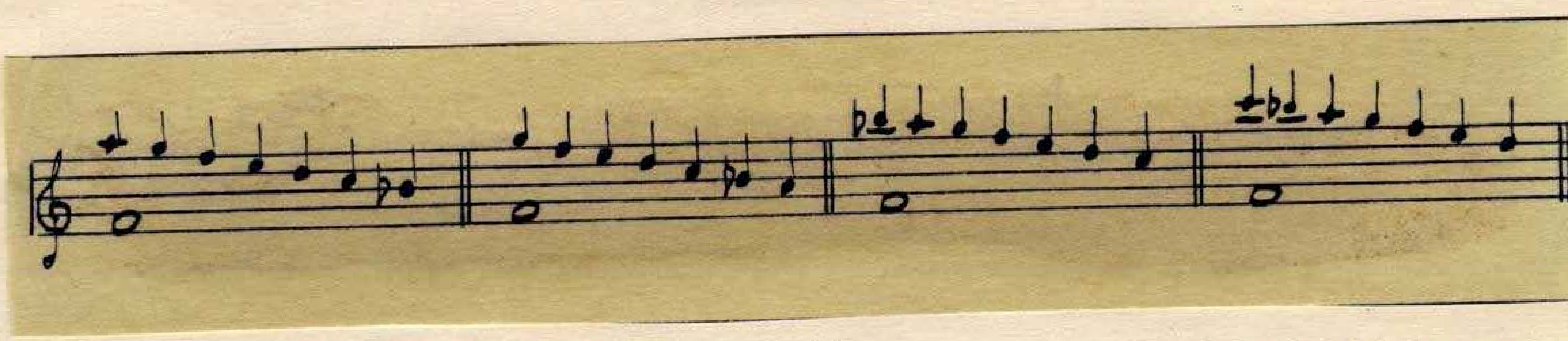
Seven attacks of CP against one of CF offer new forms of delayed resolutions. The number of new combinations grows, and it becomes quite easy to develop various melodic forms, built on parallel, converging and diverging axes.

Examples of Delayed Resolutions.

Figure XIV.

Examples of Scalewise Passages Ending with a Consonance.

Figure XV.



Examples of $\frac{CP}{CF} = 7a$

Figure XVI.

$\frac{CP}{CF} = 8a$

Eight attacks of CP against one of CF offer a great variety of melodic forms. The latter can be obtained through the technique of delayed resolutions. It is equally fruitful to devise melodic forms by means of attack-groups. For example, thinking of 8 as

$\frac{8}{8}$ series represented through its binomials and trinomials. Interference groups can be carried out in counterpoint in the same way as in the Melodization of Harmony, where such groups were used against the attacks of H.

Examples of Delayed Resolutions.

Figure XVII.

All scalewise passages ending with a consonance must start and end with the same pitch unit, as such is the property of our seven-name musical system.

Examples of Scalewise Passages Ending with a Consonance.

Figure XVIII.

Examples of $\frac{CP}{CF} = 8a$

Figure XIX.

$\frac{CP}{CF} = 8a$ gives sufficient technical equipment for any greater quantity of attacks. It is desirable to devise such cases as $\frac{CP}{CF} = 12a$ and $\frac{CP}{CF} = 16a$, as they provide very usable material for the animated forms of passage-like obligato. Under usual (traditional) treatment, such groups with many attacks of CP against CF remain uniform or nearly

uniform in durations.

The most important conditions for obtaining an expressive counterpoint:

(1) abundance of dissonances;

(2) delayed resolutions;

(3) interference attack-groups.

J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross   Subject: Music

Lesson CLIV.

Composition of the Attack-Groups in Two-Part Counterpoint.

In all the previous forms of counterpoint the attack-group of CP against each attack of CF was constant: $\frac{CP}{CF} = A$ const.

The monomial attack group consisted of any desirable number of attacks: $A = a, 2a, 3a, \ldots ma$.

Now we arrive at binomial attack-groups for CP. This can be expressed as $\frac{CP}{CF} = A_1 + A_2$, i.e., counterpoint written to two successive attacks of the cantus firmus consists of two different attack-groups.

For instance:

(1) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{2a}{a} + \frac{a}{a}$;   (2) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{3a}{a} + \frac{2a}{a}$;

(3) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{5a}{a} + \frac{3a}{a}$;   (4) $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{a}{a} \quad \frac{8a}{a}$; ...

The selection of number values for the attacks of CP against the attacks of CF depends on the amount of contrast desired in the two successive attack-groups of CP.

All further details pertaining tothis matter

are in the respective chapter of the Theory of Melodization.

Binomial attack-groups are subject to permutations.

For example: $\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} = \frac{4a}{a} + \frac{2a}{a}$. This binomial attack group can be varied further through the permutations of the higher order. Suppose CF has 8a. Then the whole contrapuntal continuity will acquire the following distribution of the attack-groups:

$$\frac{CP_1}{CF_1} + \frac{CP_2}{CF_2} + \frac{CP_3}{CF_3} + \frac{CP_4}{CF_4} + \frac{CP_5}{CF_5} + \frac{CP_6}{CF_6} + \frac{CP_7}{CF_7} + \frac{CP_8}{CF_8}, \text{ or}$$

$$\frac{CP_{1-8}}{CF_{1-8}} = \frac{4a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{4a}{a} + \frac{2a}{a}.$$

Polynomial attack-groups of CP against CF can be devised in a similar fashion.

The resultants of interference, their variations, involution groups and series of variable velocities can be used as material for this purpose.

Examples of polynomial attack-groups of $\frac{CP}{CF}$:

(1) $\frac{CP_{1-6}}{CF_{1-6}} = \frac{3a}{a} + \frac{a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{a}{a} + \frac{3a}{a}$;

(2) $\frac{CP_{1-9}}{CF_{1-9}} = \frac{2a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{2a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{a}{a} + \frac{2a}{a}$;

(3) $\frac{CP_{1-5}}{CF_{1-5}} = \frac{a}{a} + \frac{2a}{a} + \frac{3a}{a} + \frac{5a}{a} + \frac{8a}{a}$;

(4) $\frac{CP_{1-4}}{CF_{1-4}} = \frac{9a}{a} + \frac{6a}{a} + \frac{6a}{a} + \frac{4a}{a}$.

Simplest duration-equivalents of attacks will be used in the following examples.

Figure XX.

Examples of counterpoint with $\frac{CP}{CF}$ = A var.

$$\frac{CP}{CF} = \frac{a}{a} + \frac{6a}{a} + \frac{3a}{a}.$$

$$\frac{CP}{CF} = \frac{a}{a} + \frac{5a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{3a}{a} + \frac{3a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{5a}{a} + \frac{a}{a}.$$

$$\frac{CP}{CF} = (1+2)^3 = \frac{a}{a} + \frac{2a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{2a}{a} + \frac{4a}{a} + \frac{4a}{a} + \frac{8a}{a}.$$

E, d_4.

At this stage it should not be difficult to develop the technique of writing one attack of CP to a group of attacks of CF. In an exercise CF must be so constructed as to permit the matching of one attack against a given attack-group. In a given melody, when composing a counterpart, it is necessary to compose the attack-groups first. This should be accomplished with a view upon the possibilities of the treatment of harmonic intervals. Whenever the assumed group does not permit to carry out the resolution requirements (such as expanding of the second, contracting of the seventh or the ninth, etc.), the attack-group itself must be reconstructed.

As it was mentioned before, it is quite practical to re-write the given melody into uniform durations first, and then to assign the advantageous attack-groups. After the counterpoint is written, the original scheme of durations can be reconstructed.

With the present equipment, only such melodies can be used as cantus firmus which are built on one scale at a time, and the scale itself must belong to the First Group (see Theory of Pitch Scales).

The procedure itself of distributing the attack-groups of a given melody is analogous to that used in the branch of Harmonization of Melody, where

attacks of a given melody were distributed in relation to the quantity of chords accompanying them.

The following is a melody subjected to different attack treatments for the purpose of writing a counterpart to it.

Figure XXI.

(please see next page)

Fig. XXI.

$$\frac{CP}{CF} = \frac{a}{3a} + \frac{a}{a} + \frac{a}{2a} + \frac{a}{2a} + \frac{a}{a}$$

$$\frac{CP}{CF} = \frac{a}{6a} + \frac{a}{3a} + \frac{a}{2a} + \frac{a}{4a} + \frac{a}{3a}$$

Lesson CLV.

In writing a counterpart to a given melody (but without any considerations of the given harmonic accompaniment) it is important to consider:

(1) the composition of attacks, and

(2) the composition of durations.

Composition of attacks depends upon the degree of animation of the given melody. If a lively melody is to be compensated, the countermelody should be devised on the basis of reciprocation of attacks and, finally, durations. All the techniques pertaining to variations of two elements serve as material for the two part compensation (counterbalancing).

If a lively melody is to be contrasted, the countermelody should be devised by summing up groups of attacks together with their durations. The sums of durations of the given melody, with the specified number of attacks against each attack of the countermelody, define the durations of the counterpart.

If a slow melody is to be compensated (counterbalanced) by a slow counterpart, the technique of reciprocation of attacks and durations should take place. Variations of two elements provide such a technique.

If a slow melody is to be contrasted, the countermelody should be devised first by defining the

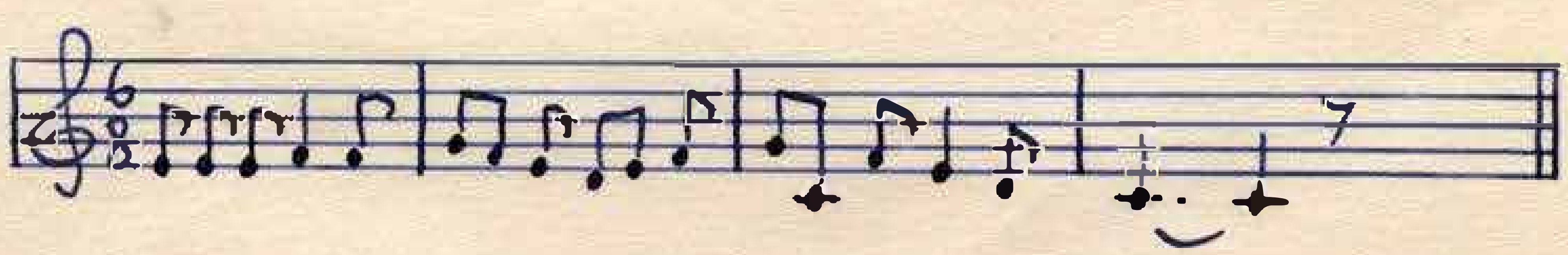
number of attacks in the countermelody against each individual attack of the given melody, after which the sum of the attacks of the counterpart will represent the duration, equivalent to the duration of one attack of the given melody.

Melodies where animated portions alternate with the slow ones, or with cadences, are particularly suited for the compensation method. In such a case when one melody stops, the other moves and vice versa.

We shall analyze now the problem of writing the counterpart to a given melody.

Let us take Ben Jonson's "Drink to Me Only With Thine Eyes".

The melody reads as follows:



Reconstruction of this melody into a CF gives it the following appearance:

This is a fairly animated type of melody.

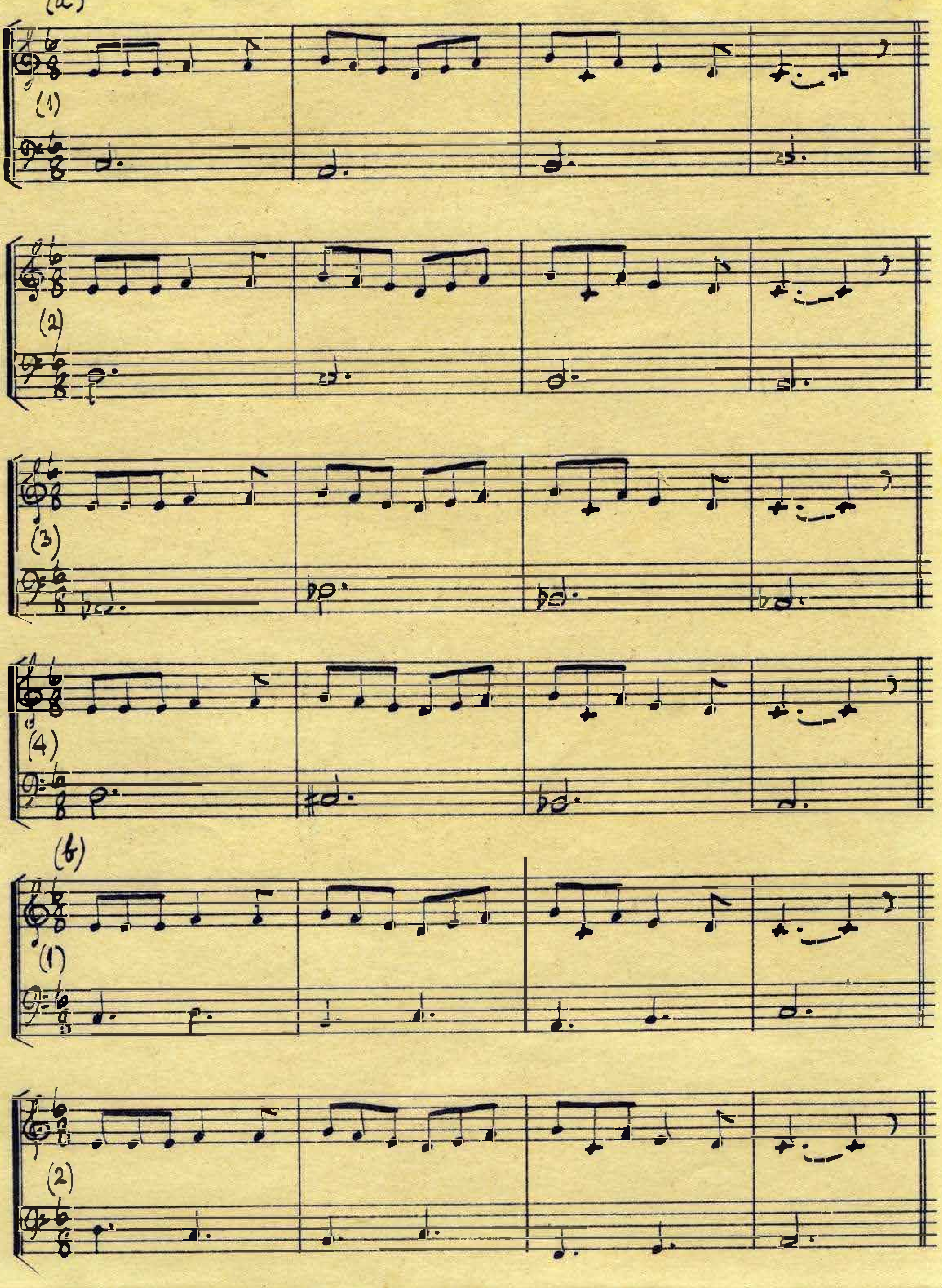
Let us devise the scheme of durations for CP. One of the simplest solutions for a contrasting CP would be to make each attack of CP correspond to T. Thus we would obtain CP = 4a and a = 6t. For a less moderate contrast we could assign CP = 8a and a = 3t. To obtain CP of the counterbalancing type would require the assignment of two contrasting elements, if such can be found in CF. As $T_1 = 2a$ and $T_2 = 6a$, and as $T_3 = 5a$ and $T_4 = a$, this CF provides sufficient material for assigning two elements and for compensating them in CP. There is of course no way to counterbalance the original version of this melody.

Thus, we have obtained the following three solutions, each different but equally acceptable.

Figure XXII.

(please see next page)

Fig. XXII.




(3)
(4)
(c)
(1)
(2)
(3)
(4)

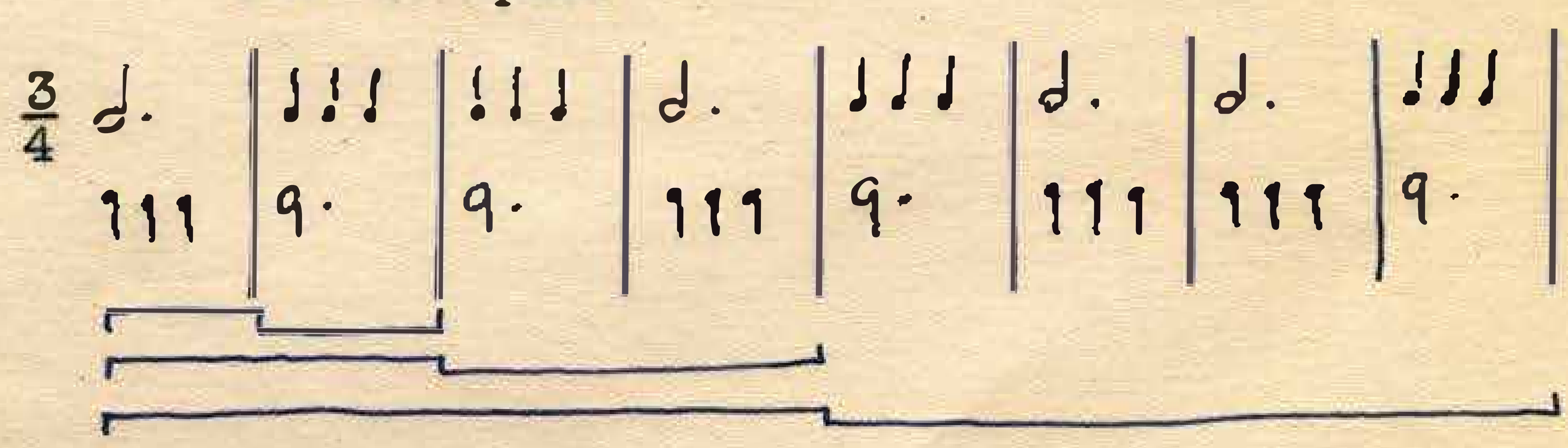
Lesson CLVI.

Direct Composition of Durations in Two-Part Counterpoint.

In composing an original two-part counterpoint it is often desirable to compose the two counterparts rhythmically first. The entire technique concerning binomials and their variations (see Theory of Rhythm) is applicable in this case.

Counterbalancing (compensation) is achieved through the permutation of binomials, and this may follow through the higher orders.

For example:

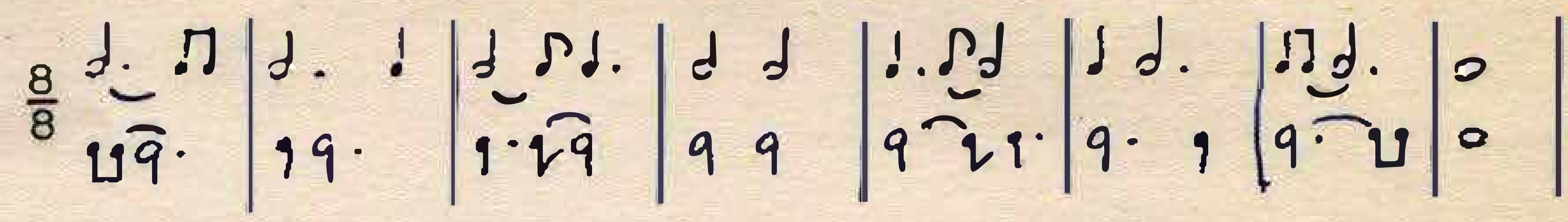


Which part is written first (thus becoming CF) is not essential in such a case. It is essential, however, to write one part completely, and not section by section. CP must be written after CF is completed.

For more diversified rhythmic continuity,

resultants with an even number of terms can be used. The binomials constantly reciprocate in such a case.

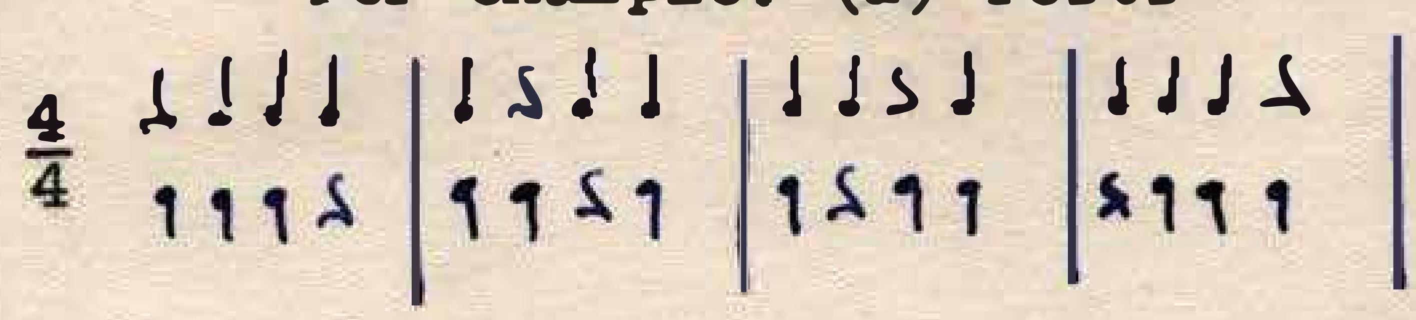
For example: $T = r_{8\div7}(+8t)$.



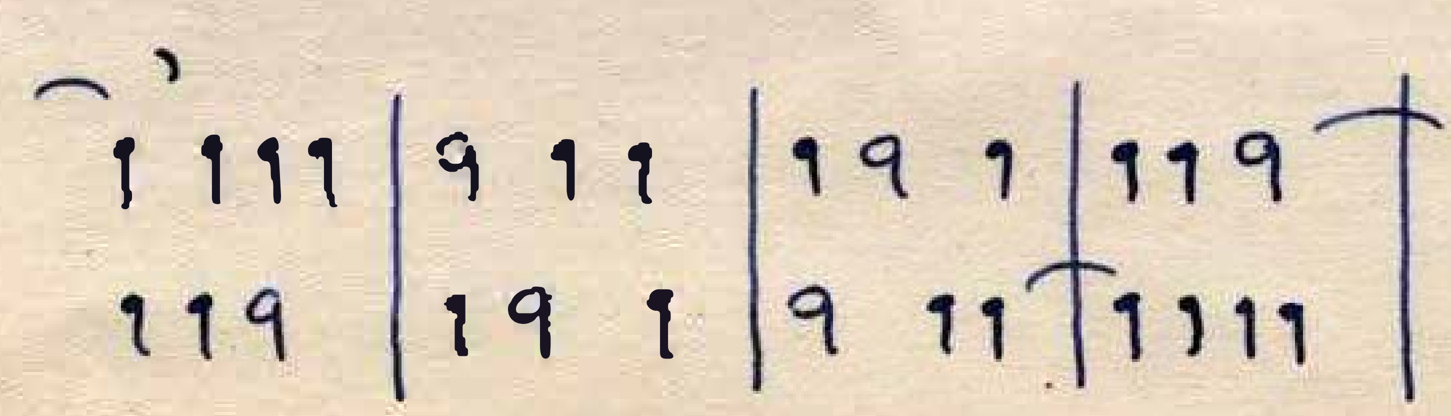
In all such cases (continuous reciprocation of the variable binomials), the number of attacks of CP against CF remains constant, while the durations vary.

Still more homogeneous effects of rhythm in both counterparts may be achieved through the use of variations of rests or split-unit groups. The groups themselves do not have to be binomials; the two best of any polynomial groups take place.

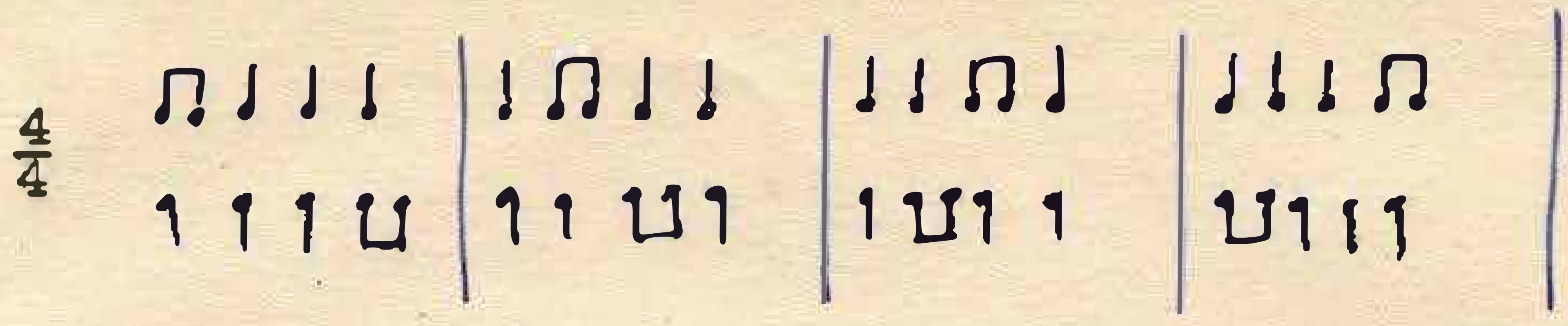
For example: (a) rests



or: (b) tied rests

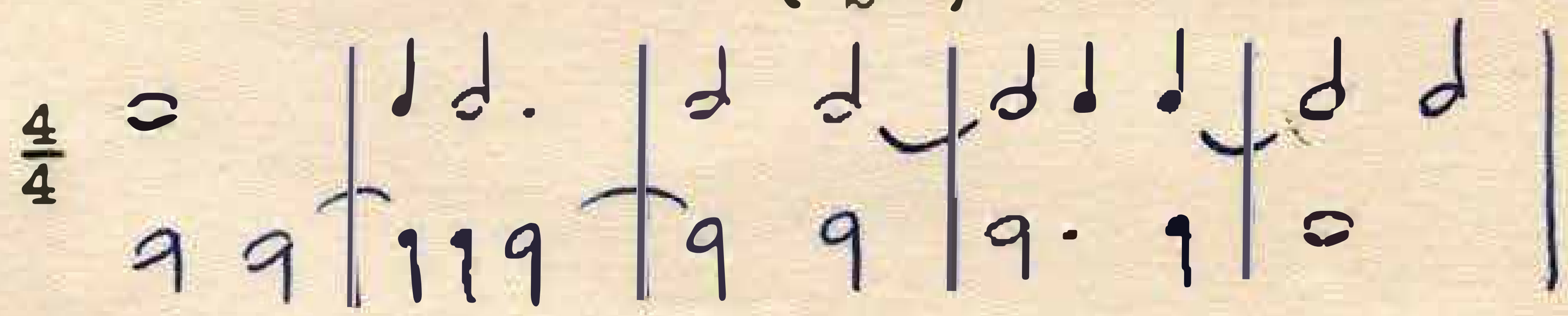


(c) split-unit groups



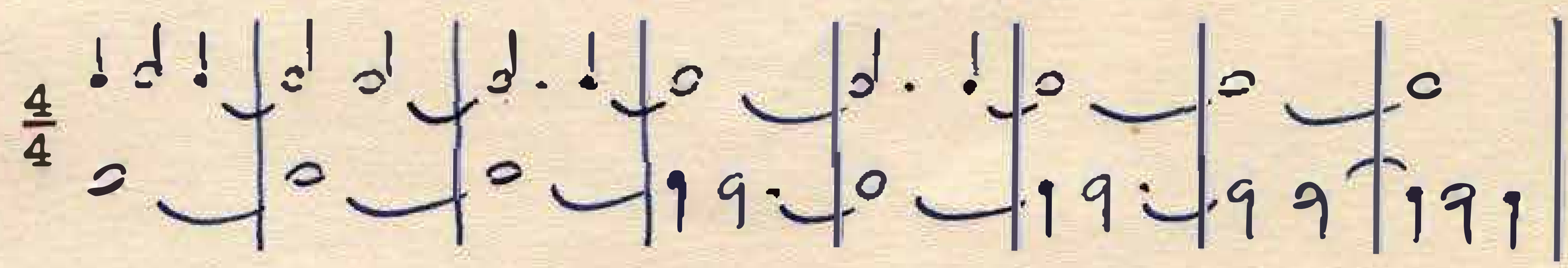
Any rhythmic group set against its converse provides satisfactory counterparts.

For example: $2\left(\frac{r_{5\div4}}{2}\right)$; T = 4t.



Any of the series of variable velocities can be used for such a purpose.

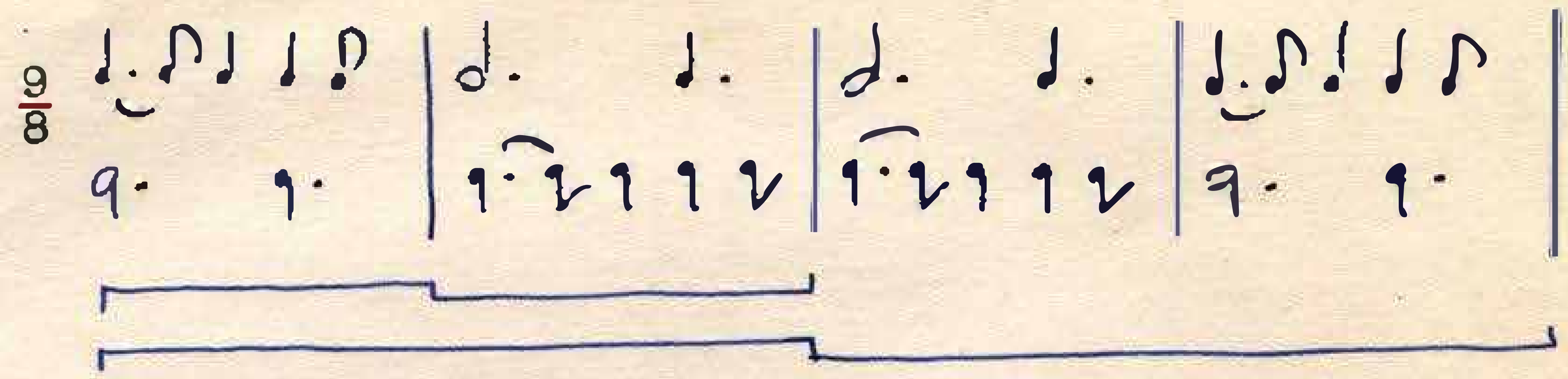
For example: summation Series I:



Adjacent contrasts for two mutually compensating parts can be achieved by any synchronized involution-groups placed in sequence. The two powers supply the a and b elements, and thus are treated through the permutations of two elements (any order).

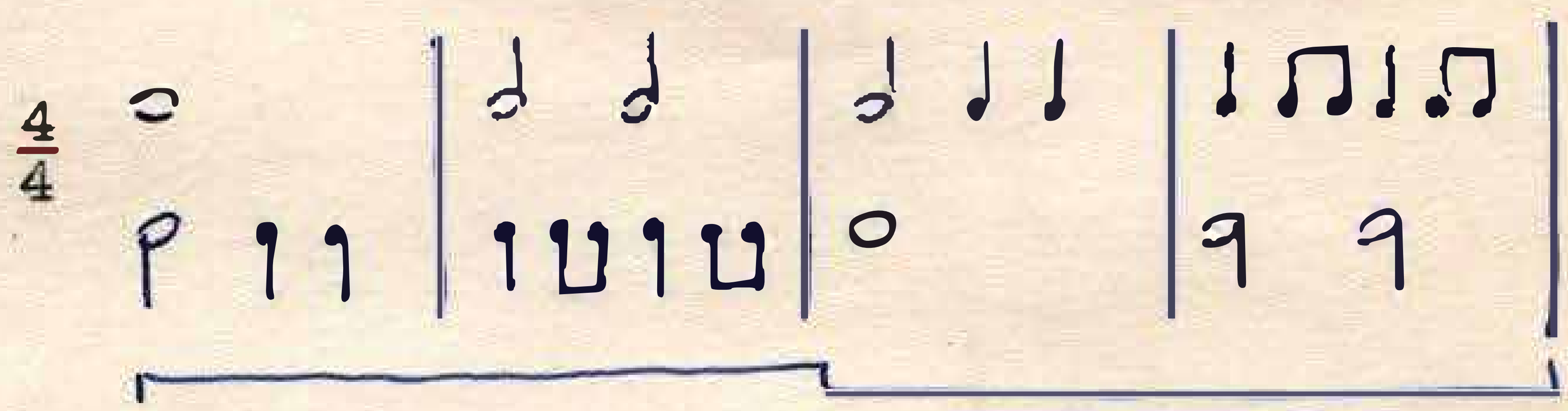
For example: $(2+1)^2 + 3(2+1)$.

$a = (2+1)^2$; $b = 3(2+1)$



For example: $4(2+1+1) + (2+1+1)^2$.

$a = 4(2+1+1)$; $b = (2+1+1)^2$.



All the above described devices permit to start with the composition of either part as CF, and they all refer to counterbalancing (compensation).

The technique of simultaneous harmonic contrasts between CF and CP is based on the distributive involution for the two synchronized parts used simultaneously. Any number of terms can be used as a group. The limitation of <u>two parts</u> corresponds to the <u>two power-groups</u> (adjacent or non-adjacent

powers). In all such cases the number of attacks of CP against CF is constant, and such a number equals the quantity of terms in the polynomial.

Thus a binomial squared gives $\frac{CP}{CF} = 2a$, a trinomial squared gives $\frac{CP}{CF} = 3a$, etc.

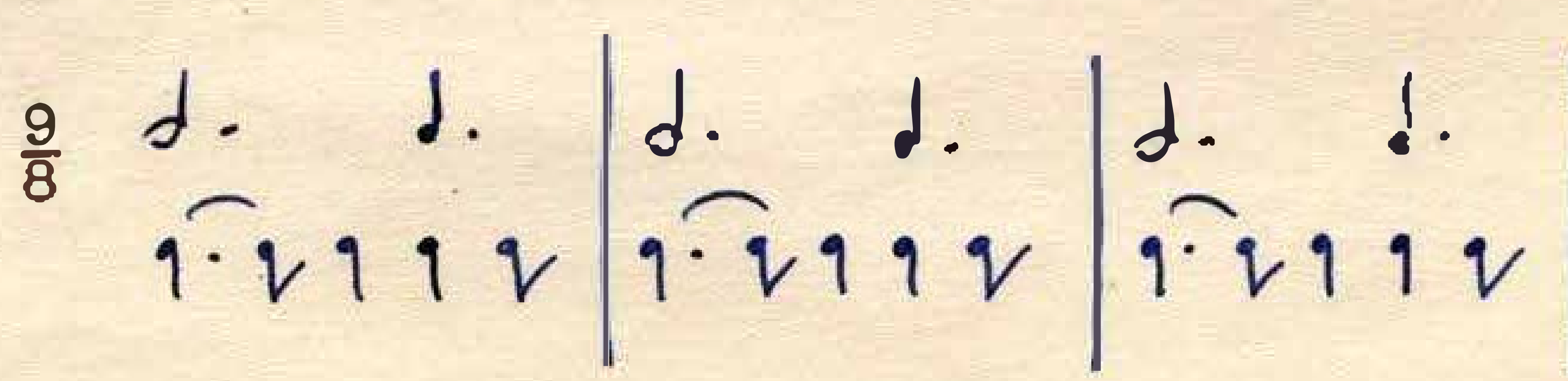
Still greater contrasts can be achieved either by using larger polynomials, or by synchronizing non-adjacent powers. In the latter case a binomial cubed and used against its synchronized first power gives $\frac{CP}{CF} = 4a$, i.e., 2^2 ; a trinomial cubed and used against its synchronized first power gives $\frac{CP}{CF} = 9a$, i.e., 3^3, etc.

Nothing prevents the composer from using adjacent higher powers, like cubes against squares, fourth power groups against cubes, etc.

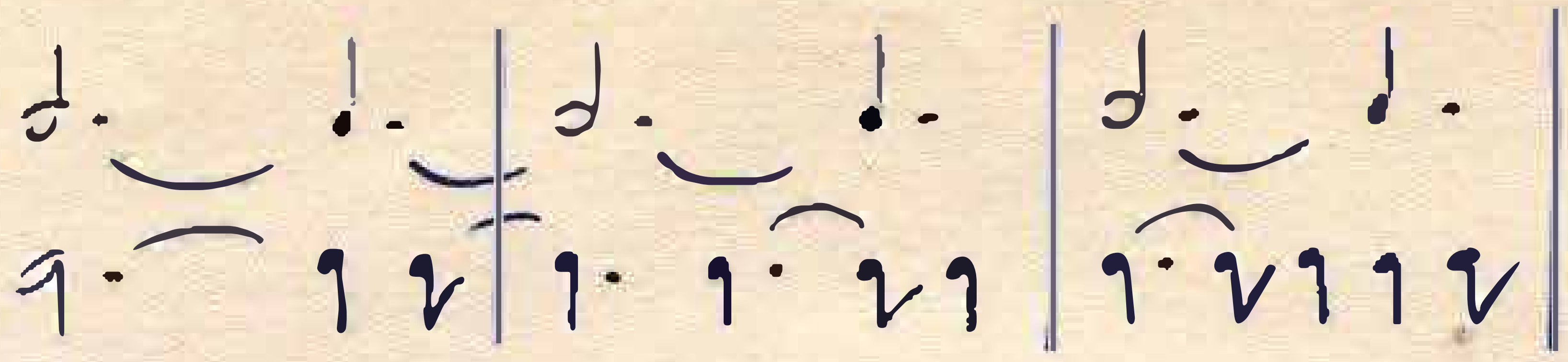
In all these cases the lower power employed represents CF, as it is easier to match several attacks against a given one attack, than vice versa.

Examples:

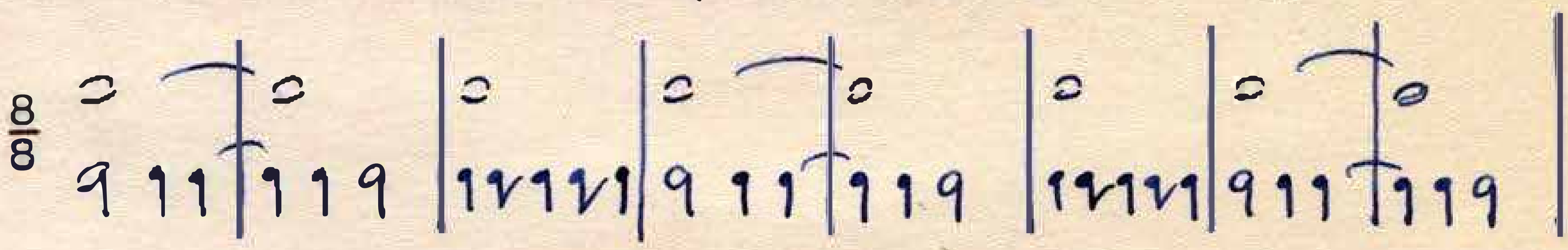
(a) $CF = 3(2+1)$; $CP = (2+1)^2$.



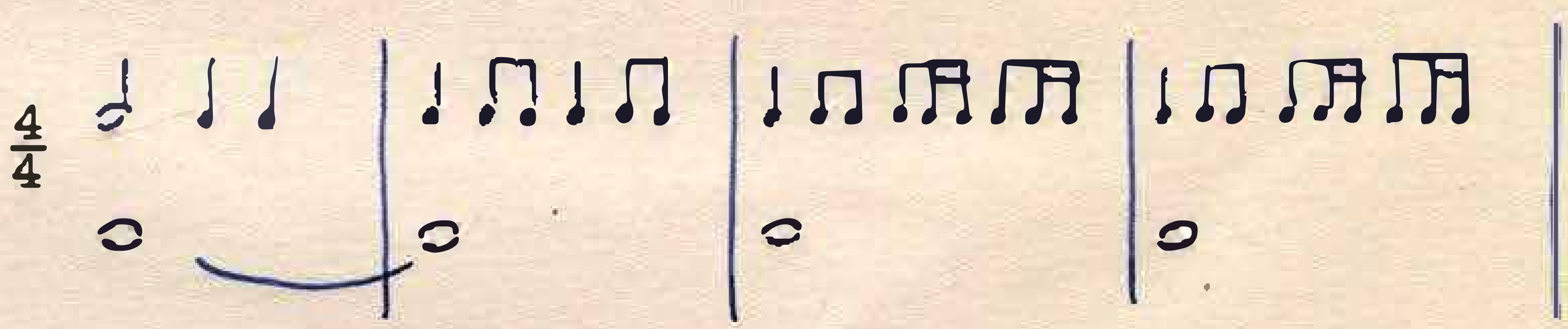
(b) CF = 9(2+1) ; CP = $(2+1)^3$.



(c) CF = 8(2+1+2+1+2) ; CP = $(2+1+2+1+2)^2$.



(d) CF = 16(2+1+1) ; CP = $(2+1+1)^3$.



In addition to involution-groups, <u>coefficients of duration</u> can be used, like $\frac{CP}{CF} =$

$$= \frac{2(r_{4\div3})}{r_{8\div6}} = \frac{(3+1+2+2+1+3) + (3+1+2+2+1+3)}{6+2+4+4+2+6}$$, as well

as the resultants of instrumental interference composed for two parts.

Figure XXIII.

Examples of Two-Part Counterpoint with Pre-Composed Duration-Groups.

Fig. XXIII.

In all the following examples the intonation of CF was composed first.

COEFFICIENTS OF DURATION = R $\underline{4 \div 3}$

THE RESULTANTS OF INSTRUMENTAL INTERFERENCE: R $\underline{3 \div 2}$

Lesson CLVII.

Chromatization of the Diatonic Counterpoint

It seems to be easy to write a chromatic counterpart to any diatonic melody, as any suitable pitch-units can be chosen from the entire chromatic scale. Such countermelodies, however, contain one general defect: the neutral character which comes with a uniform scale. To an average listener it sounds as if any pitch-unit would be equally as acceptable in place of the ones already set. This peculiarity of musical perception is due to the inherited and cultivated diatonic orientation.

An average listener hears chromatic units as an ornamental supplement to a diatonic scale. Such chromatic units are commonly used as auxiliary tones moving into the diatonic units of a given scale, thus forming directional units. Diatonic units are perceived as independent pitches (though in a certain grouping in sequence). Chromatic units are perceived as dependent pitches leading into diatonic pitches. Music constructed entirely chromatically, i.e., without diatonic dependence usually belongs to a different category than the diatonic music with directional units. It is known under the name of "atonal", or the "twelve-tone" music.

For this reason, we shall use chromatic counterpoint with diatonic dependence only. Such a counterpoint can be devised at its best by means of inserting the passing or the auxiliary chromatic units post factum.

This technique is applicable to all four types of harmonic relations. It is important that the conversion of a diatonic counterpoint into chromatic does not affect the established forms of resolutions.

The remodeling of durations can be accomplished by means of split-unit groups. This device allows to preserve the character of rhythm which was originally set.

Figure XXIV.

Examples of Chromatic Variations of the Diatonic Counterpoint.

(please see next page)

ORIGINAL

CHROMATIC VARIATION

ORIGINAL

CHROMATIC VARIATION

ORIGINAL

CHROMATIC VARIATION (BOTH PARTS ARE CHROMATIZED)

J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross   Subject: Music

Lesson CLVIII.

Composition of Contrapuntal Continuity.

Extension of any given contrapuntal continuity is based on geometrical mutations.

The fundamental technique of geometrical mutations for the two-part counterpoint is the interchange of music assigned to CF and CP. Assuming that CF represents the actual part and CP -- the actual counterpart, we obtain the two variants for each voice: $\frac{CP}{CF} + \frac{CF}{CP}$, where both CF and both CP are identical, but appear in a different octave.

In the old systems of counterpoint it was known as "vertical convertibility in octave". We shall look upon it merely as two variants of exposition for any counterpoint and consider such a convertibility to be an inherent property of counterpoint as such.

By applying the principle of variation of two elements ad infinitum, i.e., through permutations of the higher orders, we can compose an entire piece of music from one contrapuntal exposition.

Figure XXV.

Example of Contrapuntal Continuity of the Third Order Produced Through the Permutation of Parts of the Original Exposition.

(Fig. XXV)

1. 2.

(Fig. XXV, cont.)

As any musical exposition, when conceived geometrically, becomes subject to quadrant rotation (see: Geometrical Projections of Music), we obtain the four variations of the geometrical positions: (a), (b), (c), (d).

Through the vertical permutation of parts two-part exposition yields two variants. As each variant has four rotational positions, the total number of variants for one two-part contrapuntal exposition is eight:

$$\frac{CF}{CP}(a),\ \frac{CP}{CF}(a),\ \frac{CF}{CP}(b),\ \frac{CP}{CF}(b),\ \frac{CF}{CP}(c),\ \frac{CP}{CF}(c),\ \frac{CF}{CP}(d),\ \frac{CP}{CF}(d).$$

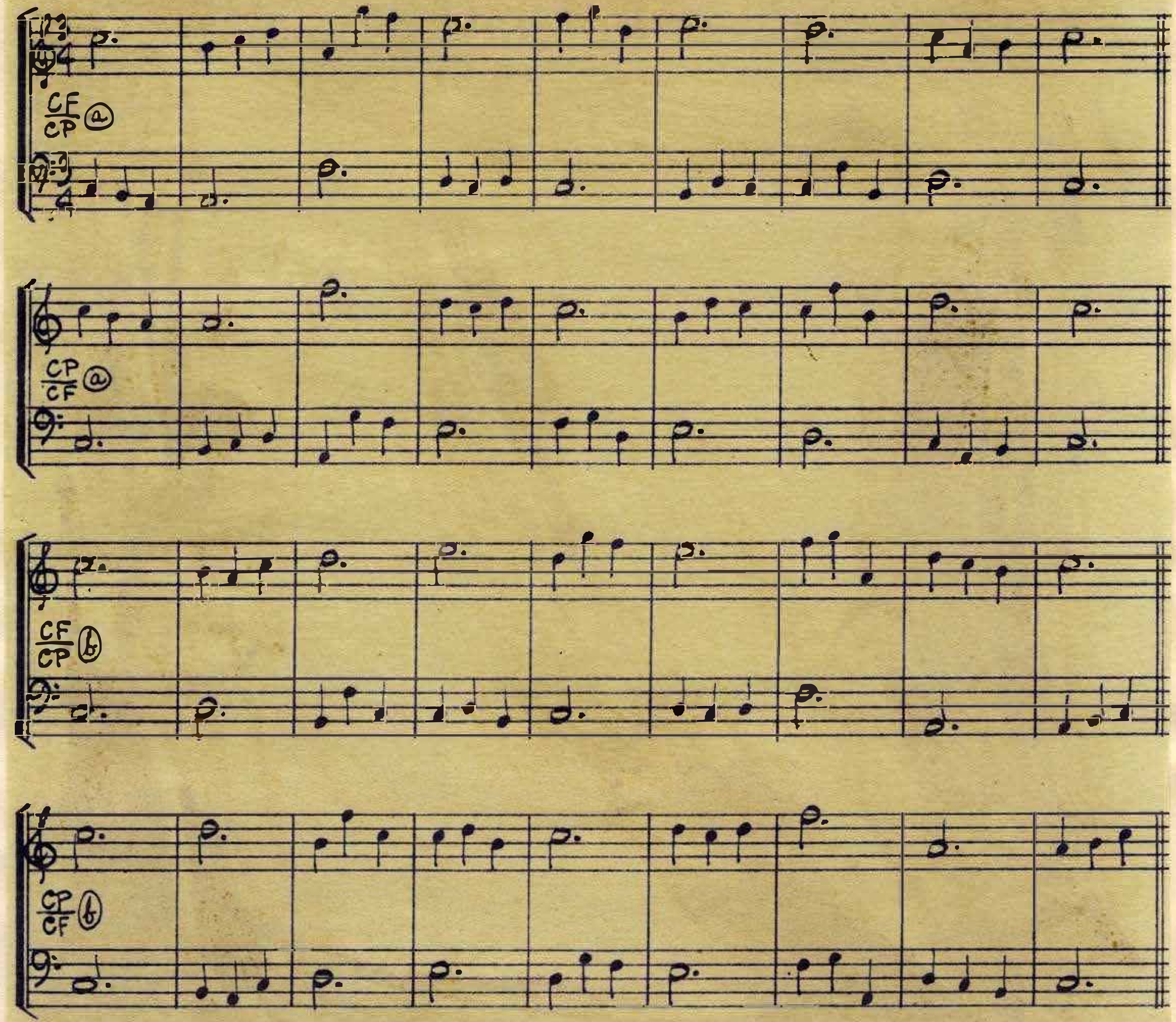
When making a transition from one form into another in the same part, place the respective pitch-unit in its nearest pitch position. This is true of both: the octave and the geometrical inversion. The axis of inversion for (c) and (d) is the axis of CF (or the part assumed to bear its meaning).

Figure XXVI.

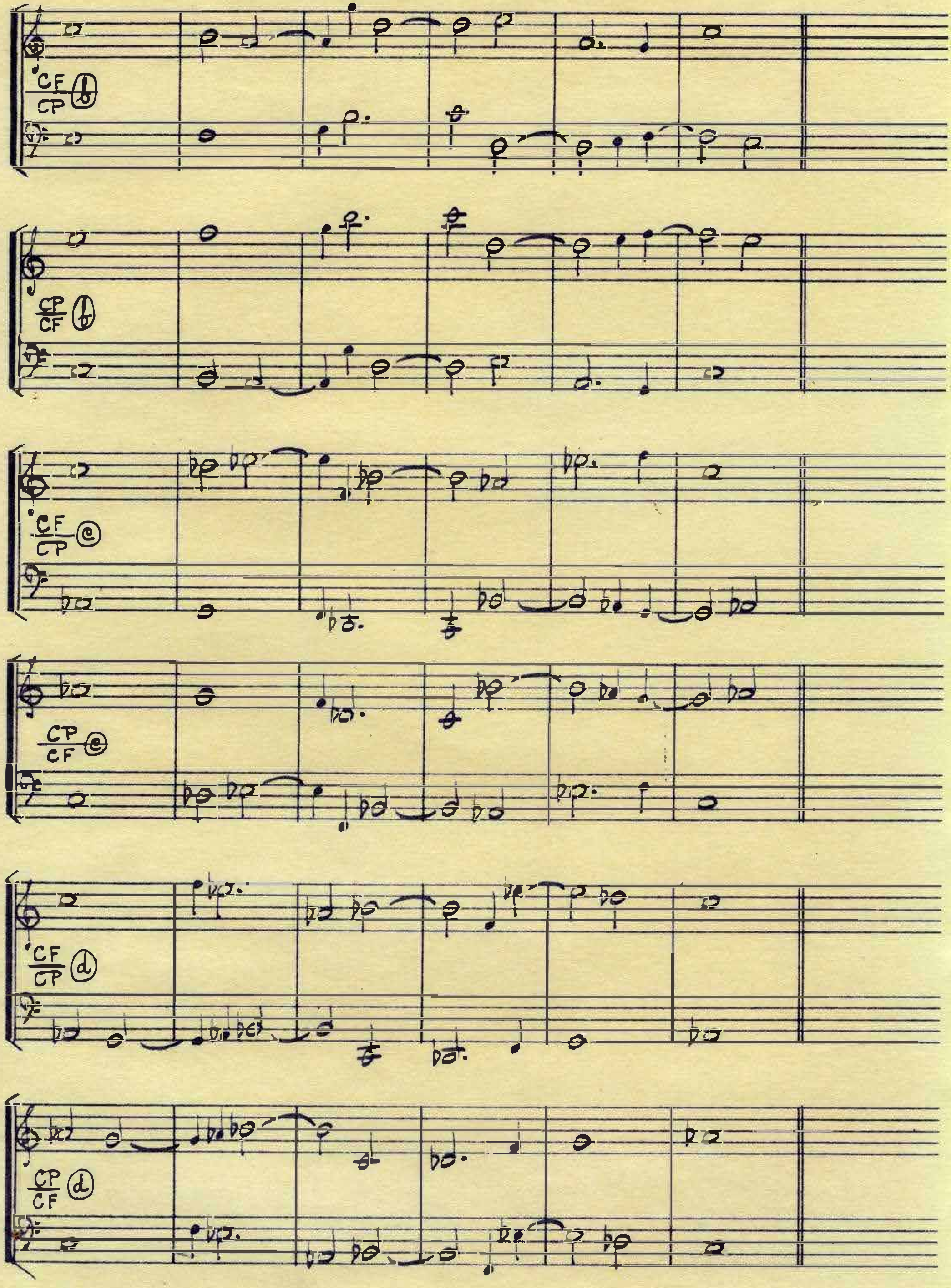
Examples of the Variants of One Exposition.

(Fig. XXVI)

TYPE I.




CF/CP ⓒ
CP/CF ⓒ
CF/CP ⓓ
CP/CF ⓓ
TYPE II
CF/CP ⓐ
CP/CF ⓐ


CF
CP
b
CP
CF
b
CF
CP
c
CP
CF
c
CF
CP
d
CP
CF
d


CF
CP
(a)
CP
CF
(a)
CF
CP
(b)
CP
CF
(b)
CF
CP
(c)
CP
CF
(c)

(Fig. XXVI, cont.)

The eight variants of contrapuntal exposition can be selected in any desirable combination. Any combination of the selected variants produces a complete form of continuity, i.e., a whole composition.

The selection of various geometrical inversions must be guided by a definite tendency with regard to the amount and distribution of contrasts. All the considerations pertaining to this matter were discussed in the Geometrical Projections of Music.

The most important principle to remember is:

(1) a and b are identical in intonation and converse in temporal structure;

(2) c and d are identical in intonation and converse

in temporal structure;

(3) a and d are converse in intonation and identical in temporal structure;

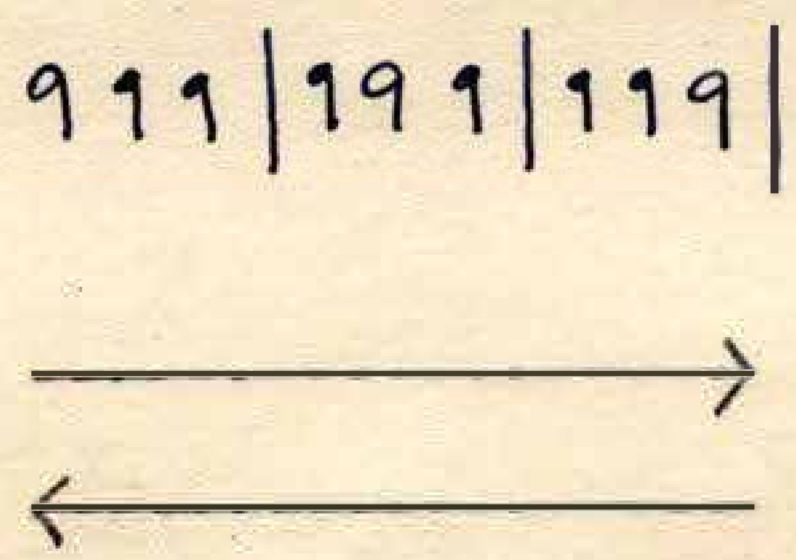
(4) a and c are converse in intonation and converse in temporal structure;

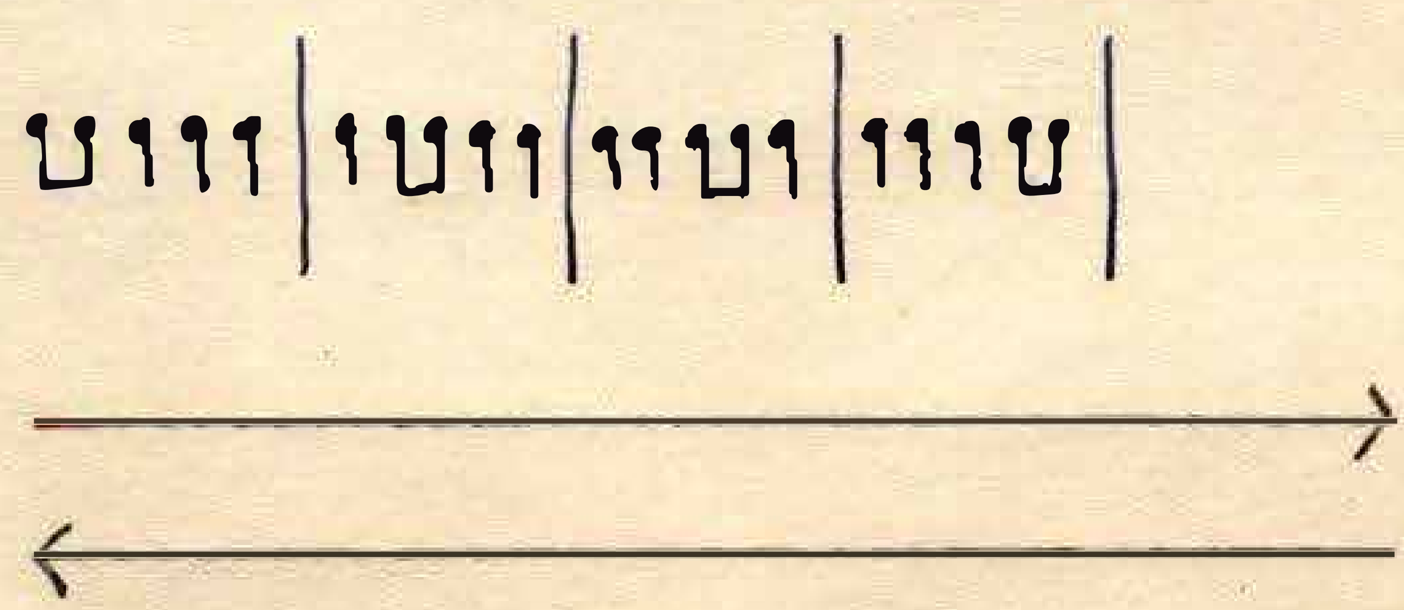
(5) b and c are converse in intonation and identical in temporal structure;

(6) b and d are converse in intonation and converse in temporal structure.

There is a way to obtain identical temporal structures for all geometrical inversions: any symmetrical group is identical with its converse. For instance:

(1) $r_{5 \div 4} = 4 + 1 + 3 + 2 + 2 + 3 + 1 + 4$

(2) 

(3) 

There is also a way to obtain an identical pitch-scale for all geometrical inversions, when desirable. The original scale must be symmetrically constructed (which does not necessarily place it into the Third or the Fourth Group). In such a case the pitch units in (a) and (d) are not identical but the scale structure (that is, the set of intervals) is. For instance:

(a) $c - e^{\flat} - f - g - b^{\flat}$ (3 + 2 + 2 + 3) ↑
(b) $b^{\flat} - g - f - e^{\flat} - c$ (3 + 2 + 2 + 3) ↓
(c) d - f - g - a - c (3 + 2 + 2 + 3) ↑
(d) c - a - g - f - d (3 + 2 + 2 + 3) ↓

Examples of complete forms of contrapuntal continuity based on geometrical inversions:

(1) $\frac{CP}{CF}(a) + \frac{CF}{CP}(a) + \frac{CF}{CP}(d) + \frac{CP}{CF}(b) + \frac{CF}{CP}(a) + \frac{CF}{CP}(c) + \frac{CP}{CF}(b)$;

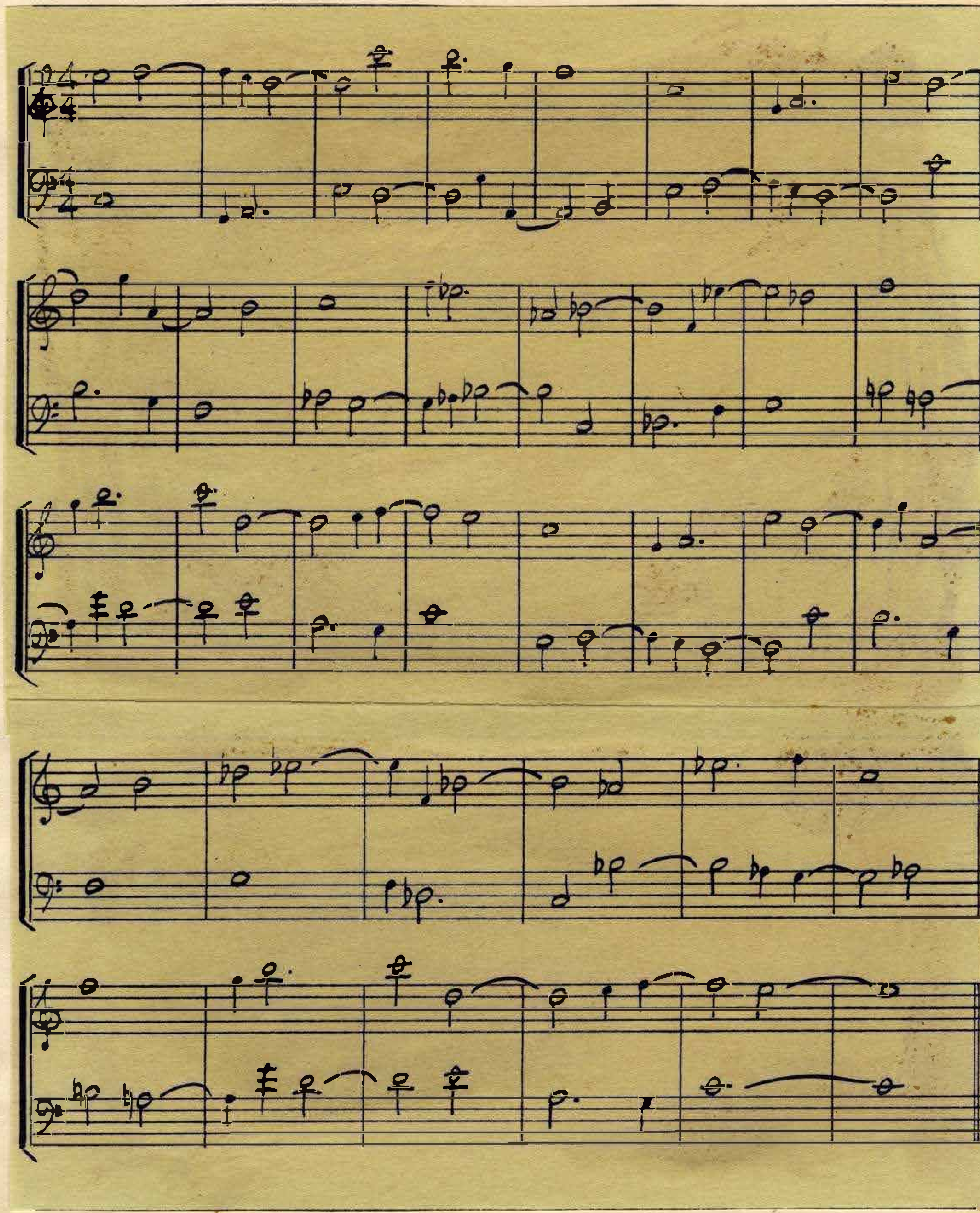
(2) $\frac{CF}{CP}(a) + \frac{CP}{CF}(b) + \frac{CF}{CP}(c) + \frac{CP}{CF}(d) + \frac{CF}{CP}(a)$;

(3) $\frac{CP}{CF}(c) + \frac{CF}{CP}(b) + \frac{CP}{CF}(a) + \frac{CF}{CP}(d) + \frac{CF}{CP}(a)$;

(4) $\frac{CF}{CP}(a) + \frac{CF}{CP}(c) + \frac{CP}{CF}(b) + \frac{CP}{CF}(d) + \frac{CF}{CP}(a)$.

We shall apply the first of the above schemes of continuity to the theme based on the exposition in type II of Fig. XXVI. The theme will be used in its original ST version (i.e., without the added balance).

Figure XXVII.



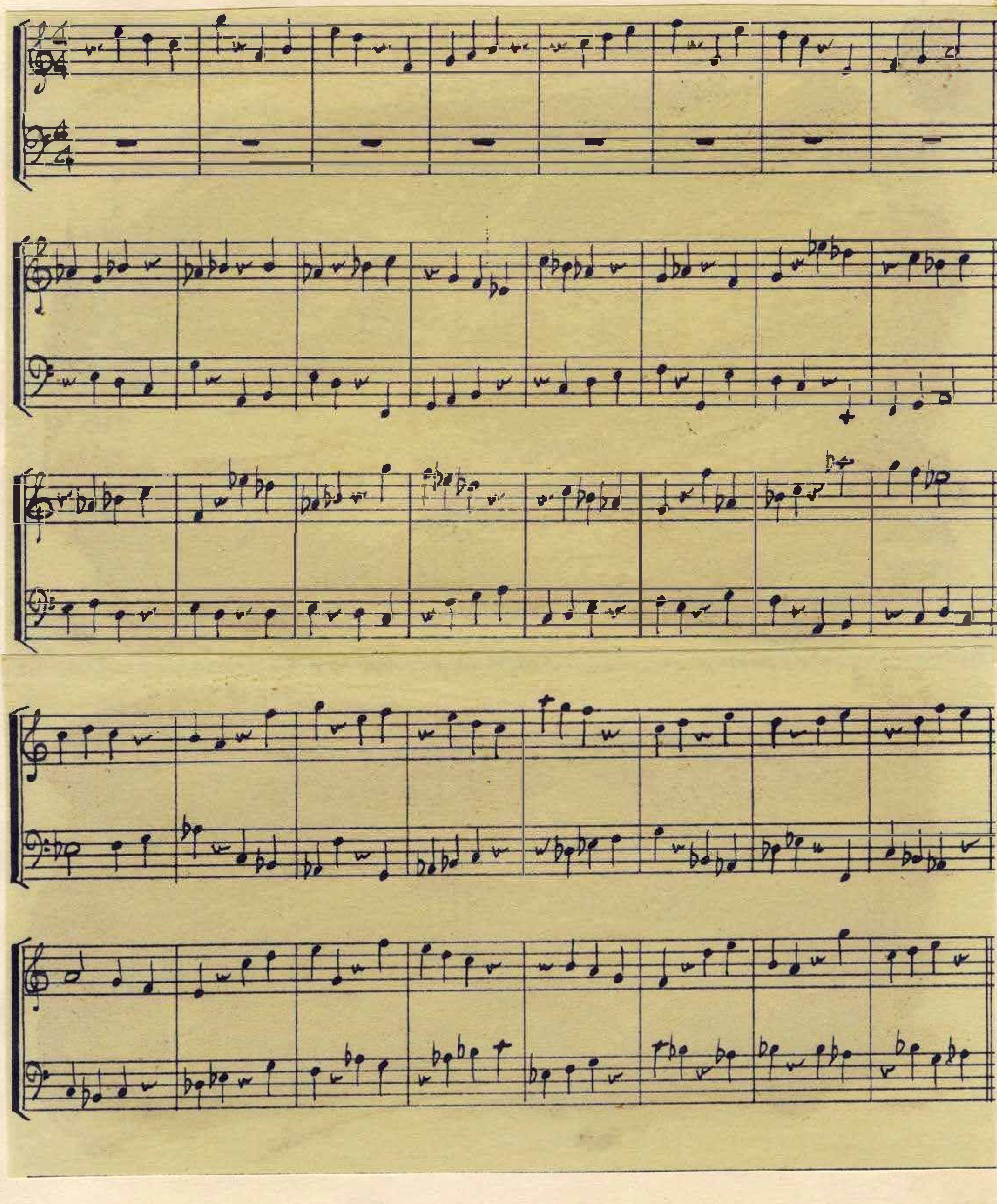
As we have seen before, the interchangeability of CF and CP produces two forms for each geometrical position. This property can be utilized for the purpose of producing continuity based on <u>imitation.</u> The two reciprocal expositions following one another are planned in such a manner, that the first one consists of an unaccompanied CF only, while the second has both parts. When CF exchanges its positions, the resulting effect is imitation.

In the following example, Fig. XXVI type III, will serve as a theme.

The complete continuity will follow this scheme: $CF\,(a) + \frac{CP}{CF}(a) + \frac{CF}{CP}(d) + \frac{CP}{CF}(c) + \frac{CF}{CP}(b)$.

<u>Figure XXVIII.</u>

(please see next page)



Lesson CLIX.

Correlation of Melodic Forms in Two-Part Counterpoint

We have achieved so far the harmonic and the temporal correlation of two melodic parts. Melodic forms have been planned in some general way, and many details were merely the outcome of the harmonic treatment of intervals.

Now we arrive at the point where systematic treatment in correlating melodic forms becomes necessary. As melody is expressed fundamentally by means of an axial combination, the correlation of two melodies becomes essentially the problem of coordination between the two axial groups.

We shall start this analytical survey with monomial axes for both CF and CP.

Under such conditions the following 25 forms become possible.

$$\frac{CF}{CF} = \frac{0}{0};\ \frac{a}{0};\ \frac{0}{a};\ \frac{b}{o};\ \frac{0}{b};\ \frac{c}{0};\ \frac{0}{c};\ \frac{d}{0};\ \frac{0}{d};$$

$$\frac{a}{a};\ \frac{b}{a};\ \frac{a}{b};\ \frac{c}{a};\ \frac{a}{c};\ \frac{d}{a};\ \frac{a}{d};\ \frac{b}{b};\ \frac{c}{b};\ \frac{b}{c};$$

$$\frac{d}{b};\ \frac{b}{d};\ \frac{c}{c};\ \frac{d}{c};\ \frac{c}{d};\ \frac{d}{d}.$$

It is important to note that the various forms of balancing and unbalancing are inherent with the above combinations. The analysis of two parts being parallel or contrary is not sufficient, as, under either conditions, one voice may be balancing and the other may

be unbalancing, or both voices may be balancing as well as unbalancing.

For example: $\frac{CF}{CP} = \frac{b}{b}$; $\frac{d}{b}$; $\frac{b}{c}$; $\frac{a}{d}$.

In the first case both voices are parallel and balancing; in the second case both voices are parallel, but CF is unbalancing and CP is balancing; in the third case both voices are contrary, but both are balancing; in the fourth case both voices are contrary, but both are unbalancing.

It follows from the above considerations, that in order to achieve continuous motion in two-part counterpoint, it is necessary to introduce an unbalancing axis in one of the parts when the other part is moving toward balance, unless a cadence is desired. J.S. Bach had more of parallel motion than it is usually believed to be, but he always managed to avoid cadencing, except where it is obviously intended. On the other hand, many academic theoreticians advocate an abundance of contrary motion as being essentially contrapuntal. This in itself is of little importance, and becomes a source of monotony, unless coupled with the composition of balance relations between CF and CP.

Thus, the selection of axial combinations for the two counterparts (or for one counterpart to a given part) depends upon the form of expression.

Axial relations with regard to their directions are: (1) parallel; (2) contrary; (3) oblique.

Axial relations with regard to their balancing tendencies are:

(1) $\frac{U}{U}$; (2) $\frac{U}{B}$; (3) $\frac{B}{U}$; (4) $\frac{B}{B}$.

In addition to this, the zero-axis expresses a continuous state of balance.

All further development of correlating axial combinations of two melodies follows the ratio development of the quantities of axes in one part in relation to another.

Under such conditions, all the above described cases refer to one category only: $\frac{CP}{CF} = ax$, i.e., one secondary axis of counterpoint corresponds to one secondary axis of cantus firmus; ax is an abbreviation of the word axis.

Now we arrive at the binomial relations of axial groups of the counterpoint in relation to the cantus firmus:

$$\frac{CP}{CF} = \frac{2ax}{ax} \text{ and } \frac{ax}{2ax}$$

Under such conditions, a monomial axis of one part corresponds to a binomial axial combination of another.

For instance:

$$\frac{CP}{CF} = \frac{0+a}{0} ; \frac{a+b}{b} ; \frac{c+d}{a} ; \frac{b+0}{c} ; \frac{d+a}{0} ; \ldots$$

$$\frac{CP}{CF} = \frac{0}{0+a} ; \frac{b}{a+b} ; \frac{a}{c+d} ; \frac{c}{b+0} ; \frac{0}{d+a} ; \ldots$$

It is easy to see that there are 200 such simultaneous combinations, as there are 10 original binomial axial combinations, each having 2 permutations. 20 combinations are now combined vertically with 5 monomials (0, a, b, c, d). This produces $20 \cdot 5 = 100$. Finally, 100 must be multiplied by 2, as each simultaneous combination can be inverted.

The period of duration of one axis equals to the sum of durations of the two axes constituting the binomial. Thus, in a combination:

$$\frac{CP}{CF} = \frac{2ax}{ax} = \frac{axmt + axnt}{axpt} = \frac{T}{T} = 1,$$ the time period

for both parts is the same.

Time ratios for the binomial axes must be selected in accordance with the series which the monomial axis represent.

For instance, the duration of ax of CF is 8T; then, CP can be matched as any binomial of $\frac{8}{8}$ series. Let us select the 5+3 binomial of this series. Now we can define the simultaneous temporal relations as follows:

$$\frac{CP}{CF} = \frac{ax5T + ax3T}{ax8T}$$

In a simultaneous combination of a binomial

versus monomial axial combination it acquires the following significance: during the period of duration of a monomial axis (balanced, balancing or unbalancing) its counterpart has two phases which may be: U+U; U+B; B+U; B+B. If we single out a continuous balance (O-axis) as an independent form, we obtain 12 forms of balance relations between CP and CF, when one of them is a binomial and the other a monomial.

$$\frac{CF}{CP} = \frac{ax}{2ax} = \frac{0}{U+U} ; \frac{0}{U+B} ; \frac{0}{B+U} ; \frac{0}{B+B} ;$$

$$\frac{U}{U+U} ; \frac{U}{U+B} ; \frac{U}{B+U} ; \frac{U}{B+B} ;$$

$$\frac{B}{U+U} ; \frac{B}{U+B} ; \frac{B}{B+U} ; \frac{B}{B+B} .$$

The same quantity is available for $\frac{CP}{CF} = \frac{ax}{2ax}$.

If O-axis participates in a binomial, there are 15 more combinations, as O+U, O+B, B+O, O+O would have to be multiplied by 3.

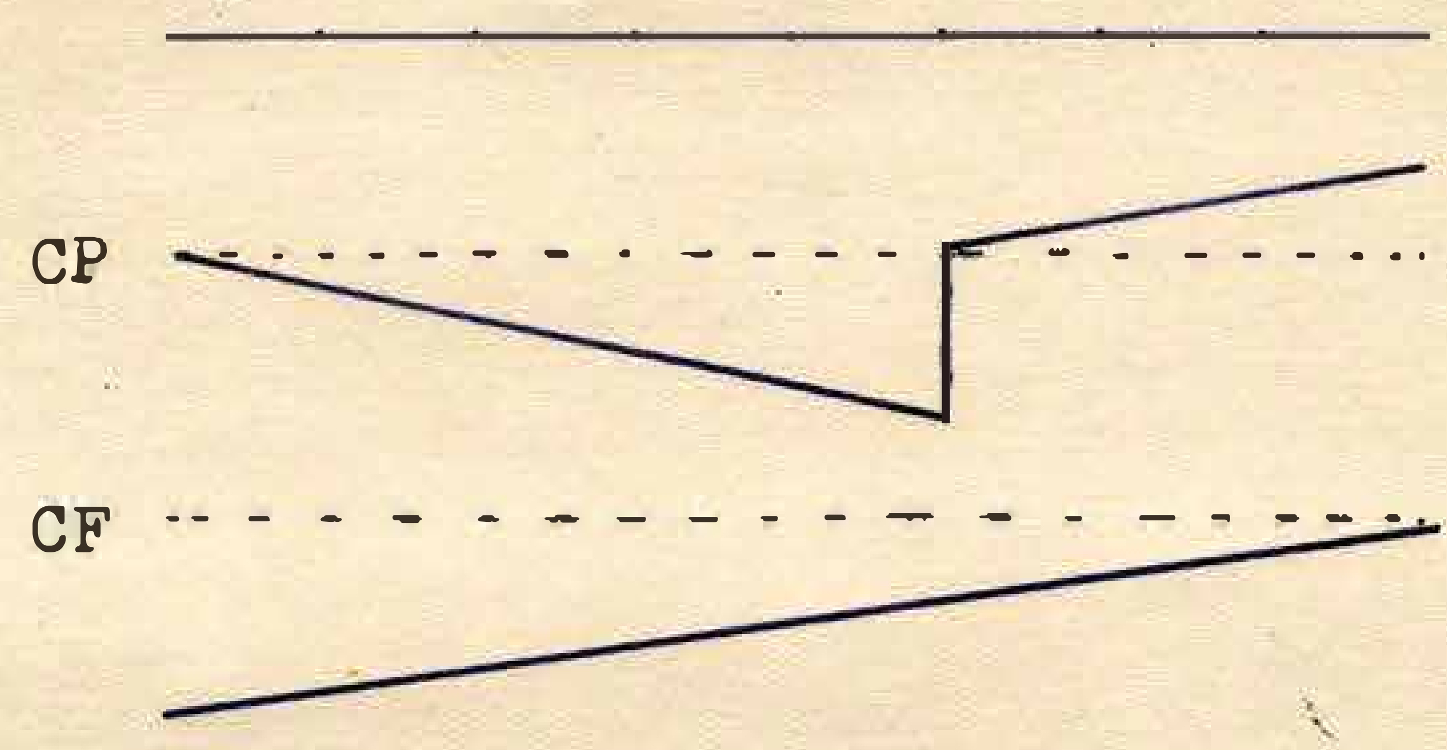
Let us select one of the possible combinations, and let it be: $\frac{CP}{CF} = \frac{2ax}{ax} = \frac{U+U}{B} = \frac{d+a}{c}$.

Suppose CF = 8T and we match the previously selected time-ratio for CP. Then the correlation of $\frac{CP}{CF}$ appears as follows: $\frac{CP}{CF} = \frac{d5T + a3T}{c8T}$.

In this case CP unbalances for 5T in the direction below its P.A., and unbalances still further

in the direction above its P.A. for 3T. While this happens, CF moves steadily toward its own P.A. in the upward direction, during the course of 8T.

Figure XXIX.



In the same fashion, trinomial axial combinations of one part can be correlated with a monomial axis of another. The quantities of simultaneous combinations equal the number of trinomials times 5.

There are 60 trinomials with two identical terms (see Theory of Melody) and 60 trinomials with all terms different. This yields: $120 \cdot 5 = 600$ for $\frac{CP}{CF}$ and the same quantity for $\frac{CF}{CP}$.

As the number of axes in one part is three and in the other part -- one, we can write:

$$\frac{CP}{CF} = \frac{3ax}{ax} \text{ or } \frac{CP}{CF} = \frac{ax}{3ax}.$$

In each case, the trinomial requires three temporal coefficients, the sum of which equals to that of monomial.

$$\frac{CP}{CF} = \frac{3ax}{ax} = \frac{axmt + axnt + axpt}{axT}, \text{ where } mt + nt +$$

$+ pt = T.$

Let T equal 5. Then, by selecting 2+2+1, which is one of the trinomials of $\frac{5}{5}$ series, we obtain:

$$\frac{CP}{CF} = \frac{ax2T + ax2T + axT}{ax5T}.$$

The trinomial distribution of the O, U and B gives the following number of the forms of balance.

O+O+U; O+O+B; U+U+O; U+U+B; B+B+O; B+B+U.

Each of the above 6 combinations has 3 permutations, giving the total of $6 \cdot 3 = 18$. When each of these variations is placed against O, U or B in the counterpart, the number of forms becomes tripled: $18 \cdot 3 = 54$.

Thus both $\frac{CP}{CF}$ and $\frac{CF}{CP}$ have 54 forms each.

But the above forms contain trinomials with two identical terms. The addition of trinomials without identical terms produces one combination: O+U+B, which has 6 permutations. These 6 forms, being placed against the three possible forms of the counterpart, produce $6 \cdot 3 = 18$.

$\frac{CP}{CF}$ and $\frac{CF}{CP}$ have 18 forms each.

The total of trinomial combinations of balance of $\frac{CP}{CF}$ is $54 + 18 = 72$, and the same number for $\frac{CF}{CP}$.

When secondary axes are substituted for the forms of balance, each case gives more than one solution. For example: $\frac{CP}{CF} = \frac{U+0+B}{U}$;

CP	CF
(1) U = a; U = d;	U = a; U = d.
(2) 0 = 0	
(3) B = b; B = c;	

Then the following solutions are available:

$$\frac{CP}{CF} = \frac{a+0+b}{a}\ ;\ \frac{a+0+b}{d}\ ;\ \frac{a+0+c}{a}\ ;\ \frac{a+0+c}{d}\ ;$$

$$\frac{d+0+b}{a}\ ;\ \frac{d+0+b}{d}\ ;\ \frac{d+0+c}{a}\ ;\ \frac{d+0+c}{d}\ .$$

Let us assign the previously discussed $\frac{5}{5}$ series trinomial time ratio. We yield the following solutions:

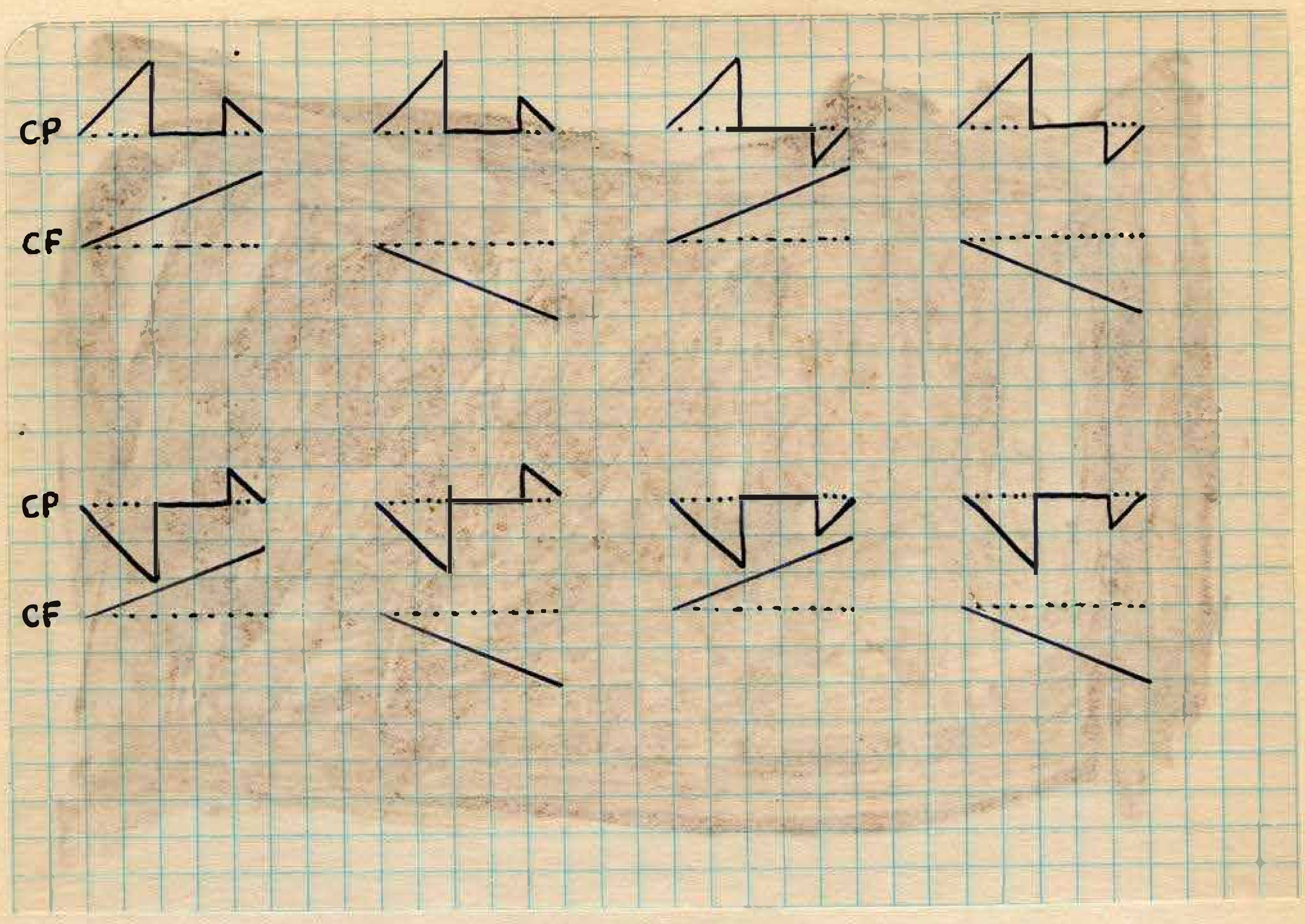
$$\frac{CP}{CF} = \frac{a2T + 02T + bT}{a5T}\ ;\ \frac{a2T + 02T + bt}{d5T}\ ;\ \frac{a2T + 02T + cT}{a5T}\ ;$$

$$\frac{a2T + 02T + cT}{d5T}\ ;\ \frac{d2T + 02T + bT}{a5T}\ ;\ \frac{d2T + 02T + bT}{d5T}\ ;$$

$$\frac{d2T + 02T + cT}{a5T}\ ;\ \frac{d2T + 02T + cT}{d5T}\ .$$

Figure XXX.

(please see next page)


CP
CF
CP
CF

Lesson CLX.

Ultimately a polynomial axial combination can serve as the counterpart to a monomial axis. The effect of such a correlation is instability (polynomial) versus stability (monomial). The selection of forms of O, U and B depends upon the effects of balance necessary in each particular case. The abundance of unbalancing axes results in restless, disquieting, unstable melodies. Such melodies are termed as dramatic, passionate, ecstatic, etc. The abundance of balancing and the O-axes produces the restful, quiet, stable melodies. They are usually termed as contemplative, epical, serene.

Examples of composition of $\frac{CP}{CF} = \frac{max}{ax}$.

Let $m = 5$; then: $\frac{CP}{CF} = \frac{5ax}{ax}$.

Let us consider the following balance-group: U+B+U+B+U. Let us assume that the two extreme terms are identical, but different from the middle one. Then the possibilities for the U's are:

(1) a+d+a and (2) d+a+d

Let us select the first combination. Let us assume that both B's are identical but on the opposite side of P.A. from the two identical U's. Then we get: c+c for the B+B. The entire axial combination for the CP appears as follows:

$CP = a+c+d+c+a$

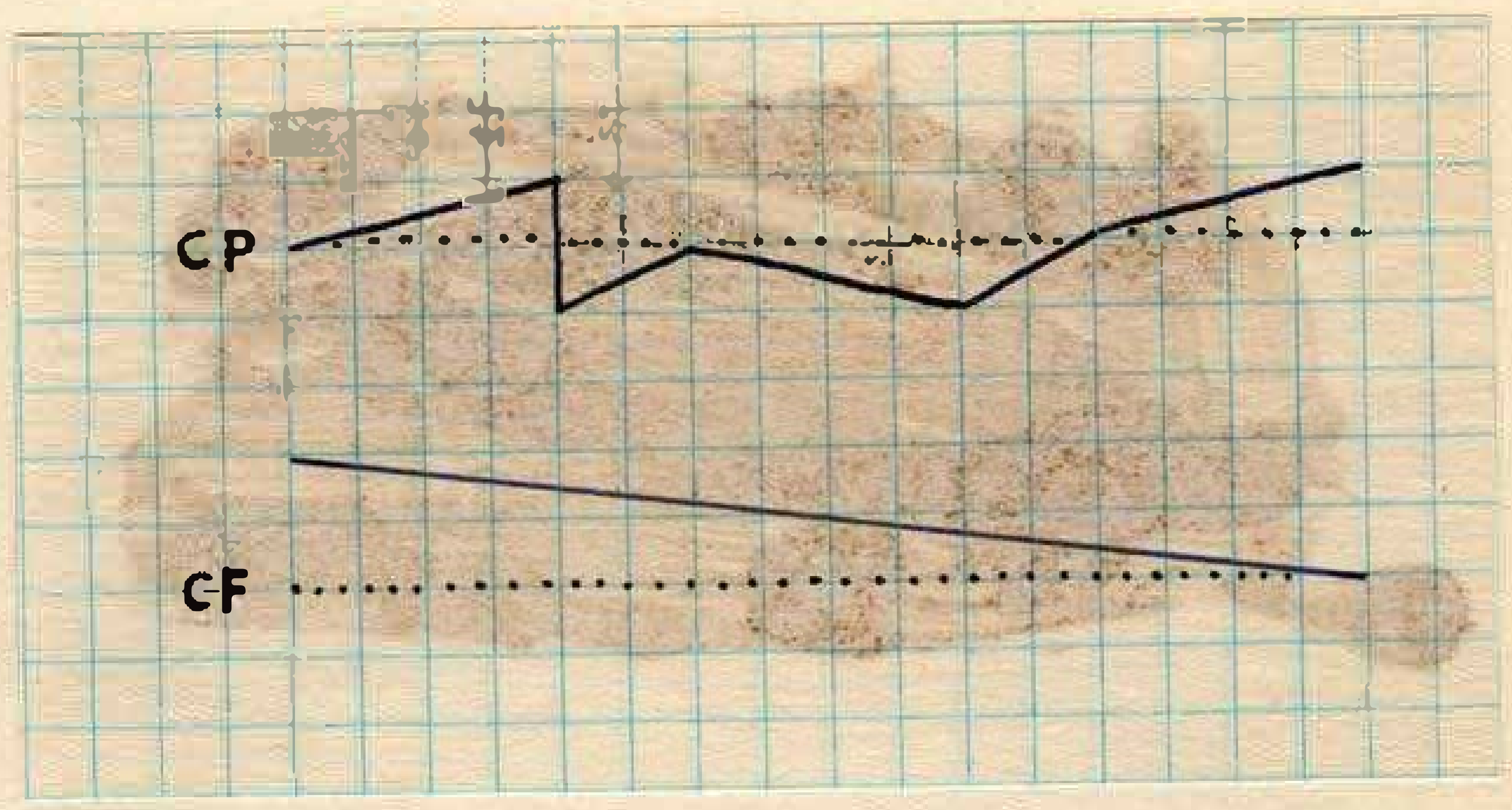
Let CF be represented by B, and let it be b, in order to achieve greater variety of balancing forms of CP in relation to CF.

$$\frac{CP}{CF} = \frac{a+c+d+c+a}{b}$$

Let the duration of the entire group be 16T. Let the temporal coefficients correspond to $\frac{8}{8}$ series on the basis of t = 2T. Then, by selecting a quintinomial (for the five axes of CP), we obtain the following temporal scheme:

$$\frac{CP}{CF} = \frac{a4T + c2T + d4T + c2T + a4T}{b16T}$$

Figure XXXI.



The temporal ratios, discussed so far, referred to the form $\frac{CP}{CF} = 1, 2, 3, \ldots m$.

Such axial relations can be further developed into polynomial groups in both CF and CP:

(1) Through the technique previously applied to the composition of attack-groups (see Melodization of Harmony);

(2) By the direct application of ratios producing interference.

The first technique makes it possible to match any desirable number of axes of the CP against each axis of the CF.

Let us take CF with 4 axes. We can match 2, 3 or more axes of CP against each axis of CF and in any desirable sequence.

For example: $\frac{CP}{CF} = \frac{2ax}{ax} + \frac{2ax}{ax} + \frac{2ax}{ax} + \frac{2ax}{ax}$.

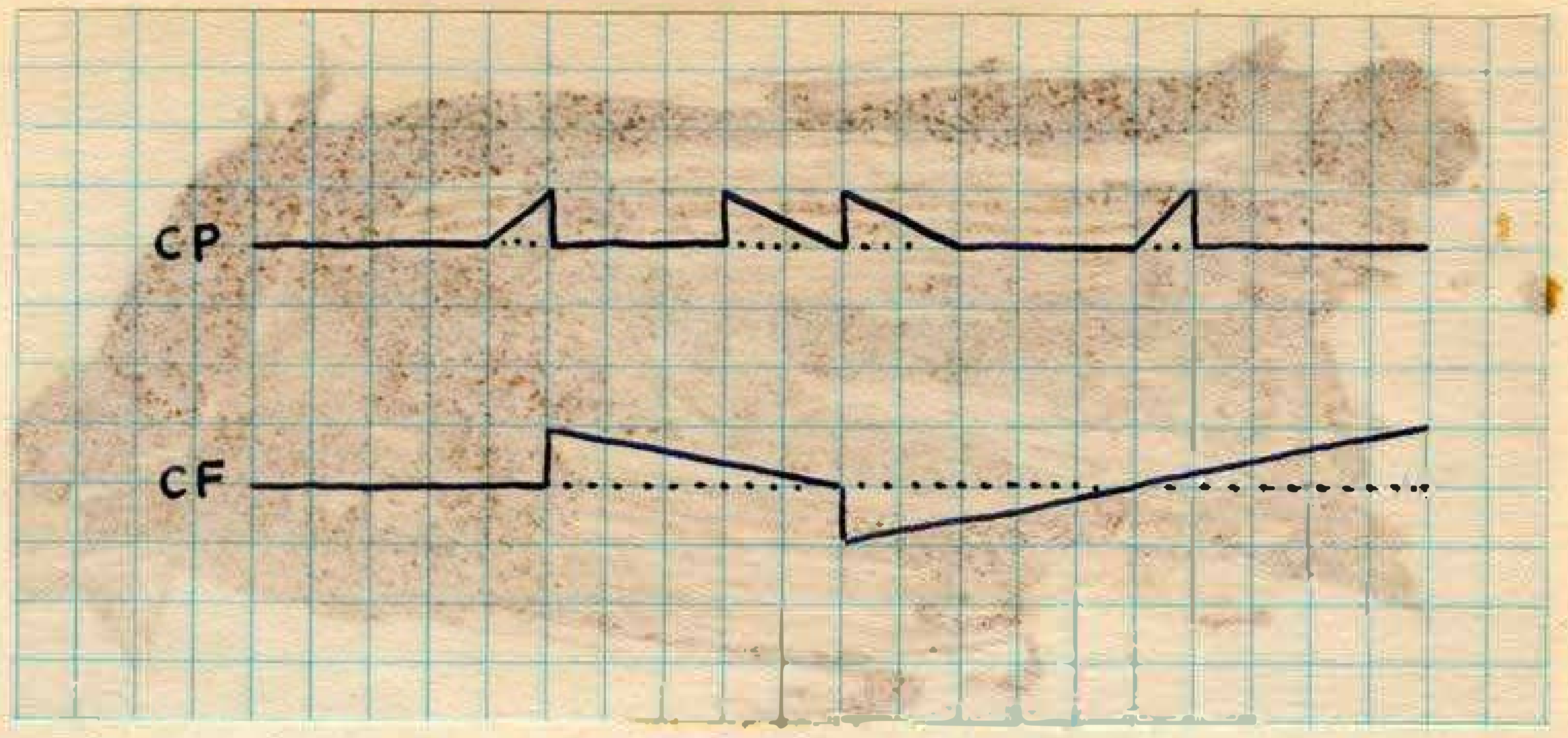
By assigning temporal coefficients in such a way that the sum of durations in each 2ax of CP corresponds to the duration of ax of CF, we acquire a synchronized $\frac{CP}{CF}$. With the temporal coefficients based on $r_{5 \div 4}$, for instance, we obtain the following correlation:

$$\frac{CP}{CF} = \frac{ax4T + axT}{ax5T} + \frac{ax3T + ax2T}{ax5T} + \frac{ax2T + ax3T}{ax5T} + \frac{axT + ax4T}{ax5T}$$

Let 0+b+c+a be the axial combination of CF, and (0+a) + (0+b) + (b+0) + (a+0) -- the axial combination of CP. Then $\frac{CP}{CF}$ acquires the following appearance.

$$\frac{CP}{CF} = \frac{04T + aT}{05T} + \frac{03T + b2T}{b5T} + \frac{b2T + 03T}{c5T} + \frac{aT + 04T}{a5T}$$

Figure XXXII.



When proportionate relations of the temporal coefficients of $\frac{CP}{CF}$ are desirable and a constant number of the axes of CP is assigned against each axis of CF, the technique of distributive involution solves the problem.

For example: $\frac{CP}{CF} = \frac{9ax}{3ax} = \frac{3ax}{ax} + \frac{3ax}{ax} + \frac{3ax}{ax}$.

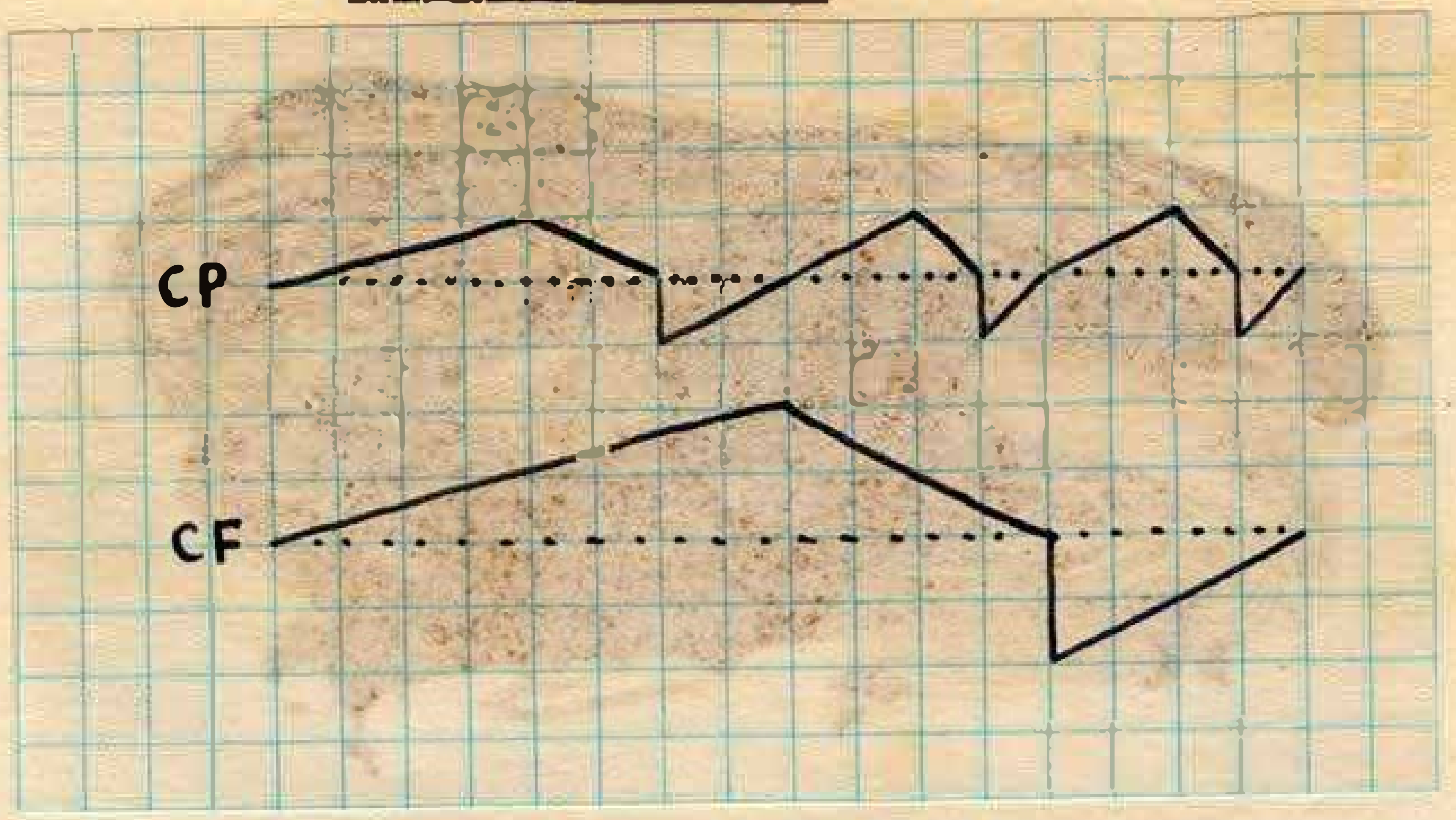
To carry out this form of correlation in proportions, we shall select the square of 2+1+1 of the $\frac{4}{4}$ series.

$$\frac{CP}{CF} = \frac{ax4T + ax2T + ax2T}{ax8T} + \frac{ax2T + axT + axT}{ax4T} + \frac{ax2T + axT + axT}{ax4T}$$

Let the axial combination for both CP and CF be the trinomial a+b+c. Then:

$$\frac{CP}{CF} = \frac{a4T + b2T + c2T}{a8T} + \frac{a2T + bT + cT}{b4T} + \frac{a2T + bT + cT}{c4T} .$$

Figure XXXIII.



Most complex temporal relations result from the quantities of axes in CP and CF, which produce interference ratios. We shall discuss here only the simplest forms of such interference, which require uniform temporal coefficients for both CP and CF, only different in value. This corresponds to Binary Synchronization as described in the Theory of Rhythm. In this sense an $\frac{a}{b}$ ratio represents the number of secondary axes in the two counterparts.

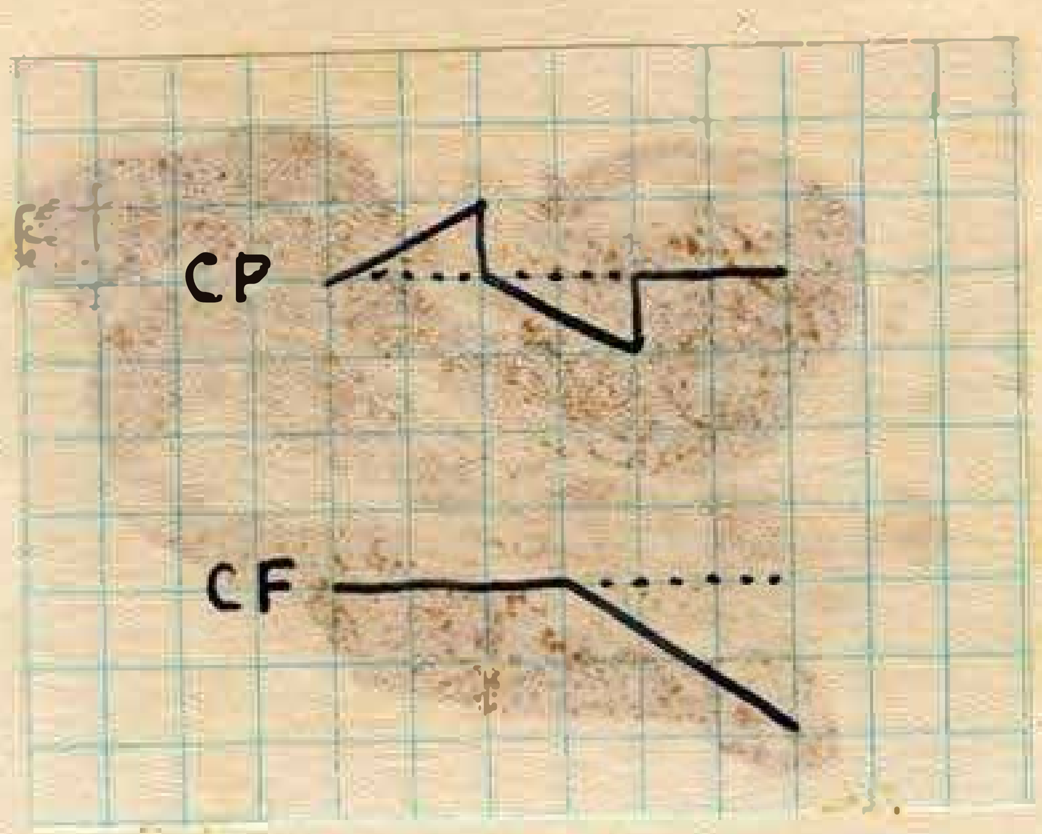
Let us take $\frac{3}{2}$ ratio. Under such conditions $\frac{CP}{CF} = \frac{3ax}{2ax}$ or $\frac{CP}{CF} = \frac{2ax}{3ax}$. After synchronization, the first expression appears as follows:

$$\frac{CP}{CF} = \frac{ax2T + ax2T + ax2T}{ax3T + ax3T}$$

Let CF consist of 0+d and CP -- of a+d+0. Then:

$$\frac{CP}{CF} = \frac{a2T + d2T + 02T}{03T + d3T}$$

Figure XXXIV.



Series of accelerations used in their reciprocal directions serve as another material for the temporal coefficients of $\frac{CP}{CF}$. This technique produces two counterparts in the form of growth versus decline.

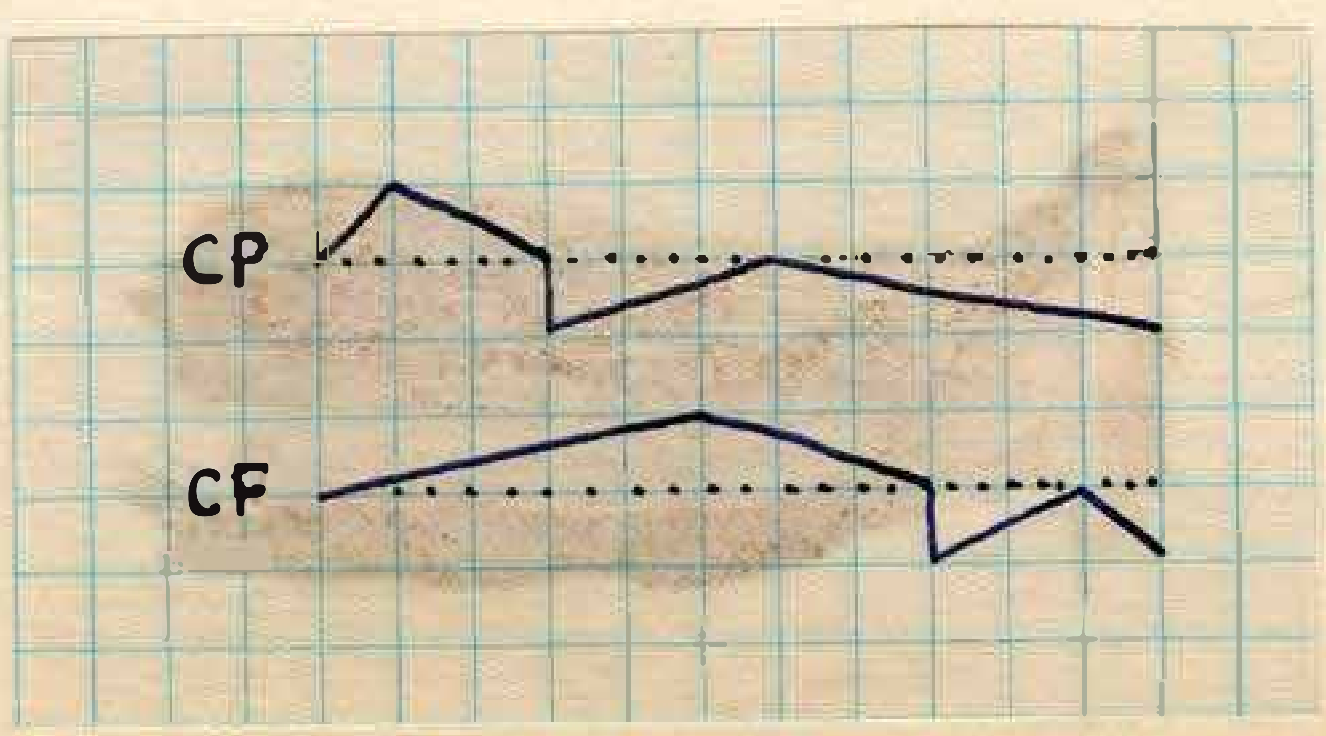
An example:

$$\frac{CP}{CF} = \frac{axT + ax2T + ax3T + ax5T}{ax5T + ax3T + ax2T + axT}$$

Axial combinations: $\frac{CP}{CF} = \frac{a+b+c+d}{a+b+c+d}$. Hence:

$$\frac{CP}{CF} = \frac{aT + b2T + c3T + d5T}{a5T + b3T + c2T + dT}$$

Figure XXXV.



This case illustrates the fact that even identical axial combinations in both counterparts can be made contrasting by the reciprocation of temporal coefficients.

An obvious contrast of some axial combinations against their own magnified versions can be achieved by means of the coefficients of duration applied to the original group of temporal coefficients.

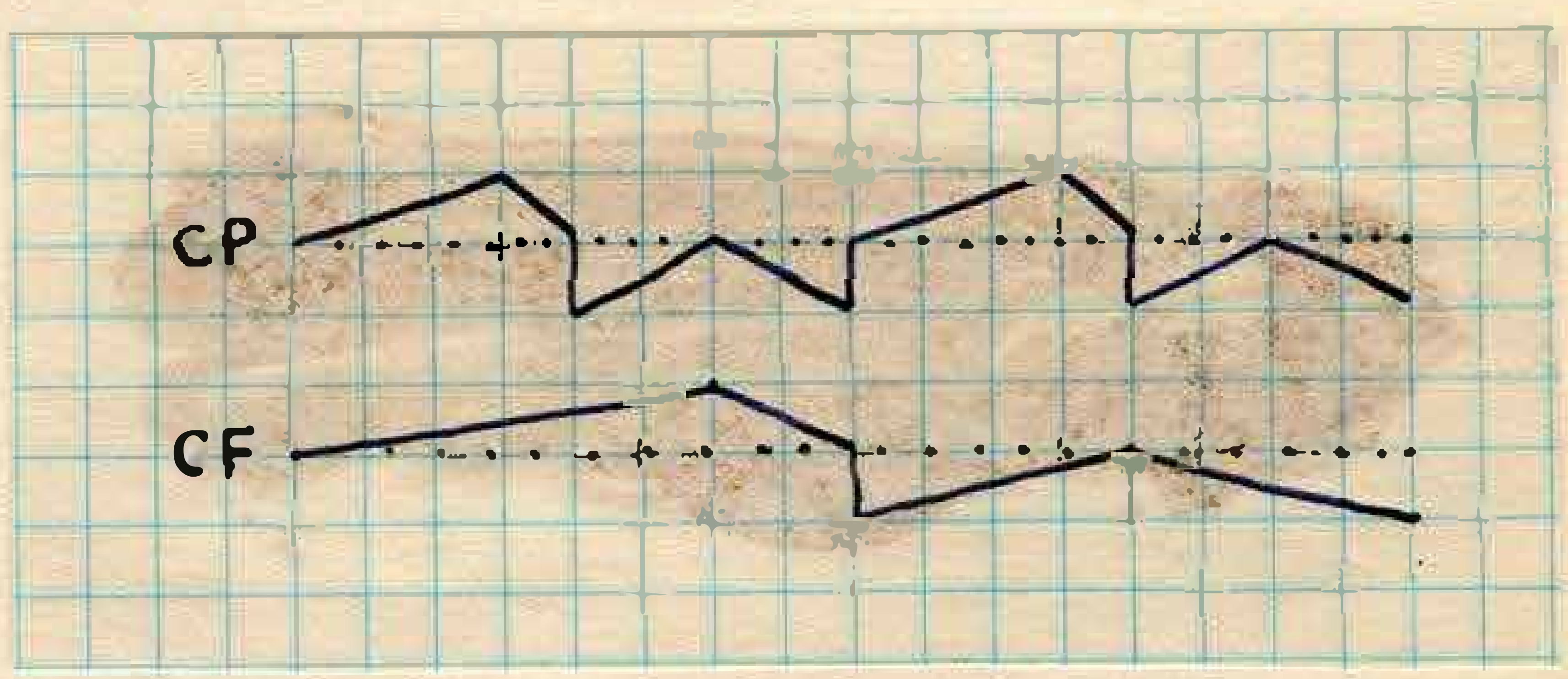
An example:

$$\frac{CP}{CF} = \frac{2(ax3T + axT + ax2T + ax2T)}{ax6T + ax2T + ax4T + ax4T}$$

Axial combination: $\frac{CP}{CF} = \frac{a+b+c+d}{a+b+c+d}$. Hence:

$$\frac{CP}{CF} = \frac{a3T + bT + c2T + d2T + a3T + bT + c2T + d2T}{a\underset{6}{b}T + b2T + c4T + d4T}$$

Figure XXXVI.



Lesson CLXI.

After the correlation of temporal coefficients has been established, the correlation of pitch ranges of both counterparts must follow.

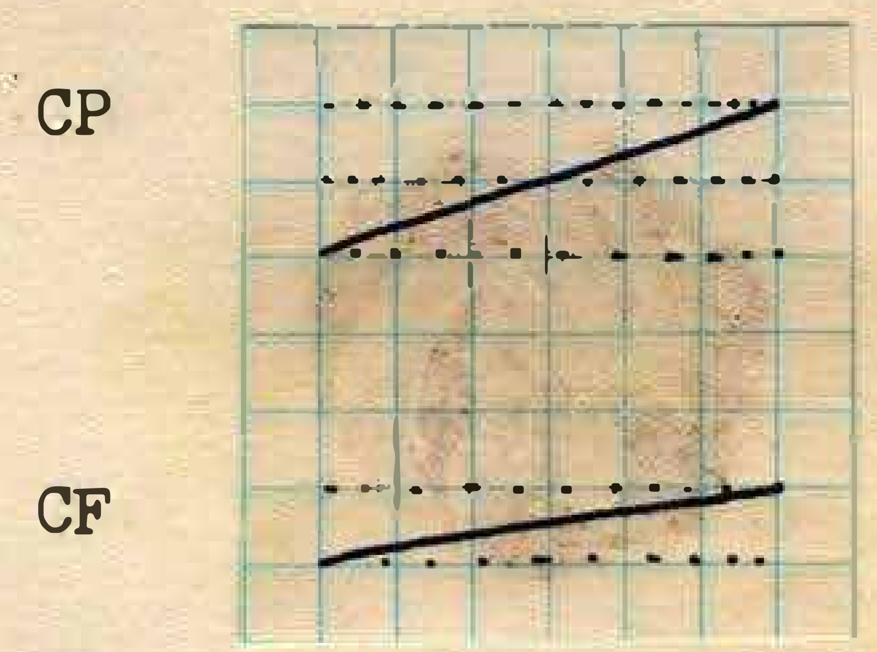
Identical secondary axes may have a different rate of speed. In terms of pitch ranges it means that a greater range may be covered in the same period of time as the smaller range.

Identical axes having different pitch-ranges produce noticeable amount of contrast.

$\frac{CP}{CF} = \frac{axT2P}{axTP}$. Let a be the axis in both parts.

Then: $\frac{CP}{CF} = \frac{aT2P}{aTP}$.

Figure XXXVII.

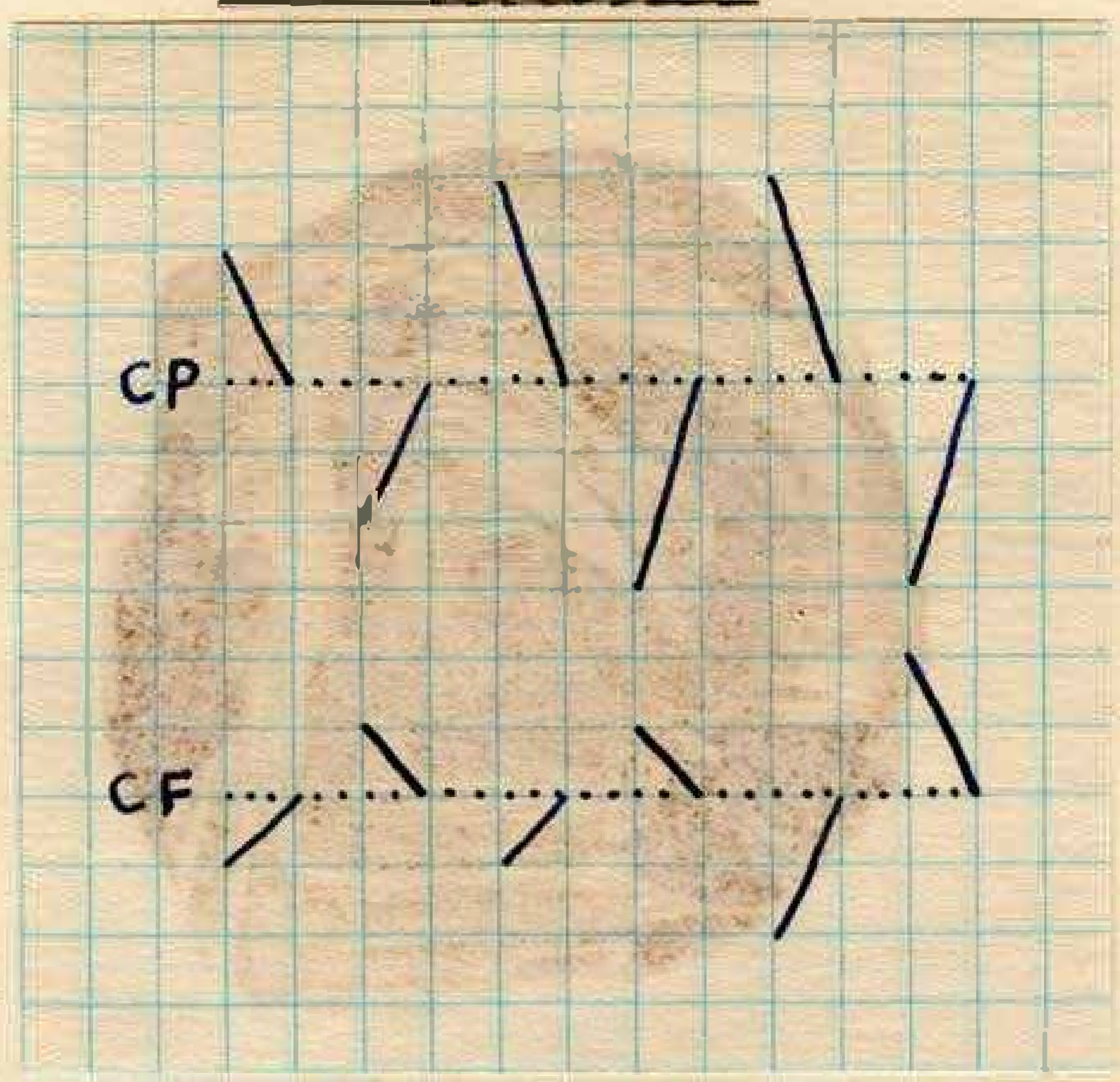


When the two counterparts are represented by the axes identical with respect to balance, but non-identical in structure, the contrast becomes still more obvious.

(1) $\frac{CP}{CF} = \frac{B}{B}$.

$$\frac{CP}{CF} = \frac{b2P}{cP} ; \frac{c2P}{bP} ; \frac{b3P}{cP} ; \frac{c3P}{bP} ; \frac{b3P}{c2P} ; \frac{c3P}{b2P} ; \ldots$$

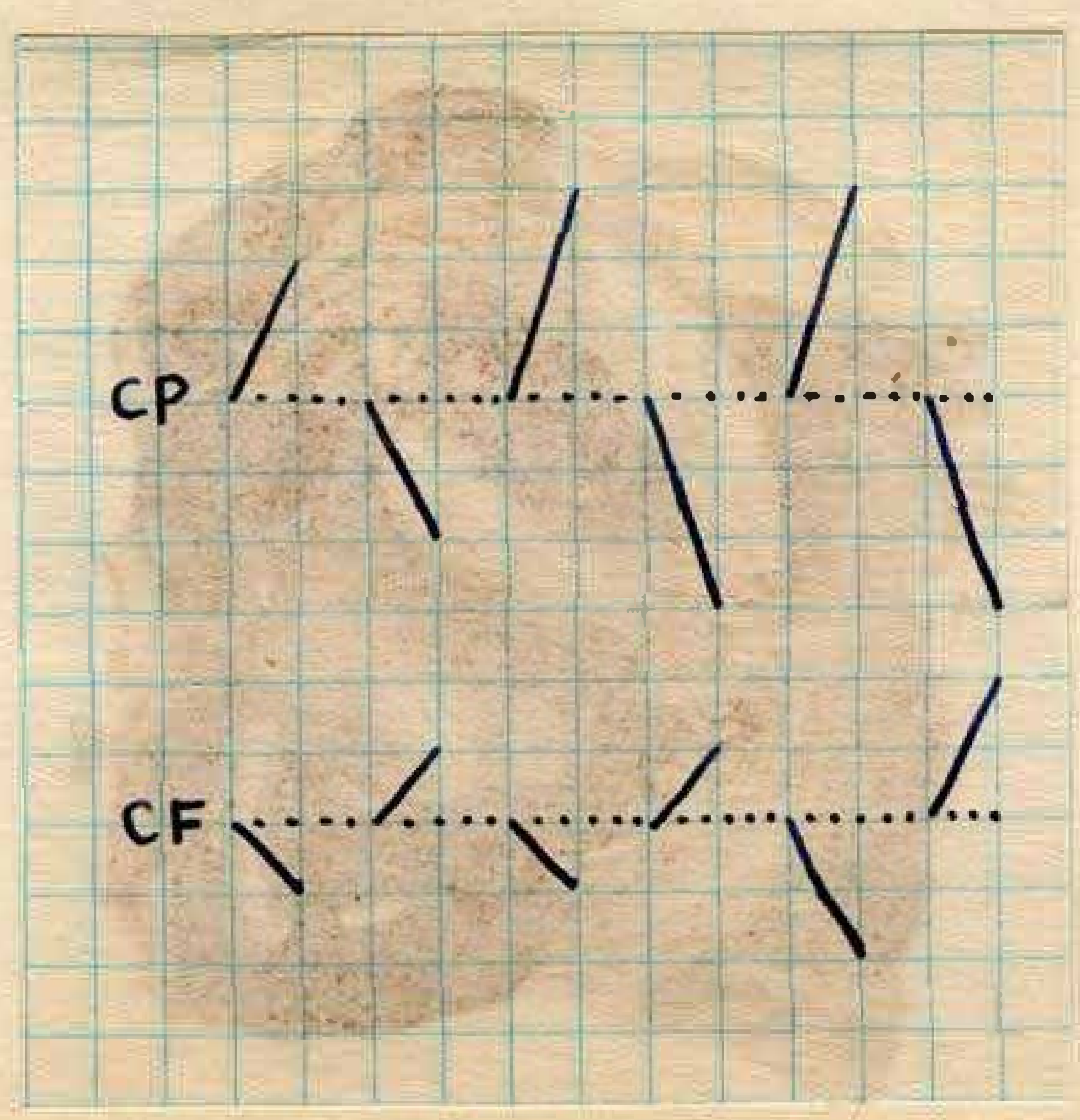
Figure XXXVIII.



(2) $\frac{CP}{CF} = \frac{U}{U}$.

$$\frac{CP}{CF} = \frac{a2P}{dP} ; \frac{d2P}{aP} ; \frac{a3P}{dP} ; \frac{d3P}{aP} ; \frac{a3P}{d2P} ; \frac{d3P}{a2P} ; \ldots$$

Figure XXXIX.

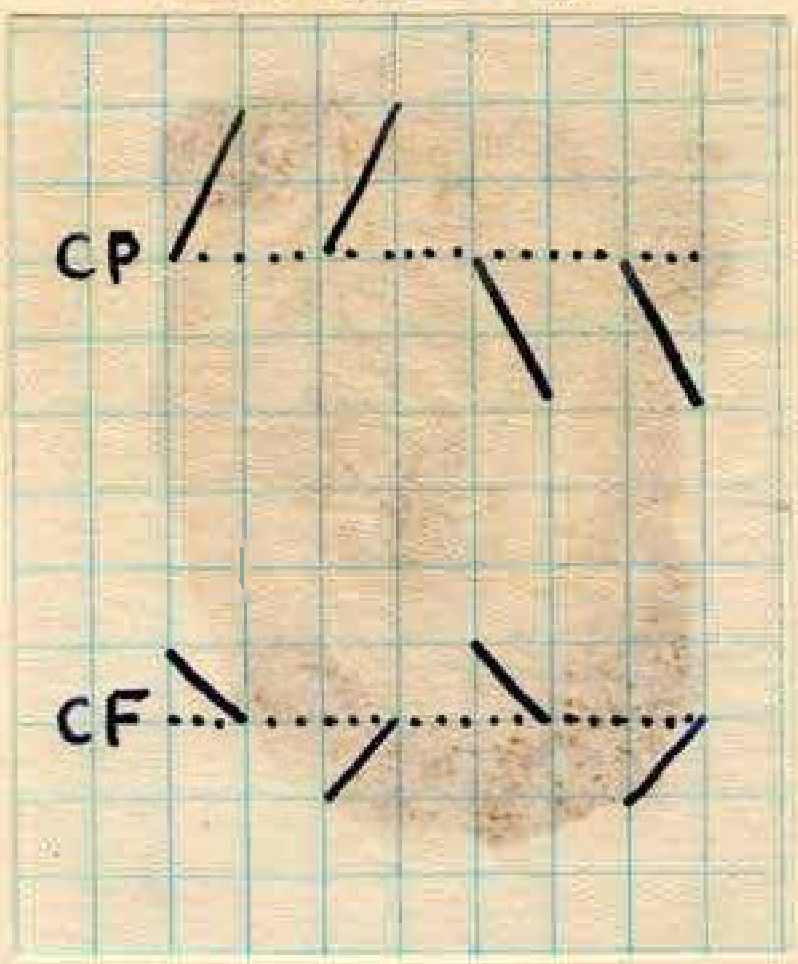


Still greater contrasts result from juxtaposition of pitch ranges of the two counterparts, when the axial structures differ with respect to balance.

$$\frac{CP}{CF} = \frac{U}{B} .$$

$$\frac{CP}{CF} = \frac{a2P}{bP} ; \frac{a2P}{cP} ; \frac{d2P}{bP} ; \frac{d2P}{cP} ; \ldots$$

Figure XL.



O-axis is not to be concerned with, when correlating pitch-ranges of the two counterparts.

As pitch-ratios may be in direct, oblique or inverse relations with the time-ratios in each part, correlation of the two counterparts offers the following fundamental possibilities:

$$\frac{CP}{CF} = \frac{T\div P \text{ direct}}{T\div P \text{ direct}} ; \quad \frac{T\div P \text{ oblique}}{T\div P \text{ direct}} ; \quad \frac{T\div P \text{ inverse}}{T\div P \text{ direct}} ;$$

$$\frac{T\div P \text{ oblique}}{T\div P \text{ oblique}} ; \quad \frac{T\div P \text{ inverse}}{T\div P \text{ oblique}} ; \quad \frac{T\div P \text{ inverse}}{T\div P \text{ inverse}} ,$$

The second, the third and the fifth forms have another variant each (by inversion). Thus, the

total number of the above relations is 6+3 = 9.

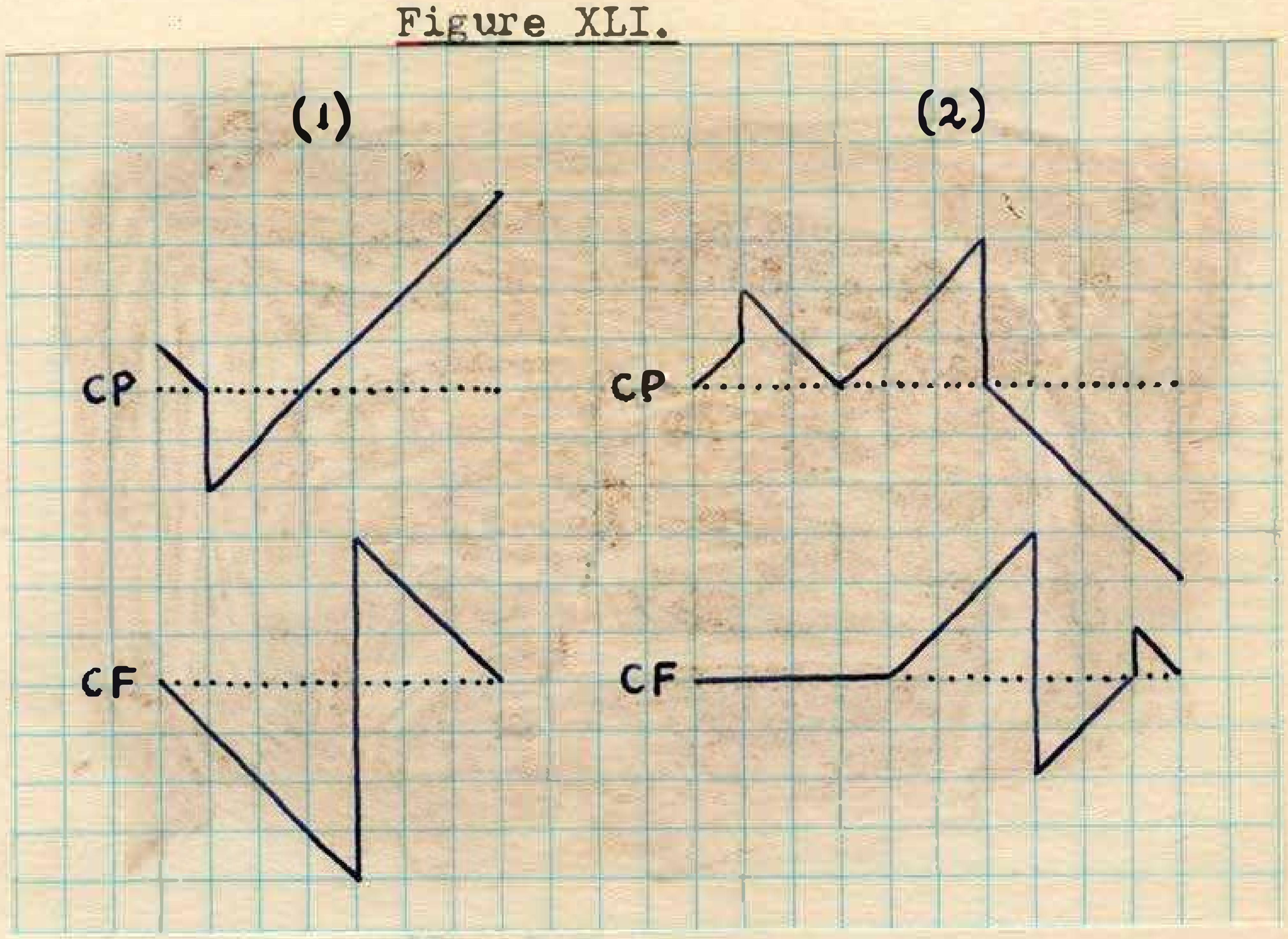
Examples:

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ direct}}$$

(1) $\frac{CP}{CF} = \frac{bTP + c2T2P + a4T4P}{d4T4P + b3T3P}$;

(2) $\frac{CP}{CF} = \frac{aTP + b2T2P + a3T3P + d4T4P}{04T + a3T3P + c2T2P + bTP}$.

Figure XLI.

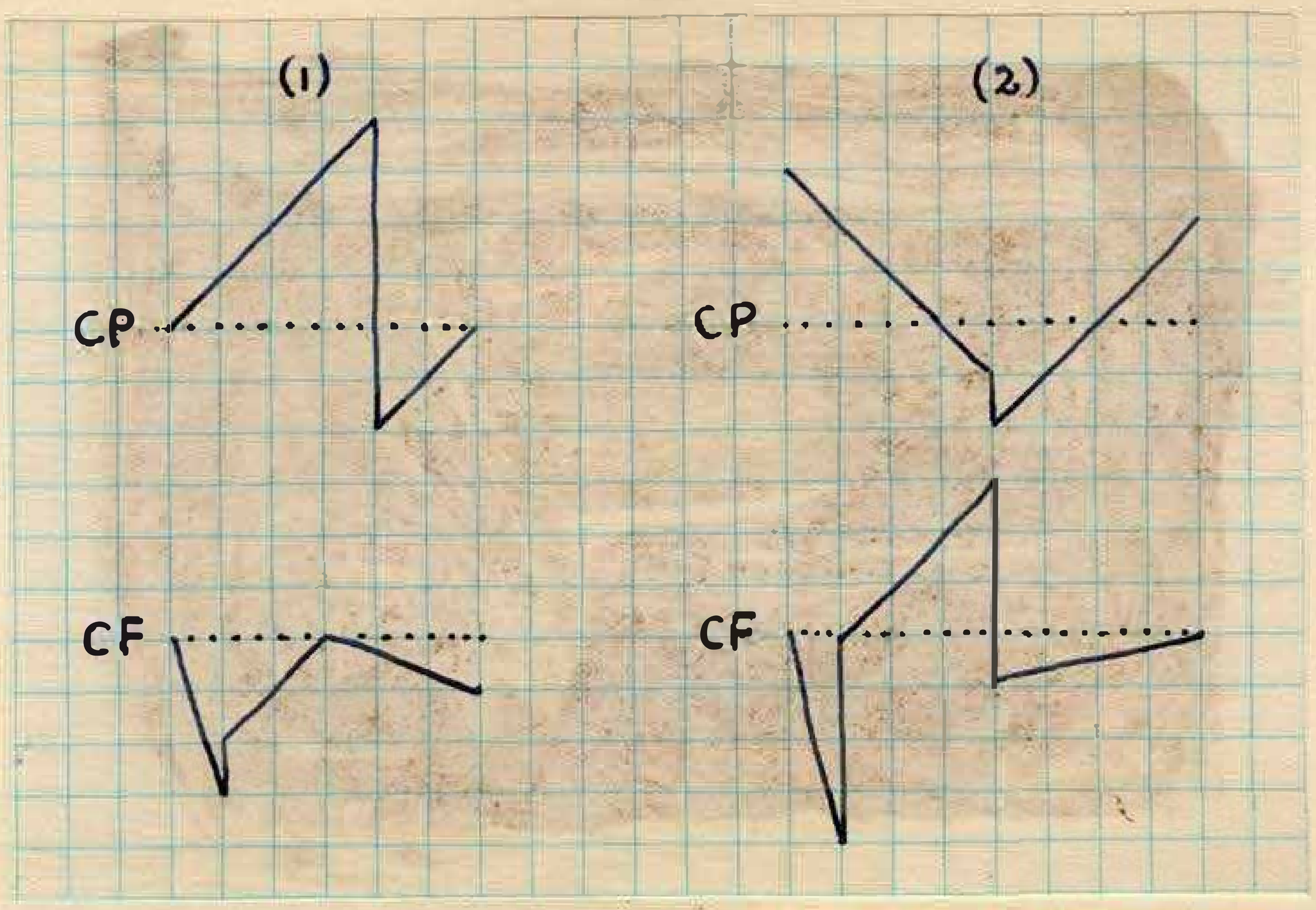


$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ oblique}}$$

(1) $\frac{CP}{CF} = \frac{a4T4P + c2T2P}{dT3P + c2T2P + d2T1P}$;

(2) $\frac{CP}{CF} = \frac{b3T3P + dTP + c2T2P + a2T2P}{dT4P + b3T3P + c4T1P}$

Figure XLII.

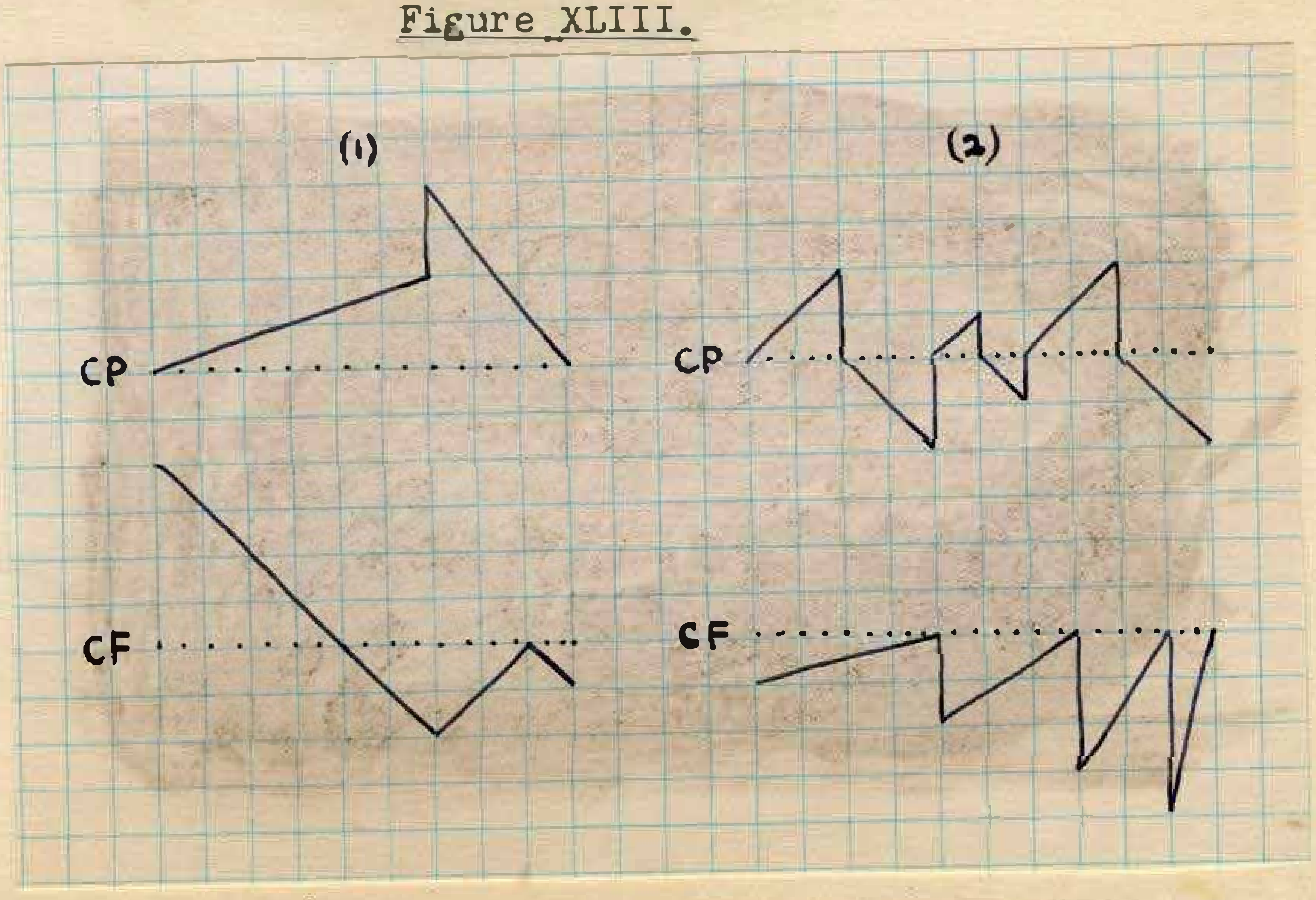


$$\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ direct}}$$

(1) $$\frac{CP}{CF} = \frac{a6T2P + b3T4P}{b4T4P + d2T2P + c2T2P + dTP} \; ;$$

(2) $$\frac{CP}{CF} = \frac{aT2P + d2T2P + aTP + dTP + a2T2P + d2T2P}{c4T1P + c3T2P + c2T3P + cT4P} \; .$$

Figure XLIII.

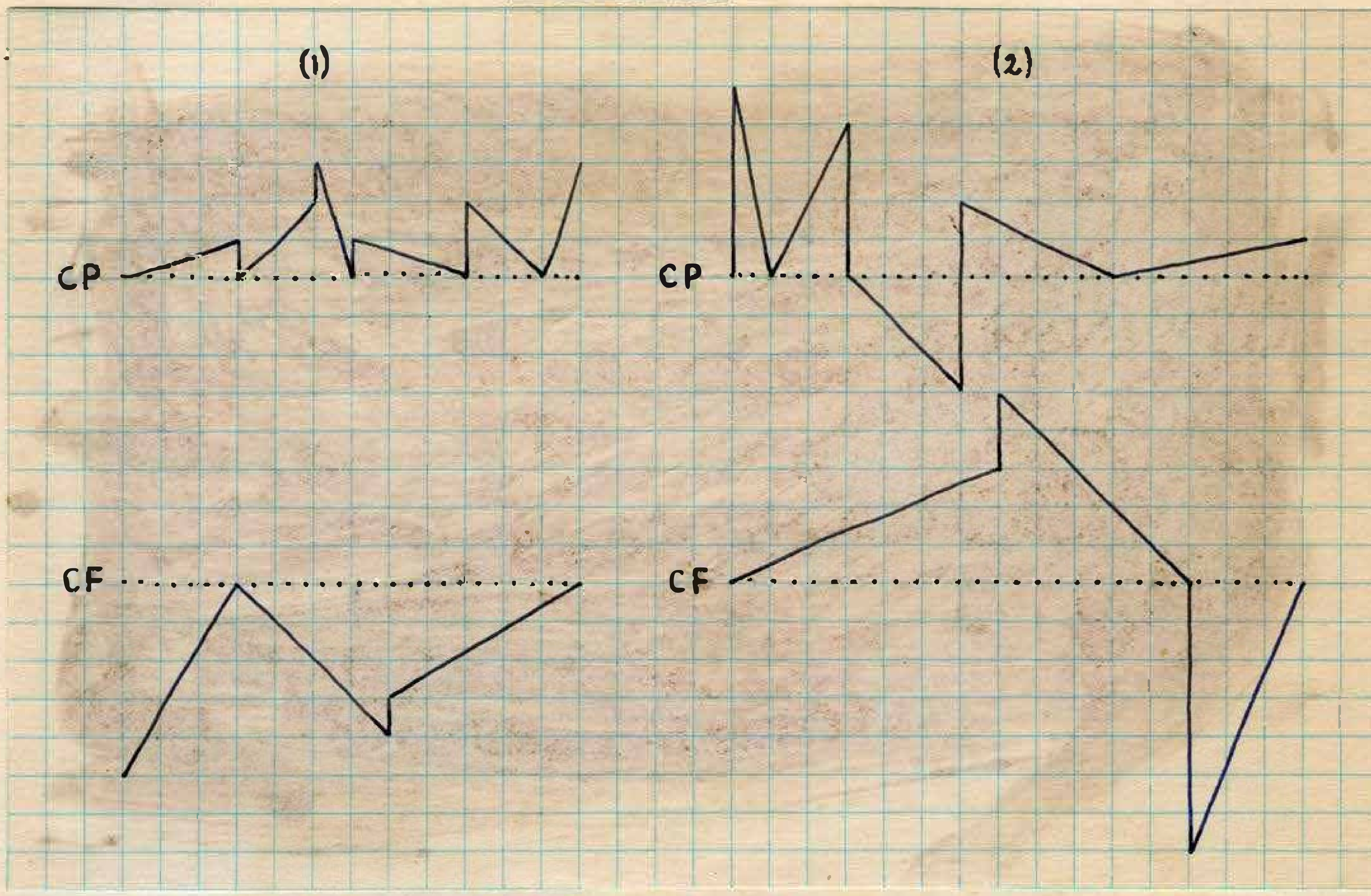


$$\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ oblique}}$$

(1) $$\frac{CP}{CF} = \frac{a3T1P + a2T2P + bT3P + b3T1P + b2T2P + aT3P}{c3T5P + d4T4P + c5T3P}$$

(2) $$\frac{CP}{CF} = \frac{bT5P + a2T4P + d3T3P + b4T2P + a5T1P}{a7T3P + b5T5P + c3T7P}$$

Figure XLIV.

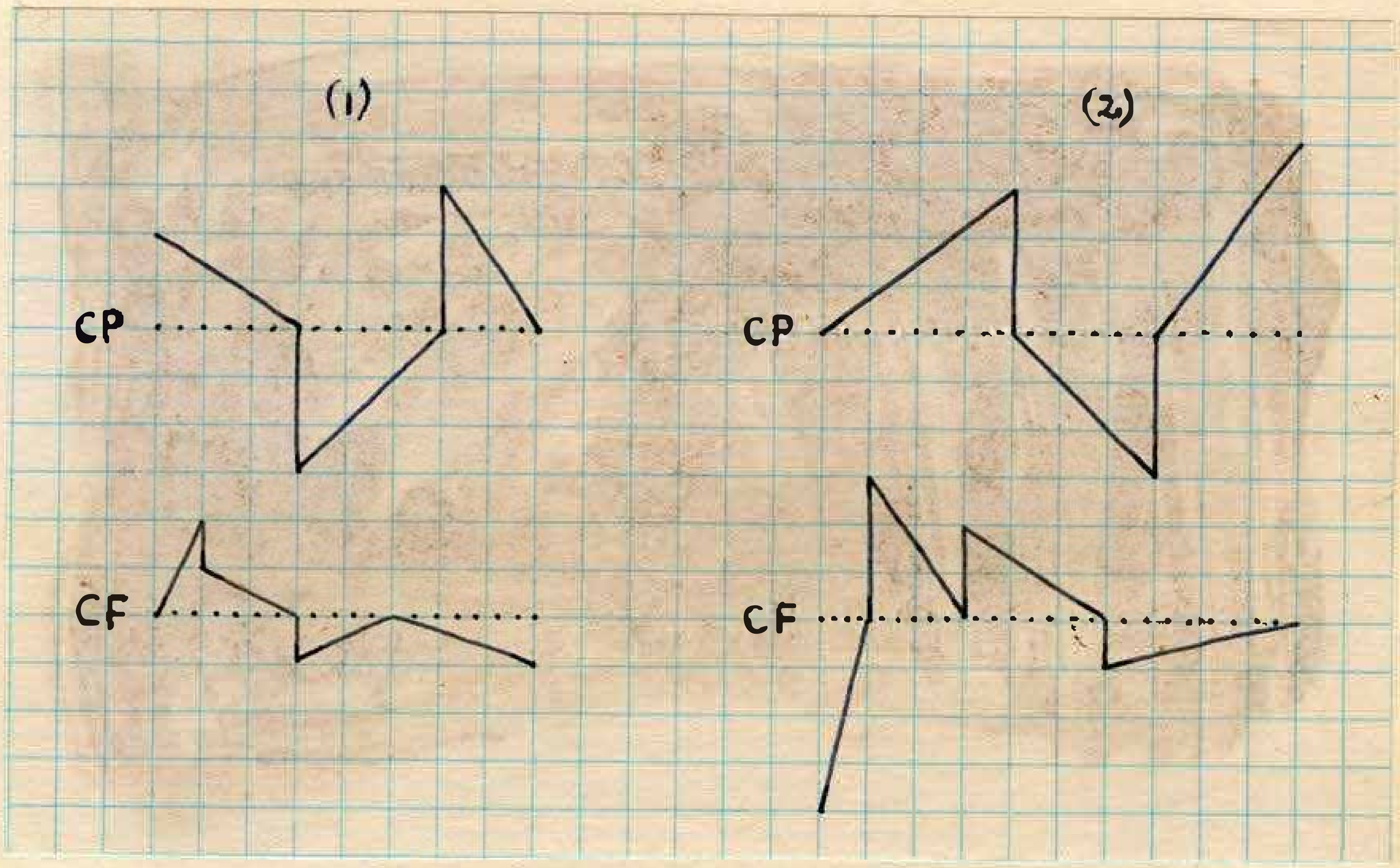


$$\frac{CP}{CF} = \frac{T \div P \text{ oblique}}{T \div P \text{ inverse}}$$

(1) $$\frac{CP}{CF} = \frac{b3T2P + c3T3P + b2T3P}{aT2P + b2T1P + c2T1P + d3T1P};$$

(2) $$\frac{CP}{CF} = \frac{a4T3P + d3T3P + a3T4P}{cT4P + b2T3P + b3T2P + c4T1P}$$

Figure XLV.

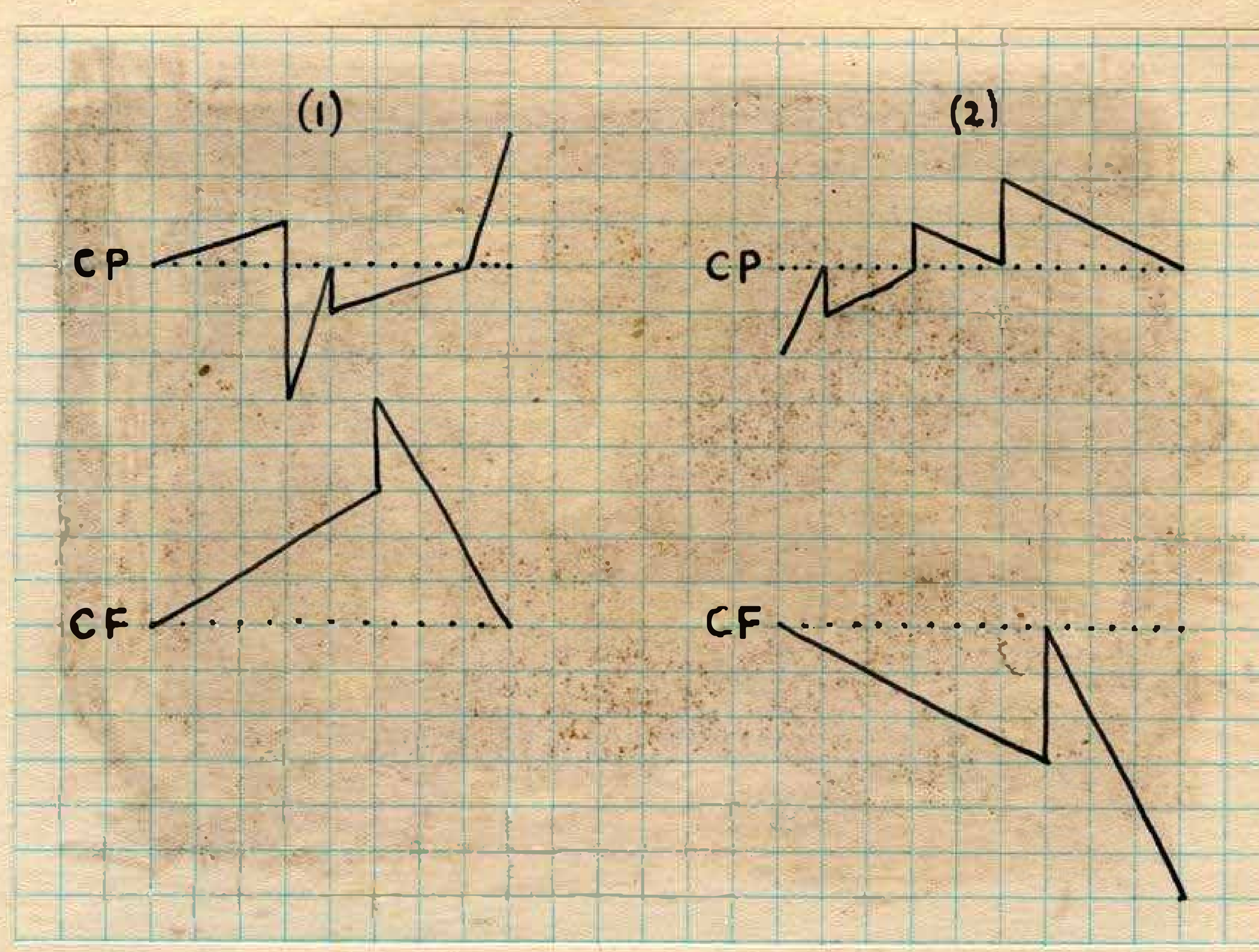


$$\frac{CP}{CF} = \frac{T \div P \text{ inverse}}{T \div P \text{ inverse}}$$

(1) $$\frac{CP}{CF} = \frac{a3T1P + cT3P + c3T1P + aT3P}{a5T3P + b3T5P}$$

(2) $$\frac{CP}{CF} = \frac{cT2P + c2T1P + b2T1P + b4T2P}{d6T3P + d3T6P}$$

Figure XLVI.



Example of Application

$$\frac{CP}{CF} = \frac{T \div P \text{ direct}}{T \div P \text{ inverse}}$$

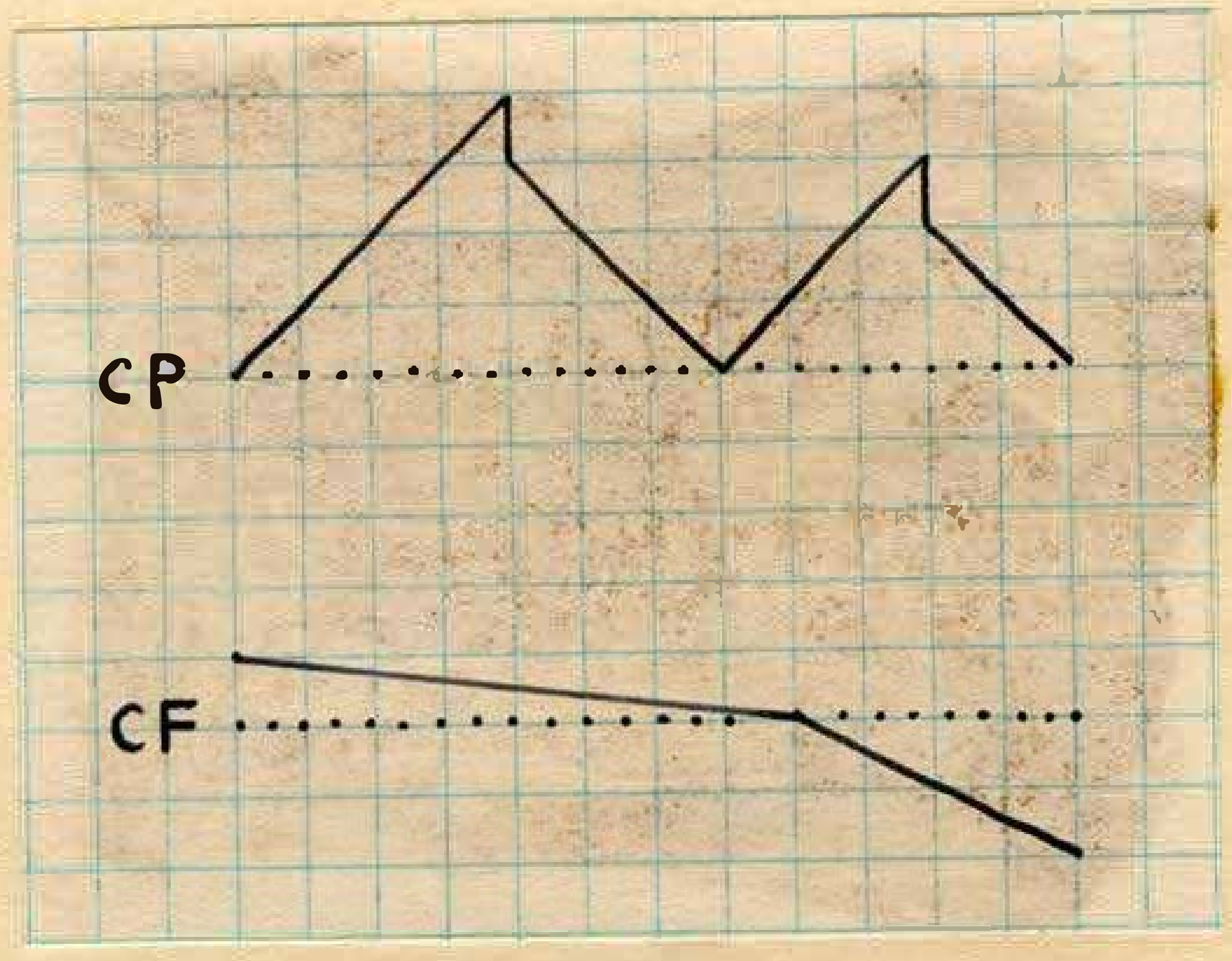
$$\frac{CP}{CF} = \frac{a4T4P + b3T3P + a3T3P + b2T2P}{b8T1P + d4T2P}$$

$$T(CF) = (4+3+3+2)^2 = (16+12+12+8) + (12+9+9+6) +$$

$$+ (12+9+9+6) + (8+6+6+4).$$

$$T(CP) = (\boxed{1} +1+1+1+1+1+1+1+1+1+1+1)$$

Axial combination of $\frac{CP}{CF}$ in its general form:

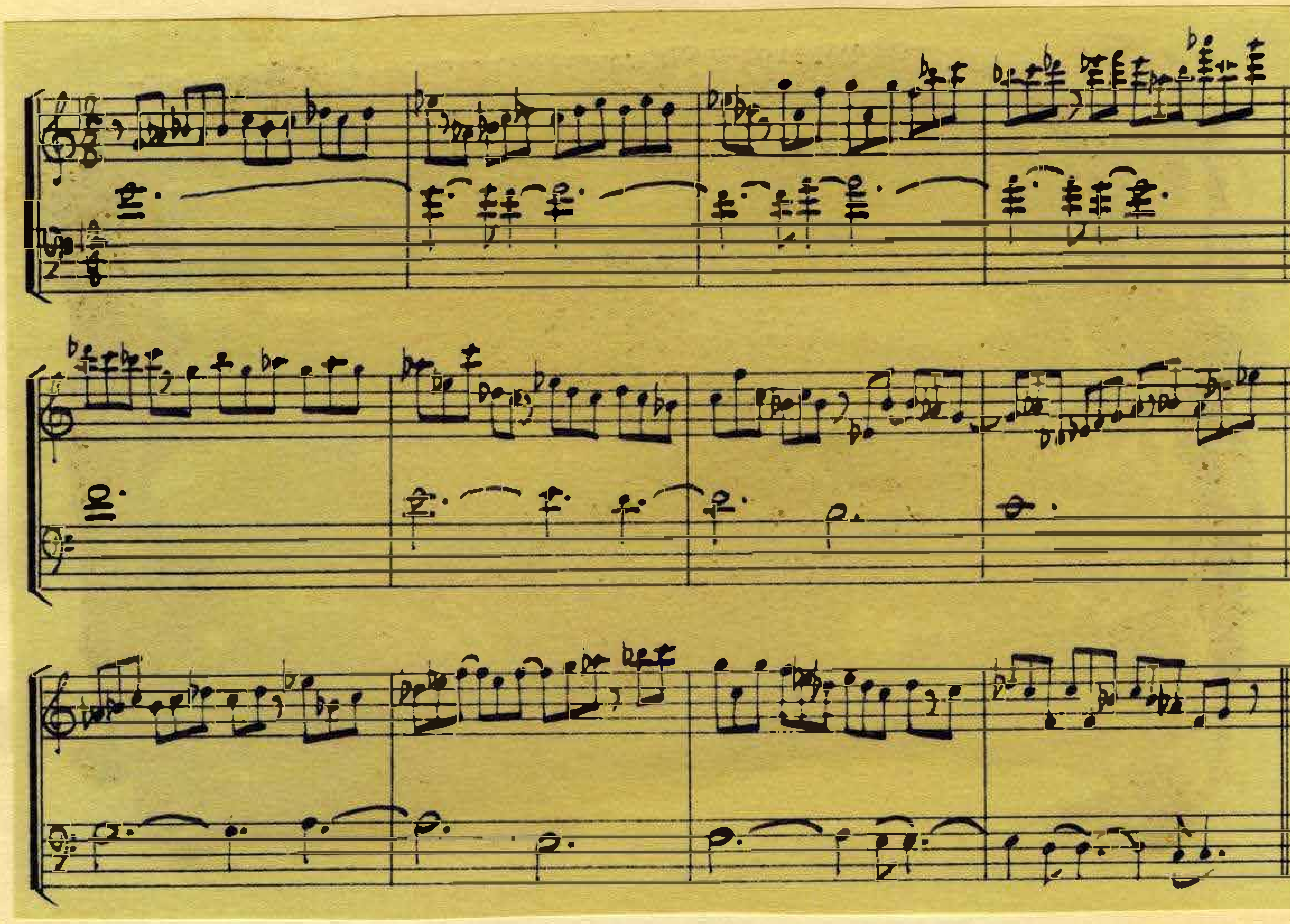


Let CF be constructed from C-maj. nat. d_0 scale and CP -- from $A^{\flat}$- maj. nat. d_4 scale. Let P = 5p with approximation. Under such conditions, the range of CF will be about an octave and a half, and the range of CP -- about two octaves.

Figure XLVII.

(please see next page)

(Fig. XLVII)



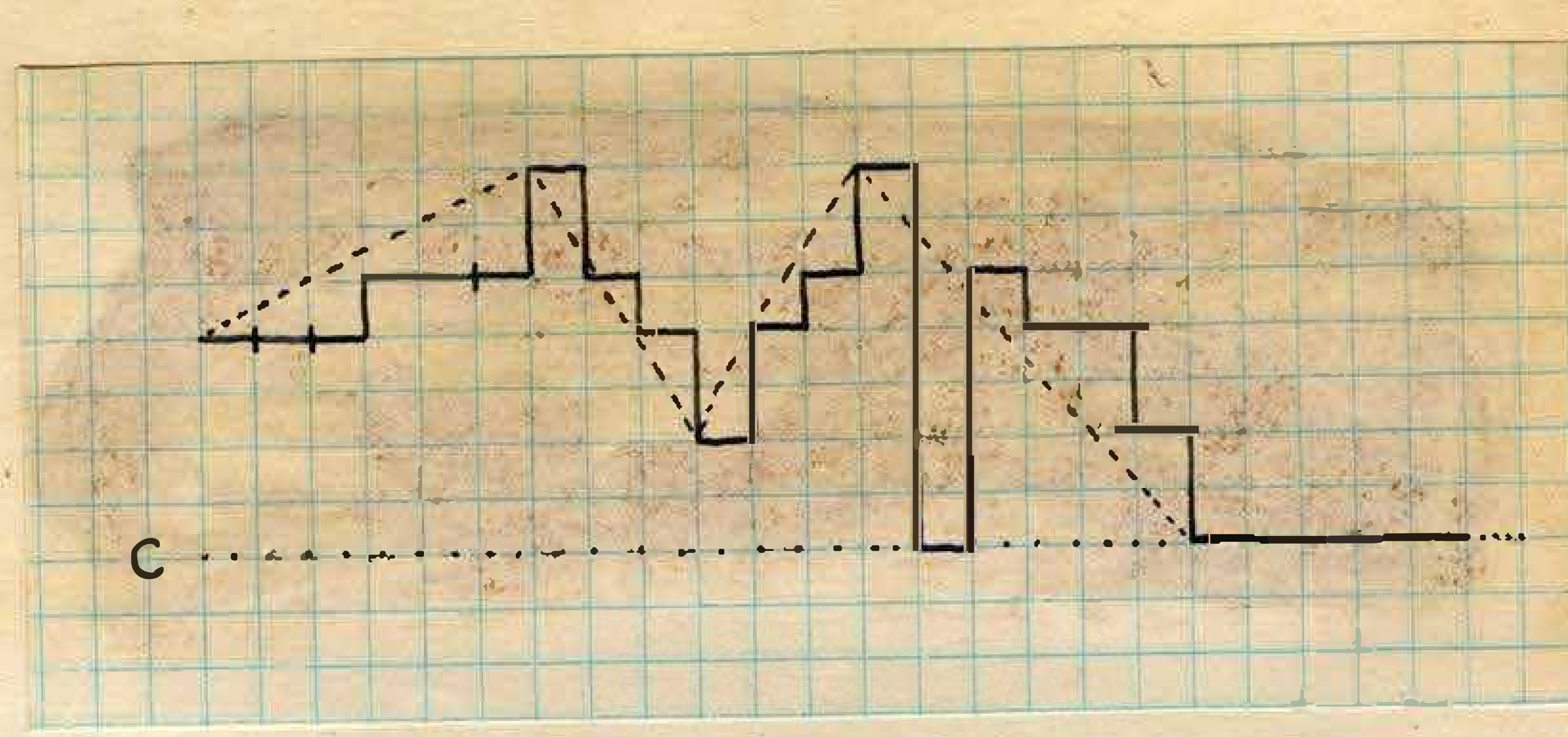
Composition of the counterpart to a given melody by means of axial correlation.

In order to accomplish the process of correlation of counterparts by means of axial correlation, it is necessary to reconstruct the axial group of the given melody first. After the analysis of TP ratios of CF has been accomplished, it is

important to detect whether the T÷P is of direct, oblique or inverse form. After this, the general planning of the CP axial combination must follow. First -- with respect to T÷P correlation, and second -- with respect to the axial combination itself and its T÷P ratios.

The following graph is a transcription of Ben Jonson's "Drink to Me Only With Thine Eyes".

Figure XLVIII.



This melody contains a modal modulation. P.A.$_1$ is Phrygian (d_2) and P.A.$_2$ is Ionian (d_0). The entire axial group gradually gravitates toward P.A.$_2$, where it forms its absolute balance. If we take into account all the minute crossings, analysis of the axial group appears as follows.

$$P.A._1 = a6t + b2t + dt + ct + a2t + b3t + d3t$$

$$P.A._2 = b3t + 05t + \boxed{t}\ .$$

The modulation is performed by establishing the correspondence between d3t ($P.A._1$) and b3t ($P.A._2$). We can say that: d3t ($P.A._1$) $\equiv$ b3t ($P.A._2$). As pitch ranges are approximately equal, the P- ratio may be regarded as constant.

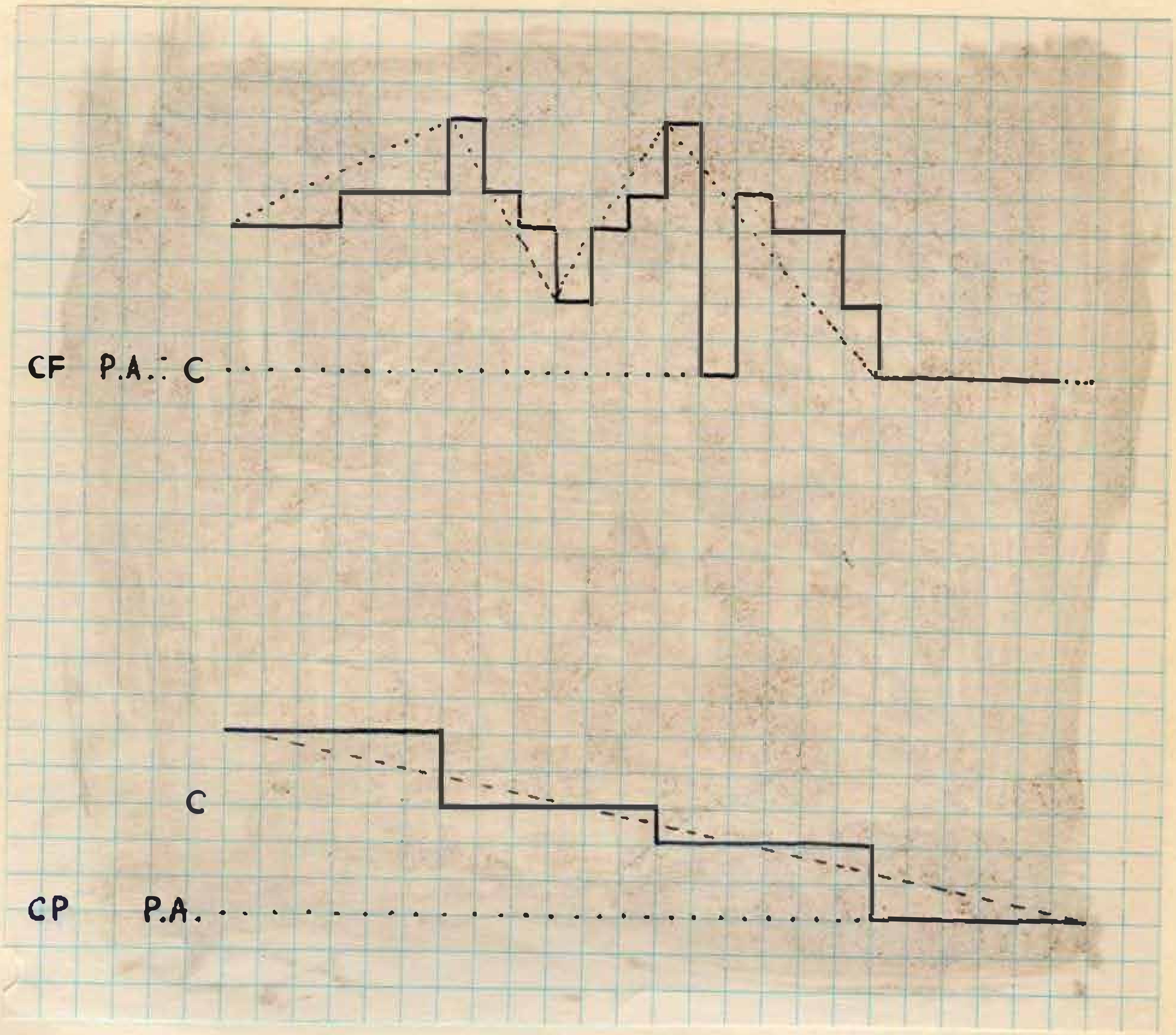
Let us devise a counterpart in 1÷4 time-ratio. This would mean that CP would have only one secondary axis. As the general tendency of CF is gradual gravitation toward balance in the course of two oscillations (which correspond to four directions and eight individual axes), we shall introduce b-axis for the counterpart. Then CP will consist of one direction, consistently gravitating toward balance. Under such conditions $\frac{CP}{CF}$ represents a complete cycle of development.

This counterpart corresponds to the case (2) in group (a) of Fig. XXII, where CP has an Aeolian P.A. (d_5).

Figure XLIX.

(please see next page)

(Fig. XLIX)



J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross   Subject: Music

Lesson CLXII.

The Use of Symmetric Scales in Two-Part Counterpoint

The unity of style requires that both counterparts are based on symmetric scales, if one of them is.

Scales of the Third Group and scales of the Fourth Group, mostly in contracted form, serve as material for counterpoint. It is acceptable to have one counterpart in the Third Group and another either in the Third or in the Fourth Group. When the two counterparts belong to the different groups, two cases can be observed:

(1) both scales have identical set of pitches;

(2) both scales have different set of pitches.

Example:

(1)

$$S_1 = \overset{T_1}{c} - f - \overset{T_2}{a\flat} - d\flat - \overset{T_3}{e} - a\natural - \overset{T_1}{c}$$

$$S_2 = \overset{T_1}{c} - d\flat - \overset{T_2}{e} - f - \overset{T_3}{g\sharp} - a - \overset{T_1}{c}$$

(2)

$$S_1 = \overset{T_1}{c} - d - e\flat - f - \overset{T_2}{f\sharp} - g\sharp - a - b - \overset{T_1}{c}$$

$$S_2 = \overset{T_1}{c} - d - f - g - \overset{T_2}{a\flat} - b\flat - d\flat - e\flat - \overset{T_3}{e\natural} - f\sharp - a - b - \overset{T_1}{c}$$

Relations of the harmonic axes of the two counterparts can be carried out in all four forms previously used. Their meaning with regard to symmetric scales appears as follows:

Type I (U.U.) : both scales have the same T_1, the same number of tonics and an identical set of pitch-units;

Type II (U.P.) : both scales have the same number of tonics, their sets of pitch-units are identical, but their harmonic axes are on different tonics;

Type III (P.U.) : both scales have an identical form of symmetry (the quantity of tonics) and an identical set of pitch-units; none of the tonics of one scale have common pitches with the tonics of the other, i.e., the two sets of tonics belong to the mutually exclusive sets of pitches;

Type IV (P.P.) : the two scales belong to either identical or non-identical forms of symmetry; their sectional scales are of non-identical structure, yet belonging to one family (according to the classification offered in the First Group of Scales); the two sets of tonics belong to the mutually exclusive sets of pitches.

Examples of two-part counterpoint executed

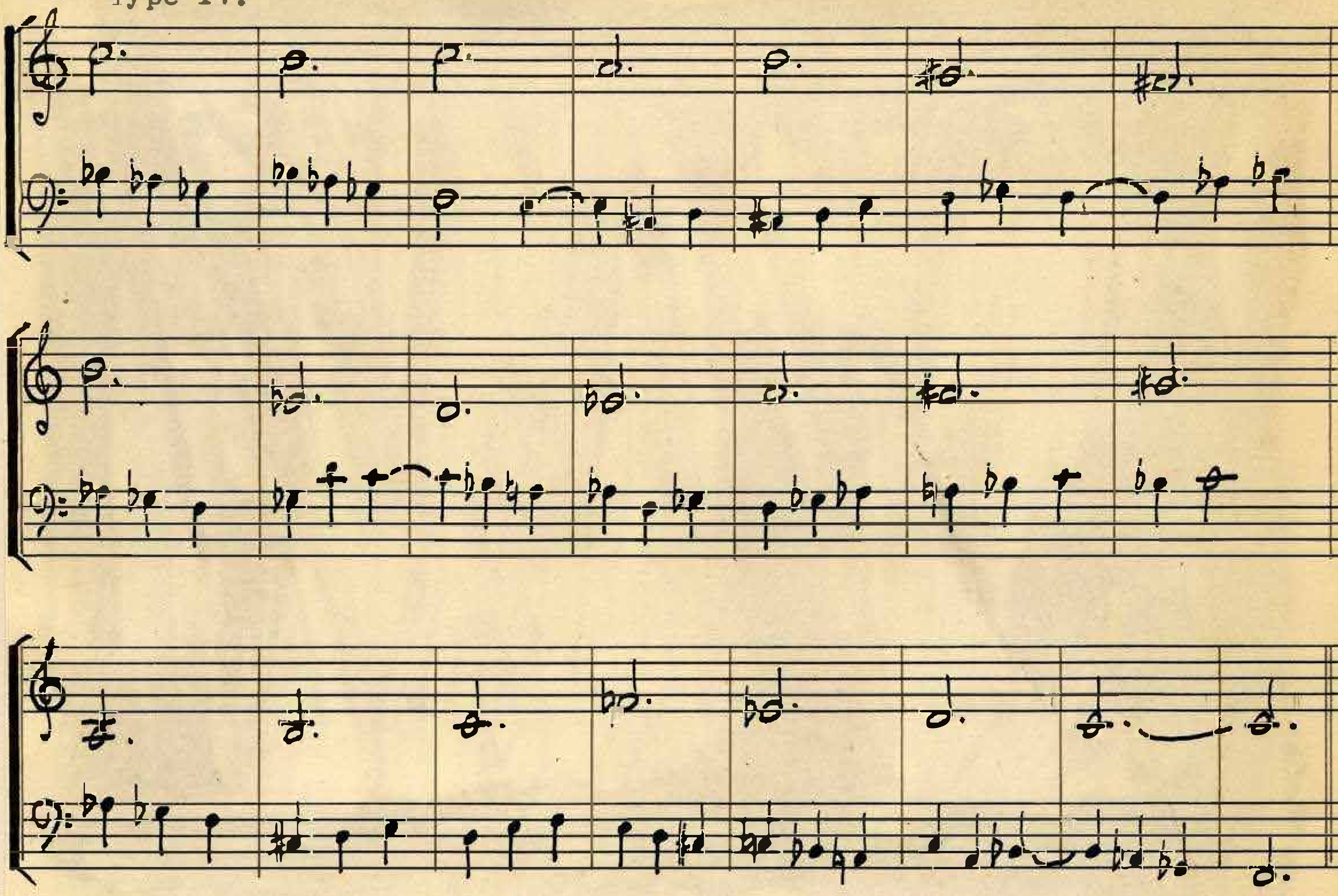
in the scales of the Third and the Fourth Group

Figure L.


Scale:
axis of CF
axis of CP
Type III
CP
CF
Scale of CF:
Scale of CP:
Scale of CP contracted and transposed to F - axis:

(Fig. L, cont.)

Type IV.



Lesson CLXIII.

Continuous (Canonic) Imitation.

The source of Continuous Imitation, usually known as Canonic, is a well known phenomenon of acoustical resonance, bearing the name of Hellenic nymph Echo. Before any composer existed on this planet, nature created by chance a quintuple echo "Lorelei" (which can be justly called five-part canon) discovered on the Rhein. Admiral Wrangel (Russian) describes a place in Siberia, where the river Lena enters a canyon about 600 feet high and where a pistol shot rapidly repeats itself more than a hundred times.

How would you like that for a canon?

Music theorists, which is typical of their species, think canon to be a purely esthetic development. Whatever they think, it is a natural phenomenon and the most ancient form of musical continuity.

There is a common belief that it requires a great skill to write a canon. In reality, the real cause of any difficulty in writing in this form is methodological incompetence. Both music theorists and composers are guilty, because they have not been able to formulate the principles of continuous imitation.

I will not discuss the case of Sergei Ivanovich Taneiev, as his interpretation of the canon

requires the knowledge of his "Convertible Counterpoint of Strict Style", which is a highly complicated system and deals with the Strict Style only. Besides, it does not bring the solution to melodic and rhythmic forms, being mostly preoccupied with the vertical and horizontal convertibility of intervals in the harmonic sense.

Canon is a complete composition written in the form of continuous imitation.

The usual academic approach to this form is such that the student is taught first how to write an "ordinary" imitation (scientifically: discontinuous imitation). After not getting anywhere with this form of imitation, he begins to struggle with the canon. As from the start the principles of any imitation are not disclosed to him, it does not make any difference whether the imitation is discontinuous or continuous. Once such principles are defined and the technique is specified, it becomes obvious that the discontinuous imitation is merely a special case of continuous imitation.

With this in view, we shall establish the principles of continuous imitation.

Continuous imitation consists of one melody, coexisting in two different parts in its different phases and at a constant velocity.

This melody, being of identical structure in both parts, may vary in intonation. The latter condition takes place only when the scale-structure itself varies.

The temporal organization of continuous imitation has no direct influence on the duration of a canon. Longer rhythmic groups are preferable, however, as continuous recurrence of the same rhythmic structure becomes, eventually, monotonous.

The main source of continuous self-stimulation in a canon is its melodic form, i.e., the axial group. With the devices offered in the Theory of Melody (see Chapter II) it is possible to evolve an axial group of great extension and, if necessary, without any repetitions. Thus, the continuance of melodic flow becomes completely protected.

The correlation of harmonic types and the treatment of harmonic intervals remains the same as for all other forms of contrapuntal technique. This permits to compose canons in unitonal as well as in polytonal types.

Temporal Structure of Continuous Imitation.

A complete composition based on continuous imitation is known as canon.

The duration of continuous imitation or of a canon is the multiple of its temporal structure. The temporal structure of a two-part canon is related to the

theme of the canon as 3÷1. The first third is the announcement, the second third is the imitation of announcement in the first voice and the counterpoint in the second voice, and the last third is the imitation of the first portion of counterpoint in the second voice and the second portion of counterpoint in the first voice. After the temporal scheme is exhausted, it begins to repeat itself with new intonations.

If we designate the first entering voice as $\vec{P}_{I}$ (whether upper or lower), the second entering voice as $\vec{P}_{II}$, the first announcement as CP_1, the first portion of counterpoint as CP_2, the second portion of counterpoint as CP_3 etc., the temporal structure of a canon appears as follows:

$$\frac{\vec{P}_{I}}{\vec{P}_{II}} = \frac{CP_1 + CP_2 + CP_3}{CP_1 + CP_2}.$$ The continuation of the temporal structure does not alter the process, merely increasing the subnumerals of CP in the original relation:

$$\frac{\vec{P}_{I}}{\vec{P}_{II}} = \frac{CP_1 + CP_2 + CP_3}{CP_1 + CP_2} + \frac{CP_4 + CP_5}{CP_3 + CP_4} + \frac{CP_6 + CP_7}{CP_5 + CP_6} + \dots$$

The temporal structure of any two-part canon is based on two elements, which appear as reciprocal permutations. All forms of variation of two elements are applicable therefore to two part canons (see Theory

of Rhythm). Let a and b be two elements representing any kind of duration-groups. Then,

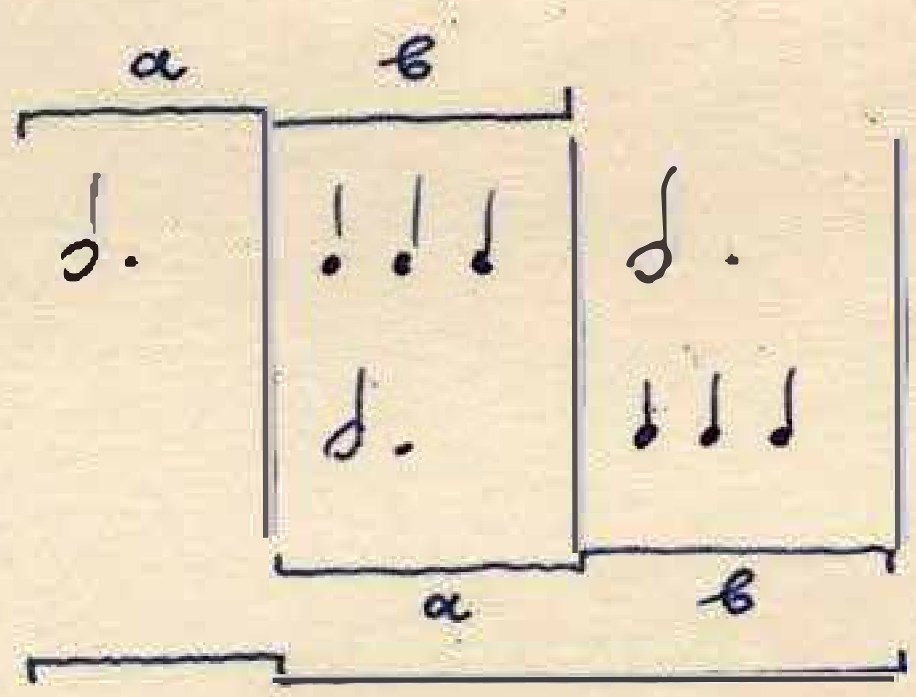
$$\frac{\vec{P}_I}{\vec{P}_{II}} = \frac{a + b + a}{a + b}$$, and the continuation of the temporal structure assumes the following appearance:

$$\frac{\vec{P}_I}{\vec{P}_{II}} = \frac{a + b + a}{a + b} + \frac{b + a}{a + b} + \frac{b + a}{a + b} + \ldots$$

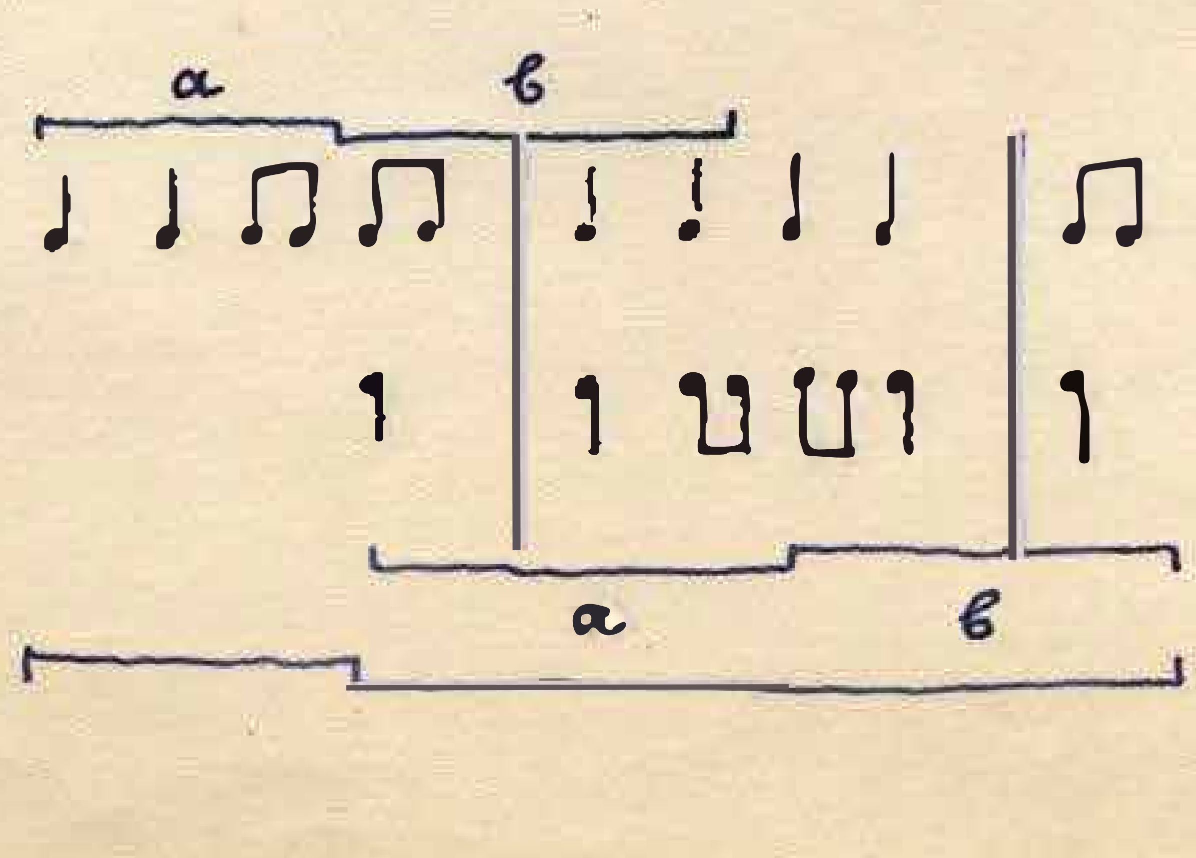
The duration of a temporal structure is the real factor controlling the flow of the canon. The longer the structure (not by speed, by the quantity of attacks), the greater the fluidity of the canon. Duration-groups of all kinds are acceptable as temporal structures for continuous imitation and for the canon.

A. Temporal structures composed from the parts of resultants.

(1) $\frac{3}{4}$

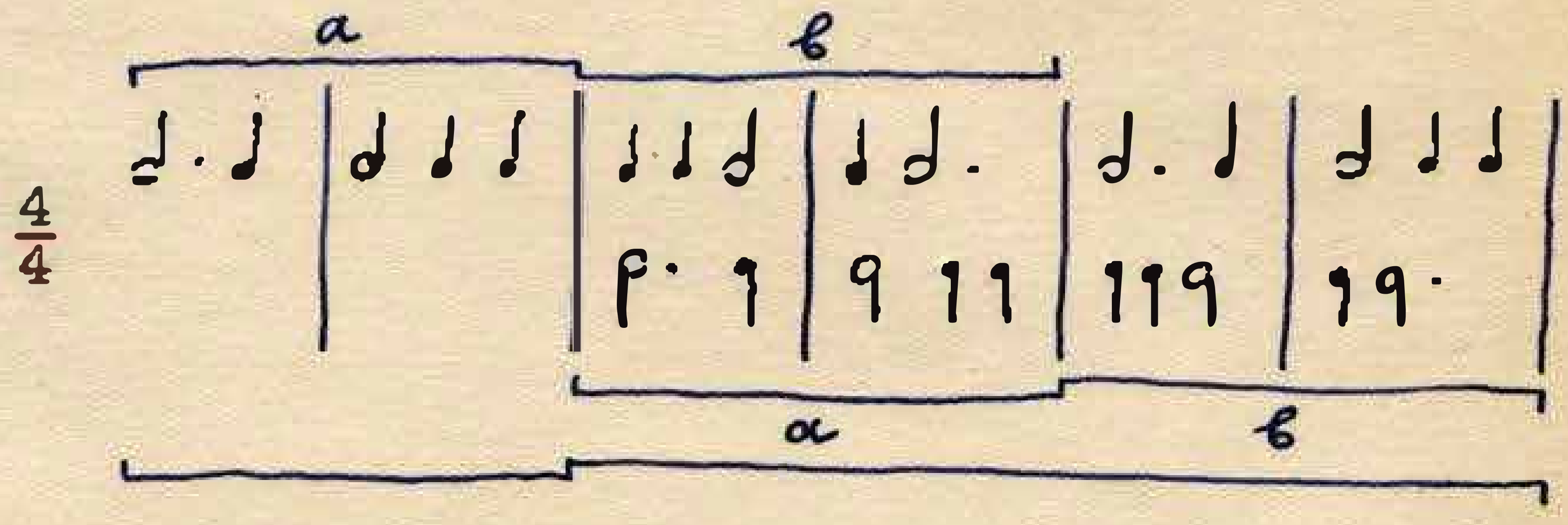


(2) $\frac{4}{4}$

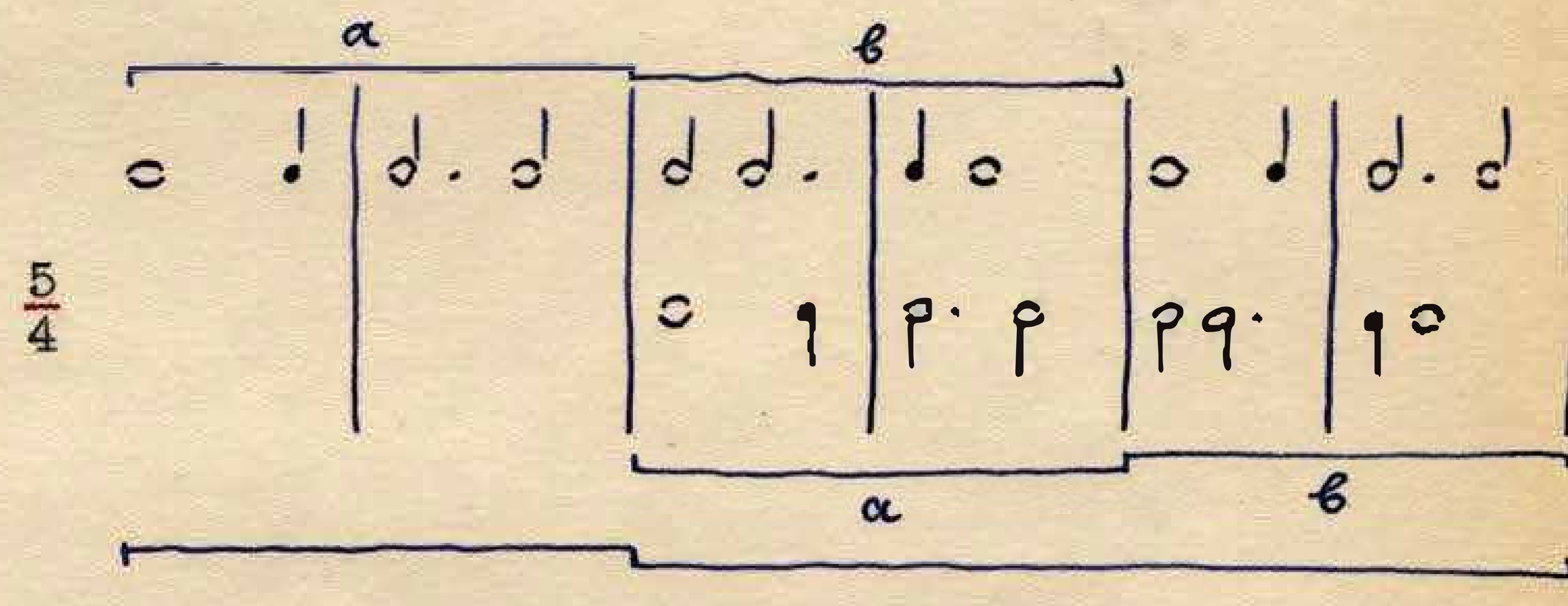


B. Temporal structures composed from complete resultants.

(1) $r_{4\div3}$



(2) $r_{5\div4}$

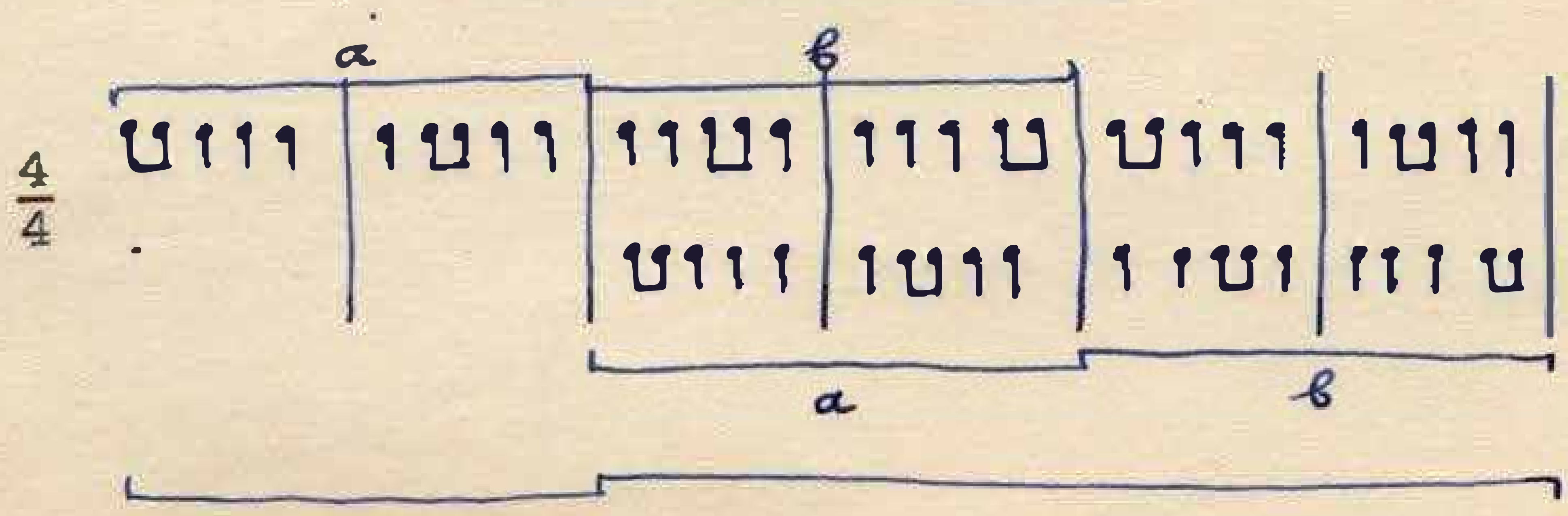


(3) $r_{6\div5}$

(4) $r_{8 \div 7}$

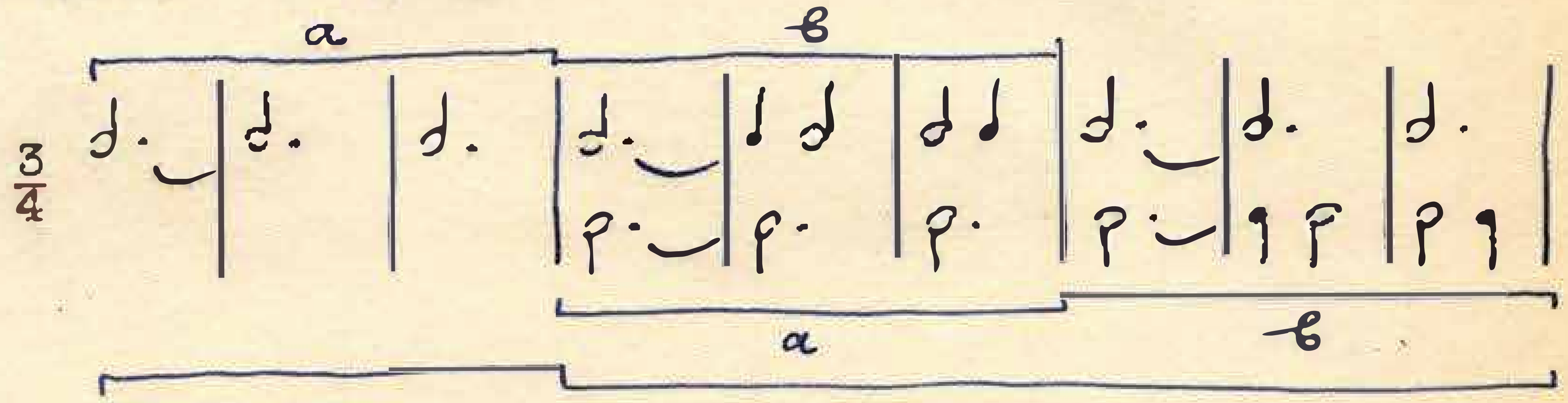
(5) $r_{9 \div 8}$

C. Temporal structures evolved by means of permutations.

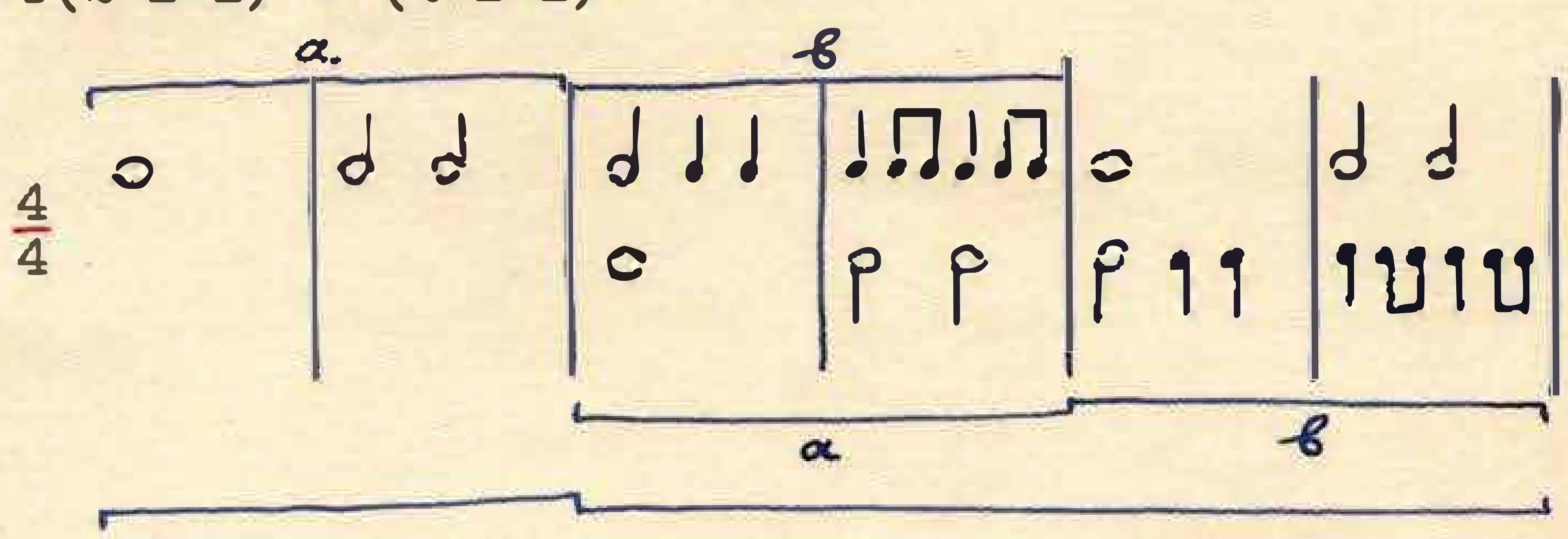


D. <u>Temporal structures composed from synchronized involution-groups.</u>

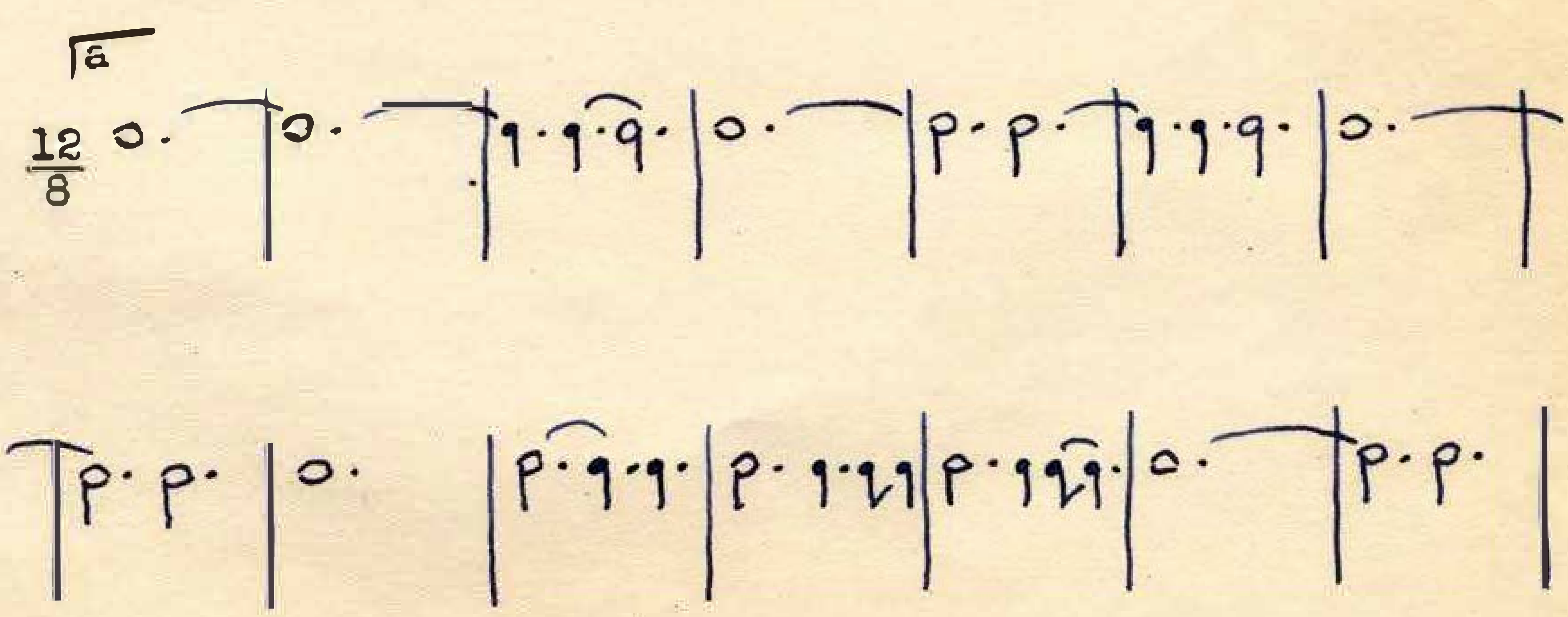
(1) $3(2+1) + (2+1)^2$

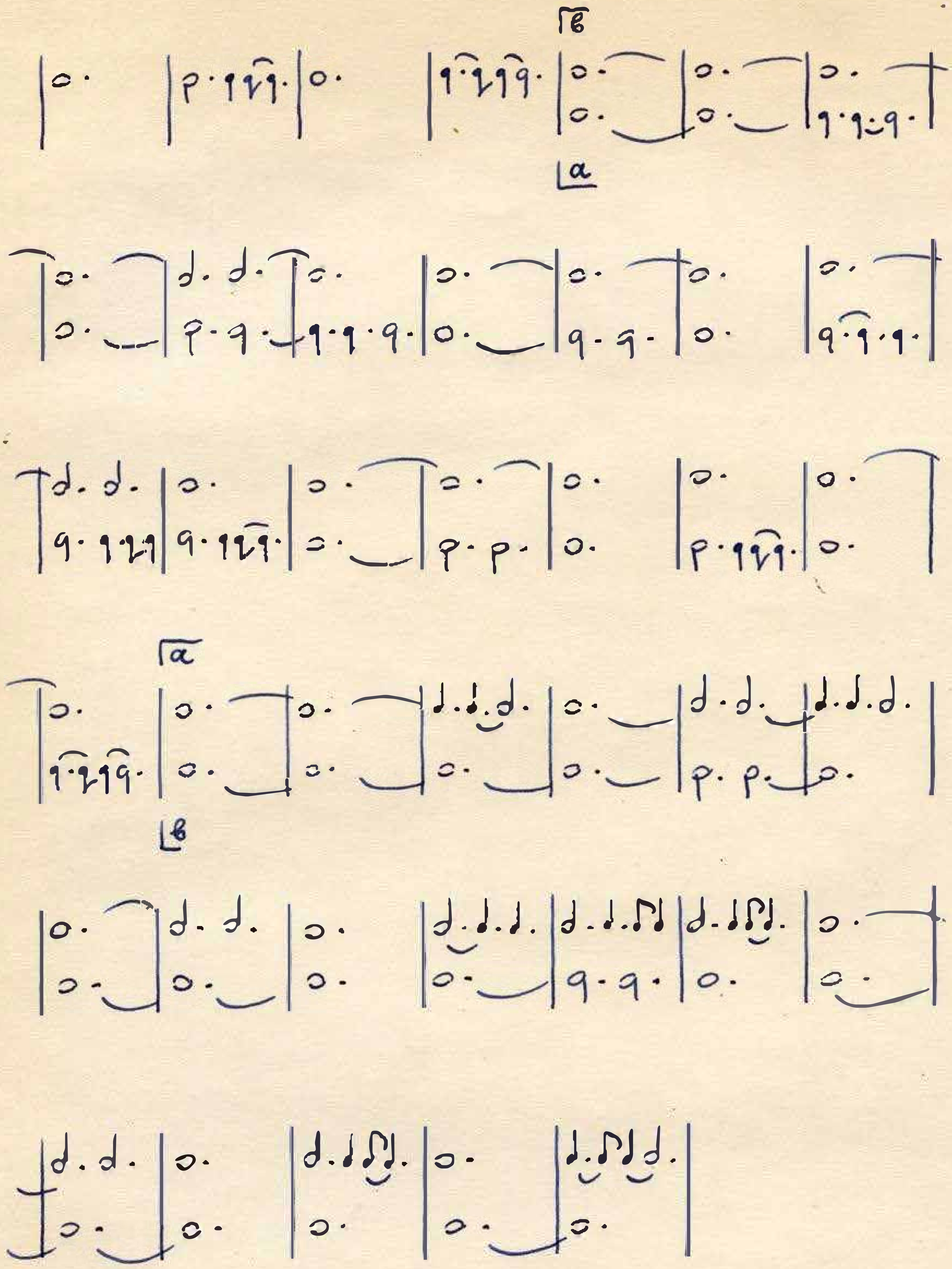


(2) $4(2+1+1) + (2+1+1)^2$

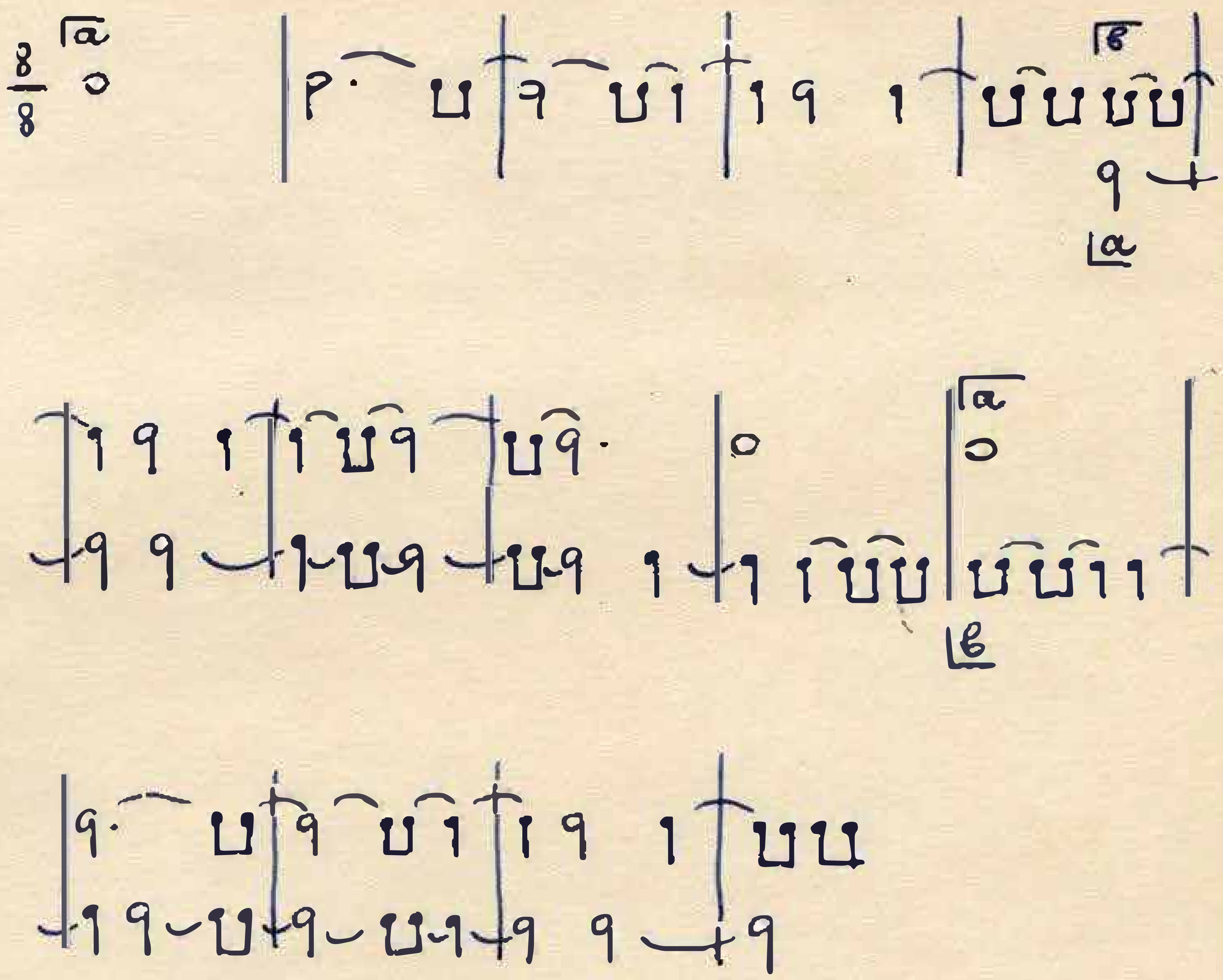


(3) $(3+1+2)^3 + 6(3+1+2)^2$




b
a
a
b

E. Temporal structures composed from acceleration-groups and their inversions.



Lesson CLXIV.

Composition of canons in all four types of harmonic correlation.

As canon is a duplication of melody at a certain time interval, the differences of intonation in the two counterparts are due to scale-structures. Thus, type I produces identical intonations, type II -- non-identical intonations, type III -- identical intonations and type IV -- non-identical intonations. The choice of axes in all four forms of correlation remains based on the original principle: consonance between the axes of two counterparts. In types II and IV the starting P.A. can be in a dissonant relation with the P.A. of the first voice, but it must end on a consonance.

As continuous imitation can go on indefinitely, we have to know the exact technique of bringing it to a close. Cadences are produced by the leading tones moving into their primary axis. As the first moving voice defines what happens to the second voice, all that is necessary is to produce a leading tone in the first moving voice. When this portion of melody is transferred to the second voice, the first voice produces its own leading tone, after which both voices close on their primary axes.

The use of symmetric pitch-scales is applicable to canons as well.

Examples of two-part canons in all four types of harmonic correlation.

Figure LI.

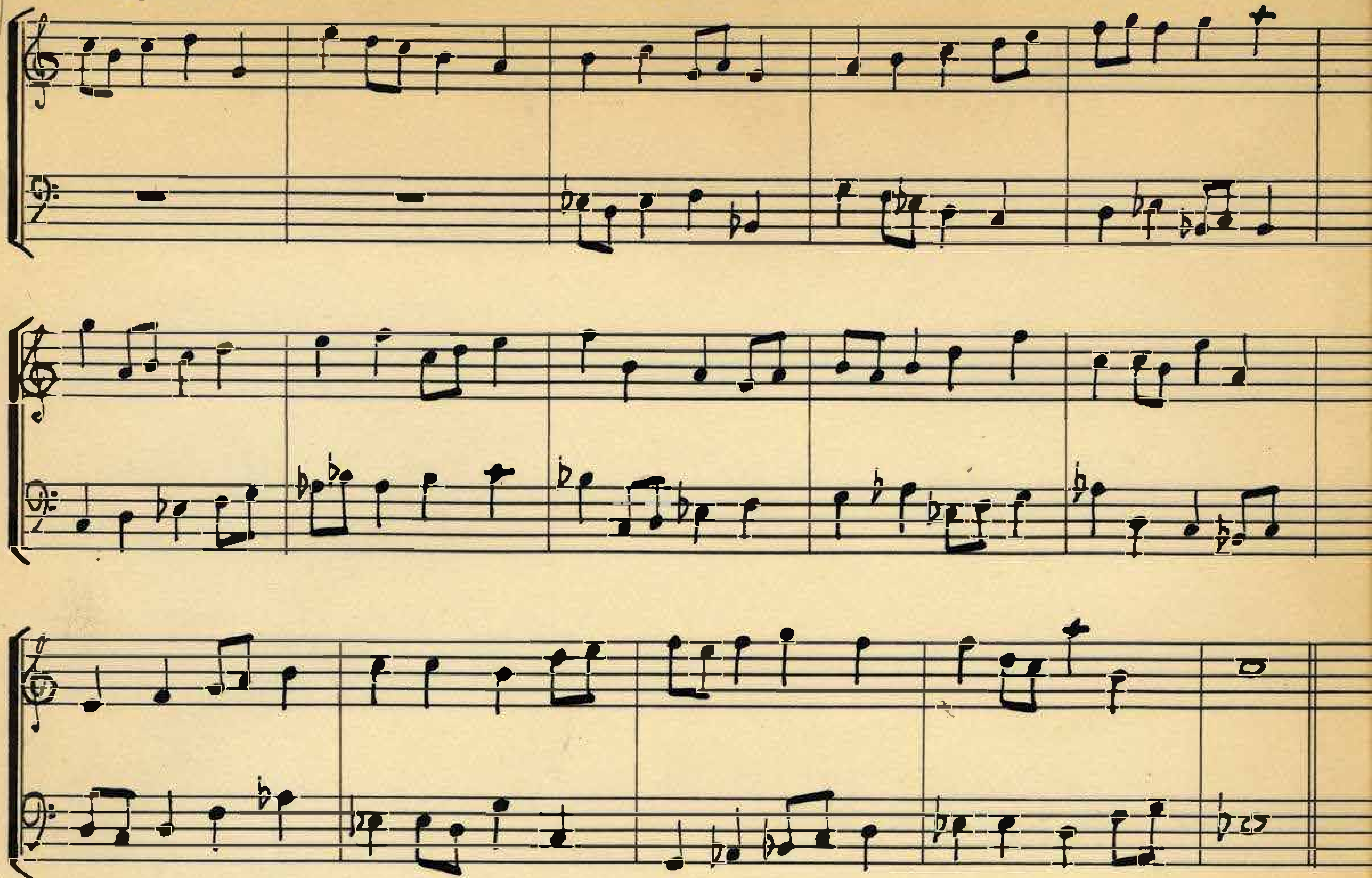
(cont. on following pages)

Type III

Type IV
A, d_6

Type II

Type III



Type IV

(4)

Type I

Type I symmetric

(Fig. LI, cont.)

Composition of Canonic Continuity by means of Geometrical Inversions.

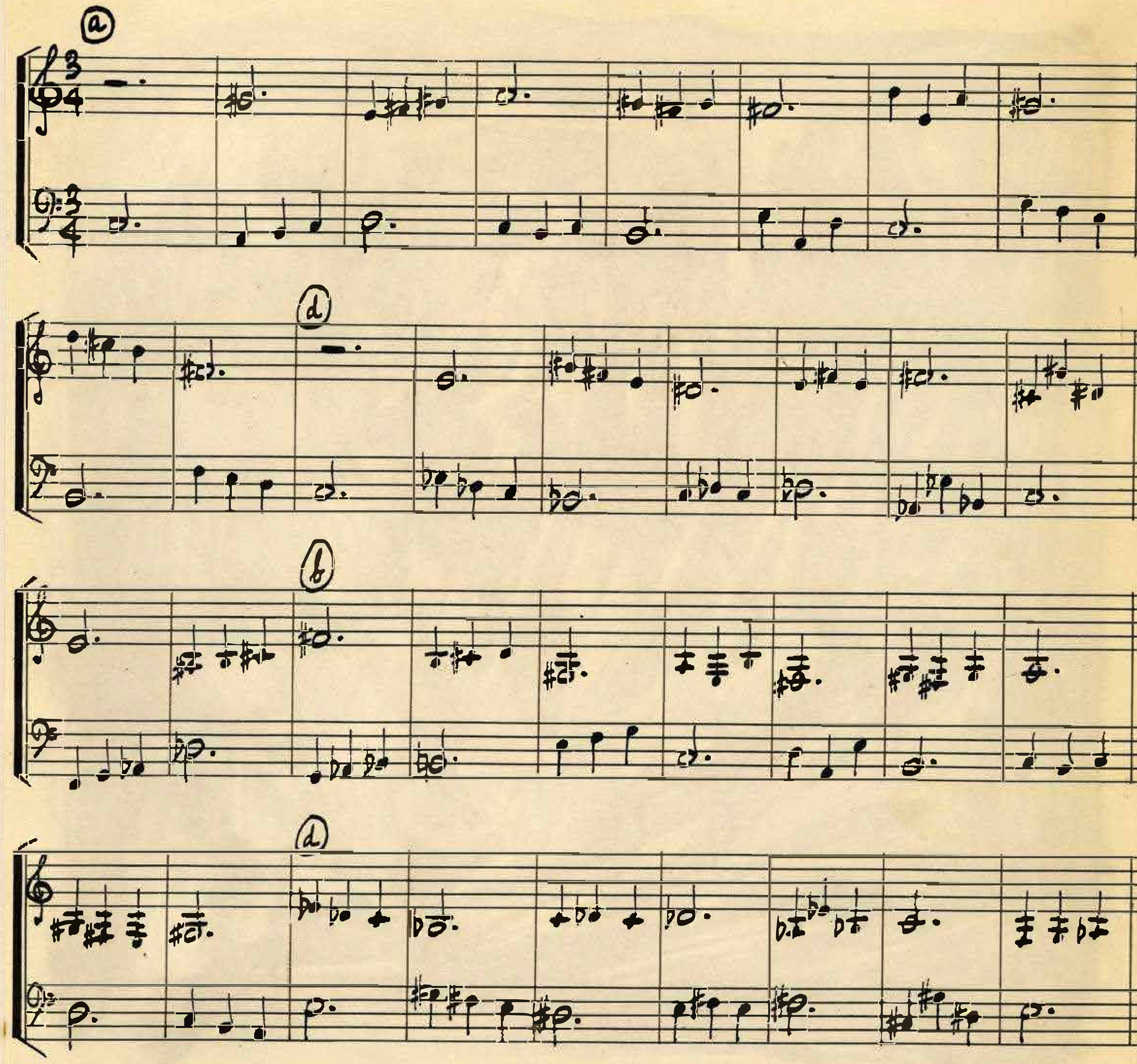
The original canon can be considerably extended by means of geometrical inversion.

The voice entering first produces the axis of inversion for the positions (c) and (d) . The final balance of the last cadence must not participate in the sequence of inversions, as this would disrupt the continuous flow of the canon. It must be used only at the very end of the composition, if the canon ends in position (a) or (d) . Otherwise a new balance must be added.

Under such conditions, the canon consists of several contrasting and independent sections of continuous imitation.

Example of a canon developed through the use of geometrical inversions.

Figure LII.



(Fig. LII, cont.)

Each geometrical inversion allows the use of two vertical permutations of the counterparts. Octave readjustment of the parts becomes a necessity under such conditions.

Lesson CLXV.

Fugue

A complete composition based on discontinuous imitation constitutes a Fugue.

A fragmentary (incomplete) composition based on discontinuous imitation constitutes a Fugato.

All other names established in the past, like Sinfonia, Invention, Praeludium, Fughetta refer to the same fundamental form, i.e., Fugue. The difference is mostly in the magnitude of the composition (Fugue, Fughetta) or in the type of harmonic correlation of the counterparts. Thus a Fugue which is unitonal-unimodal is called Invention, Praeludium or Sinfonia. Praeludium being the loosest term of all, as in many cases it has nothing in common with the Fugue. A Fugue which is unitonal-polymodal (and of a specified polymodality) is called Fugue.

As in my opinion the presence or absence of polymodality as well as of polytonality is a matter of harmonic specifications, which vary with time and place, any complete composition based on discontinuous imitation can rightly be called Fugue.

Fugato usually appears as a polyphonic episode in a homophonic composition.

<u>The Form of a Fugue</u>

The temporal structure of a fugue depends on the quantity of themes (subjects). It is customary to call the fugue with one theme a "single fugue" and the fugue with two themes a "double fugue". Triple fugues are very rare, and a real triple fugue requires many parts (voices), otherwise the idea that each part is a theme becomes nonsensical.

For this reason it is expedient to confine the two-part counterpoint to fugues with one theme only.

The general characteristic of all fugues is the appearance of the theme in all parts in sequence. The complete thematic cycle is known as an <u>exposition.</u> In two-part counterpoint the first entering voice announces the theme (we shall call it CF, for the sake of unity in terminology), after which the second voice enters with the imitation. This imitation is usually called "reply" and might as well have been called "echo". In fact, it is the same theme, with the possible difference caused by the form of harmonic correlation. Thus, <u>reply</u> in the types I and III is identical with the <u>theme,</u> whereas in the types II and IV it is non-identical, insofar as the scale-structure is modified.

At the time the second entering voice makes its announcement (CF), the first entering voice evolves a counterpart (CP) to it. Thus the form of <u>the first</u>

exposition (E_1) is as follows:

$$E_1 = \frac{P_I}{P_{II}} = \frac{CF + CP}{CF}$$ and the form of any other

exposition (E_n) is: $E_n = \frac{P_I}{P_{II}} = \frac{CF + CP}{CP + CF}$.

In both cases the definition of the first entering voice (P_I) and second entering voice (P_{II}) can be inverted.

In a fugue where CF and CP represent the only thematic material and no interludes are used, the entire composition acquires the following form:

$$F = E_1 + E_2 + E_3 + \ldots + E_n.$$

In homophonic music this corresponds to a theme with variations. In the fugue the variation technique consists of geometrical inversions of the original exposition.

The counterpoint to the theme can be either constant (i.e., the CP is carried out thru the entire fugue), or variable (i.e., a new CP is composed for each exposition). Statistically, the use of constant or variable CP is about fifty-fifty. In the XVII and XVIII Centuries constant CP was somewhat of a luxury, as the counterpoint which we consider to be general technique, at that time was known as vertically convertible counterpoint, which was believed to be more difficult to

execute. On the other hand, old composers did not know the technique of geometrical inversions, but used tonal inversions instead and merely as a trick, on some special occasions. With the systematic use of geometrical inversions, fugue becomes greatly diversified. Under such a condition, constant CP becomes merely a practical convenience. Once the theme and the counterpoint are composed (preparation of one exposition), you get the entire fugue by means of quadrant rotation arranged in any desirable sequence. If rotations refer to the entire E, the fugue assumes the following appearance:

$F = E_{1(m)} + E_{2(n)} + E_{3(p)} + \ldots$, where m, n and p are any of the geometrical inversions.

For example:

(1) $F = E_{1(a)} + E_{2(d)} + E_{3(c)} + E_{4(b)} + E_{5(a)}$

(2) $F = E_{1(a)} + E_{2(b)} + E_{3(a)} + E_{4(d)} + E_{5(a)} + E_{6(c)} +$
$+ E_{7(a)}$

(3) $F = E_{1(a)} + E_{2(a)} + E_{3(c)} + E_{4(b)} + E_{5(b)} + E_{6(d)}$

Such schemes are subject to composers' inventiveness.

Quadrant rotation may affect each appearance of the theme, then theme and reply appear in the different geometrical positions.

For example:

$$(1)\quad E = \frac{P_I}{P_{II}} = \frac{CF\,a + CP}{CF\,d}$$

$$(2)\quad E = \frac{P_I}{P_{II}} = \frac{CF\,b + CP}{CP + CF\,a}$$

$$(3)\quad E = \frac{P_{II}}{P_I} = \frac{CF\,b}{CF\,c + CP}$$

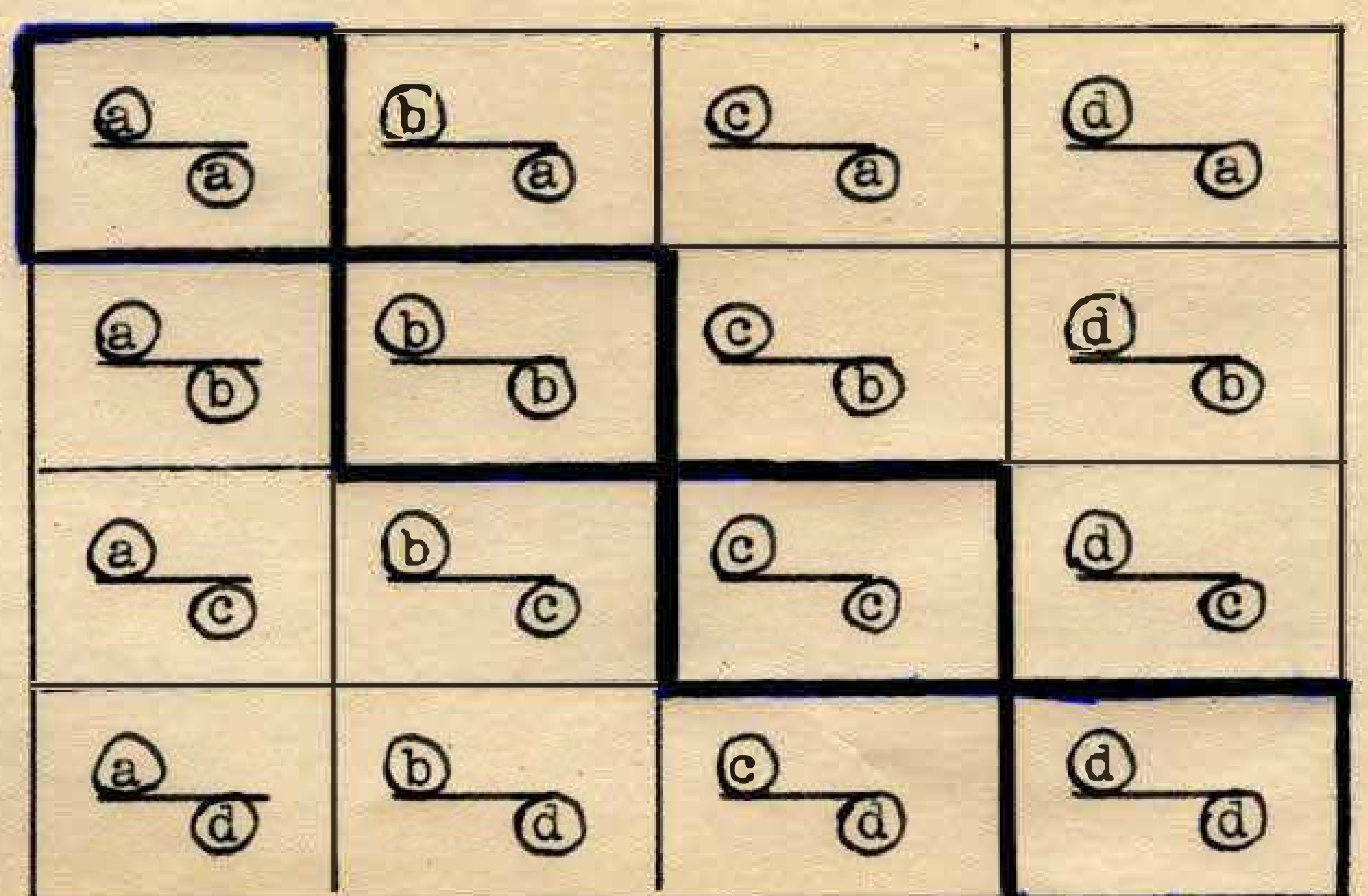
It is important to note that position is always identical for two simultaneous parts. Thus, CF d means that CP set against it is also in position d.

Quadrant rotation applied to theme and reply produces altogether 16 geometrical forms of exposition.

Forms of Imitation Evolved

Through Four Quadrants

Figure LIII.



All cases referring to one geometrical position for the entire E form the diagonal arrangement (heavily outlined) on the above table and appear to be special cases of the general rotary system.

It is easy to see that with this technique a fugue of any length can be composed without any effort.

<u>An example of fugal scheme employing quadrant rotation.</u>

$$F = \left(\frac{CF_{a} + CP_{a}}{CF_{a}}\right)E_1 + \left(\frac{CF_{d} + CP_{d}}{CP_{d} + CF_{d}}\right)E_2 +$$

$$+ \left(\frac{CF_{a} + CP_{d}}{CP_{a} + CF_{d}}\right)E_3 + \left(\frac{CF_{d} + CP_{a}}{CP_{d} + CF_{a}}\right)E_4 +$$

$$+ \left(\frac{CF_{c} + CP_{c}}{CP_{c} + CF_{c}}\right)E_5 + \left(\frac{CF_{a} + CP_{c}}{CP_{a} + CF_{c}}\right)E_6 +$$

$$+ \left(\frac{CP_{b} + CF_{d}}{CF_{b} + CP_{d}}\right)E_7 + \left(\frac{CP_{b} + CF_{c}}{CF_{b} + CP_{c}}\right)E_8 +$$

$$+ \left(\frac{CF_{d} + CP_{b}}{CP_{d} + CF_{b}}\right)E_9$$

As this example shows, CF may appear in the same voice successively, when its geometrical position alters.

The form of fugue where counterpoint is varied with some or with each of the expositions can

also be subjected to quadrant rotation.

The general scheme of such a fugue appears as follows:

$$F = \left(\frac{CF + CP_1}{CF}\right) E_1 + \left(\frac{CF + CP_1}{CP_1 + CF}\right) E_2 + \left(\frac{CF + CP_2}{CP_2 + CF}\right) E_3 +$$

$$+ \left(\frac{CF + CP_3}{CP_3 + CF_3}\right) E_4 + \ldots$$

<u>An example with application of the quadrant rotation</u>

$$F = \left(\frac{CF + CP_1}{CF}\right) a\, E_1 + \left(\frac{CF + CP_2}{CP_1 + CF}\right) d\, E_2 +$$

$$+ \left(\frac{CF\, b + CP_3\, a}{CP_2\, b + CF\, a}\right) E_3 + \left(\frac{CF + CP_2}{CP_1 + CF}\right) b\, E_4 +$$

$$+ \left(\frac{CF + CP_3}{CP_2 + CF}\right) c\, E_5 + \left(\frac{CP_2\, b + CF\, c}{CF\, b + CP_1\, c}\right) E_6$$

In the old fugue the elimination of monotony was usually achieved by means of <u>Interludes.</u> An interlude (we shall term it: I) is a contrapuntal sequence of the imitation or of the general type. Statistics show that about 50 out of 100 interludes are thematic (i.e., based on elements of CF or CP) and the rest neutral (i.e., using thematic elements of its own).

As in the case of counterpoint itself, I may be composed once and rotated afterwards. In other cases a new I may be composed each time. In the

old classical fugues interludes served mostly as a bridge between the E's, and leading into new key. In our fugues of types I and II they can serve the same purpose, whereas in types III and IV the interludes are hardly necessary, as the key variety is already inherent with the group of different symmetric tonics. As we shall see later, the fact that we have two parts does not limit the quantity of tonics.

The general scheme of a fugue with interludes appears as follows:

$$F = E_1 + I_1 + E_2 + I_2 + E_3 + I_3 + \ldots + E_n + I_n.$$

This form is equivalent to the <u>First Rondo</u> of the homophonic music.

I_1, I_2, I_3, ... may be either identical (though in different geometrical positions) or totally different. I_n, i.e., the last interlude is quite a common feature in the old fugues and has the meaning of a <u>conclusion</u> (coda). By rotating the same interlude we acquire <u>new modulatory directions.</u>

The method of composing a fugue by this system consists of the following stages:

(1) Composition of the theme;

(2) Composition of the counterpoint (one or more) to the theme; this is equivalent to the preparation of an exposition;

(3) Preparation of the exposition (or of all expositions if there is more than one counterpoint) in four geometrical positions:

$$\frac{CF}{CP}\ a\ ;\quad \frac{CF}{CP}\ b\ ;\quad \frac{CF}{CP}\ c\ ;\quad \frac{CF}{CP}\ d\ ;$$

(4) Composition of the interlude(s);

(5) Preparation of the four geometrical positions of the interlude(s);

(6) Composition of the scheme of F;

(7) Assembling the fugue.

J O S E P H   S C H I L L I N G E R

C O R R E S P O N D E N C E   C O U R S E

With: Dr. Jerome Gross — Subject: Music

Lesson CLXVI.

Composition of the Theme

Theme in a fugue is of utmost importance, as it constitutes at least one half of the entire composition. Nobody yet has defined clearly the requirements for a fugal theme. A good fugal theme is usually ascribed to the composer's genius, and this is neither help nor consolation to the student of this subject. We want to know precisely, what makes the melody a suitable fugal theme, as experience shows that: (1) not every good or expressive melody makes a suitable fugal theme, and (2) not every suitable fugal theme is a good melody for any other purpose. Composers, who were outstanding melodists, failed to show important achievements as contrapuntalists (Chopin, Schumann, Liszt, Chaikovsky and others).

The first requirement for a fugal theme is that it must be an incomplete melodic form. In the best and most typical fugues by J.S. Bach we find that such incomplete melodic forms follow their completion as counterpoint evolving during the announcement of the theme in the second voice.

An incomplete melodic form in this case means that at the moment the second voice starts the theme, the

first voice does not arrive at its primary axis.

For an illustration, let us take Fugue II, Vol. I, Well Tempered Clavichord (later to be referred to as W.T.C.) by J.S. Bach.

Figure LIV.

The theme ends on the first sixteenth of the third bar, while the melodic form completes itself on the third quarter of the same bar. It is interesting to note that the theme (and the melodic form) is constructed on the binary axis: $\frac{0}{d}$. In order to present his announcement clearly, Bach uses $\frac{1}{t}$ (= 𝅘𝅥𝅯) at the point where the theme might have stopped otherwise, reserving the eighth until the reply is far on its way of developing. Thus Bach eliminates the danger of stopping, which, indeed, if realized, would have spoiled the entire fugue. Another important detail is the juxtaposition of db-axis in CP versus O-axis in CF.

All other requirements for a fugal theme really derive from the first one: all such resources of temporal rhythm and axial forms can be used which demonstrate an unfinished melodic structure in the process of its formation.

The presence of any one of the following structural characteristics, as well as of any combinations of the latter, produces a suitable fugal theme.

(1) The presence of rests; particularly a decreasing series of rests, combined with an increasing number of attacks; stop-and-go effects; gaining momentum effects.

(2) The sequence of decreasing duration-values: rhythmic acceleration in the broadest sense.

(3) Dialogue effects achieved by means of binary axes, and by means of attack-groups contrasting in form, like legato-staccato.

(4) Effects of growth, achieved by means of binary and ternary diverging axes.

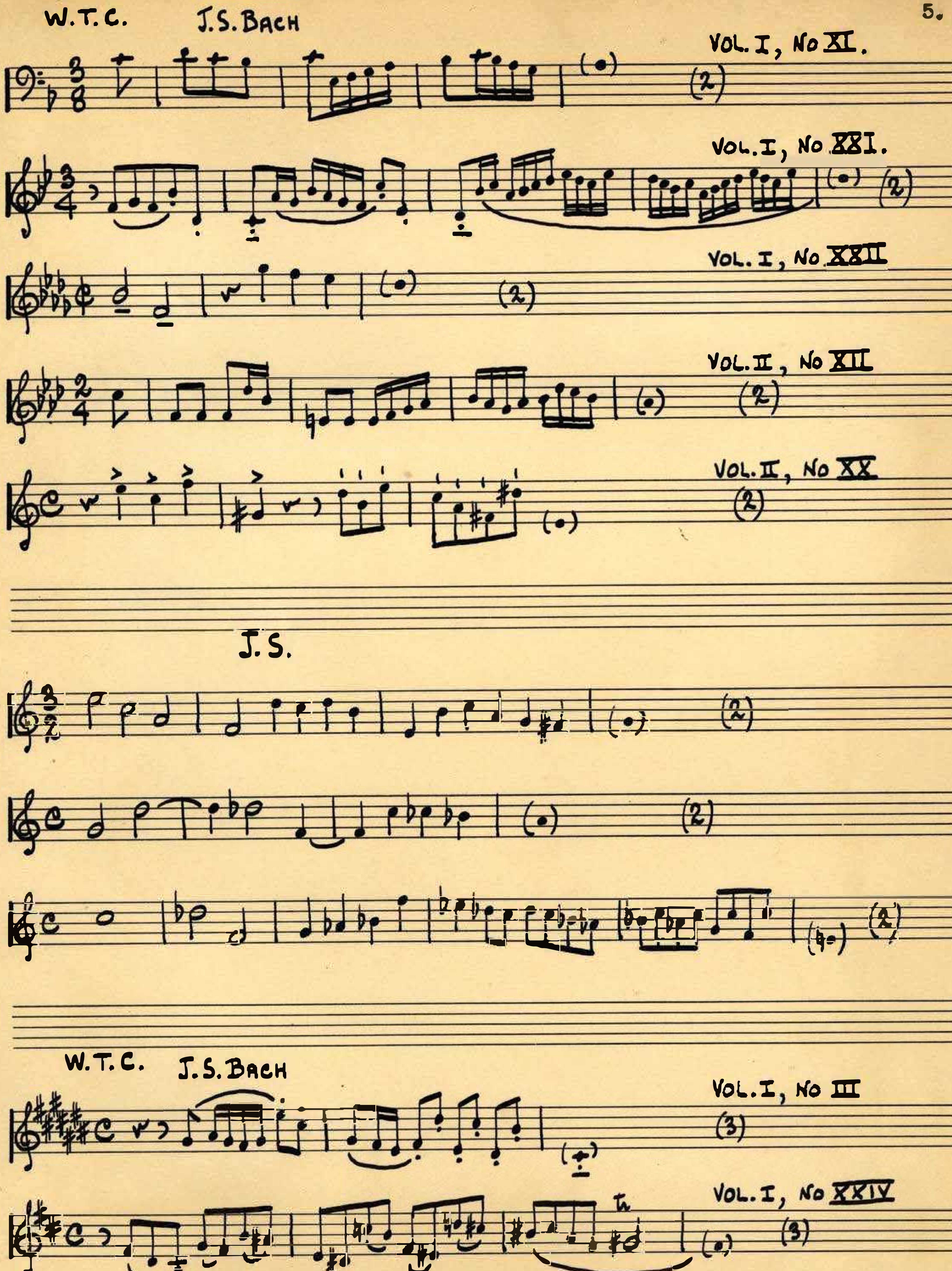
(5) The presence of resistance forms (including repetition, phasic and periodic rotation), particularly leading to climaxes.

Combinations of the above techniques applied to one theme make the latter more saturated and tense, which increases the fugal characteristic.

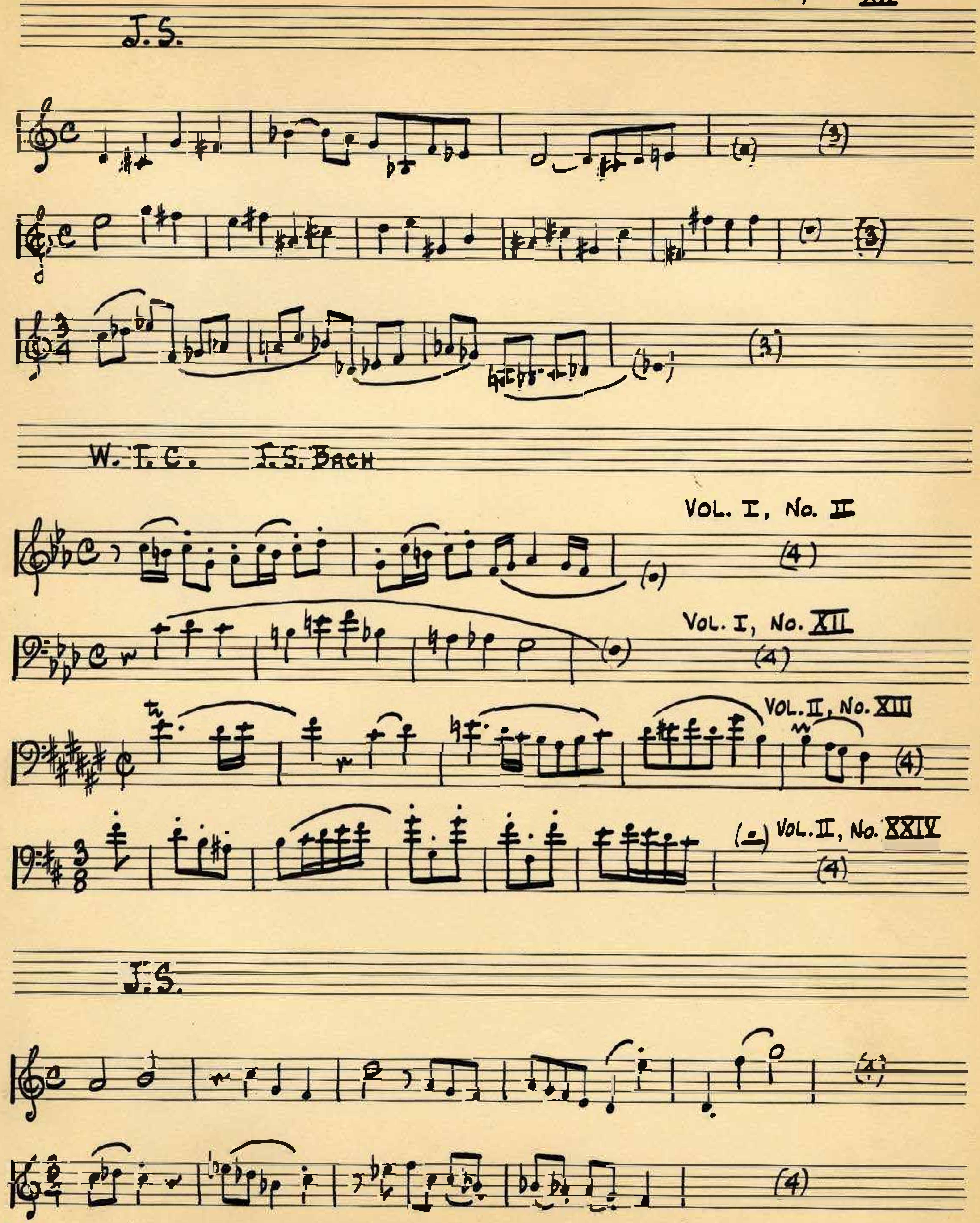
Figure LV.

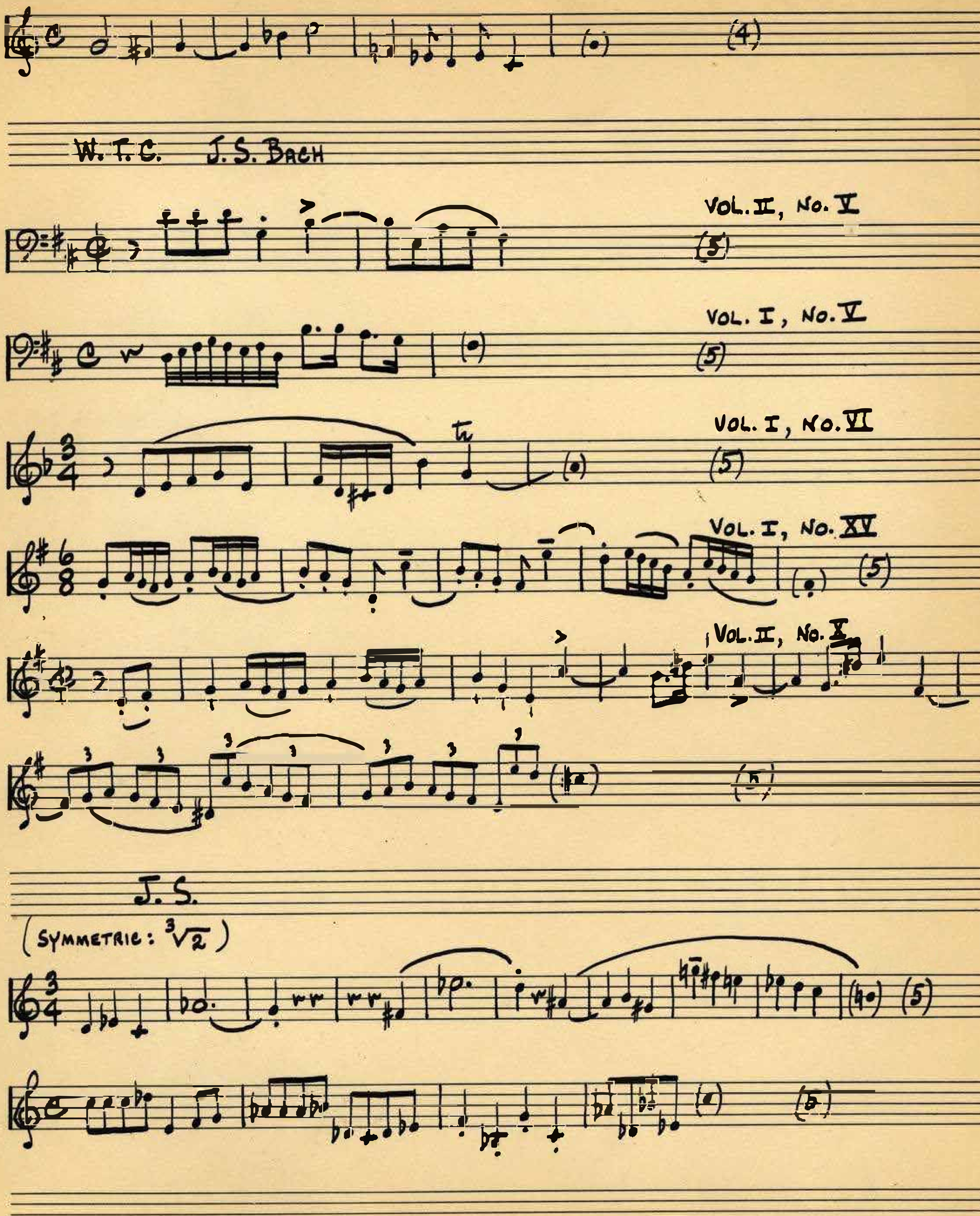
Fugal themes by J.S. Bach and by just J.S.

(Numbers in musical examples refer to the preceding classifications).


W.T.C.
J.S. BACH
VOL. I, No XI.
(2)
VOL. I, No XXI.
(2)
VOL. I, No XXII
(2)
VOL. II, No XII
(2)
VOL. II, No XX
(2)
J.S.
(2)
(2)
(2)
W.T.C.
J.S. BACH
VOL. I, No III
(3)
VOL. I, No XXIV
(3)

Also Vol. I, No. XI
" " " XXI
" II, " XII




(4)
W.T.C. J.S. BACH
VOL. II, NO. V
(5)
VOL. I, NO. V
(5)
VOL. I, NO. VI
(5)
VOL. I, NO. XV
(5)
VOL. II, NO. X
(5)
J.S.
(SYMMETRIE: $\sqrt[3]{2}$)
(5)
(5)

As it follows from the above examples, the total duration of a theme (in terms of quantities of attacks, or in terms of bars) largely depends upon the composer's decision. However, the following generalization is true for most classical fugues: the duration of the fugal theme is in inverse proportion to the number of parts.

Indeed, the first theme of Fugue IV, Vol. I, W.T.C. has only five attacks; the theme in Fugue XXII, Vol. I, W.T.C. has six attacks. Both of these fugues are written in five parts. On the other hand, Fugue X of the same volume, written in two parts, has a theme of twenty-six attacks.

It is not important that the reply enters on the same time-unit of the measure as the theme. Quite to the contrary, difference in the starting moments (in relation to the bar) adds interest to the whole composition, as it produces an element of surprise.

Themes unsuitable for fugues can be subjected to some alterations, which will make them suitable.

It can be demonstrated, by reversing the procedure, that the mere addition of 0-axis to any melodic form can render it suitable as a fugal theme. J.S. Bach's theme from the "Toccata and Fugue" in D- minor for Organ, being deprived of its 0-axis, loses all its fugal quality. When 0-axis is taken out, the

axial combination becomes: b+a+c+a. This theme seems to have nothing but rotation in relatively narrow range. The inclusion of 0-axis produces an effect of growing resistance, and the axial combination becomes:

$\frac{0}{d+c+c}$.

Figure LVI.

The number of bars in a fugal theme is an optional quantity. It may be pair or odd. It may be integral or fractional. Both odd and fractional are preferable to pair and integral, because the latter two suggest a cadence at the end of the theme. I believe one of the factors that influenced Buxtehude and all the Bachs is the awareness of <u>cantus firmus</u> (in a strict sense) as a theme. Canti firmi usually had an odd number of attacks.

Lesson CLXVII.

Preparation of the Exposition

After selecting the theme, the composer must devote himself to the preparation of fugal exposition.

As it is easy, with this method, to write four types of fugues on one theme, it becomes desirable to prepare four expositions for the future fugues. In a two-part fugue, the entire preparation of E consists merely of writing CP to CF. It is advisable that the exposition prepared for each type would be written out in all geometrical positions. This saves time during the period of assembling the fugue. Fugues of type IV often require preparation of two expositions, as when the axes exchange in $\frac{CP}{CF}$, CP may not fit, and a new counterpoint must be written (CP_{II}).

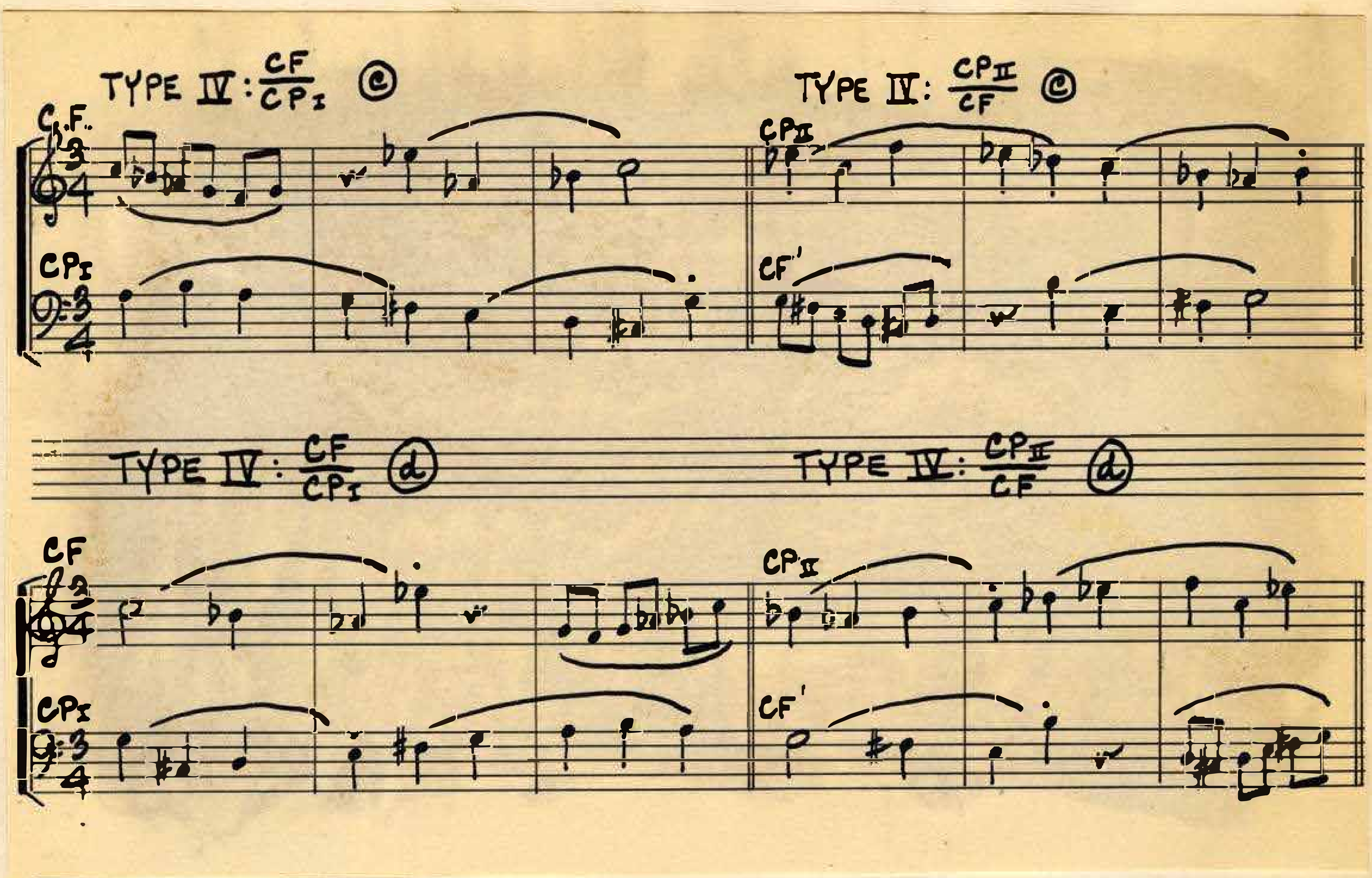
To make the demonstration of all techniques pertaining to fugue concise, we shall use a very brief theme.

Figure LVII.

(please see following pages)


CF (THEME)
TYPE I: CF/CP (a)
CF
CP
TYPE I: CF/CP (b)
TYPE I: CF/CP (d)
CF
CP
TYPE I: CF/CP (c)
TYPE II: CF/CP (a)
CF
CP
TYPE II: CF/CP (b)
TYPE II: CF/CP (d)
CF
CP
TYPE II: CF/CP (c)


TYPE III: CF/CP (a)
TYPE III: CF/CP (b)
CF
CP
TYPE III: CF/CP (d)
TYPE III: CF/CP (c)
CF
CP
TYPE IV: CF/CPI (a)
TYPE IV: CPII/CF (a)
CF
CPII
CPI
CF'
TYPE IV: CF/CPI (b)
TYPE IV: CPII/CF (b)
CF
CPII
CPI
CF'



Composition of the Expositions

Composition of the expositions in type I does not require any special considerations, as both parts have an identical P.A.

In type II, the modal modulations of CF, and its respectively related CP, must be in one system of modal sequence. For example, if P.A. of CF_1 is c and P.A. of CP_1 is e, the axis of CF_2 (reply) must be a and CP_2 (counterpoint to reply) must have P.A. on c, in order to retain the axial unity in the first part for the course of one exposition, and in order to preserve

the vertical relation of $\frac{CP}{CF}$ as it was originally conceived.

The entire structure of the fugue (from the above relations) appears as follows:

$$F = \left[\frac{(CF_1 + CP_1)\underline{c}}{CF_2\ \underline{a}}\right] E_1 + \left[\frac{(CF_3 + CP_3)\underline{a}}{CP_2\underline{c} + CF_4\ \underline{f}}\right] E_2 + \left[\frac{(CF_5 + CP_5)\underline{f}}{CP_4\ \underline{a} + CF_6\ \underline{d}}\right] E_3 + \dots$$

where $\underline{c}$, $\underline{a}$, $\underline{f}$, $\underline{d}$, ... are the primary axes of the respective parts.

Likewise if $\frac{CF_1}{CP_1} = \frac{\underline{c}}{\underline{f}}$, the sequence of P.A.'s becomes: $\frac{c}{g} + \frac{g}{c+d} + \frac{d}{g+a} + \dots$

In type III, the tonal (key) modulations of CF, and its respectively related CP, must be in one system of symmetric sequence. This sequence preserves its constant $\frac{CP}{CF}$ relation only when CP_2 (the reply) forms its P.A. in symmetric inversion to the original setting. Let us take the symmetry of $\sqrt[4]{2}$.

For example: $\frac{CP}{CF} = \frac{e\flat}{\underline{c}}$. In order to preserve the axial relation where CP is 3 semitones above CF, the reply must appear from the opposite equidistant point, i.e., from $\underline{a}$. This allows the relative stability of both parts, as CP_1 being three semitones above CF requires the $\underline{c}$-axis.

The structure of such a fugue, evolved on four points of symmetry (tonics), appears as follows:

$$F = \left[\frac{(CF_1 + CP_1)\underline{c}}{CF_2\ \underline{a}}\right]E_1 + \left[\frac{(CF_3 + CP_3)\underline{a}}{CP_2\underline{c} + CF_4\ \underline{f\sharp}}\right]E_2 + \left[\frac{(CF_5 + CP_5)\underline{f\sharp}}{CP_4\ \underline{a} + CF_6\ \underline{e\flat}}\right]E_3 + \left[\frac{(CF_7 + CP_7)\underline{e\flat}}{CP_6\ \underline{f\sharp} + CF_8\ \underline{c}}\right]E_4$$

A similar case evolved from three points of symmetry ($\sqrt[3]{2}$), where $\frac{CP}{CF} = \frac{\underline{e}}{\underline{c}}$, gives the following sequence of P.A.'s:

$$\frac{\underline{c}}{\underline{a}} + \frac{\underline{a\flat}}{\underline{c}+\underline{e}} + \frac{\underline{e}}{\underline{a\flat}+\underline{c}}$$

In type IV, in order to carry out the sequence of P.A.'s in symmetric inversion of the original setting, it often becomes necessary to prepare two independent expositions:

$$E = \frac{CP_I}{CF} \text{ and } E' = \frac{CP_{II}}{CF'}$$, as CP may be in a different intervallic relation to CF_2 than it is to CF_1. The difference usually appears in variations on semitone or whole tone, which results in most disturbing relations, such as a second instead of a third. For this reason, example in Fig. LVII offers two expositions.

It is easy to see the unfitness of CP_I as a counterpoint to reply, by exchanging it with P.A. of CF.

The sequence of symmetric P.A.'s in type IV of Fig. LVII would develop on the basis of its pre-set expositions:

$$E = \frac{CP_I}{CF}\ \frac{\underline{e\sharp}}{\underline{c}} \text{ and } E' = \frac{CP_{II}}{CF'} = \frac{\underline{c}}{\underline{e\sharp}}.$$

Considering the enharmonic equality of $e^{\#}$ and f, $a^{\#}$ and $b^{\flat}$ etc., and the fact that CF is evolved in natural major d_0 and CF' in natural major d_6, we obtain the following structure for the fugue:

$$F = \left[\frac{(CF + CP_{II})\underline{c}}{CF' \quad \underline{e}^{\#}}\right]E_1 + \left[\frac{(CF + CP_{II})\underline{f}}{(CP_I + CF')\underline{g}^{\#}}\right]E_2 + \left[\frac{(CF + CP_{II})\underline{b}^{\flat}}{(CP_I + CF')\underline{d}^{\#}}\right]E_3 + \cdots$$

In the old classical fugues reply appears on the dominant (i.e., seven semitones above or five semitones below the theme). If there was a sequence of expositions before the interlude took place, the theme would usually have returned to the tonic. According to our type II, if $CF_1 = \underline{c}$ and $CF_2 = \underline{g}$, CF_3 should have been $\underline{d}$, CF_4 should have been $\underline{a}$ etc. However, this was not the case in the fugues of the classical period, and there was a good reason for it. As the tuning of <u>mean temperament</u> (the two-coordinate system: $\frac{3}{2}$ and $\frac{5}{4}$) developed abberation, while deviating from the tuning center (= 1), it was not possible to get satisfactory intonation in the course of traveling through C_5 or C_{-5} P.A.'s. And though <u>equal temperament</u> has overcome this defect, the habit remained with the composers till the end of XIX Century.

Lesson CLXVIII.

Preparation of the Interludes

Interludes (I_1, I_2, ... I_m) serve as bridges between the expositions. The last interlude, if the fugue ends with one, is a postlude (coda). Interludes serve two purposes:

(1) to divert the listener's attention from the persistence of theme;

(2) to produce a modulatory transition from one key-axis to another.

The first form is confined to one key, but may have any number of successive P.A.'s, thus producing modal modulations (U.-P.) between the two adjacent expositions having the same key-axis (U.-U. and U.-P.). The second form contains different key-axes (P.-U. and P.-P.) and connects the two adjacent expositions having different key-axes (P.-U. and P.-P.). Both forms of interludes may be either neutral or thematic. Neutral interludes are based on the material of rhythm, or intonation, or both, not appearing in any of the exposition. Thematic interludes borrow their material of rhythm, or intonation, or both from either CF or CP of the exposition. Furthermore, any of the above described types of interludes can be executed either in general or in imitative counterpoint.

The duration of an interlude depends on the duration of the exposition and the quantity of interludes. The form of an interlude itself has an influence upon its duration. In order to construct a perfect fugue, the duration of interludes must be put into some definite correspondence with the duration of expositions. Assuming one exposition as a temporal unit, we arrive at the following fundamental schemes for the temporal organization of interludes:

(1) $T\ (E) = T\ (I)$, i.e., the duration of an interlude equals to that of an exposition. This presupposes an equal duration for each of the interludes;

(2) $T\ (E) > T\ (I)$, i.e., the duration of an exposition is longer than that of an interlude. An exact ratio must be established in each case;

(3) $T\ (E) < T\ (I)$, i.e., the duration of an interlude is longer than that of an exposition. An exact ratio must be established in each case.

(4) $\overrightarrow{I} = I_1T + I_2 2T + I_3 3T + \ldots$, i.e., each successive interlude becomes longer. The durations of consecutive interludes may evolve in any desirable type of progression (natural, arithmetic, geometric, involution, summation etc.). The resulting effect of such fugue-structures is that the interludes in course of time, begin to dominate the theme. Thus the persistence of the theme diminishes.

(5) $I^{\rightarrow} = I_1 nT + I_2(n-1)T + I_3(n-2)T + \ldots$, i.e., each successive interlude becomes shorter. The resulting effect is opposite to that of (4): the domination of theme over interludes grows in the course of time.

(6) $I^{\rightarrow}$, i.e., the sequence of interludes develops along some form of rhythmic grouping.

As convertibility and quadrant rotation are general properties, the same interlude may be used several times, during the course of a fugue. This, being combined with key-transpositions, offers an enormous variety of resources, at the same time conserving the composer's energy.

Non-Modulating Interludes

(Types I and II)

Non-modulating interludes can be either neutral or thematic and they can be evolved in general or imitative counterpoint.

Figure LVIII.

(1) An example of Interlude type II executed in general counterpoint. Non-thematic (Neutral).

(2) An example of Interlude type II executed in imitative counterpoint. This one is thematic with reference to CF of Fig. LVII.

(please see next page)

(Fig. LVIII)

Modulating Interludes

I. Modulating Counterpoint Evolved through Harmonic Technique.

Contrary to the general notion, J.S. Bach's counterpoint is less "contrapuntal" than it is believed to be. And especially so when it comes to tonal (key-to-key) modulations. It is obvious that Bach as well as many other important contrapuntalists thought of key-to-key transitions in terms of modulating chords. See, for example, J.S. Bach's W.T.C., Vol. I, Fugue No. X (a two-part fugue) in E- minor. The harmonic background of this fugue is very distinct, and this fugue is rather typical and not exceptional.

It is easy to convert any modulating chord-progression written in four-part harmony into two-part harmony.

Chord structures of two-part harmony have the following functions:

(1) S(3) = 1, 3; used instead of S(5) of the three-part structure;

(2) S(5) = 1, 5; used instead of S(5) of the three-part structure;

(3) S(7) = 1, 7; used instead of S(7) of the four-part structure.

Figure LIX.

In order to obtain an interlude from a four-part chord-progression it is necessary to select the corresponding chordal functions which would translate the four-part structures into two-part structures. The voice-leading pertaining to two-part harmony will not be discussed here, as any position of two functions is equally as acceptable for the present purpose. Both parts are more or less in the vicinity of the four-part

harmony range. The final step consists of developing melodic figuration in both parts, but with somewhat contrasting rhythms of durations and attacks.

Modulating interludes can be either neutral (general counterpoint) or thematic (imitative counterpoint). In the latter case, thematic material is either borrowed from CF or CP of the expositions, or is entirely independent.

Examples of Modulating Interludes

Figure LX.

(1) Neutral and (2) Thematic.

An interlude can be used in the same fugue more than once, appearing in the different geometrical positions. It also can be transposed to any desirable key-axis, in any of the four quadrants.

II. Modulating Counterpoint Evolved through Melodic Technique.

This new technique is being offered in order to enable the composer to carry out the pure contrapuntal style, even when a key-to-key transition is desirable.

Modulating counterpoint consists of two independently modulating melodies (see modulation in the Theory of Pitch Scales), whose primary axes are in a constant simultaneous relationship at any given key-point of the sequence. After the vertical dependence has been established (the harmonic interval between CP and CF), it becomes necessary to assign to the primary axis of CP the meaning of the tonic which is nearest to CF through the scale of key-signatures.

Let the exposition end in the key of C, and let CF end on c and CP end on a. Then a becomes a- minor (as the key nearest to the key of C thru the scale of key signatures; A- major would be far more remote). Thus we have established a constant dependence where CP is the minor key three semitones below CF.

The next step consists of planning the modulation of P_I (originally: CF). Let the modulation be

to the key of f- minor.

Then:

$\overrightarrow{P_I} = C + d + G + f$

Now we assume that in order to retain the original vertical dependence between P_I and P_{II}, each axis of a major key must be reciprocated by a minor key, and vice versa. Then:

$$\frac{\overrightarrow{P_I}}{\overrightarrow{P_{II}}} = \frac{C + d + G + f}{a + F + e + A^{\flat}}$$

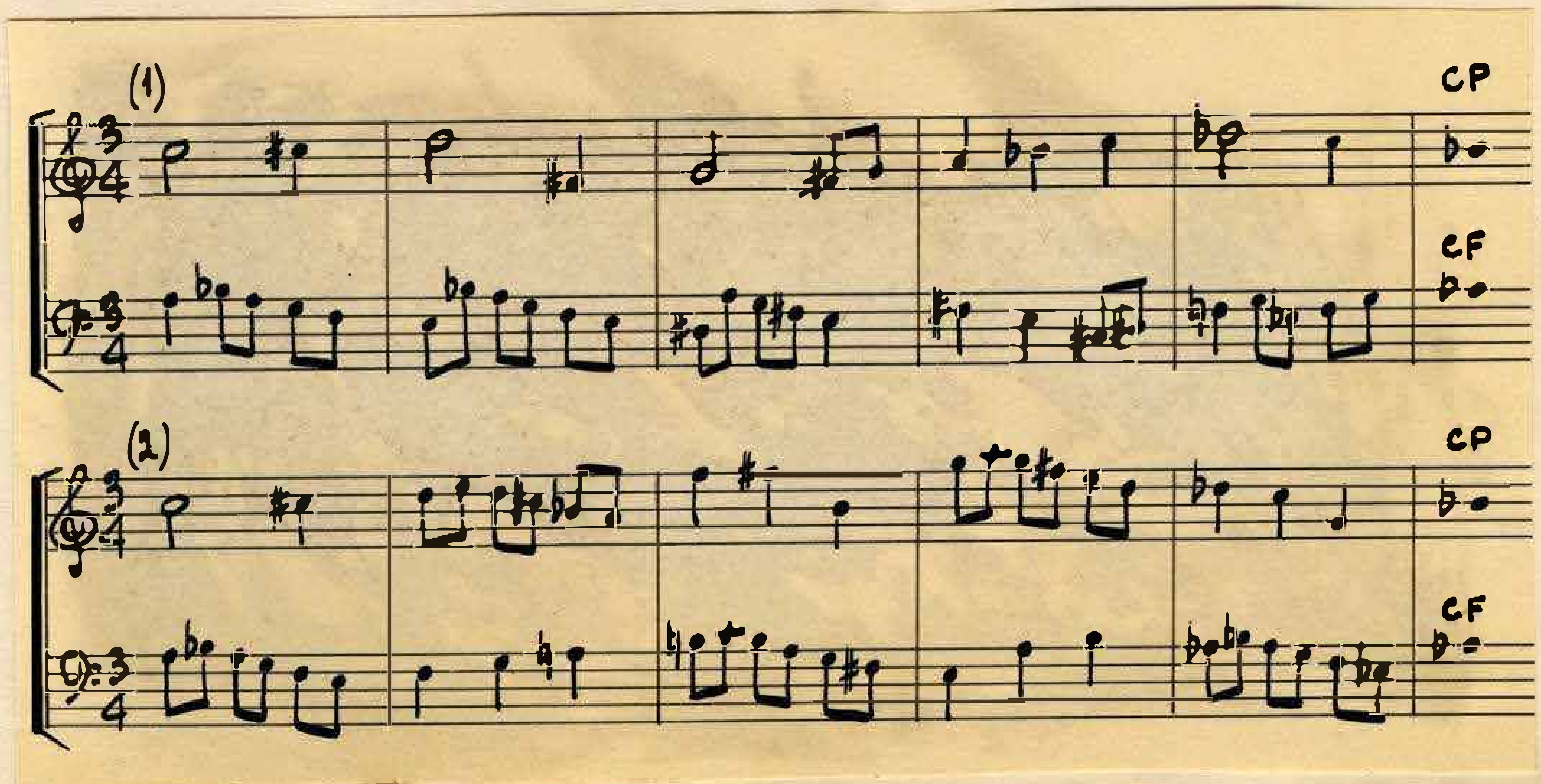
, i.e., while P_I modulates from C to d, P_{II} modulates from a to F, and when P_I modulates from d to G, P_{II} modulates from F to e; finally both parts arrive at CF having an $A^{\flat}$-axis and CP having an f-axis.

The period of modulation from key to key in both parts is approximately the same.

Examples of Modulating Interludes

Figure LXI.

(1) Neutral and (2) Thematic



The easiest way to compose modulating interludes by the contrapuntal technique is through a sequence of procedures:

(1) P_I modulates to the first intermediate key;

(2) P_{II} " " " " " "

(3) P_I " " " second " "

(4) P_{II} " " " " " "

and so on, until the entire modulation is completed.

Lesson CLXIX.

Composition of the Fugue

The process of assembling a fugue consists of planning the general sequence of expositions, interludes, their geometrical positions and their primary axes (key-axes).

In the following group of fugues only such materials were used, which were prepared in advance (see Fig. LVII, LVIII, LX and LXI).

The first three fugues have interludes (of both harmonic and melodic type), while the fourth has none, as key-variety is sufficiently great without it. The last fugue has independent counterpoints for the theme and the reply. The latter are interchanged in E_5.

The form of Fugue I (Fig. LXII):

$$E_1\,a + I_1 + E_2\,a + E_3\,d + I_2\,c + E_4\,d\,c + I_3\,b + E_5\,b$$

The form of Fugue II (Fig. LXII):

$$\overset{C}{\overline{E_1\,a + E_2\,a}} + I_1 + \overset{F}{\overline{E_3\,a + E_4\,d +}}\ \overset{G}{\overline{E_5\,d_c}} + I_2 + E_6\,a\,b$$

The form of Fugue III (Fig. LXII):

$$\overset{C}{\overline{(E_1 + E_2)}}\,a + (E_3 + E_4)\,d + I_1 + \overset{A\flat}{\overline{(E_5 + E_6)}}\,a + E_7\,\overset{A\flat\,E\flat\,C}{\overline{b_a\ \ a}}$$

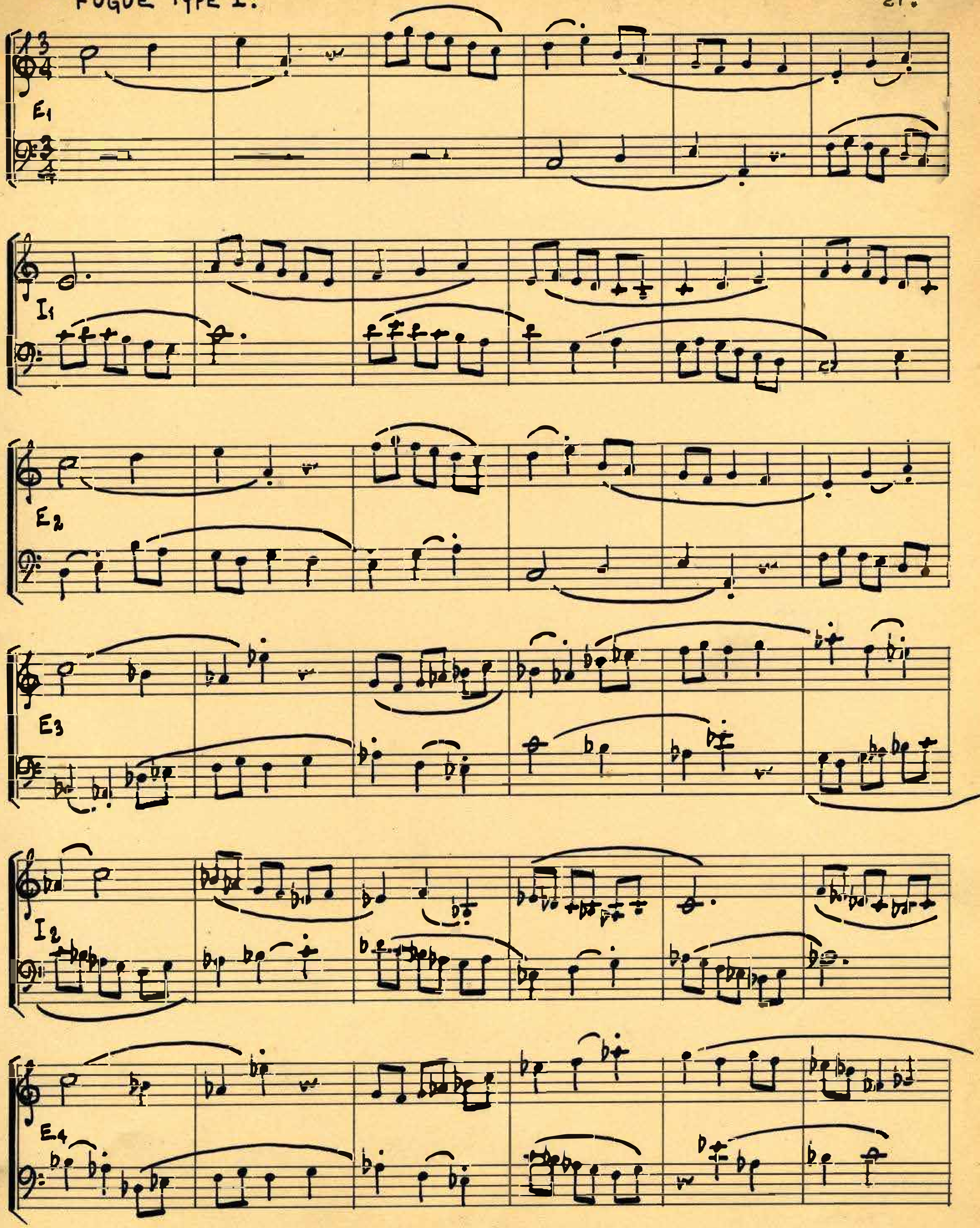
The form of Fugue IV (Fig. LXII):

$$(E_1 + E_2 + E_3 + E_4 + E_5)\,a + E_6\,c + E_7\,d + E_8\,b$$

Figure LXII.

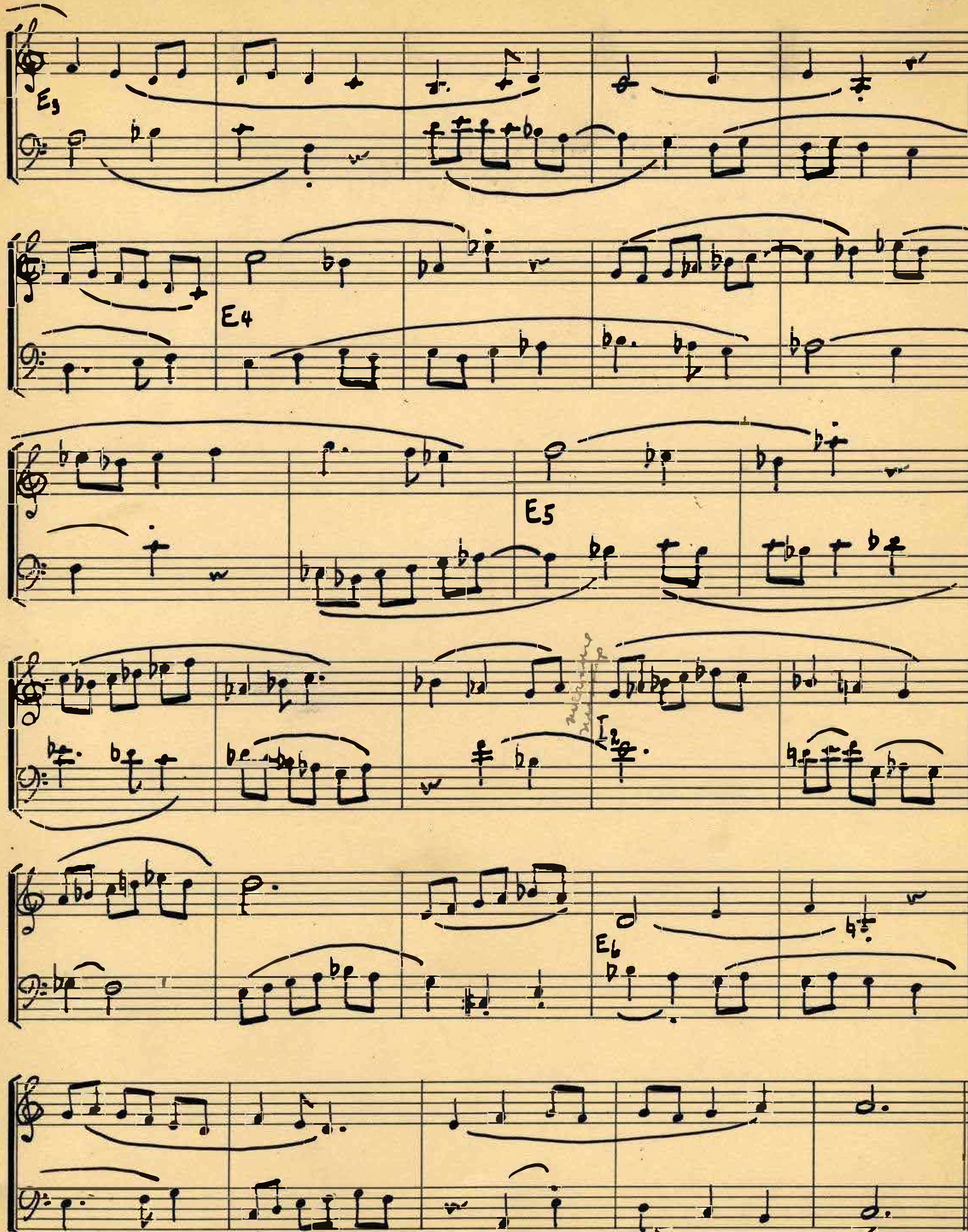
(please see following pages)

(1) FUGUE TYPE I.



(2)

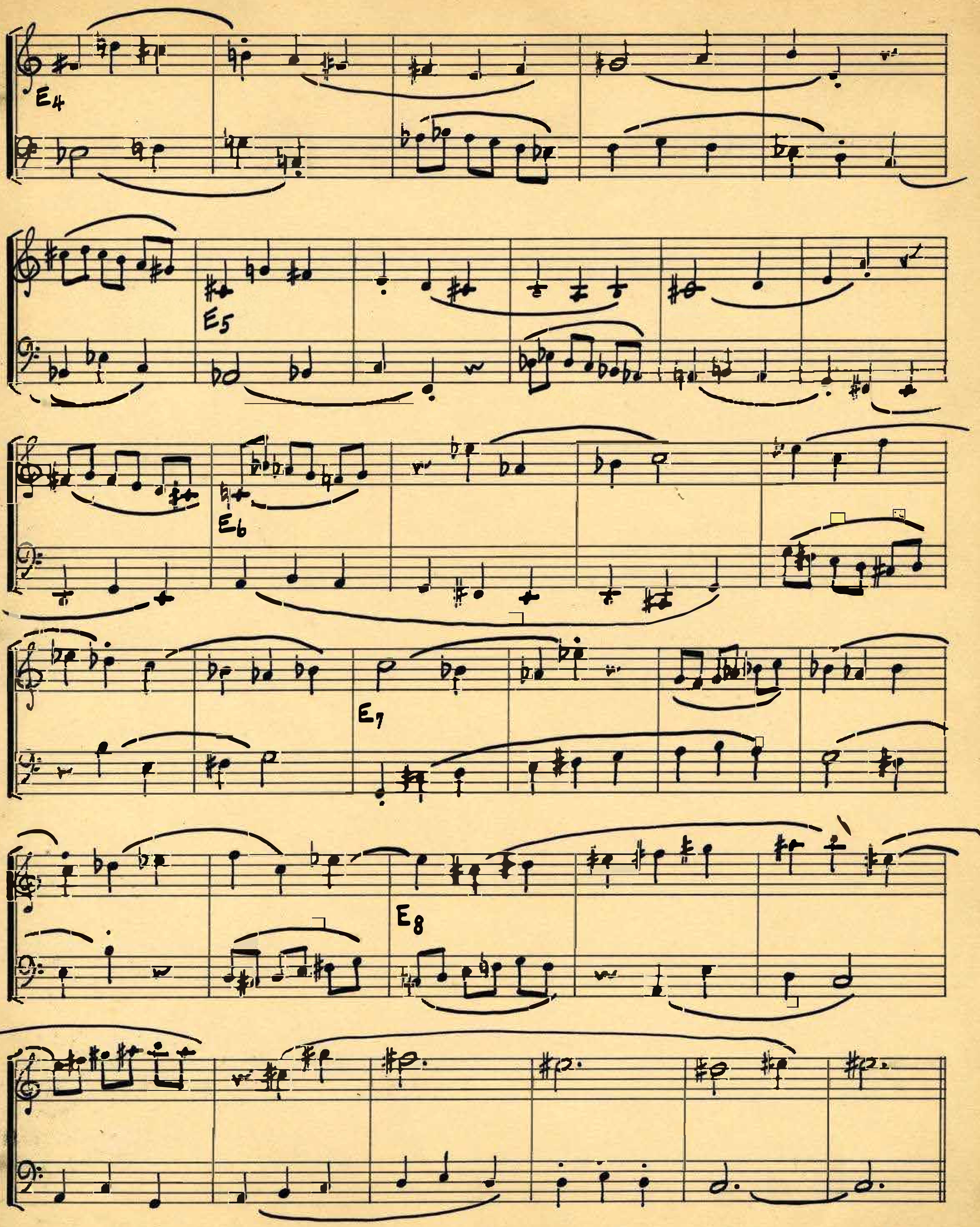
FUGUE TYPE II.


E3
E4
E5
E6

(3) FUGUE TYPE III.

(4)

FUGUE TYPE IV.


E4
E5
E6
E7
E8